Quasiparticle anisotropic hydrodynamics

Michael Strickland
Kent State University

Primary References: M. Alqahtani, M. Nopoush, R. Ryblewski, and MS
1703.05808 (to appear in PRL) and 1705.10191

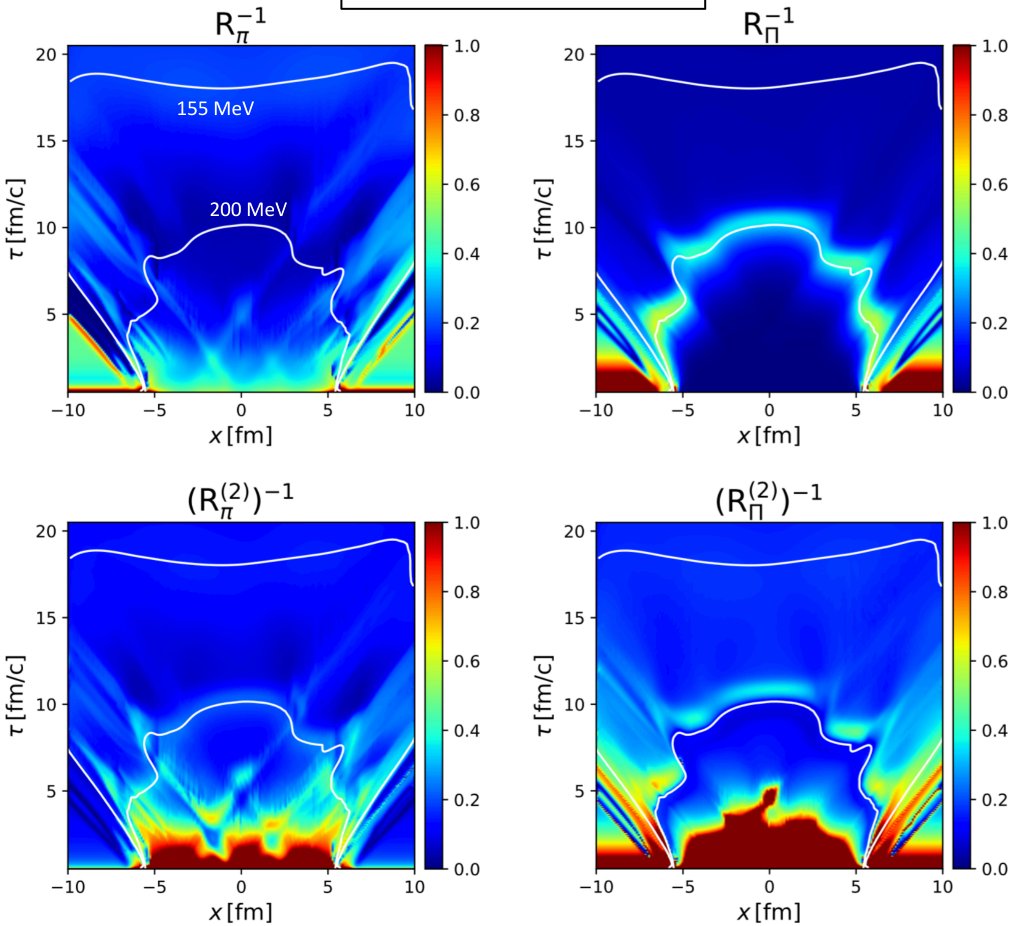
Canterbury Tales of Hot QFTs in the LHC Era
St John's College, Oxford
July 11, 2017

Motivation

- Viscous hydrodynamics is phenomenologically quite successful, however, the extreme environment generated in HICs presents a bit of a challenge to the standard formalism
- The **QGP** is born into a state of rapid longitudinal expansion which drives the system **out of equilibrium**
- There are many groups now focused on improving viscous hydrodynamics itself in order to better describe systems that are out of equilibrium, e.g. **anisotropic hydrodynamics (aHydro)**
- The goal of the aHydro program is to provide an optimized hydrodynamics(-like) framework that is **more accurate out of equilibrium**

Pb-Pb @ 2.76 TeV - Don't worry, be happy?

$$\mathrm{R}_\pi^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathscr{P}_0}, \quad \mathrm{R}_\Pi^{-1} \equiv \frac{|\Pi|}{\mathscr{P}_0}.$$



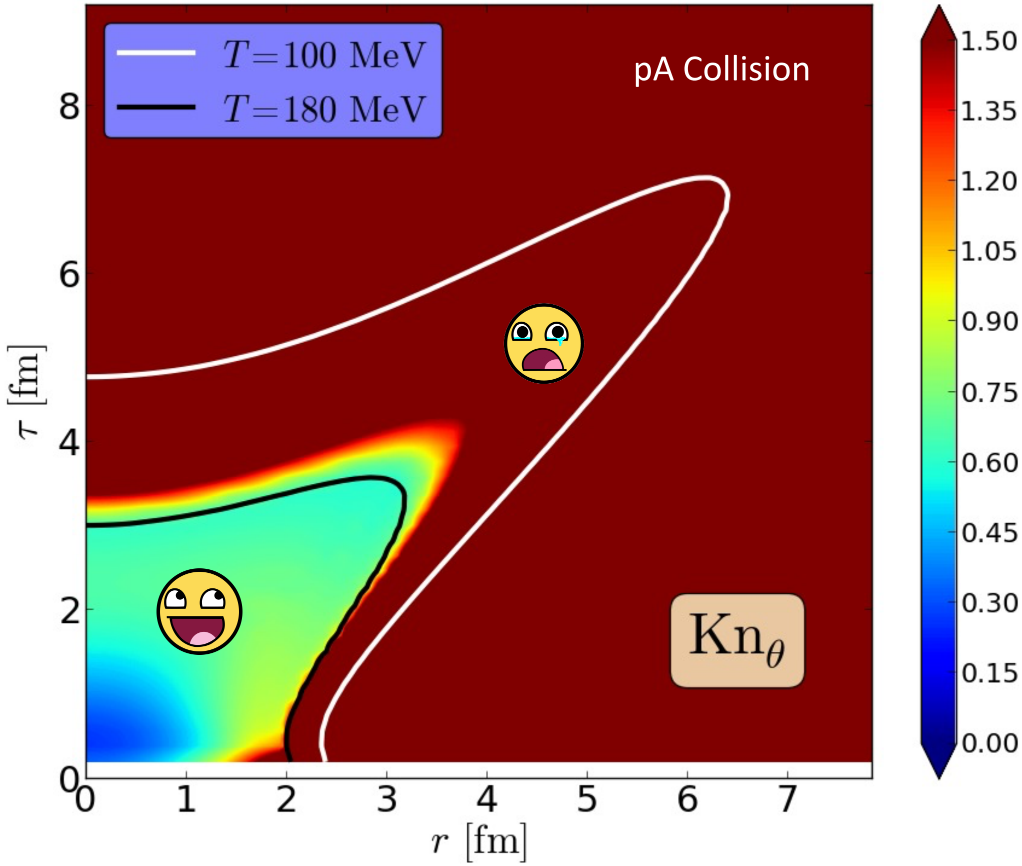
$$(\mathrm{R}_\pi^{(2)})^{-1} \equiv \frac{\sqrt{\mathscr{J}^{\mu\nu}\mathscr{J}_{\mu\nu}}}{2\eta\sqrt{\sigma^{\mu\nu}\sigma_{\mu\nu}}}, \quad (\mathrm{R}_\Pi^{(2)})^{-1} \equiv \frac{|\mathscr{J}|}{\zeta|\theta|}.$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R},$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu,$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathscr{J}^{\mu\nu} + \mathscr{K}^{\mu\nu} + \mathscr{R}^{\mu\nu}.$$

- $\mathcal{J}, \mathcal{J}^\mu$, and $\mathcal{J}^{\mu\nu}$ are O(Kn R^{-1})
- $\mathcal{K}, \mathcal{K}^\mu$, and $\mathcal{K}^{\mu\nu}$ are O(Kn^2)
- $\mathcal{R}, \mathcal{R}^\mu$, and $\mathcal{R}^{\mu\nu}$ are O(R^{-2})
- DNMR derivation assumes that Kn ~ R^{-1}
- For this to be a reasonable approx, the 2nd-order terms should be smaller than the O(Kn) Navier-Stokes terms
- **Secret: In order for code to run stably, it is necessary to "dynamically regulate" the viscous corrections**

p-A @ 2.76 TeV - Don't be happy, worry!

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327

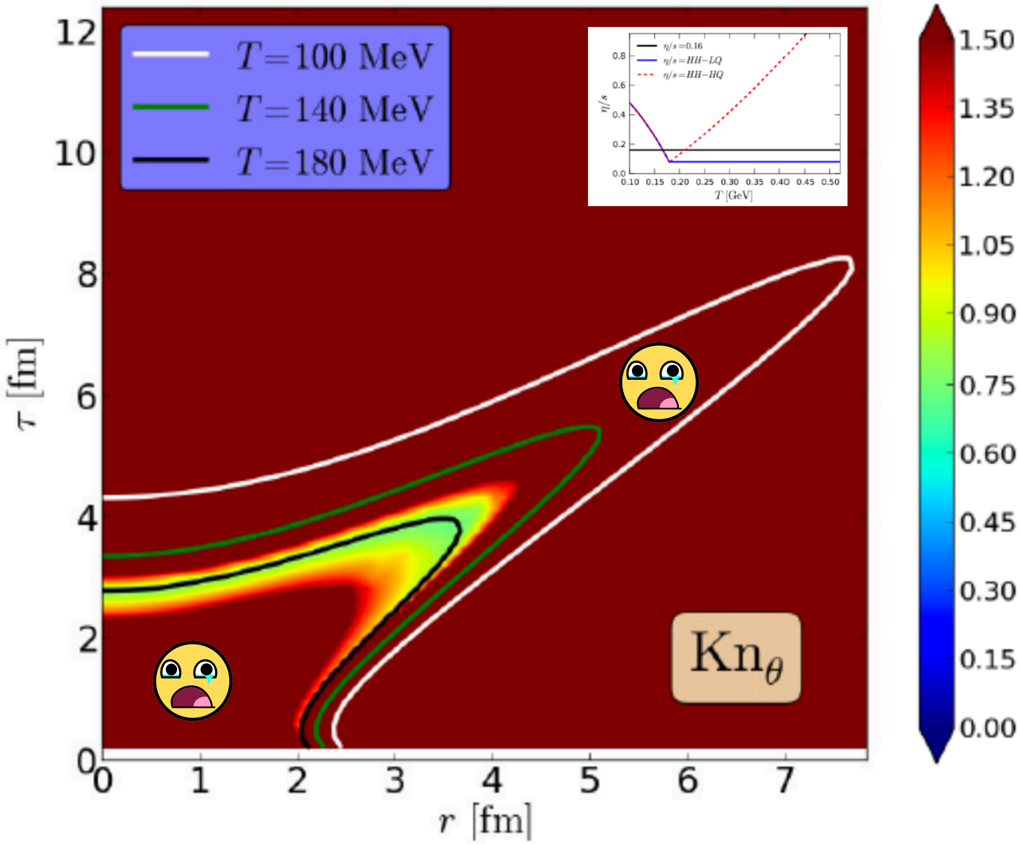


$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R}\,,$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu\,,$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathscr{J}^{\mu\nu} + \mathscr{K}^{\mu\nu} + \mathscr{R}^{\mu\nu}\,.$$

- $\mathcal{J}, \mathcal{J}^\mu$, and $\mathcal{J}^{\mu\nu}$ are O(Kn R^{-1})
- $\mathcal{K}, \mathcal{K}^\mu$, and $\mathcal{K}^{\mu\nu}$ are O(Kn^2)
- $\mathcal{R}, \mathcal{R}^\mu$, and $\mathcal{R}^{\mu\nu}$ are O(R^{-2})
- DNMR derivation assumes that Kn ~ R^{-1}
- For this to be a reasonable approx, the 2nd order terms should be smaller than the O(Kn) Navier-Stokes terms
- **In order for code to run stably, it is necessary to "dynamically regulate" the viscous corrections**

p-A @ 2.76 TeV - Don't be happy, worry!

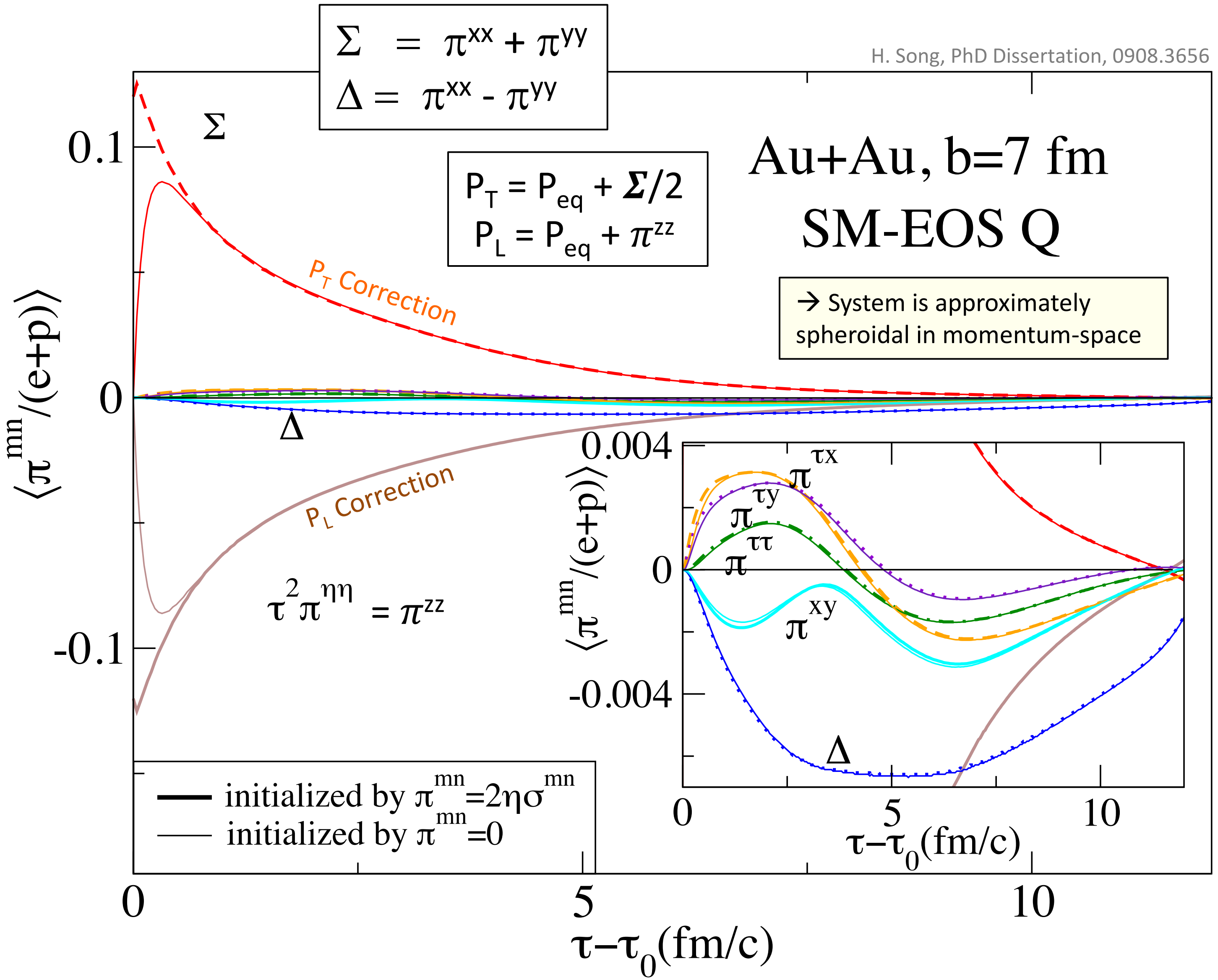
Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



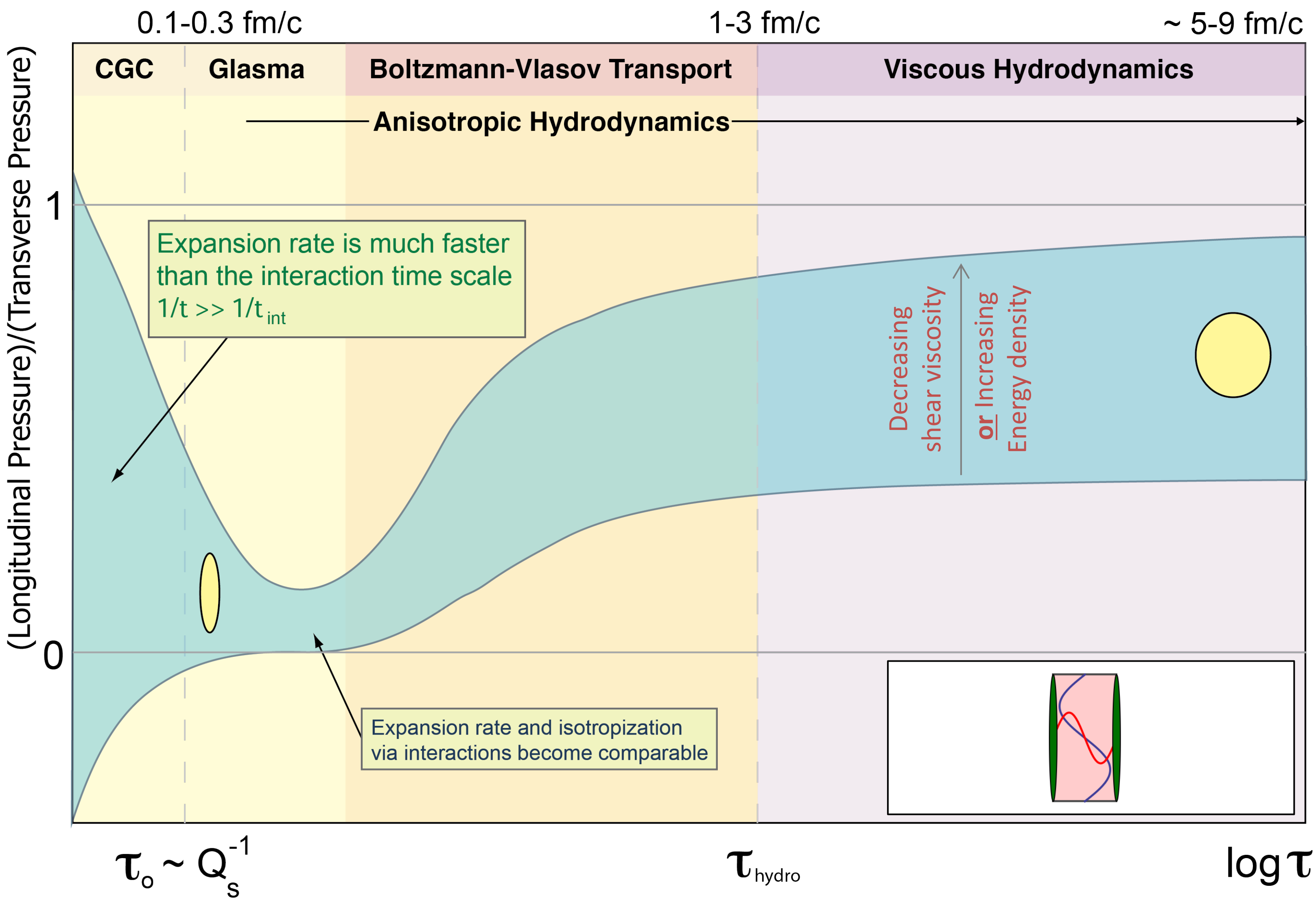
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R}\,,$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu\,,$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\,\sigma^{\mu\nu} + \mathscr{J}^{\mu\nu} + \mathscr{K}^{\mu\nu} + \mathscr{R}^{\mu\nu}\,.$$

- $\mathcal{J}, \mathcal{J}^\mu,$ and $\mathcal{J}^{\mu\nu}$ are O(Kn R^{-1})
- $\mathcal{K}, \mathcal{K}^\mu,$ and $\mathcal{K}^{\mu\nu}$ are O(Kn^2)
- $\mathcal{R}, \mathcal{R}^\mu,$ and $\mathcal{R}^{\mu\nu}$ are O(R^{-2})
- DNMR derivation assumes that Kn ~ R^{-1}
- For this to be a reasonable approx, the 2nd order terms should be smaller than the O(Kn) Navier-Stokes terms
- **In order for code to run stably, it is necessary to "dynamically regulate" the viscous corrections**

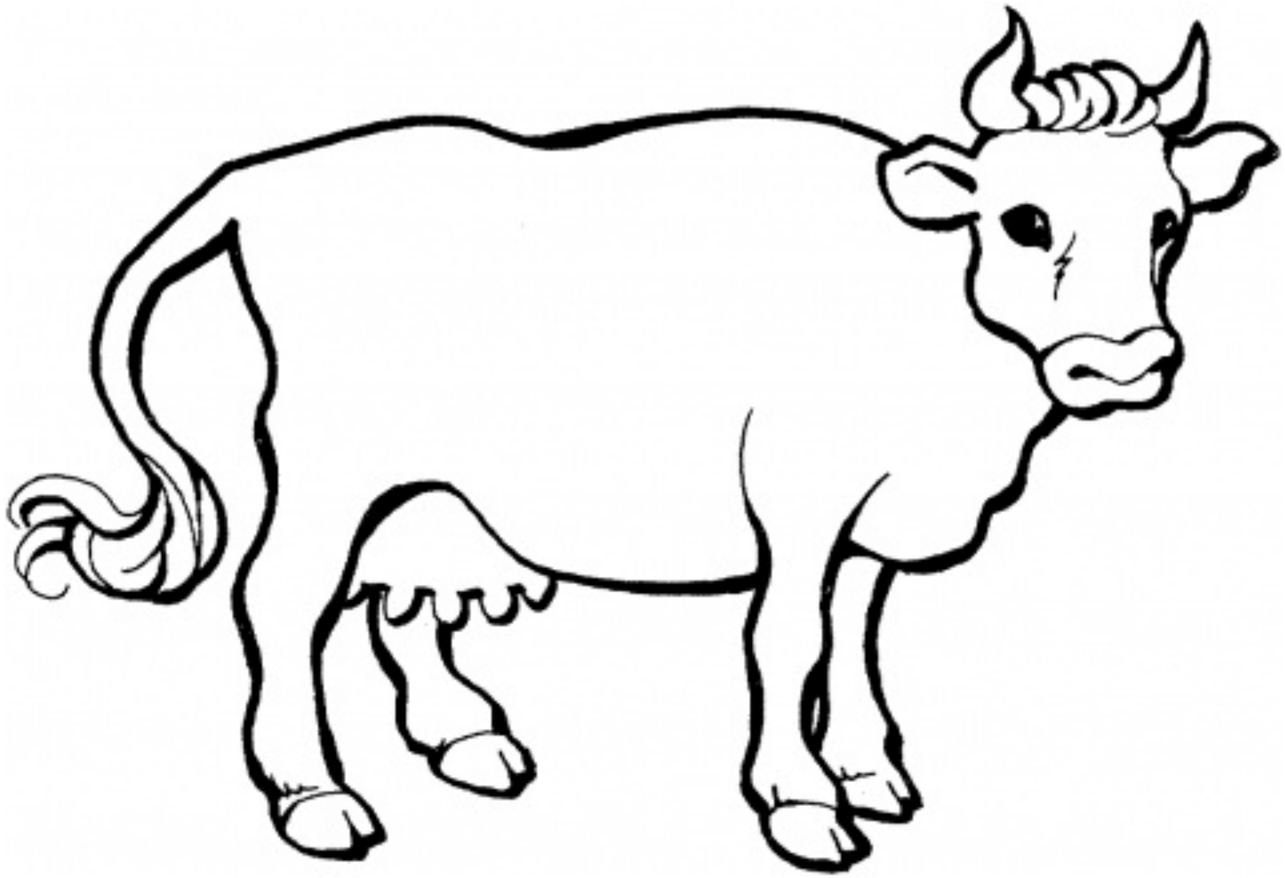
What are the largest viscous corrections?



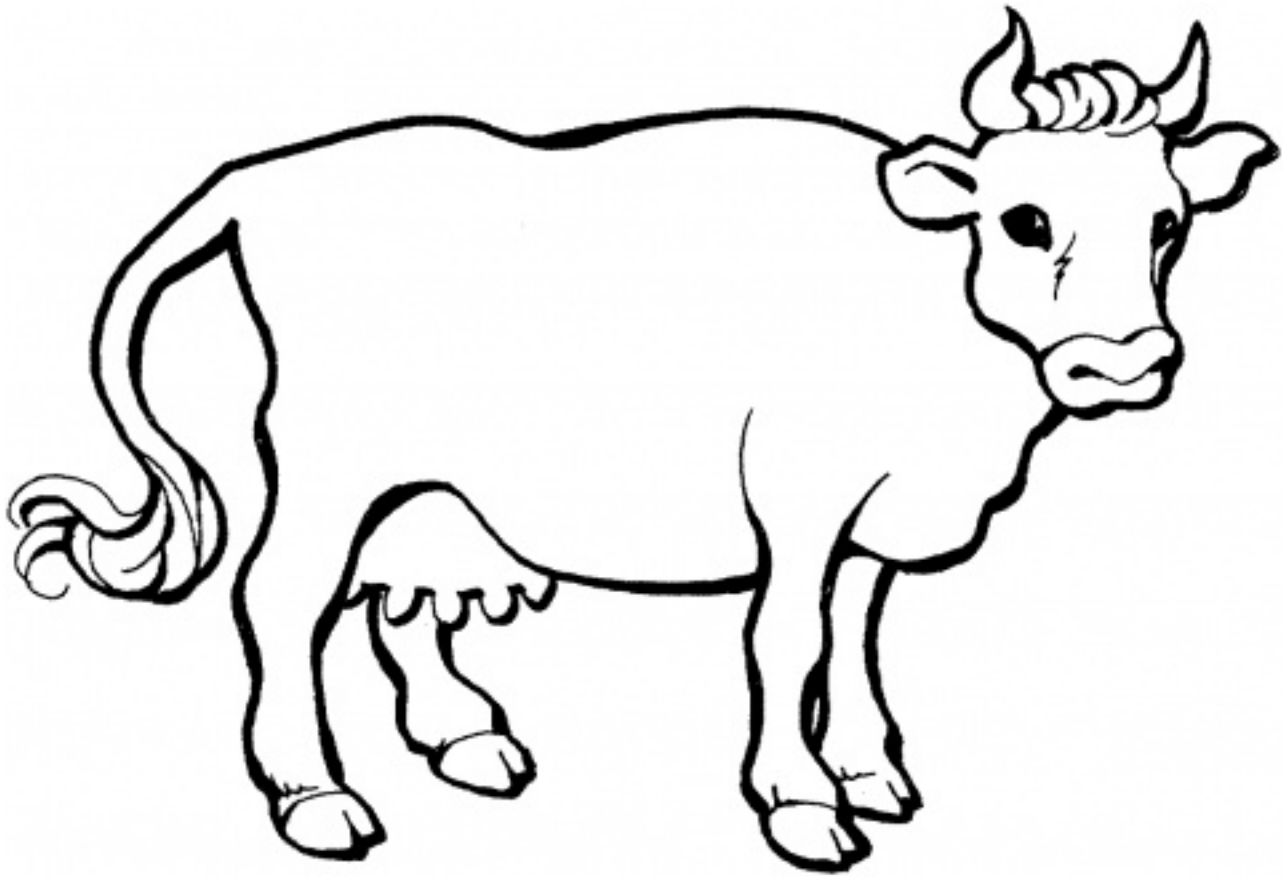
QGP momentum anisotropy cartoon



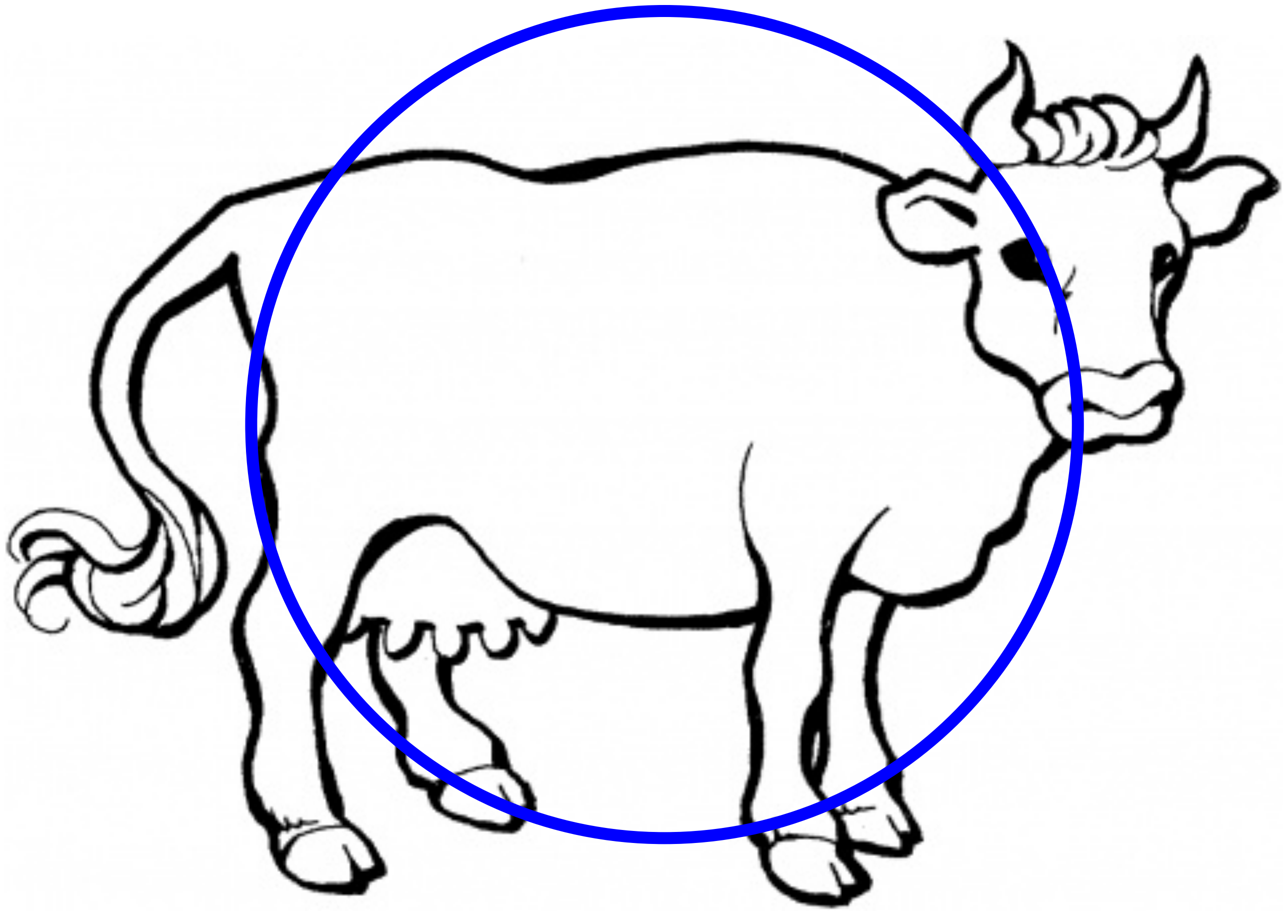
Physics 101



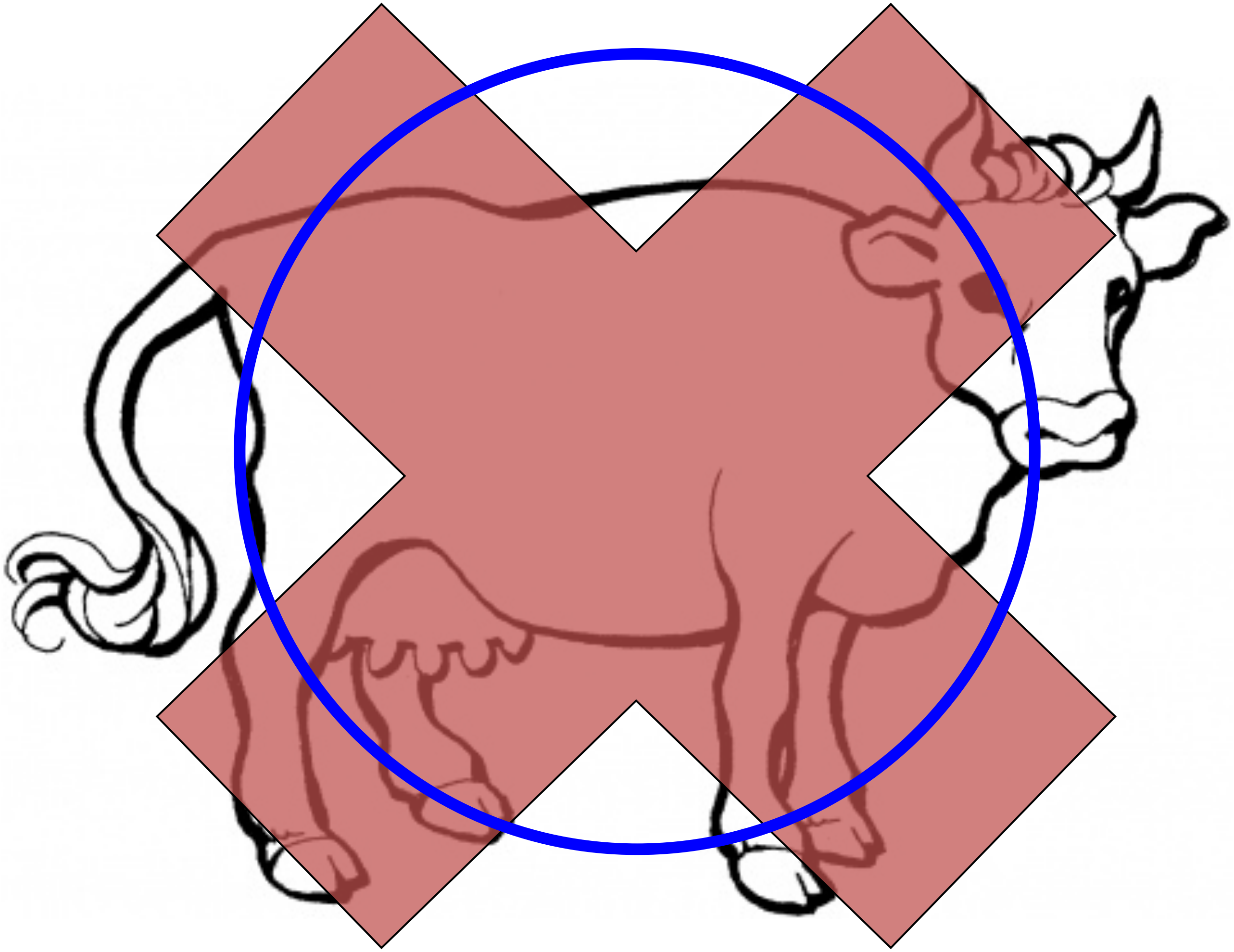
Cows are spheres?



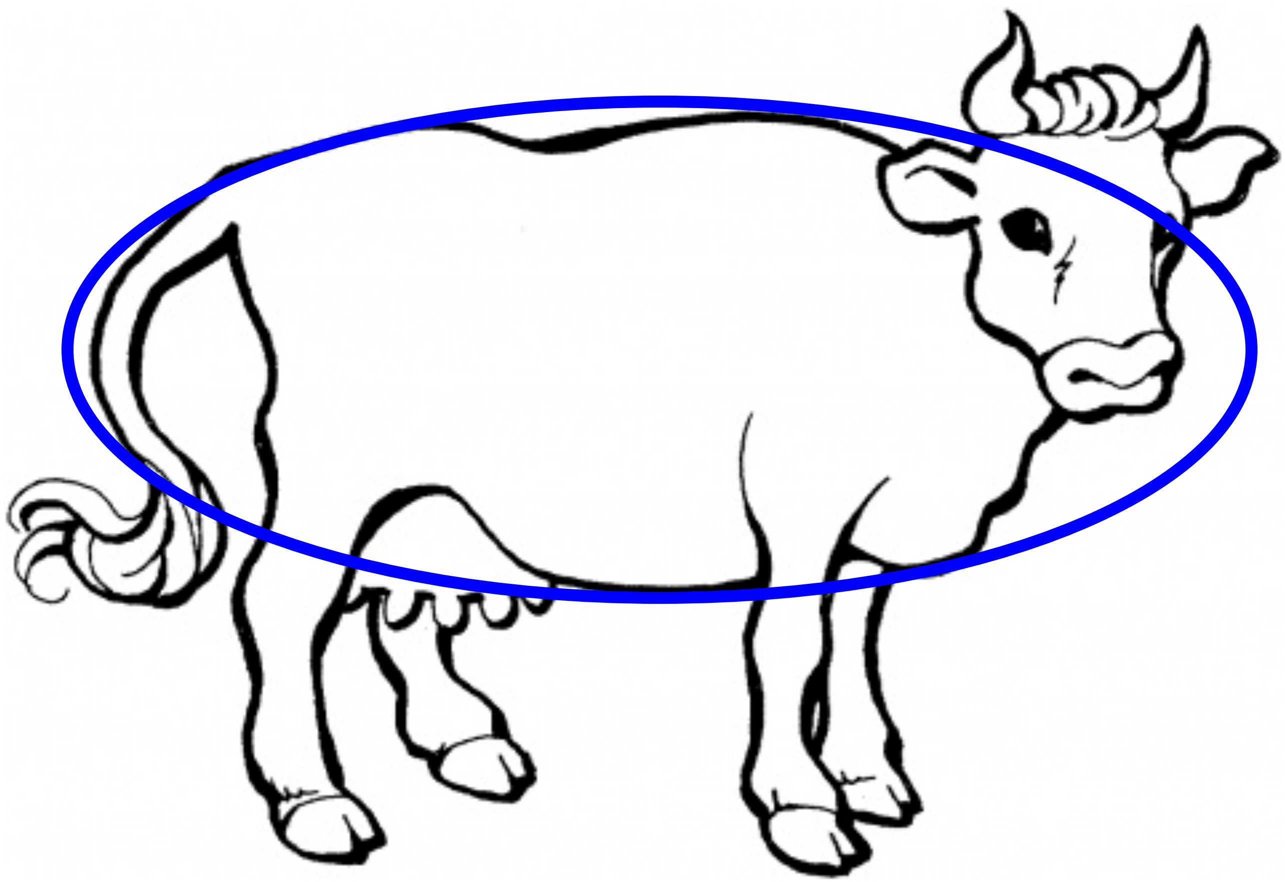
Cows are spheres?



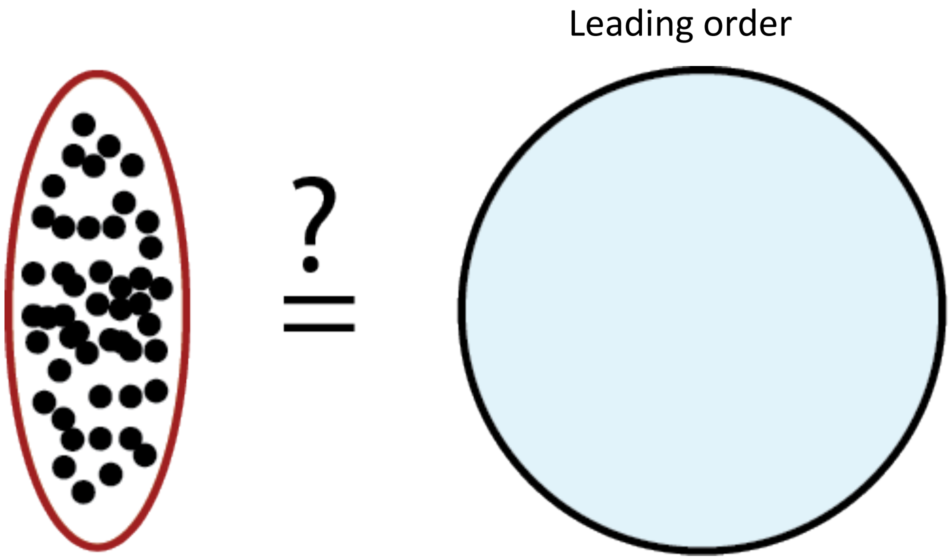
Cows are not spheres



Cows are more like ellipsoids!

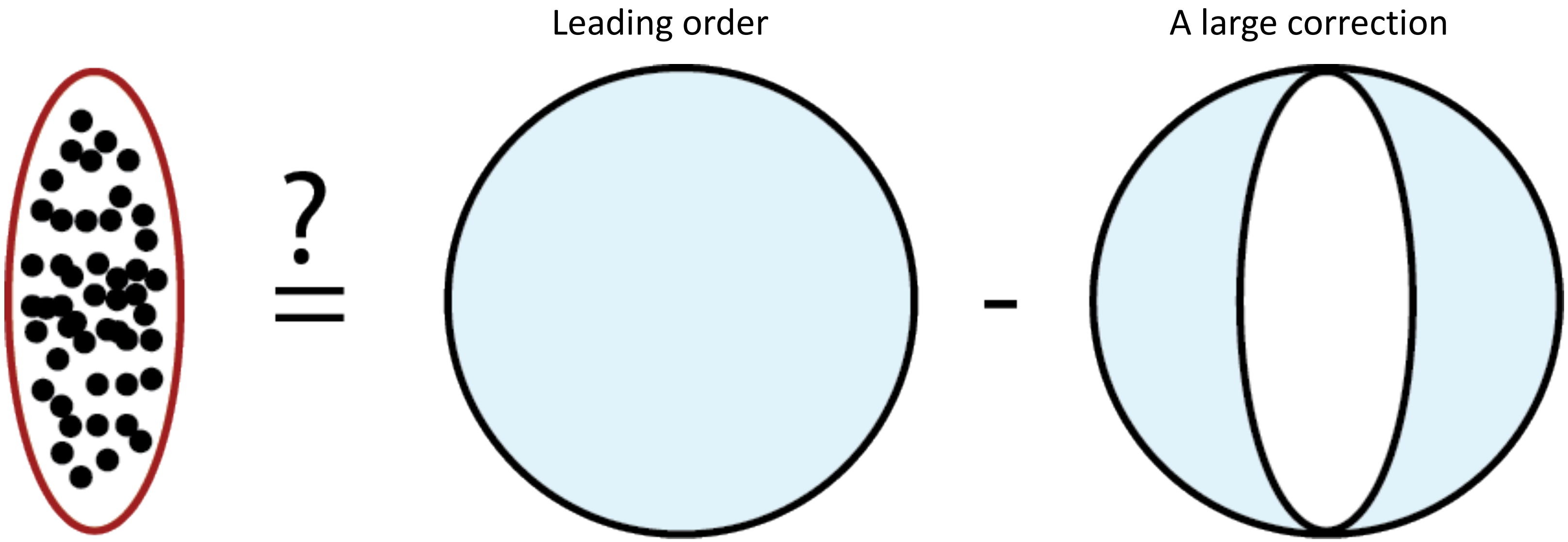


Non-spherical cows



Viscous hydro says that we should approximate our particle momentum-space distribution to first order by a sphere in momentum space. However, if the system is highly anisotropic in momentum space, this will result in large corrections…

Non-spherical cows



Viscous hydro says that we should approximate our particle momentum-space distribution to first order by a sphere in momentum space. However, if the system is highly anisotropic in momentum space, this will result in large corrections…

Spheroidal expansion method

Viscous Hydrodynamics Expansion

See e.g.
- M. Martinez and MS, 1007.0889
- W. Florkowski and R. Ryblewski, 1007.0130
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

$$f(\tau,\mathbf{x},\mathbf{p}) = \underline{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))} + \delta f$$

Isotropic in momentum space

Anisotropic Hydrodynamics (aHydro) Expansion

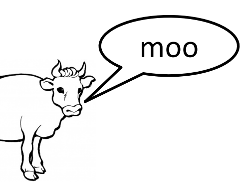
$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\rm anisotropy}\,) + \delta\tilde{f}$$

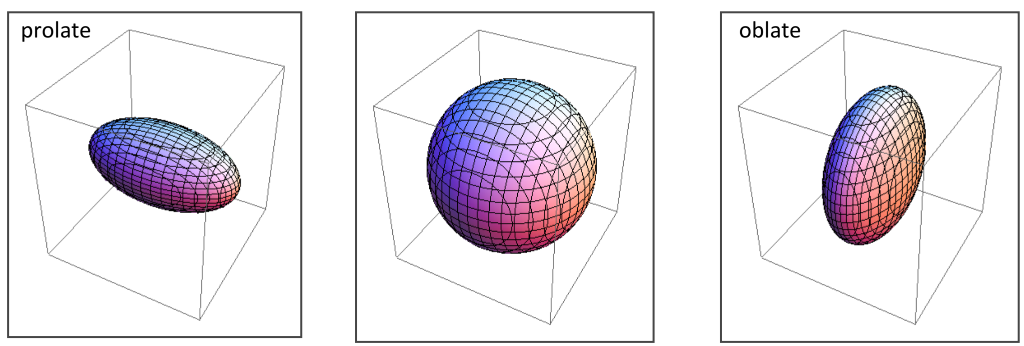
Treat this term perturbatively
→ "NLO aHydro"

→ "Romatschke-Strickland" form in LRF

$$f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

$$\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1$$





$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the ideal hydro limit when $\xi=0$ ($\Lambda \to T$)
- For longitudinal (0+1d) free streaming, the LRF distribution function is of spheroidal form; limit emerges automatically in conformal 0+1d aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in standard 2nd-order viscous hydro)
- Reduces to 2nd-order viscous hydrodynamics in limit of small anisotropies

M. Martinez and MS, 1007.0889

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

For general (3+1d) proof of equivalence to second-order viscous hydrodynamics using generalized RS form in the near-equilibrium limit see Tinti 1411.7268.

The growing anisotropic hydrodynamics family

- There are two approaches being actively followed in the literature to address this problem

  A. Linearize around a spheroidal distribution function and treat the perturbations using standard kinetic vHydro methods [“vaHydro”]
  Bazow, Heinz, Martinez, Molnar, Niemi, Rischke, MS

  B. Introduce a generalized anisotropy tensor which replaces the entire viscous stress tensor at LO and then linearize around that instead
  Tinti, Ryblewski, Martinez, Nopoush, Alqahtani, Bluhm, Florkowski, Schaefer, MS

- Each of these methods has its own advantages.

- In what I will show today, I will use the generalized method (B) at leading order.

Generalized aHydro formalism

In generalized aHydro, one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{"Bulk"}}$$

$$u^\mu u_\mu = 1$$

$$\xi^\mu{}_\mu = 0$$

$$\Delta^\mu{}_\mu = 3$$

$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

→ 11 DOFs

See e.g.
- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Equations of Motion

- Herein the EOM are obtained from moments of the Boltzmann equation in the relaxation time approximation (RTA) including temperature -dependent quasiparticle mass

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f] \qquad \mathcal{C}[f] = \frac{p^\mu u_\mu}{\tau_{\rm eq}}(f - f_{\rm eq})$$

- **It is relatively straightforward to use other collisional kernels (forthcoming)**

- 1 equation from the 0$^{\rm th}$ moment [number (non-conservation)]
- 4 equations from the 1$^{\rm st}$ moment [energy-momentum conservation]
- 6 equations from the 2$^{\rm nd}$ moment [dissipative dynamics]
- We must also specify the relation between the equilibrium (isotropic) pressure and energy density (EoS). <u>More on this later.</u>

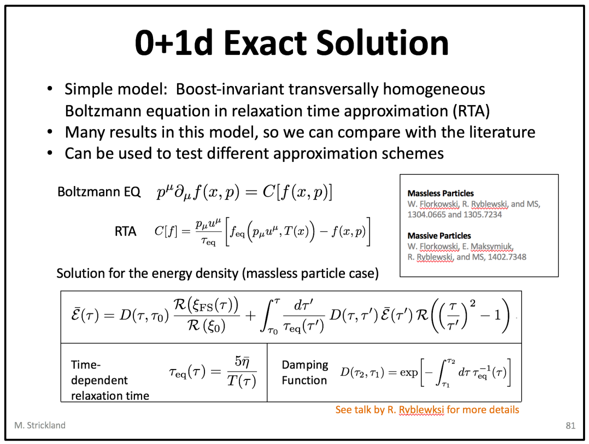
$$D_u n + n\theta_u = \frac{1}{\tau_{\rm eq}}(n_{\rm eq} - n)$$

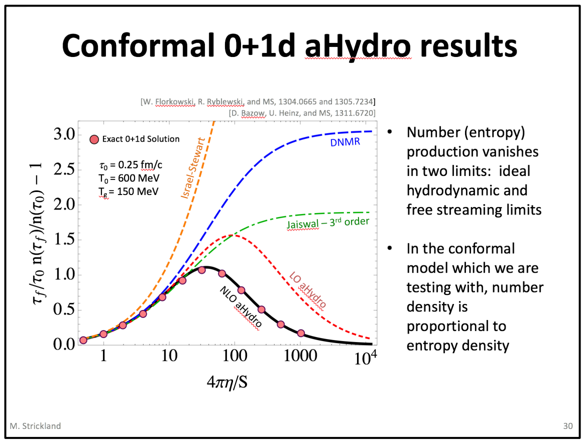
$$\partial_\mu T^{\mu\nu} = 0$$

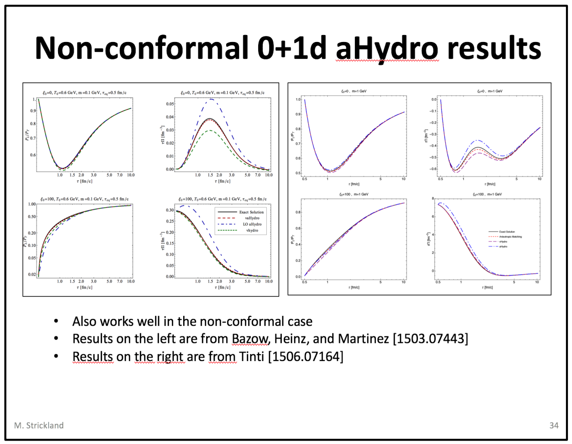
$$\partial_\mu \mathcal{I}^{\mu\nu\lambda} = \frac{1}{\tau_{\rm eq}}(u_\mu \mathcal{I}^{\mu\nu\lambda}_{\rm eq} - u_\mu \mathcal{I}^{\mu\nu\lambda}) \qquad \mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.$$

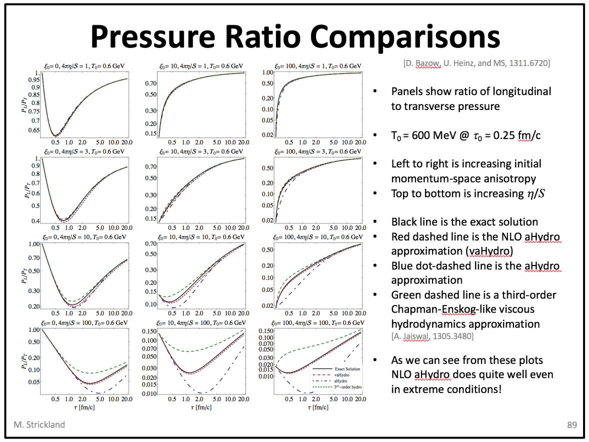
Is it really better?

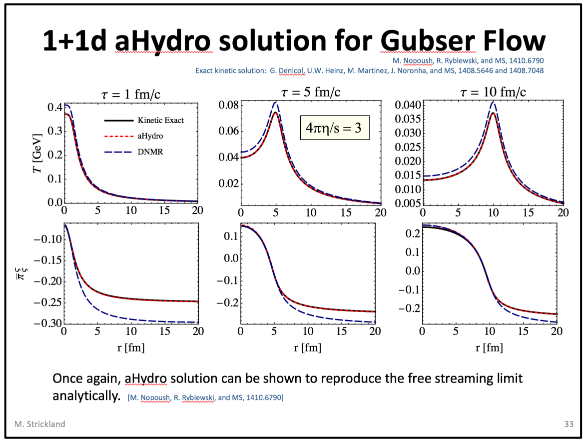
aHydro reproduces exact solutions to the Boltzmann equation in a variety of expanding backgrounds better than standard viscous hydrodynamics.

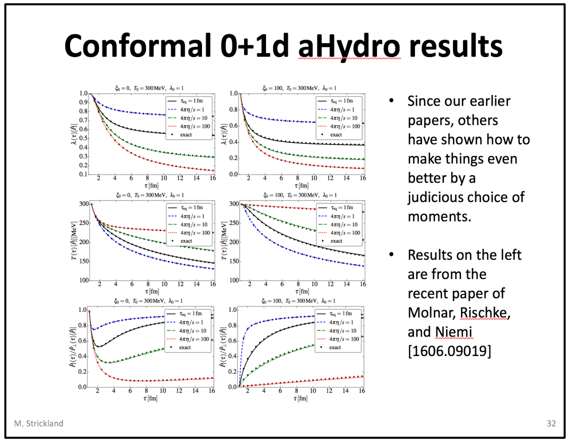






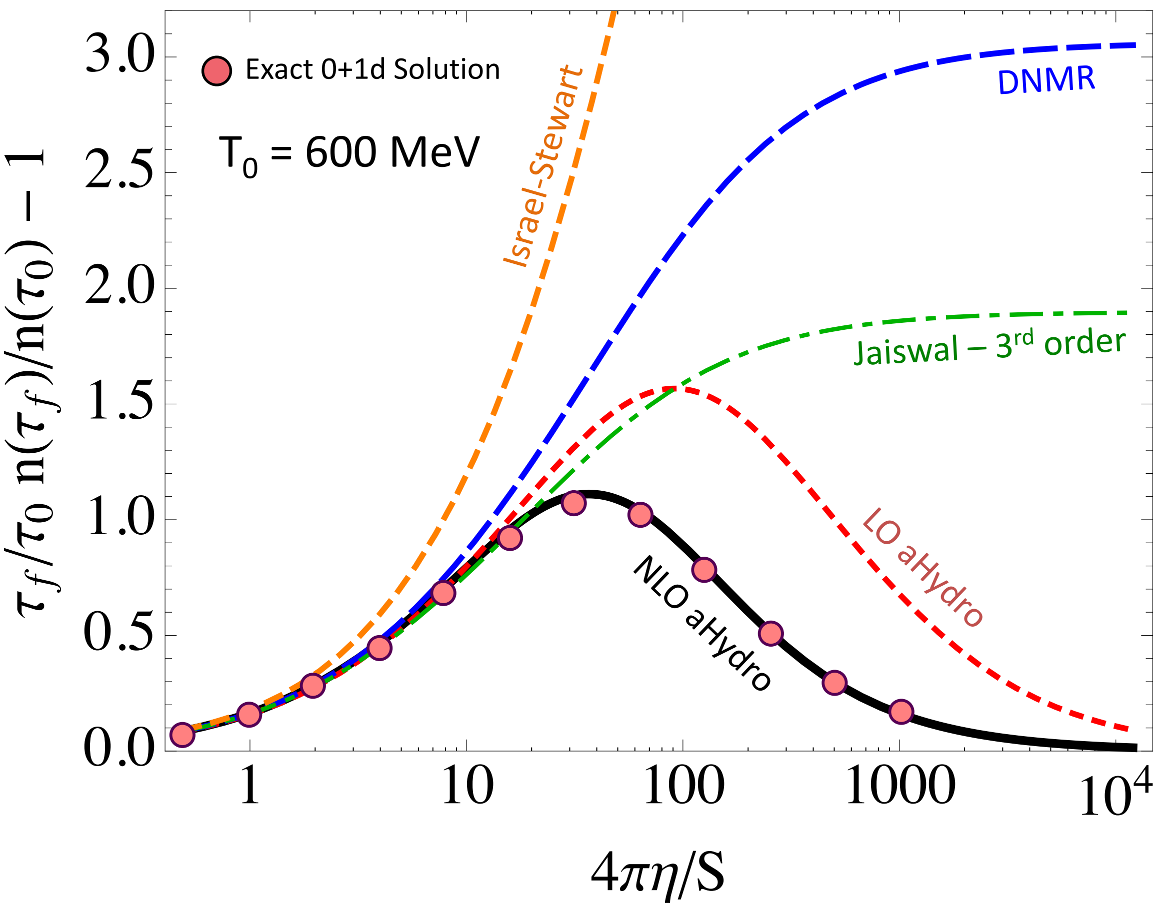






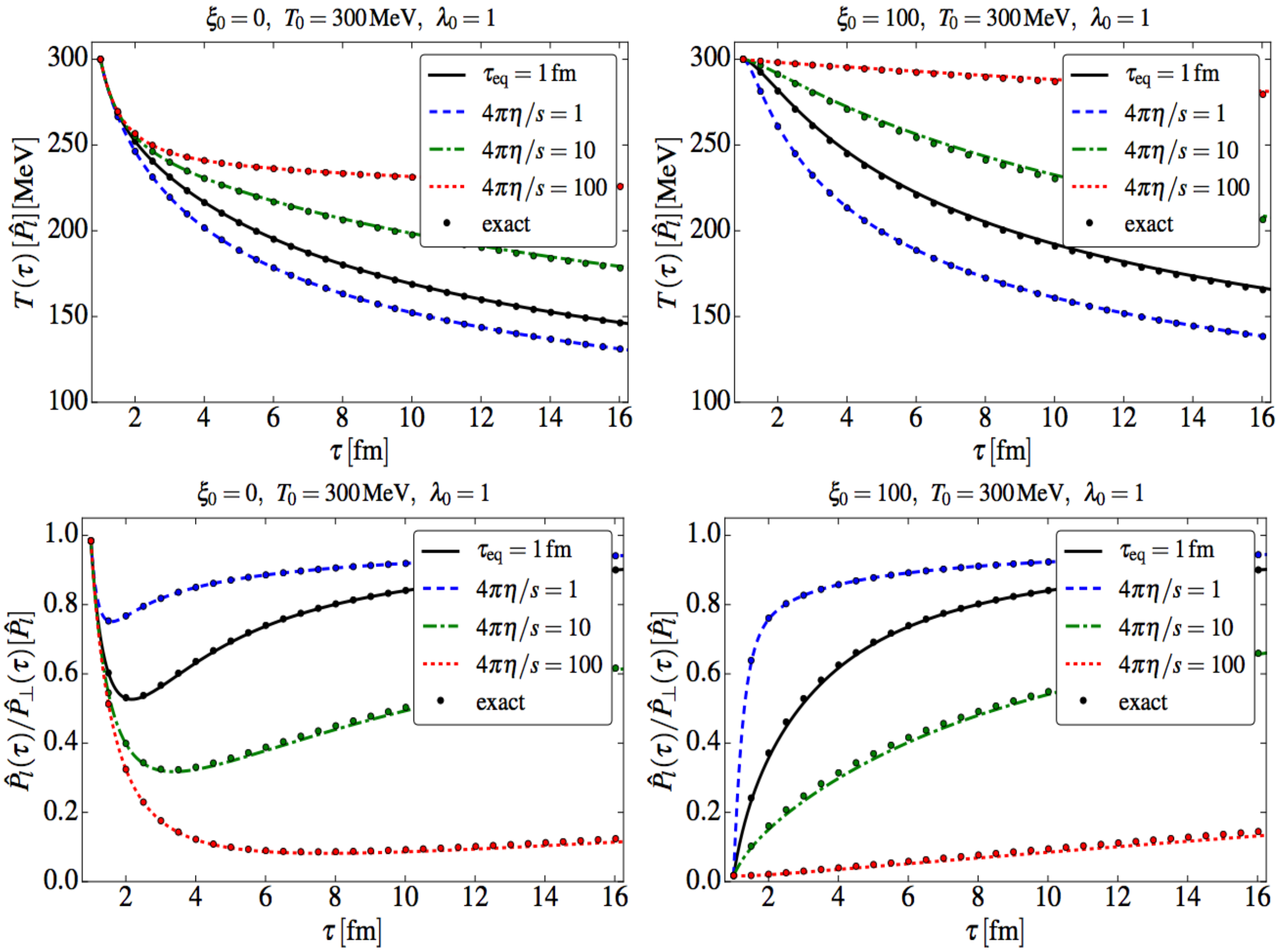
Ex. 1: Dissipative particle production

[D. Bazow, U. Heinz, and MS, 1311.6720]



- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits
- In the conformal model which we are testing with, number density is proportional to entropy density

Ex. 2: Conformal 0+1d aHydro results



- aHydro results (lines) on the left are from the recent paper of Molnar, Rischke, and Niemi [1606.09019]
- Exact solution is shown by dots [W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234]

Ex 3: Gubser Flow

S. Gubser, 1006.0006
S. Gubser and Y.Yarom, 1012.1314

Gubser flow is a cylindrically-symmetric and boost-invariant flow that possesses a high degree of symmetry when mapped to Weyl-rescaled deSitter space

$$SO(3)_q \times SO(1,1) \times Z_2$$

$SO(3)_q$	$SO(1,1)$	Z_2
rotational symmetry around beam axis + conformal symmetry	boost invariance	reflection symmetry around the collision plane

The parameter q above is an arbitrary energy scale that sets the radial extent of the system at a given proper time.

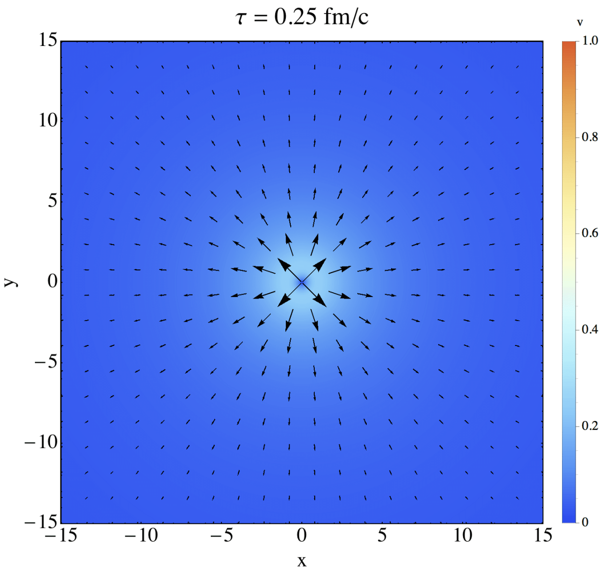
Polar Milne components

$$\tilde{u}^\tau = \cosh(\theta_\perp)$$
$$\tilde{u}^r = \sinh(\theta_\perp)$$
$$\tilde{u}^\phi = 0$$
$$\tilde{u}^\varsigma = 0$$

Transverse rapidity

$$\theta_\perp = \tanh^{-1}\left(\frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}\right)$$

This flow is quite strong: The de Sitter space velocity gradients grow exponentially e$^{|\rho|}$



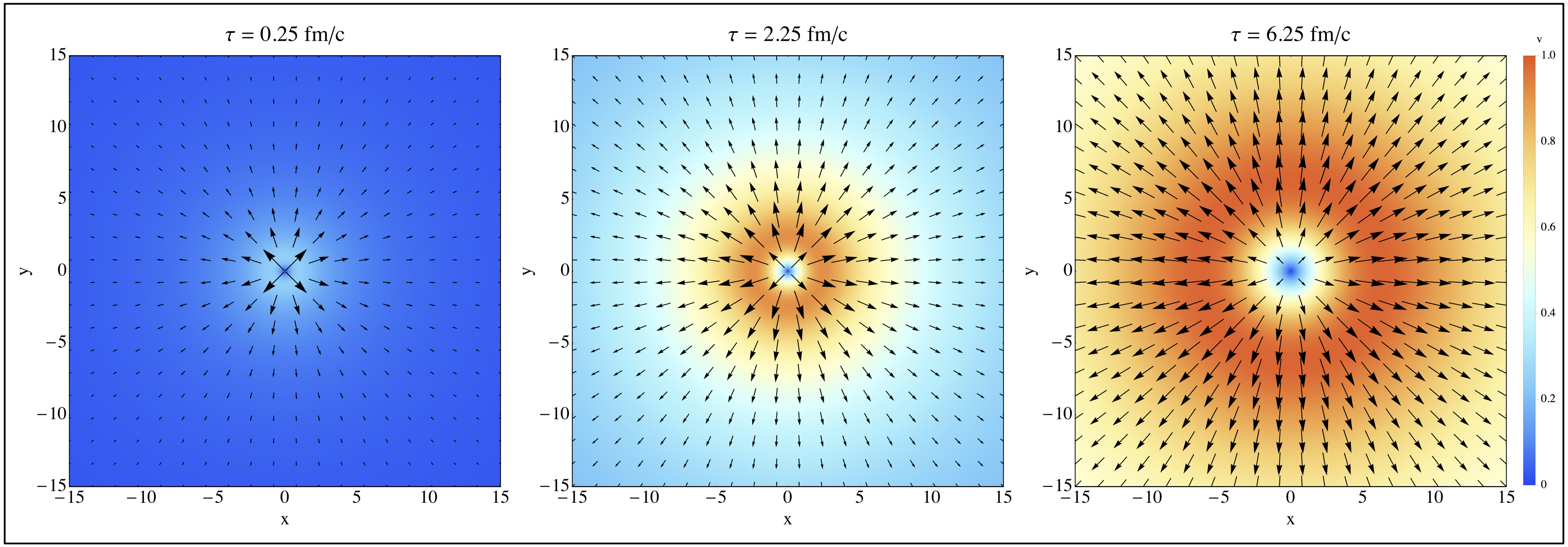
Ex 3: Gubser Flow

S. Gubser, 1006.0006
S. Gubser and Y. Yarom, 1012.1314

Gubser flow is a cylindrically-symmetric and boost-invariant flow that possesses a high degree of symmetry when mapped to Weyl-rescaled deSitter space

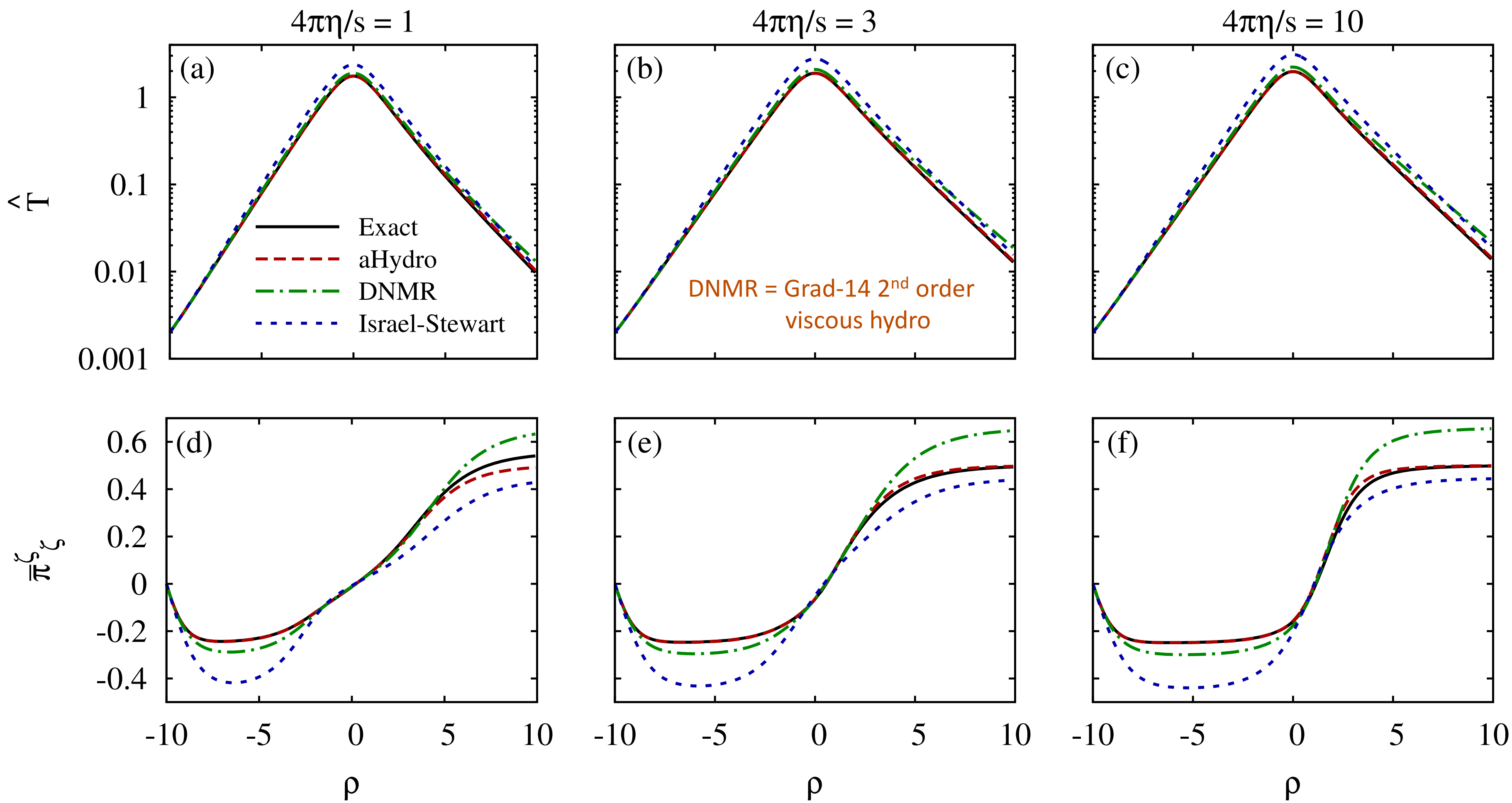
$$SO(3)_q \times SO(1,1) \times Z_2$$

$SO(3)_q$	$SO(1,1)$	Z_2
rotational symmetry around beam axis + conformal symmetry	boost invariance	reflection symmetry around the collision plane



Ex 3: LO aHydro for Gubser flow

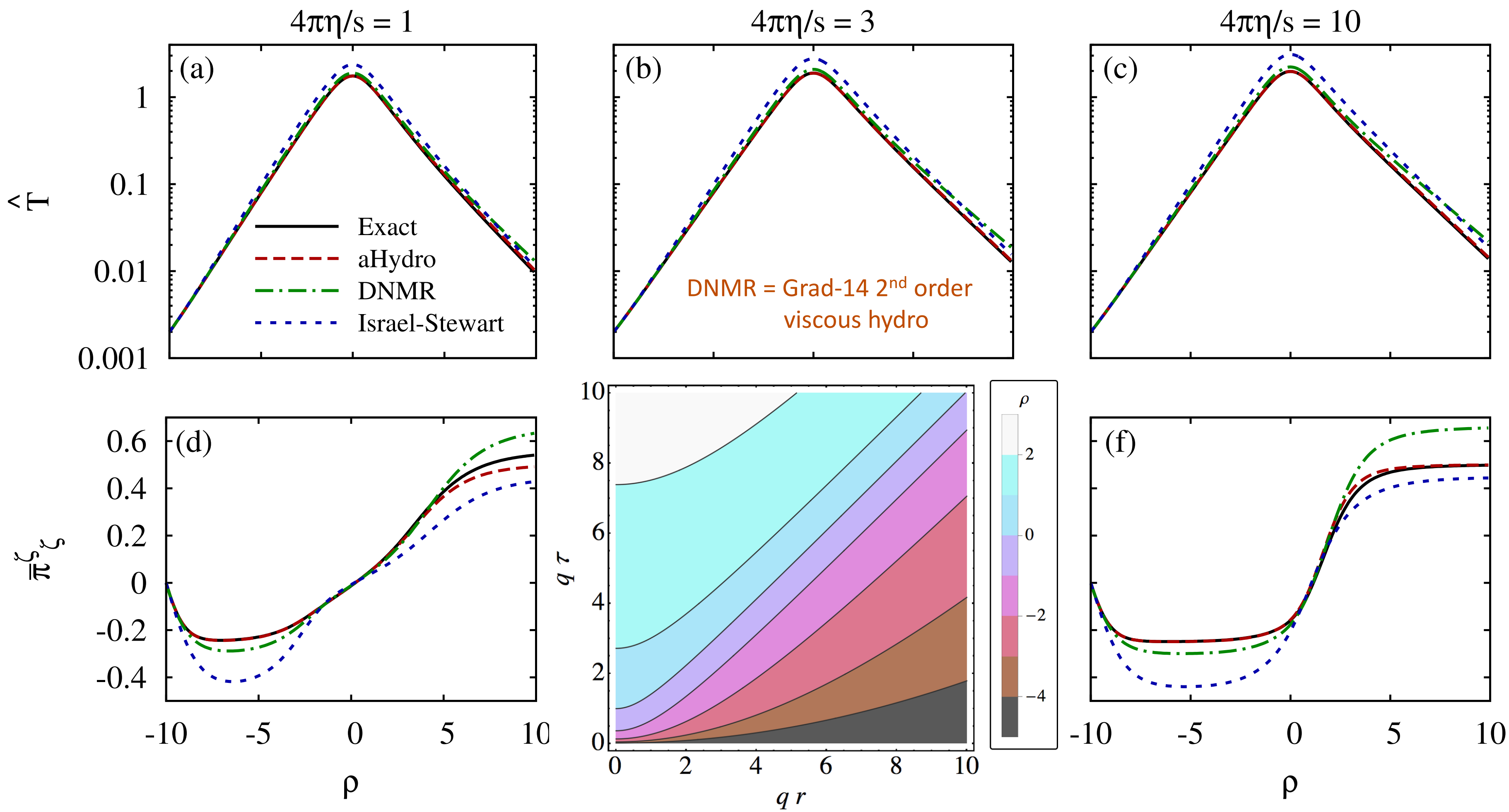
M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



Isotropic initial conditions

Ex 3: LO aHydro for Gubser flow

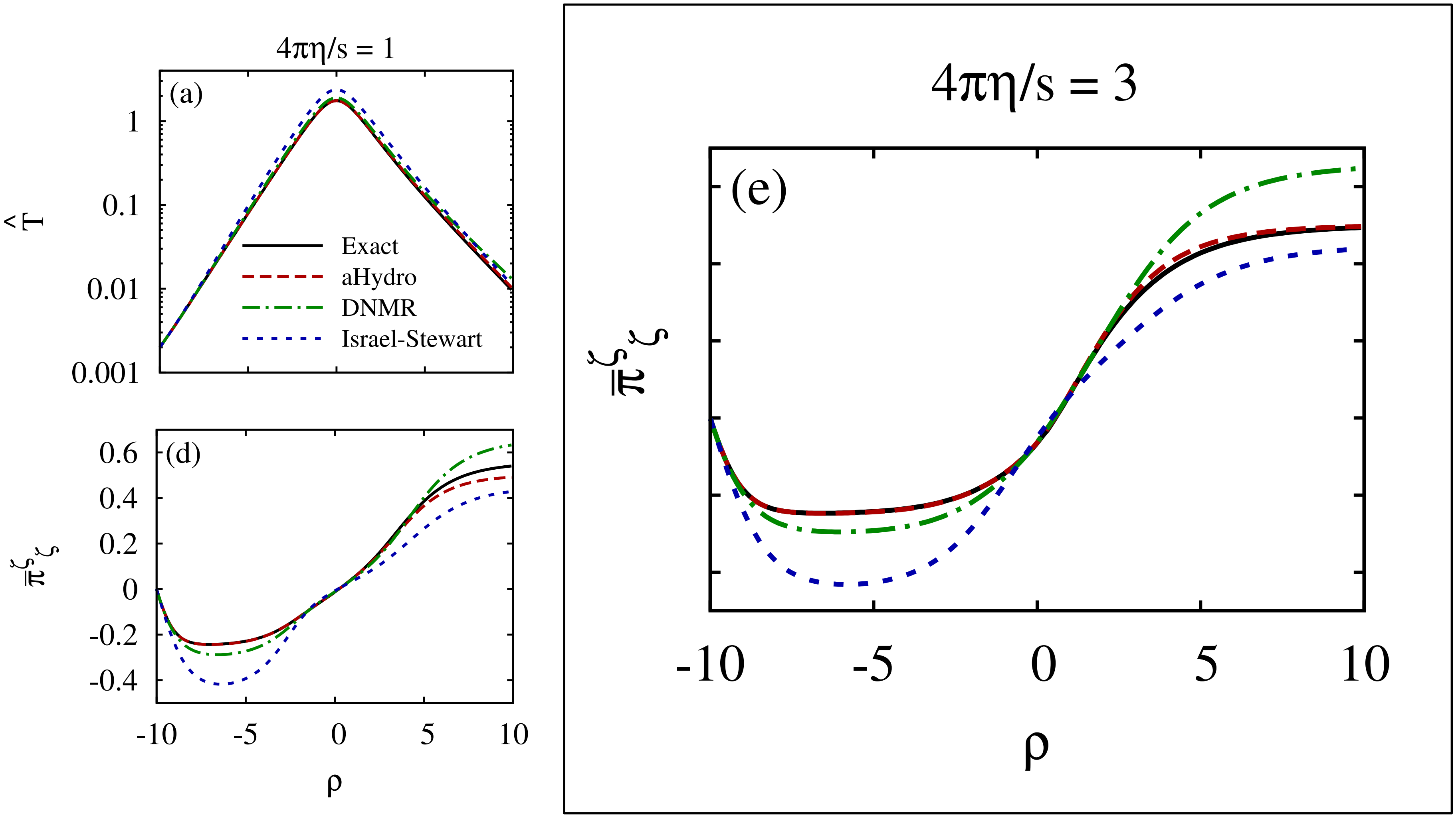
M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



Isotropic initial conditions

Ex 3: LO aHydro for Gubser flow

M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact Solution: G. Denicol, U.Heinz, M. Martinez, J. Noronha, and MS, 1408.5646; 1408.7048

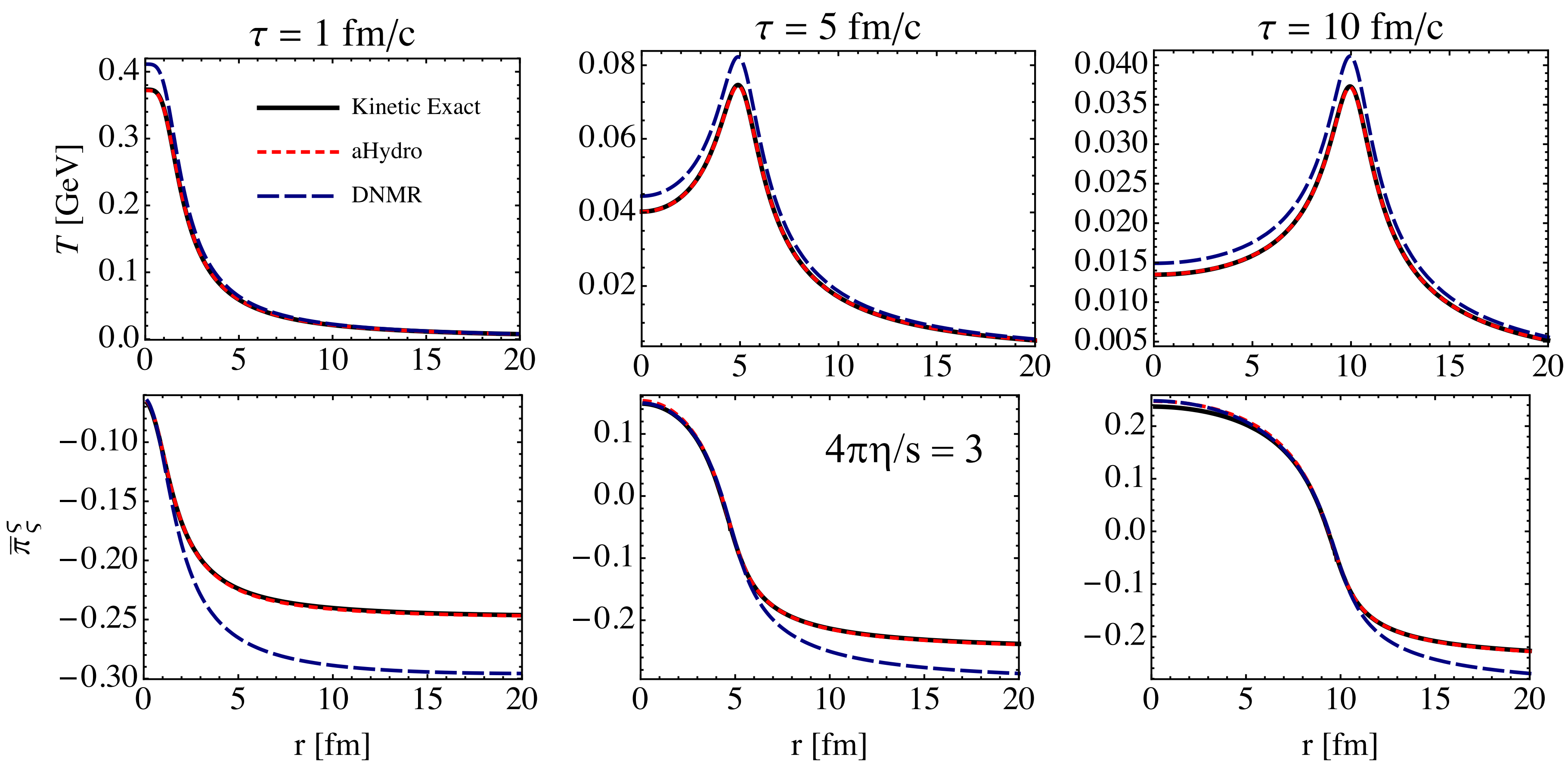


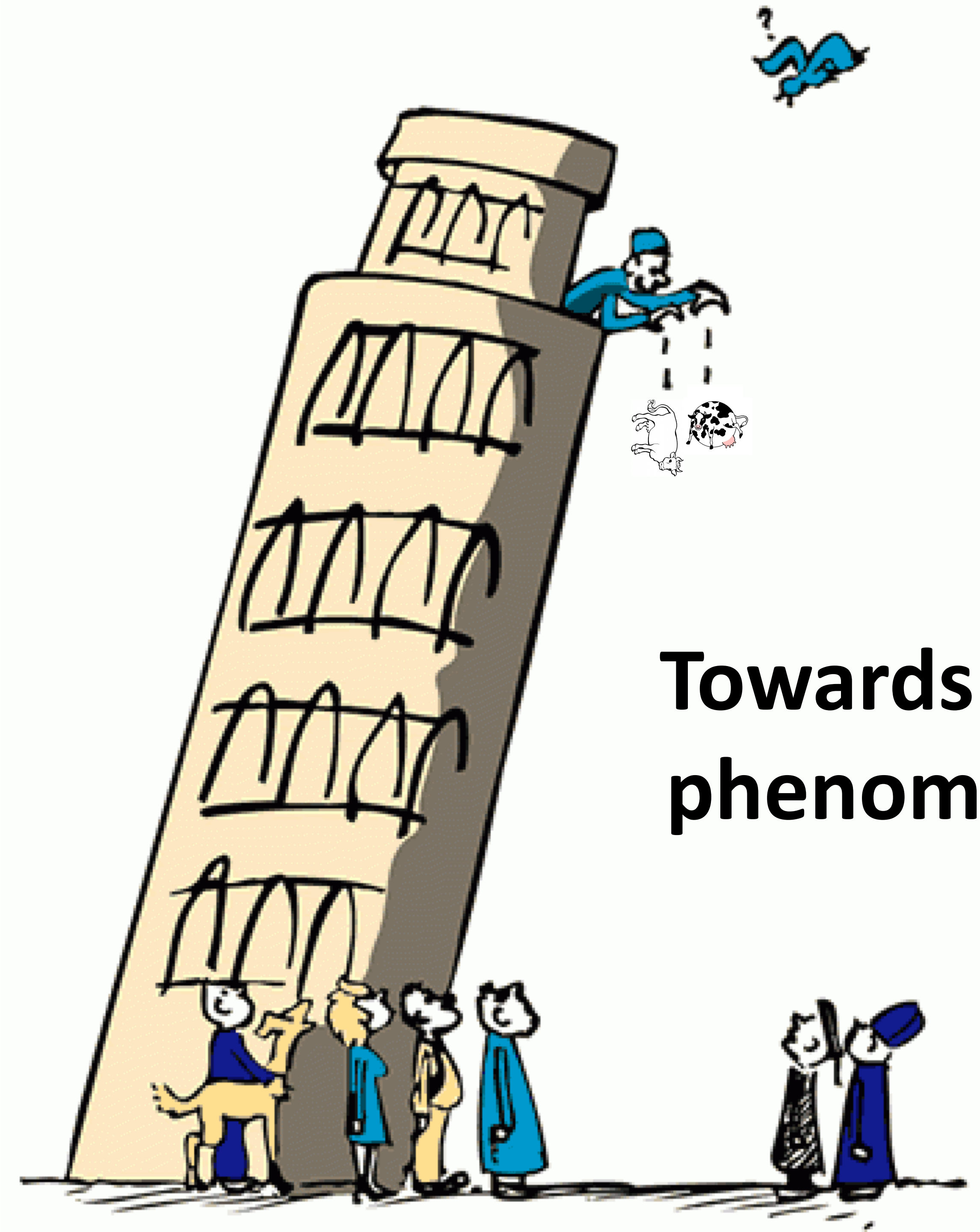
At NLO even better agreement with exact solution; see M. Martinez, M. McNelis, and U. Heinz, 1703.10955

Ex 3: aHydro for Gubser flow

M. Nopoush, R. Ryblewski, and MS, 1410.6790

Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646; 1408.7048





Towards realistic phenomenology

3+1d aHydro Equations of Motion

- Assuming an ellipsoidal form for the anisotropy tensor (ignoring off-diagonal components for now), one has seven degrees of freedom ξ_x, ξ_y, ξ_z, u_x, u_y, u_z, and λ which are all fields of space and time.
- Ignore $\delta\tilde{f}$ for now

$$
\begin{aligned}
D_u\mathcal{E} + \mathcal{E}\theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu &= 0\,,\\
D_x\mathcal{P}_x + \mathcal{P}_x\theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu &= 0\,,\\
D_y\mathcal{P}_y + \mathcal{P}_y\theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu &= 0\,,\\
D_z\mathcal{P}_z + \mathcal{P}_z\theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu &= 0\,.
\end{aligned}
$$

First Moment

$$
\mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.
$$

$$
\begin{aligned}
\mathcal{I}_i &= \alpha\, \alpha_i^2\, \mathcal{I}_{\rm eq}(\lambda,m)\,,\\
\mathcal{I}_{\rm eq}(\lambda,m) &= 4\pi \tilde{N} \lambda^5 \hat{m}^3 K_3(\hat{m})\,,
\end{aligned}
$$

$$
\begin{aligned}
D_u\mathcal{I}_x + \mathcal{I}_x(\theta_u + 2u_\mu D_x X^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_x)\,,\\
D_u\mathcal{I}_y + \mathcal{I}_y(\theta_u + 2u_\mu D_y Y^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_y)\,,\\
D_u\mathcal{I}_z + \mathcal{I}_z(\theta_u + 2u_\mu D_z Z^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_z)\,.
\end{aligned}
$$

Second Moment

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Implementing the equation of state

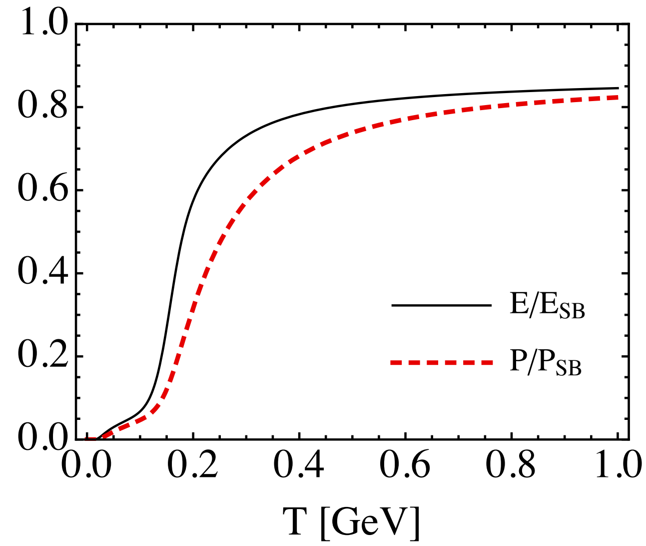
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

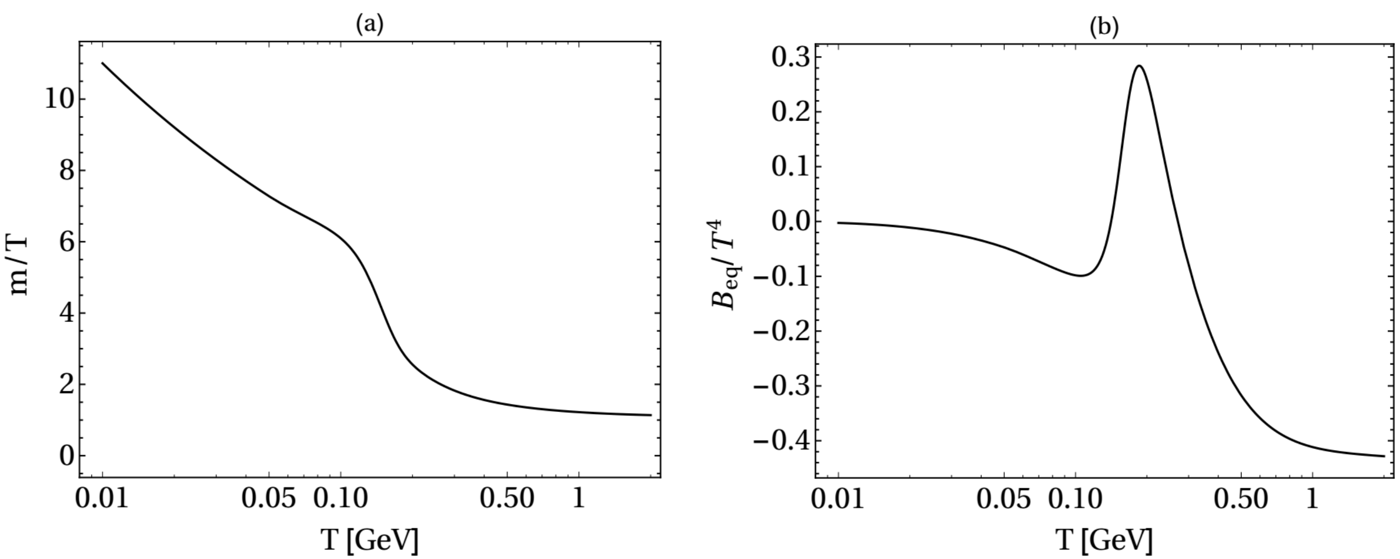
Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP f(x,p)$$





Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entry density ratio.

$$\frac{\eta}{\tau_{\rm eq}} = \frac{1}{T} I_{3,2}(\hat{m}_{\rm eq})$$

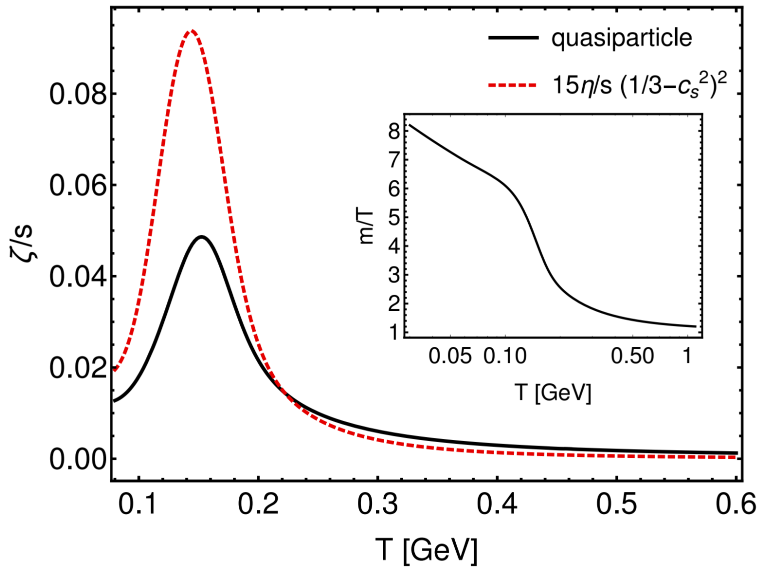
Bulk viscosity

$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,2} - c_s^2(\mathcal{E}+\mathcal{P}) + T\hat{m}^3 \frac{dm}{dT} I_{1,1}$$

$$I_{3,2}(x) = \frac{N_{\rm dof} T^5\, x^5}{30\pi^2}\left[\frac{1}{16}\Big(K_5(x) - 7K_3(x) + 22K_1(x)\Big) - K_{i,1}(x)\right],$$

$$K_{i,1}(x) = \frac{\pi}{2}\left[1 - xK_0(x)\mathcal{S}_{-1}(x) - xK_1(x)\mathcal{S}_0(x)\right],$$

$$I_{1,1} = \frac{g\, m^3}{6\pi^2}\left[\frac{1}{4}(K_3 - 5K_1) + K_{i,1}\right]$$



Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entry density ratio.

$$\frac{\eta}{\tau_{\rm eq}} = \frac{1}{T} I_{3,2}(\hat{m}_{\rm eq})$$

Bulk viscosi

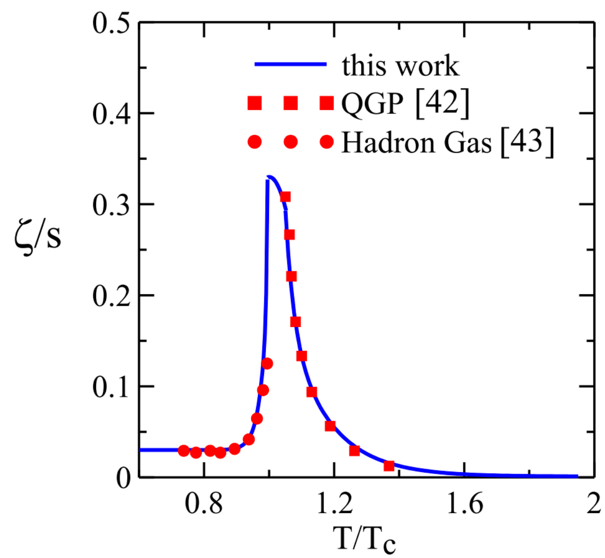
$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,$$

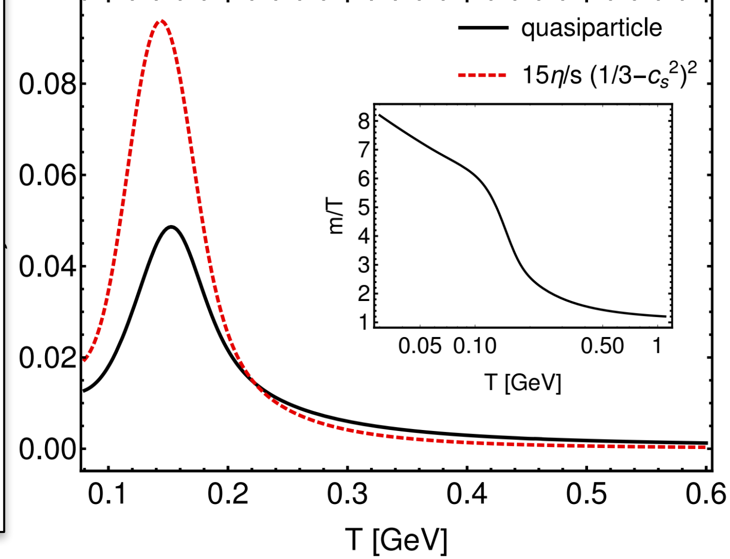
$$I_{3,2}(x) = \frac{N_{\rm dof}}{30}$$

$$K_{i,1}(x) = \frac{\pi}{2}\Big[1$$

$$I_{1,1} = \frac{g\, m}{6\pi^2}$$

Ryu et al, PRL 115 (2015) no.13, 132301





Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Quasiparticle Method

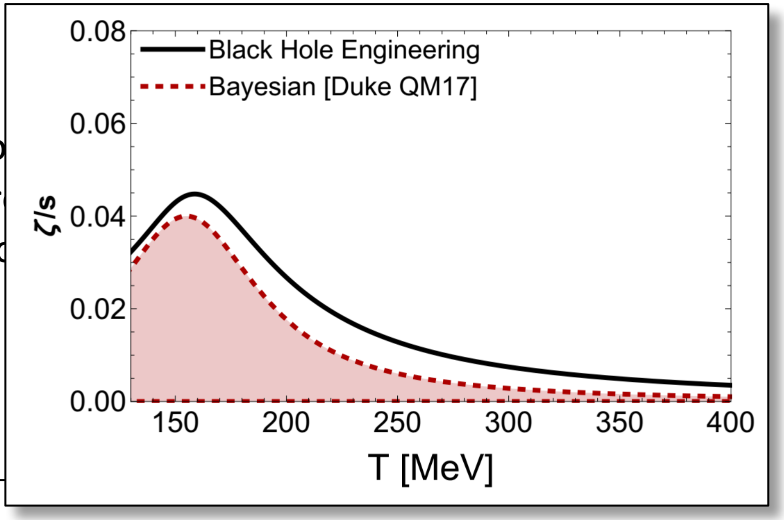
$$T^{\mu\nu} = T^{\mu\nu}_{\text{kinetic}} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$

Shear vi

Fix relaxatio
density by r
density ratio



Bulk viscosi

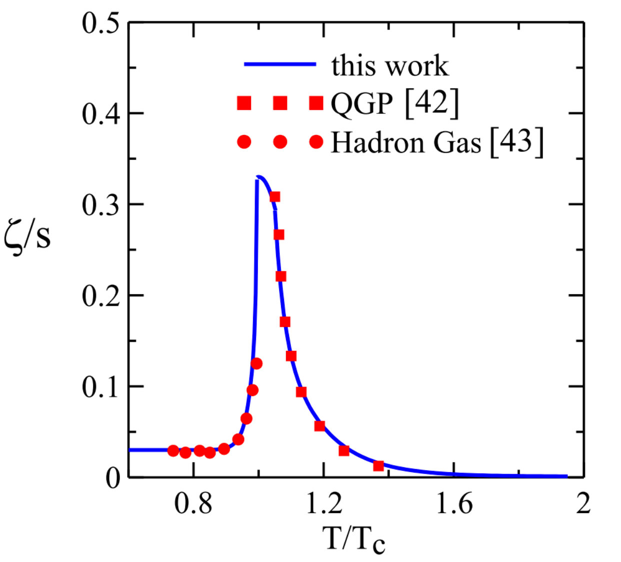
$$\frac{\zeta}{\tau_{\text{eq}}} = \frac{5}{3T} I_{3,$$

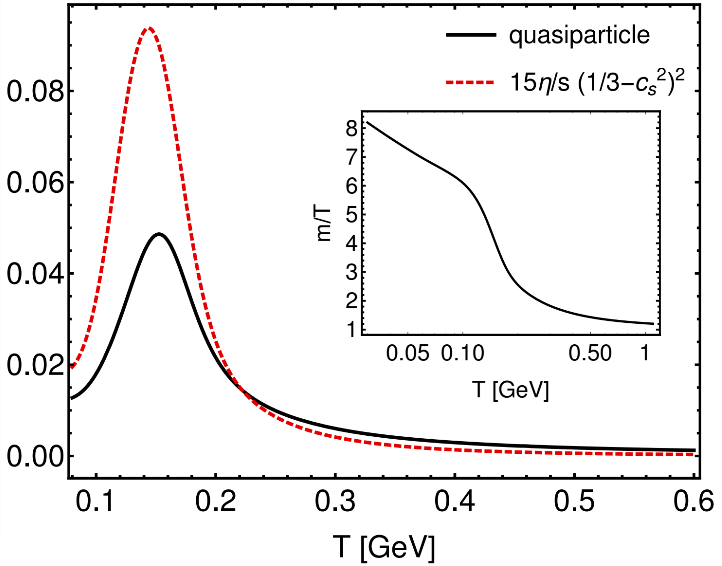
$$I_{3,2}(x) = \frac{N_{\text{dof}}}{30}$$

$$K_{i,1}(x) = \frac{\pi}{2}\Big[1$$

$$I_{1,1} = \frac{g\, m}{6\pi^2}$$

Ryu et al, PRL 115 (2015) no.13, 132301





Anisotropic Cooper-Frye Freezeout

M. Alqahtani, M. Nopoush, and MS, 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

- Use same generalized-RS form for "anisotropic freeze-out" at LO
- Form includes both shear and bulk corrections to the distribution function
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\rm eff,FO}$ = 130 MeV

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Phi\Delta^{\mu\nu}$$

isotropic / anisotropy tensor / bulk correction

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0,\xi_x,\xi_y,\xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3}\int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1-af_{\rm eq})\frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon+P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p\cdot u/T)+a] \qquad a = \text{-}1,\ +1,\ \text{or}\ 0$$

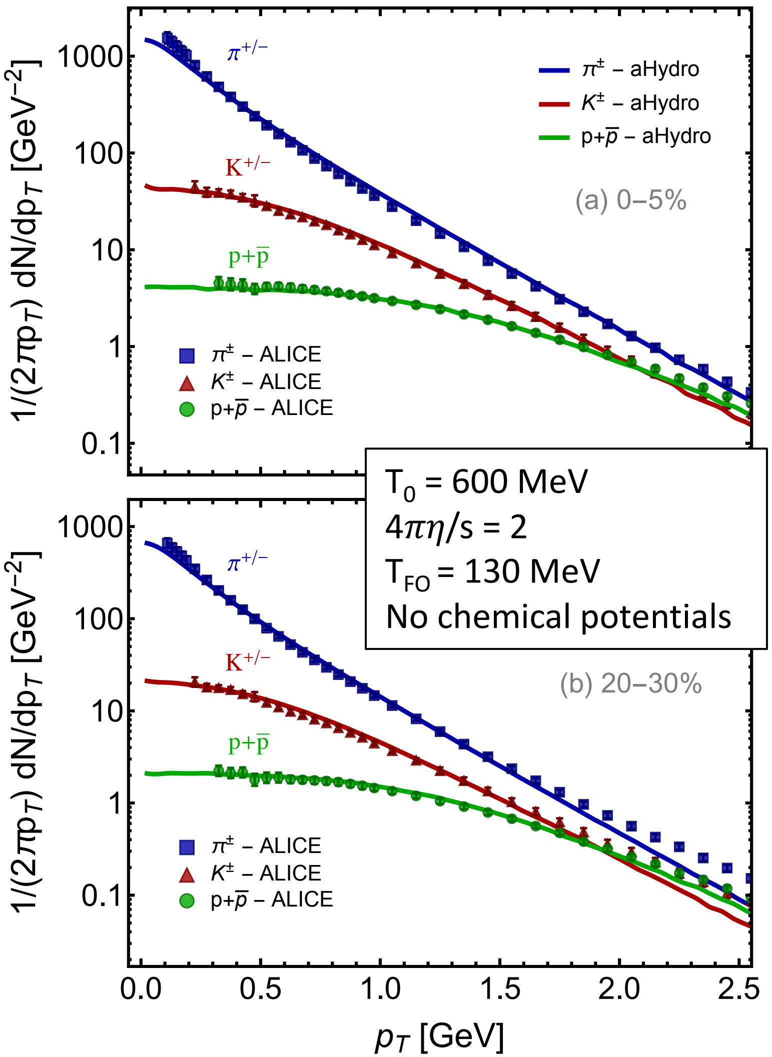
- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes worse when including the bulk viscous correction**.

The phenomenological setup

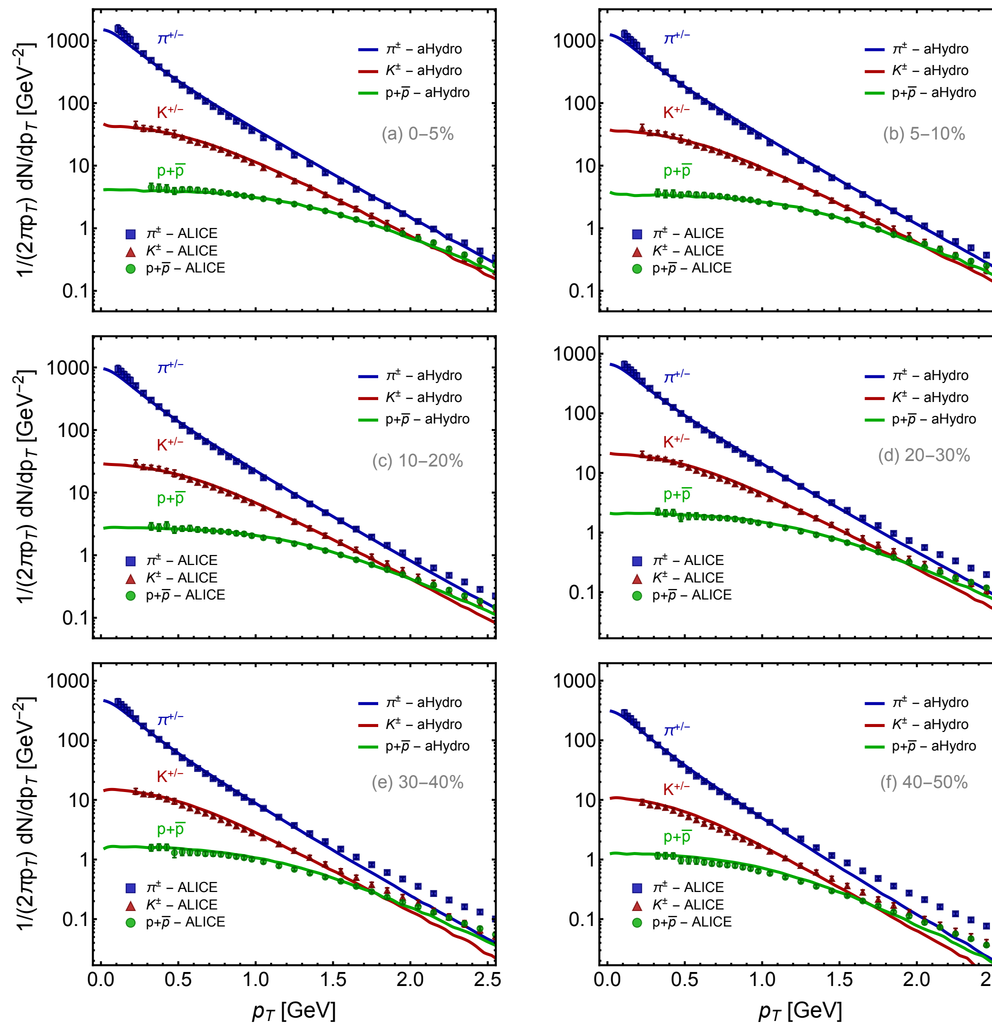
- Keep it simple at first → smooth Glauber initial conditions
- Mixture of wounded nucleon and binary collision profiles with a binary mixing fraction of 0.15 (empirically suggested from prior viscous hydro studies)
- In the rapidity direction, we use a rapidity profile with a “tilted” central plateau and Gaussian “wings”
- We take the system to be initially isotropic in momentum space
- We then run the code and extract the freeze-out hypersurface
- The primordial particle production is then Monte-Carlo sampled using the Therminator 2 [Chojnacki, Kisiel, Florkowski, and Broniowski, arXiv:1102.0273]
- Therminator also takes care of all resonance feed downs
- All data shown are from the **ALICE collaboration**

Identified particle spectra

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191

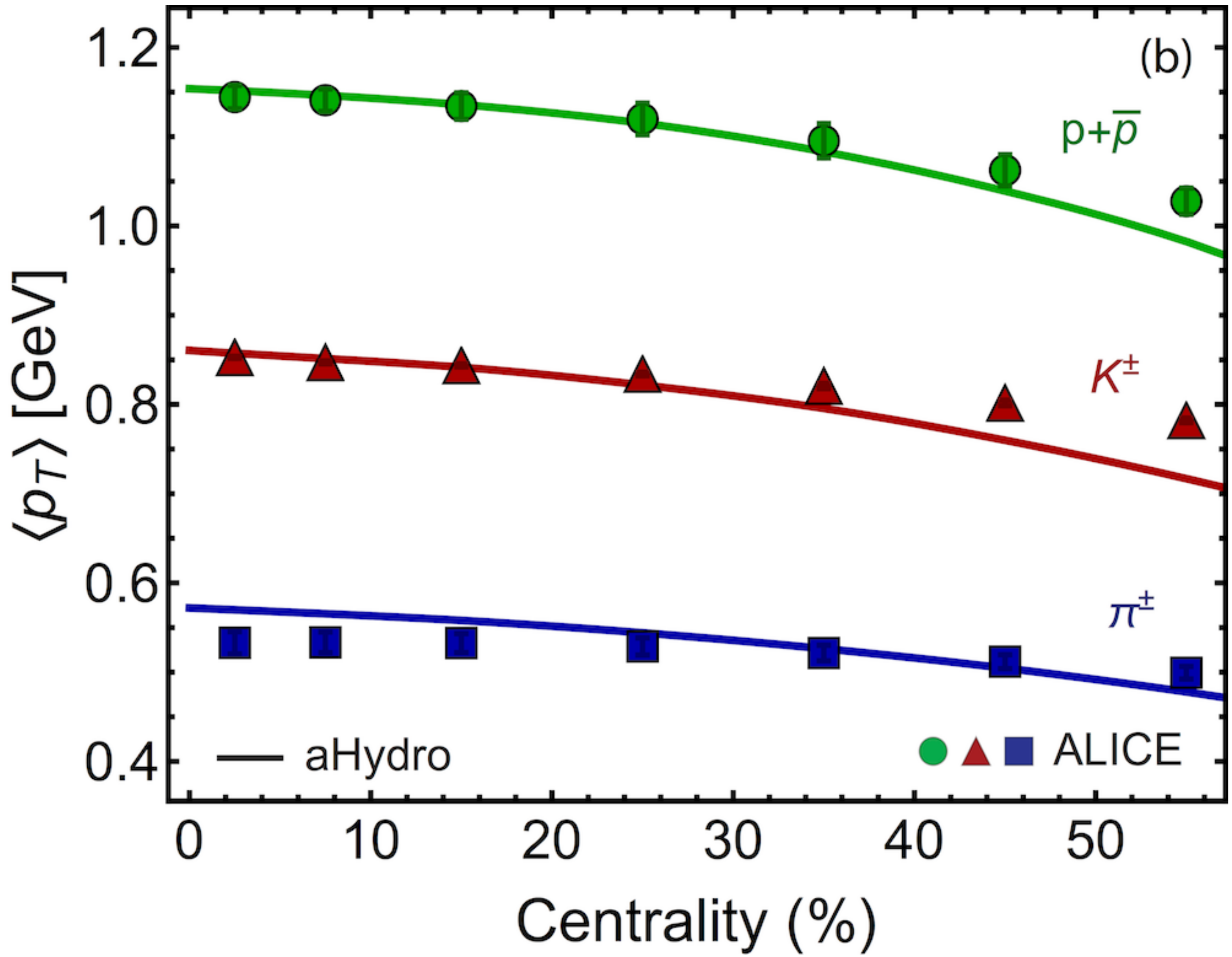


T_0 = 600 MeV
$4\pi\eta/s$ = 2
T_{FO} = 130 MeV
No chemical potentials



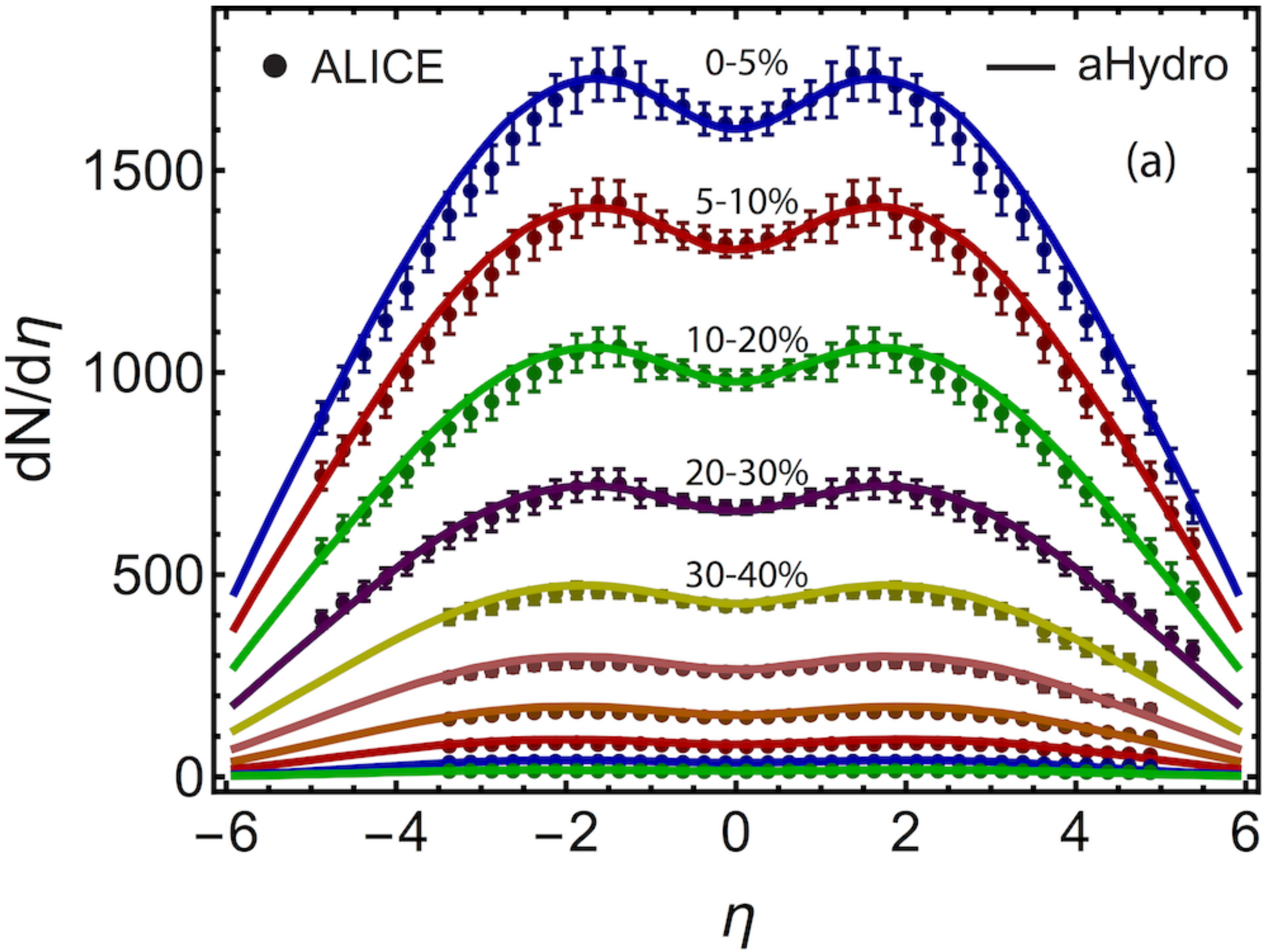
Identified particle average p_T

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



Charged particle multiplicities

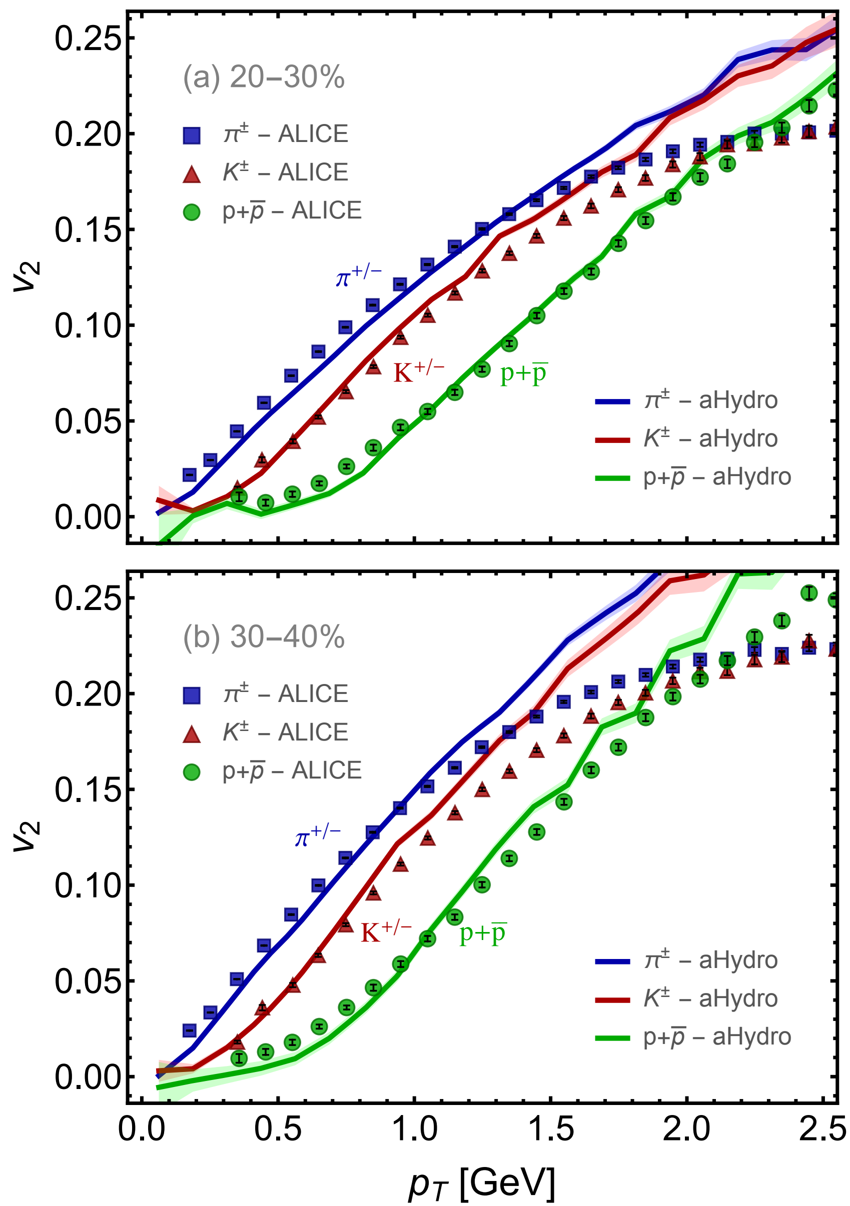
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191

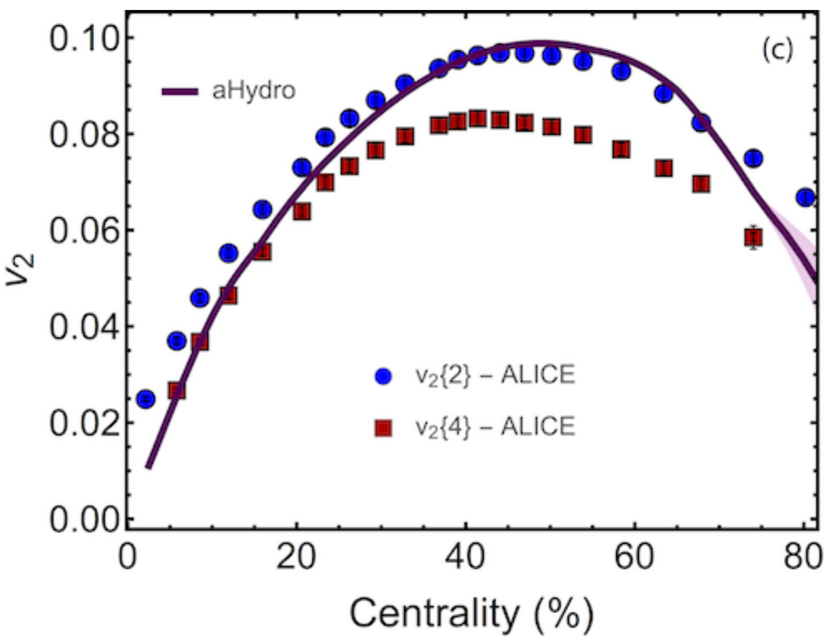


Elliptic flow

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191

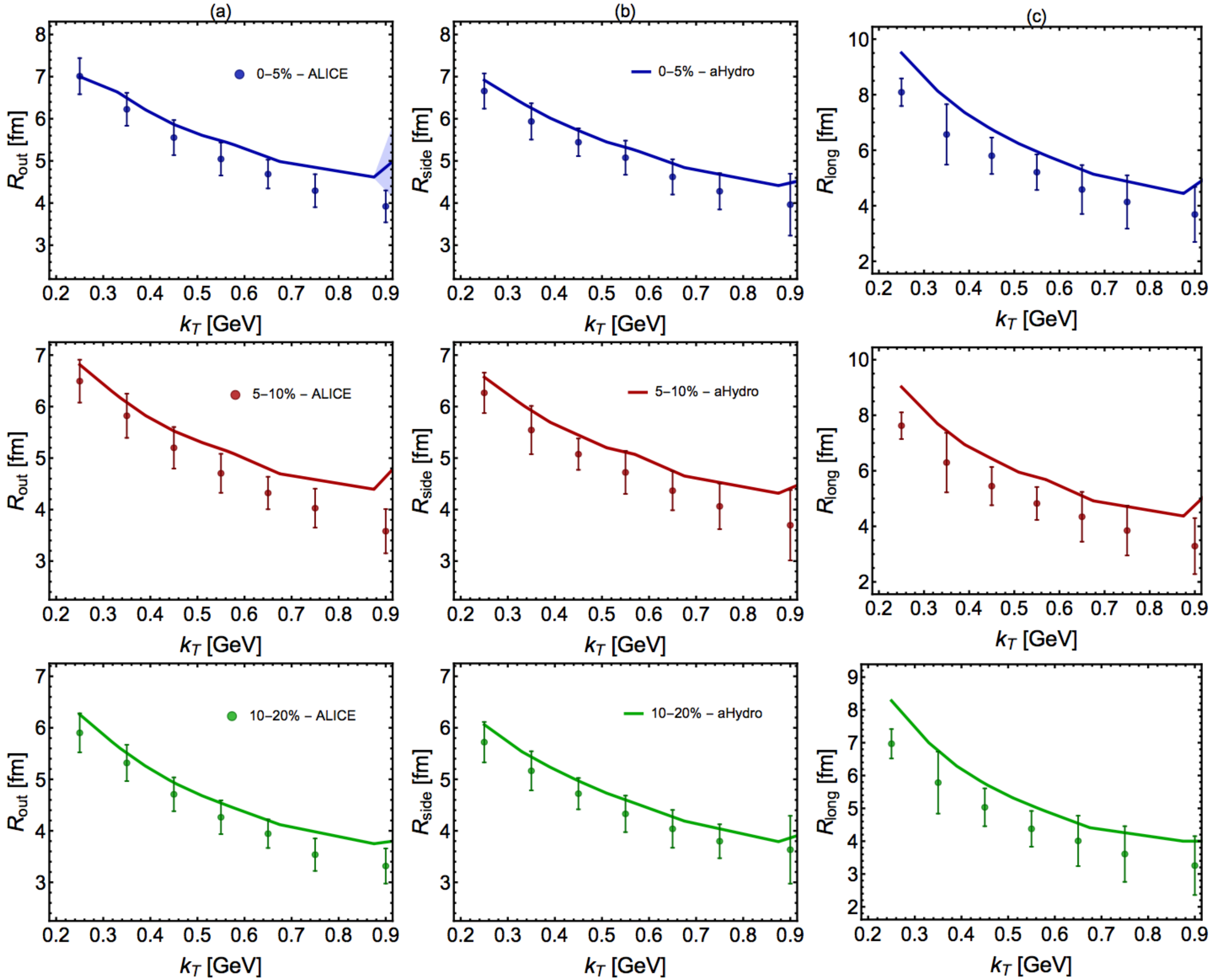
- Quite good description of elliptic flow as well
- Problems for central collisions but this is to be expected since we have not included fluctuating initial conditions yet





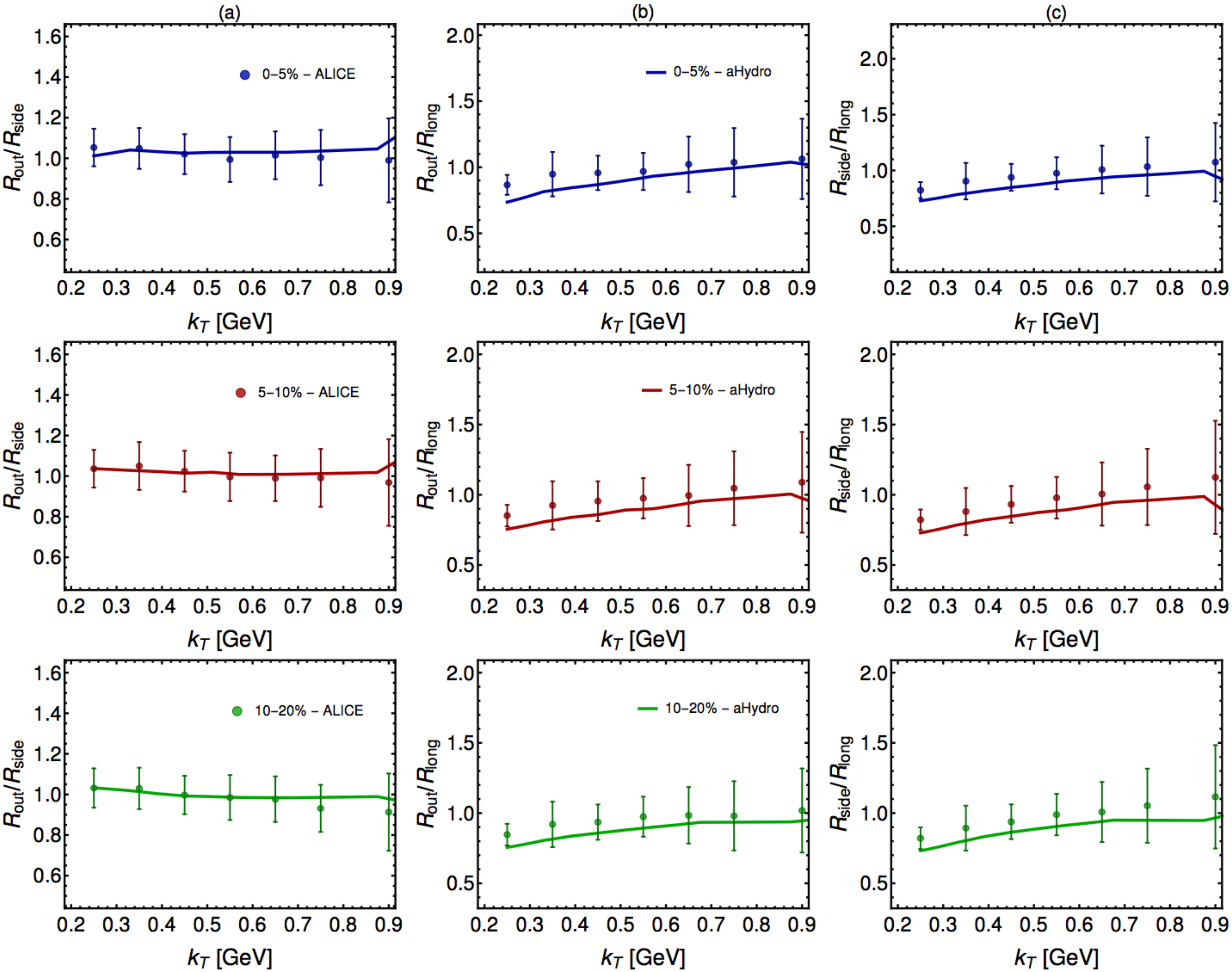
HBT Radii

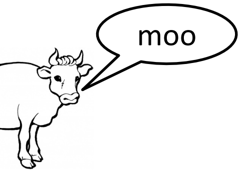
Alqahtani, Nopoush, Ryblewski, MS, 1705.10191



HBT Radii Ratios

Alqahtani, Nopoush, Ryblewski, MS, 1705.10191



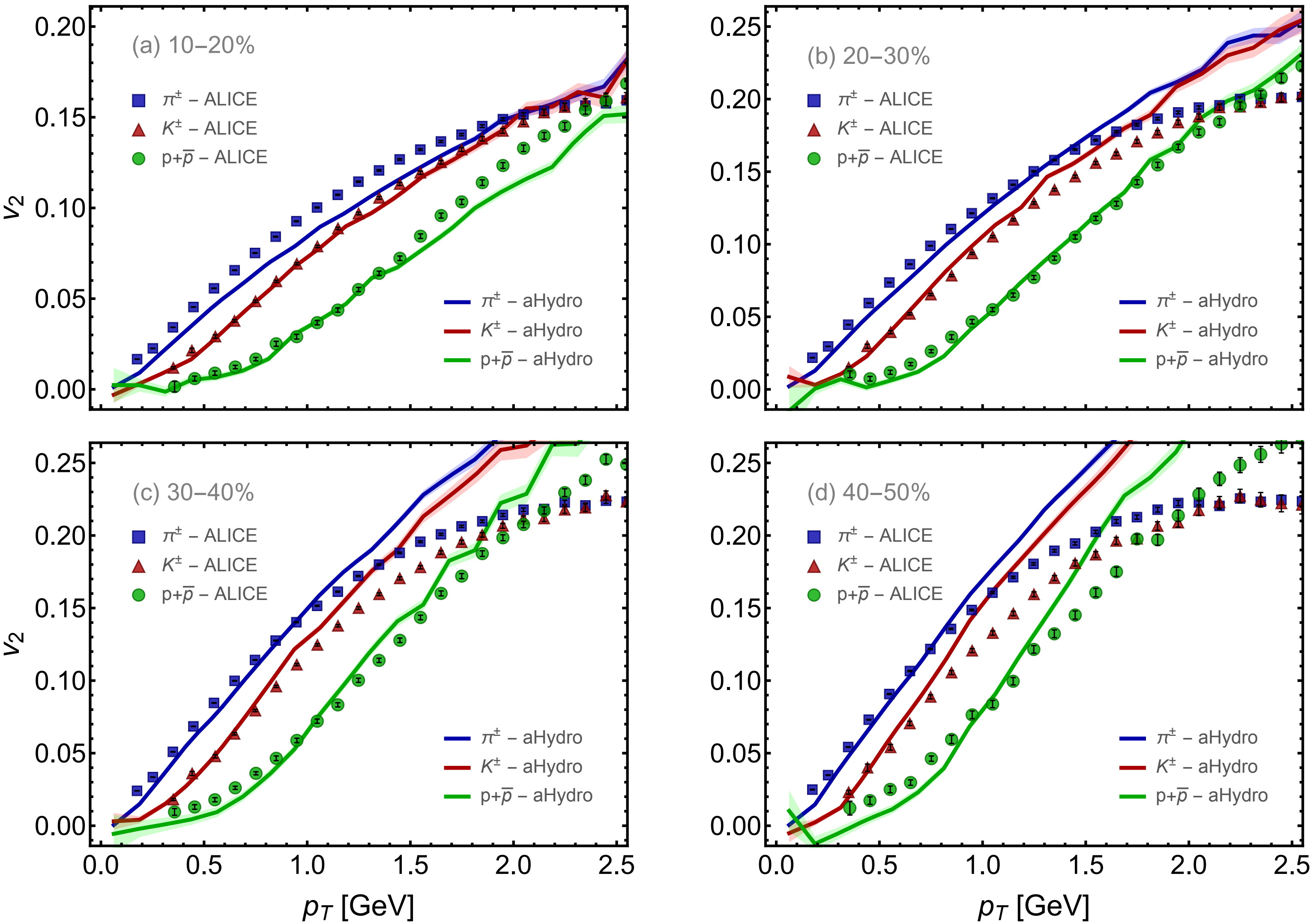


Conclusions and Outlook

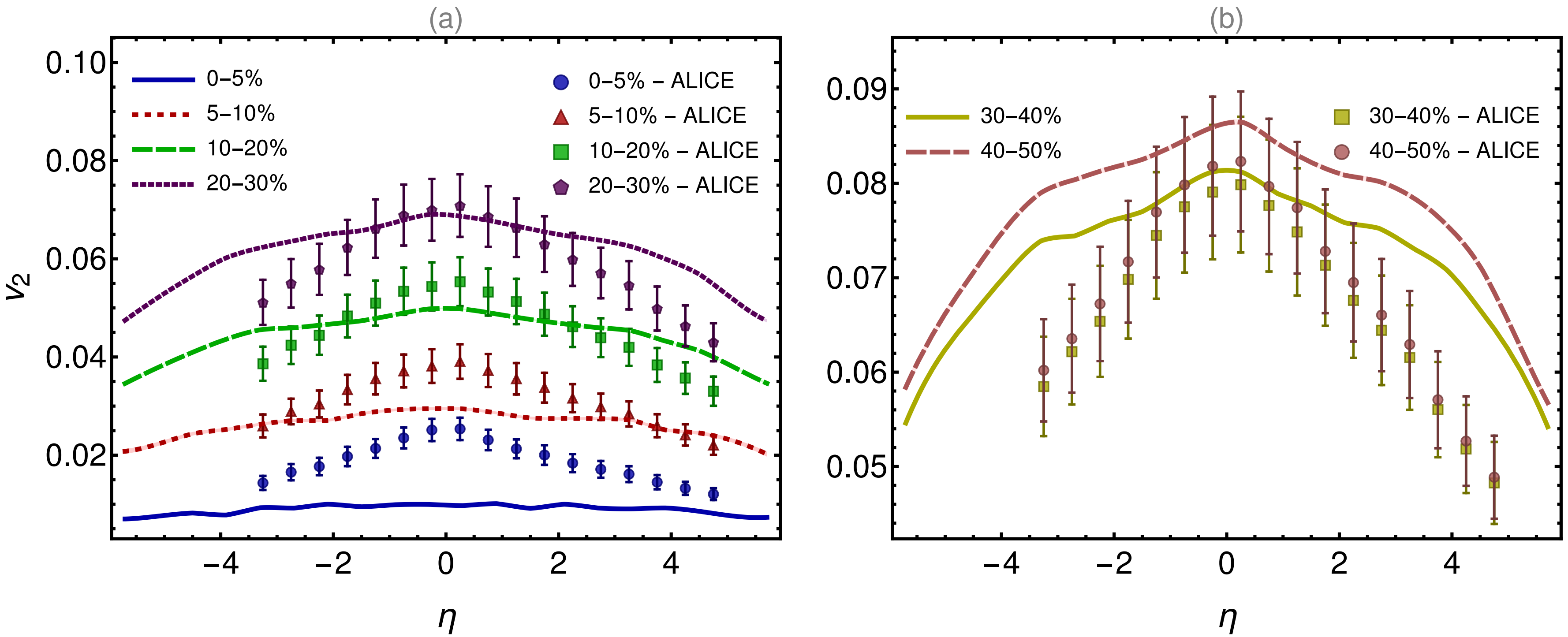
- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to create an even more quantitatively reliable model of QGP evolution.
- It incorporates some “facts of life” specific to the conditions generated in relativistic heavy ion collisions and, in doing so, **optimizes the dissipative hydrodynamics approach for HIC**.
- We now have a running 3+1d “ellipsoidal” aHydro code with realistic EoS, anisotropic freeze-out, and fluctuating initial conditions.
- Our preliminary fits to experimental data using smooth Glauber initial conditions look quite nice.
- **Future:** off-diagonal anisotropies, turn on the fluctuating initial conditions, lower-energies/finite μ_B, small systems…

Backup slides

More figures #1



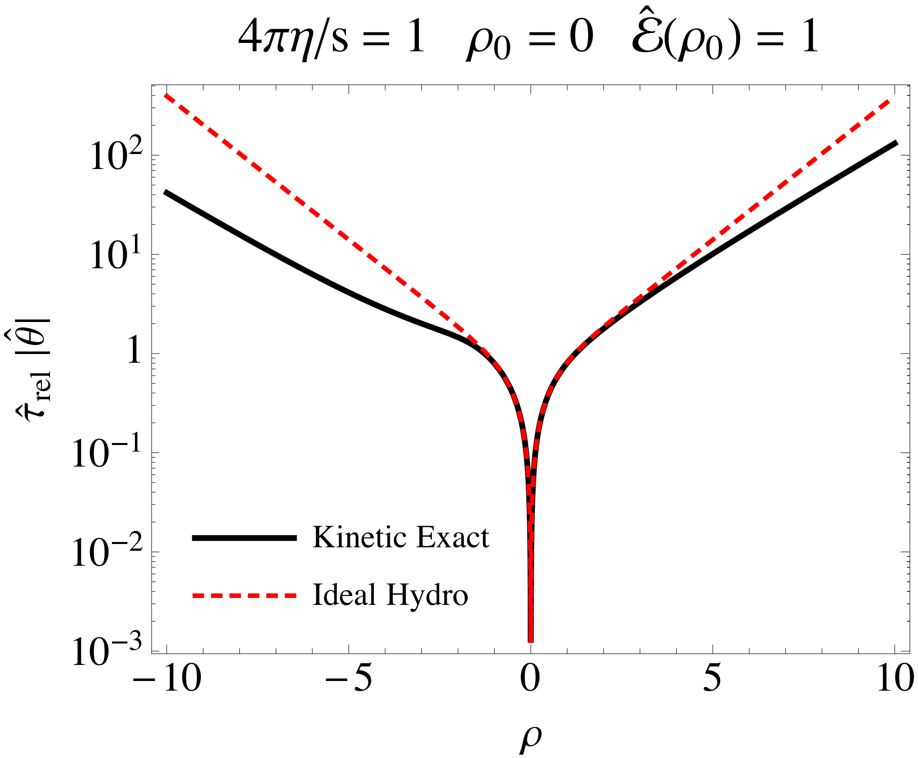
More figures #2



Gubser flow is extreme!

Knudsen number in de Sitter coordinates

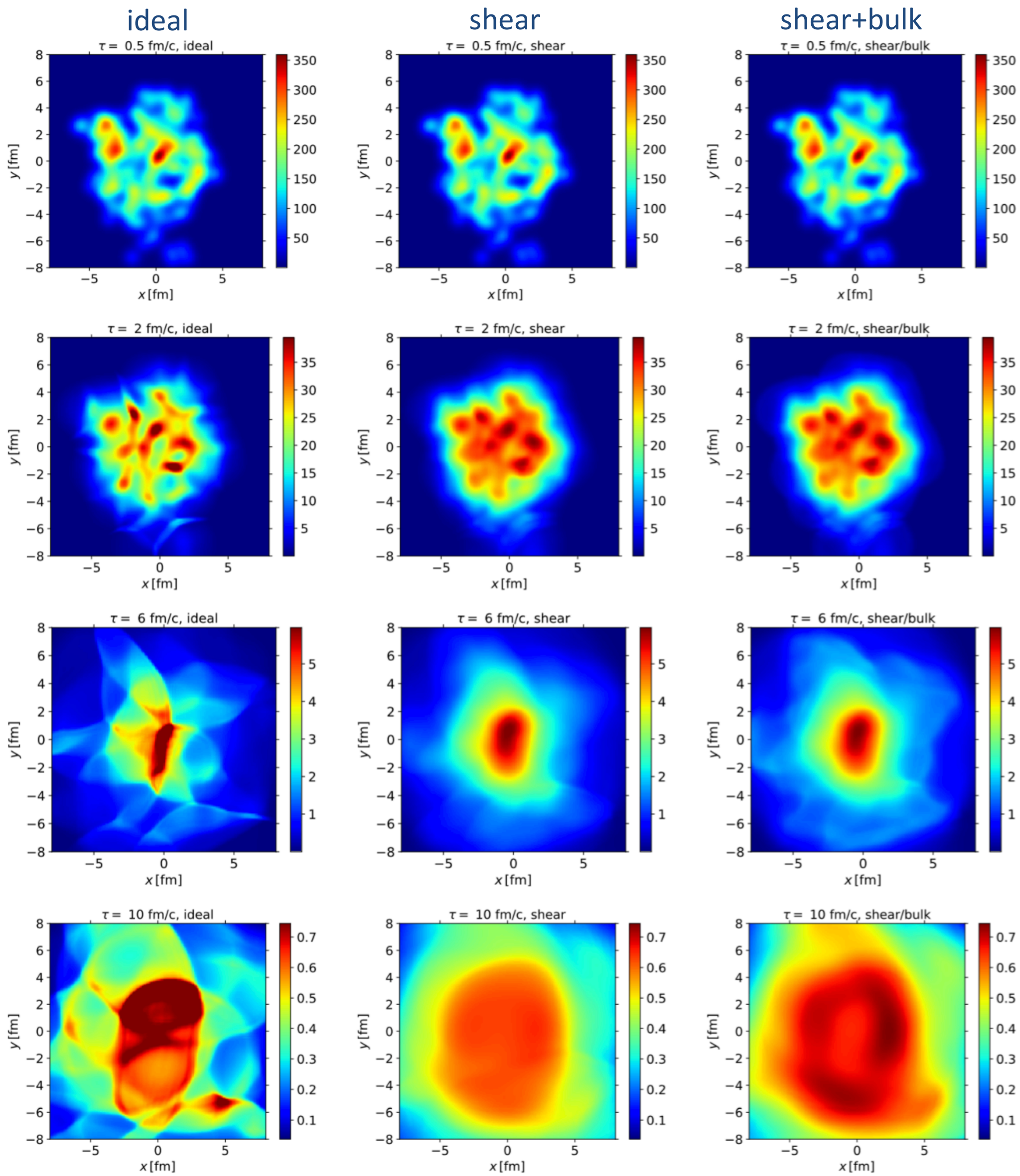
$$\mathrm{Kn} = \hat{\tau}_{\rm micro}/\hat{\tau}_{\rm macro} = \hat{\tau}_{\rm rel}\,|\hat{\theta}| \equiv \underbrace{\hat{\tau}_{\rm rel}}_{c/\hat{T}}\,\underbrace{|\hat{\nabla}\cdot\hat{u}|}_{2\tanh(\rho)}$$



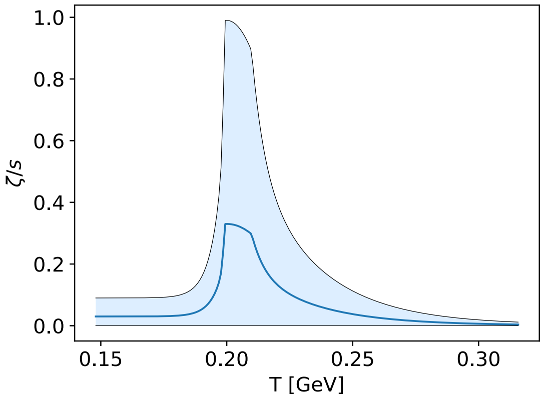
- Exponentially large gradients at early and late de Sitter times!
- Large gradients → potential problems with hydro description

Ideal Solution: S. Gubser, 1006.0006; S. Gubser and Y. Yarom, 1012.1314
Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646; 1408.7048

Some pretty pictures from 3d viscous hydro



- Left panels show output from the Ohio State/Kent State GPU-based viscous hydro code [Bazow, Heinz, and MS, 1608.06577]
- Solves the non-conformal DNMR (Denicol, Niemi, Molnar, Rischke) equations with a realistic EoS
- Parameterized ζ/s (plot below)
- $\eta/s = 0.2$
- T_0 = 600 MeV @ t_0 = 0.5 fm/c



Technicalities - A numerical challenge

- One of the most daunting challenges faced by the quasiparticle approach is that one has to evaluate a bunch of "$\mathcal{H}$" functions, e.g.

$$\mathcal{E} = \mathcal{H}_3(\boldsymbol{\alpha}, \hat{m})\, \lambda^4 + B$$

$$\mathcal{H}_3(\boldsymbol{\alpha}, \hat{m}) = \tilde{N}\alpha \int d^3\hat{p}\, \mathcal{R}(\boldsymbol{\alpha}, \hat{m})\, f_{\rm eq}\Big(\sqrt{\hat{p}^2 + \hat{m}^2}\Big)$$

$$\mathcal{R}(\boldsymbol{\alpha}, \hat{m}) = \sqrt{\alpha_x^2\, p_x^2 + \alpha_y^2\, p_y^2 + \alpha_z^2\, p_z^2 + m^2}$$

- We evaluate these efficiently by expanding the integrand around the diagonal in anisotropy space up to 12$^{\rm th}$ order.
- We do this around two points (1,1,1) and (2,2,2) and switch between these two expansions smoothly.
- With this method we were able to accelerate the evaulation of H functions by a factor of 10^5 while achieving $< 0.1\%$ accuracy.