QCD critical point, fluctuations and hydrodynamics

M. Stephanov

History

Cagniard de la Tour (1822): discovered continuos transition from liquid to vapour by heating alcohol, water, etc. in a gun barrel, glass tubes.

Name

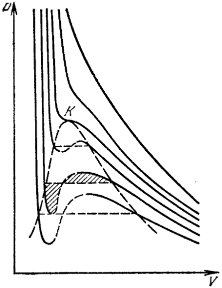
Faraday (1844) – liquefying gases:

“Cagniard de la Tour made an experiment some years ago which gave me occasion to want a new word.”

Mendeleev (1860) – measured vanishing of liquid-vapour surface tension: “Absolute boiling temperature”.

Andrews (1869) – systematic studies of many substances established continuity of vapour-liquid phases. Coined the name “critical point”.

van der Waals (1879) –
in “On the continuity of the gas and liquid state” (PhD thesis) wrote e.o.s. with a critical point.



Smoluchowski, Einstein (1908,1910) – explained critical opalescence.

Landau – classical theory of critical phenomena

Fisher, Kadanoff, Wilson – scaling, full fluctuation theory based on RG.

Critical opalescence

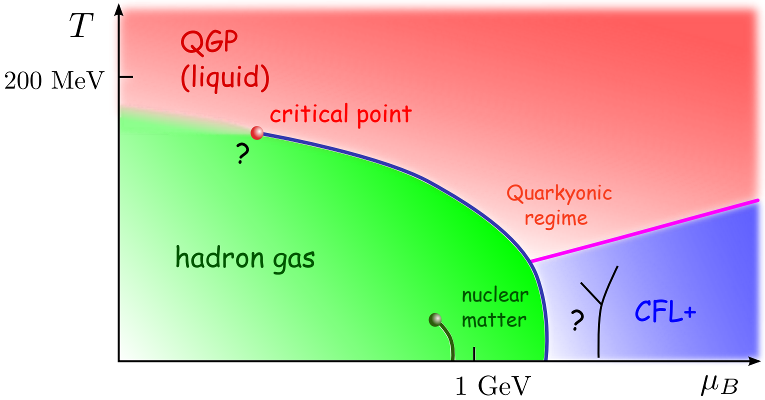
Substance[13][14]	Critical temperature	Critical pressure (absolute)
Argon	−122.4 °C (150.8 K)	48.1 atm (4,870 kPa)
Ammonia[15]	132.4 °C (405.5 K)	111.3 atm (11,280 kPa)
Bromine	310.8 °C (584.0 K)	102 atm (10,300 kPa)
Caesium	1,664.85 °C (1,938.00 K)	94 atm (9,500 kPa)
Chlorine	143.8 °C (416.9 K)	76.0 atm (7,700 kPa)
Ethanol	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
CH_4 (methane)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
CO_2	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
N_2O	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
H_2SO_4	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Water[2][16]	373.946 °C (647.096 K)	217.7 atm (22.06 MPa)

Critical point is a ubiquitous phenomenon

Critical point between the QGP and hadron gas phases?

QCD is a relativistic theory of a fundamental force.

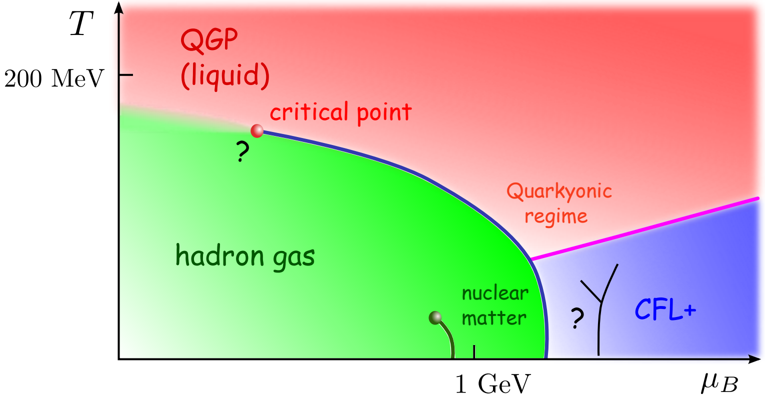
CP is a singularity of EOS, anchors the 1st order transition.



Critical point between the QGP and hadron gas phases?

QCD is a relativistic theory of a fundamental force.

CP is a singularity of EOS, anchors the 1st order transition.



Lattice QCD at $\mu_B \lesssim 2T$ – a crossover.

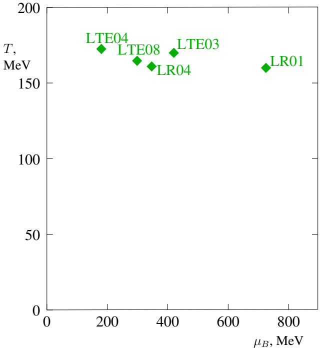
C.P. is ubiquitous in models (NJL, RM, Holog., Strong coupl. LQCD, . . .)

Essentially two approaches to discovering the QCD critical point. Each with its own challenges.

- Lattice simulations.

  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu = 0$.



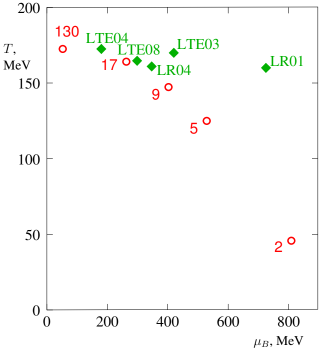
- Heavy-ion collisions.

Essentially two approaches to discovering the QCD critical point. Each with its own challenges.

- Lattice simulations.

  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu = 0$.



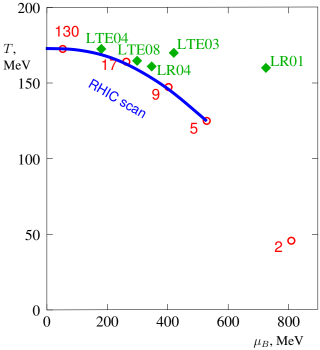
- Heavy-ion collisions.

Essentially two approaches to discovering the QCD critical point. Each with its own challenges.

- Lattice simulations.

  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu = 0$.



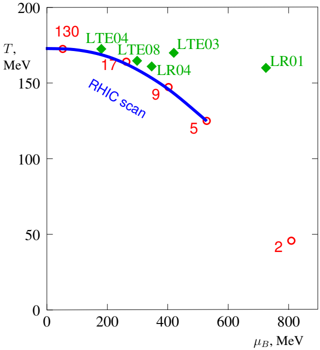
- Heavy-ion collisions.

Essentially two approaches to discovering the QCD critical point. Each with its own challenges.

- Lattice simulations.

  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu = 0$.



- Heavy-ion collisions. *Non-equilibrium.*

Outline

- Equilibrium
- Non-equilibrium

Why fluctuations are large at a critical point?

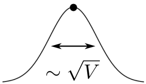
- The key equation:

$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$

Why fluctuations are large at a critical point?

- The key equation:

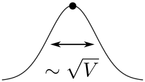
$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$



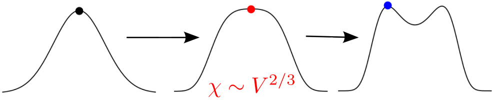
Why fluctuations are large at a critical point?

- The key equation:

$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$



- At the critical point $S(\sigma)$ “flattens”. And $\chi \equiv \langle \sigma^2 \rangle / V \to \infty$.

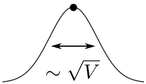


CLT?

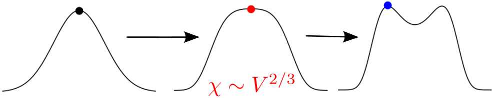
Why fluctuations are large at a critical point?

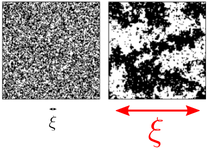
- The key equation:

$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$



- At the critical point $S(\sigma)$ "flattens". And $\chi \equiv \langle \sigma^2 \rangle / V \to \infty$.





CLT? | σ is not a sum of ∞ many *uncorrelated* contributions: $\xi \to \infty$ |

Fluctuations of order parameter and ξ

- Fluctuations at CP – conformal field theory.

  Parameter-free $\rightarrow$ universality. Only one scale $\xi = m_\sigma^{-1} < \infty$,

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right].$$

  - Width/shape of $P(\sigma_0 \equiv \int_{\boldsymbol{x}} \sigma)$ best expressed via cumulants:

- Higher cumulants (shape of $P(\sigma_0)$) depend stronger on ξ.

  Universal: $\boxed{\langle \sigma_0^k \rangle_c \sim V \xi^p}$, $p = k(3 - [\sigma]) - 3$, $[\sigma] = \beta/\nu \approx 1/2$.

  E.g., $p \approx 2$ for $k = 2$, but $p \approx 7$ for $k = 4$.

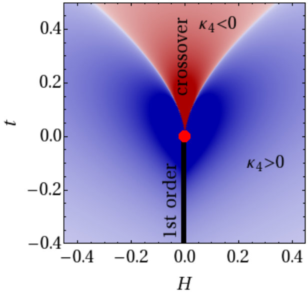
Sign

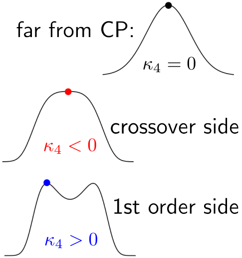
- Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma] = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \kappa_4[\sigma] = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

This dependence is also universal.

- 2 relevant directions/parameters. Using Ising model variables:





Experiments do not measure σ.

Mapping to QCD and experimental observables

Observed fluctuations are not the same as σ, but related:

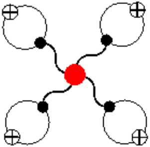
Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \kappa_4[\sigma] \times g^4 \underbrace{\left(\right.}$  $\underbrace{\left. \right)^4}_{\sim M^4} + \ldots,$

  g – coupling of the critical mode ($g = dm/d\sigma$).



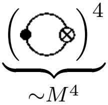
Mapping to QCD and experimental observables

Observed fluctuations are not the same as σ, but related:

Think of a collective mode described by field σ such that $m = m(\sigma)$:

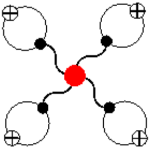
$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \kappa_4[\sigma] \times g^4$  $+ \ldots,$

  g – coupling of the critical mode ($g = dm/d\sigma$).

  - $\kappa_4[\sigma] < 0$ means $\kappa_4[M] <$ baseline



Mapping to QCD and experimental observables

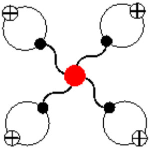
Observed fluctuations are not the same as σ, but related:

Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \kappa_4[\sigma] \times g^4 \underbrace{\left(\quad \right)^4}_{\sim M^4} + \ldots,$



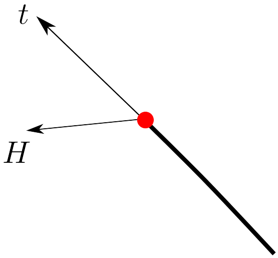
  g – coupling of the critical mode ($g = dm/d\sigma$).

  - $\kappa_4[\sigma] < 0$ means $\kappa_4[M] <$ baseline

- NB: Sensitivity to M_{accepted}: $(\kappa_4)_\sigma \sim M^4$ (number of 4-tets).

Mapping Ising to QCD phase diagram

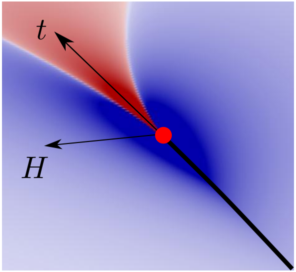
T vs μ_B:



- In QCD $(t, H) \to (\mu - \mu_{\mathrm{CP}}, T - T_{\mathrm{CP}})$

Mapping Ising to QCD phase diagram

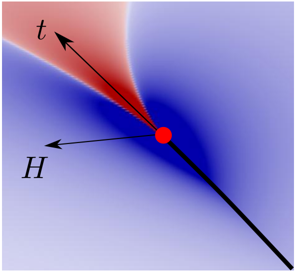
T vs μ_B:



- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

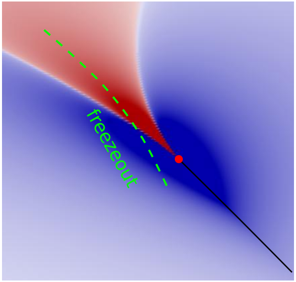
Mapping Ising to QCD phase diagram

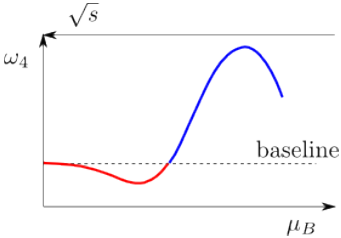
T vs μ_B:



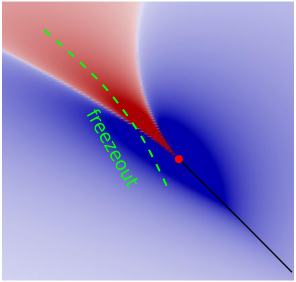
- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$
- $\kappa_n(N) = N + \mathcal{O}(\kappa_n(\sigma))$

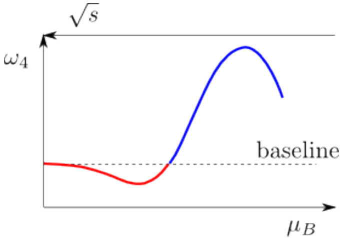
Beam Energy Scan

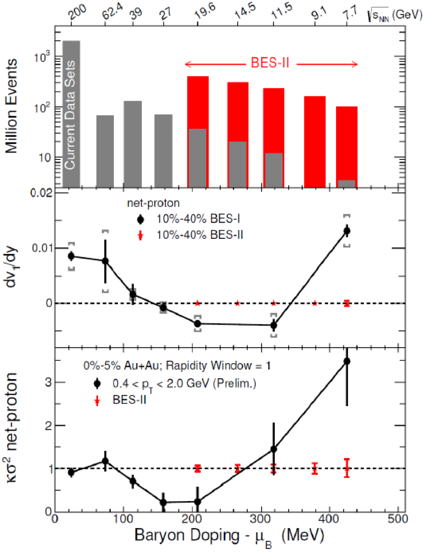




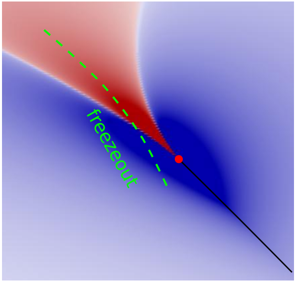
Beam Energy Scan

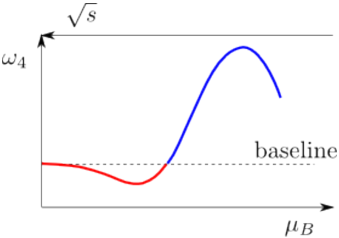


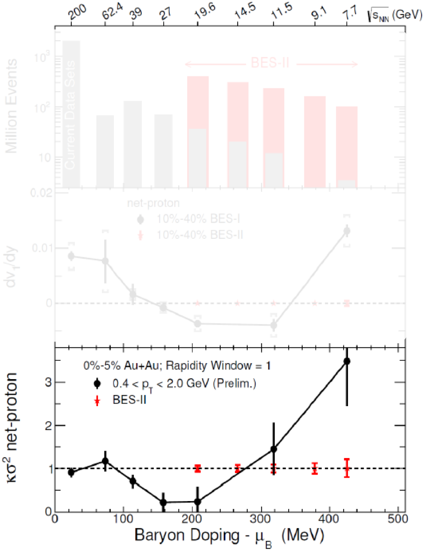




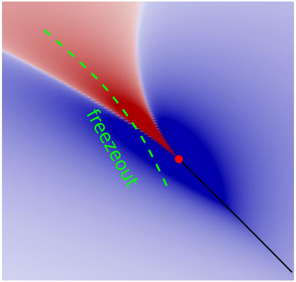
Beam Energy Scan

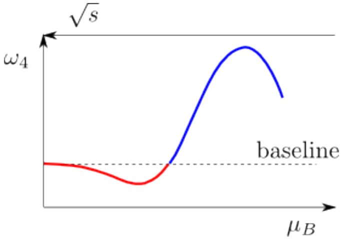


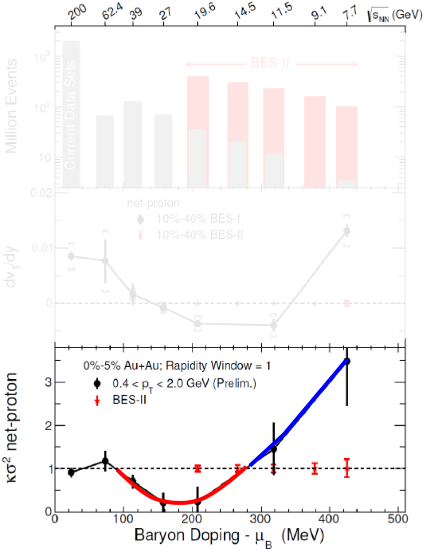




Beam Energy Scan



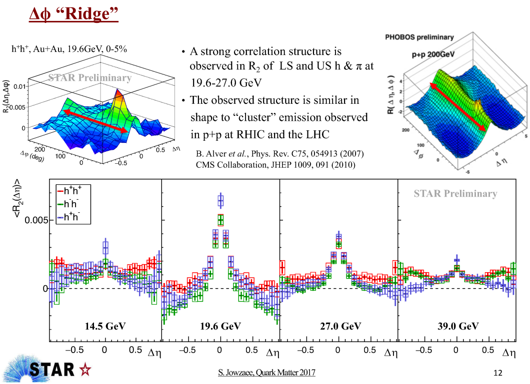




"intriguing hint" (2015 LRPNS)

QM2017 update: another intriguing hint

Preliminary, but very interesting:



- Non-monotonous $\sqrt{s}$ dependence with max near 19 GeV.
- Charge/isospin blind.
- $\Delta\phi$ (in)dependence is as expected from critical correlations.
- Width $\Delta\eta$ suggests soft thermal pions – but p_T dependence need to be checked.
- But: no signal in R_2 for K or p.

Non-equilibrium physics is essential near the critical point.

The goal for BEST COLLABORATION

Why ξ is finite

System expands and is *out of equilibrium*

Kibble-Zurek mechanism:

Critical slowing down means $\tau_{\rm relax} \sim \xi^z$.
Given $\tau_{\rm relax} \lesssim \tau$ (expansion time scale):

$$\xi \lesssim \tau^{1/z},$$

$z \approx 3$ (universal).

Why ξ is finite

System expands and is *out of equilibrium*

Kibble-Zurek mechanism:

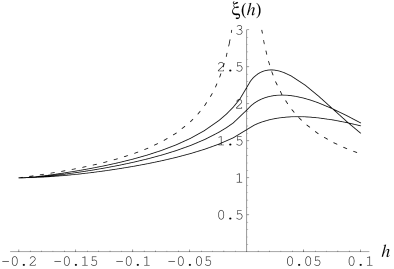
Critical slowing down means $\tau_{\rm relax} \sim \xi^z$.
Given $\tau_{\rm relax} \lesssim \tau$ (expansion time scale):

$$\xi \lesssim \tau^{1/z},$$

$z \approx 3$ (universal).

Estimates: $\xi \sim 2 - 3$ fm
(Berdnikov-Rajagopal)

KZ scaling for $\xi(t)$
and cumulants
(Mukherjee-Venugopalan-Yin)



Lessons

$$\kappa_n \sim \xi^p \quad \text{and} \quad \xi_{\max} \sim \tau^{1/z}$$

- Therefore, the magnitude of fluctuation signals is determined by non-equilibrium physics.

Lessons

$$\kappa_n \sim \xi^p \quad \text{and} \quad \xi_{\max} \sim \tau^{1/z}$$

- Therefore, the magnitude of fluctuation signals is determined by non-equilibrium physics.
- Logic so far:

  Equilibrium fluctuations + a non-equilibrium effect (finite ξ)

  $\longrightarrow$ Observable critical fluctuations

Lessons

$$\kappa_n \sim \xi^p \quad \text{and} \quad \xi_{\max} \sim \tau^{1/z}$$

- Therefore, the magnitude of fluctuation signals is determined by non-equilibrium physics.
- Logic so far:

  Equilibrium fluctuations + a non-equilibrium effect (finite ξ)

  $\longrightarrow$ Observable critical fluctuations
- Can we get *critical* fluctuations from hydrodynamics *directly*?

Time evolution of cumulants (memory)

Mukherjee-Venugopalan-Yin

Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$

Time evolution of cumulants (memory)

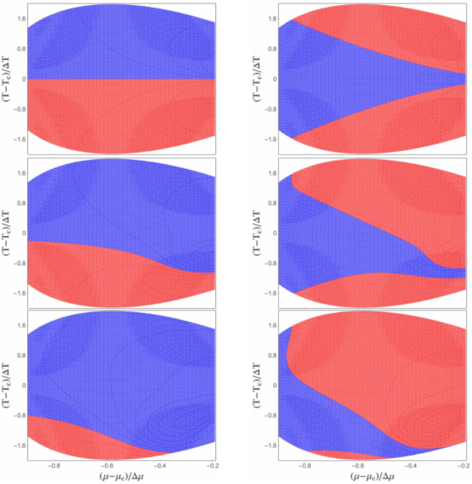
Mukherjee-Venugopalan-Yin

Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$



κ_3 κ_4

Signs of cumulants also depend on off-equilibrium dynamics.

Time evolution of cumulants (memory)

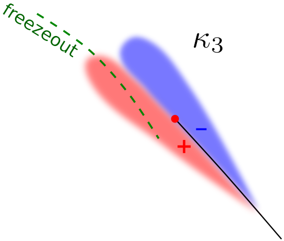
Mukherjee-Venugopalan-Yin

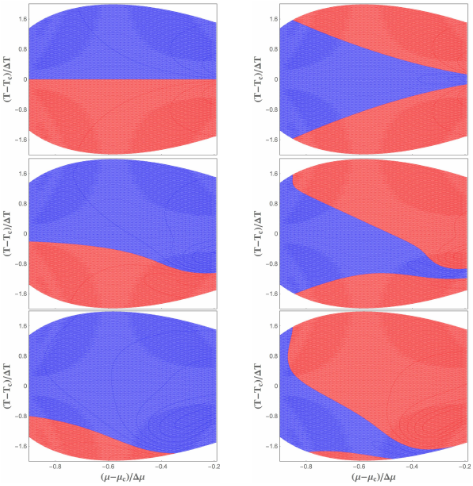
Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





κ_3 κ_4

Signs of cumulants also depend on off-equilibrium dynamics.

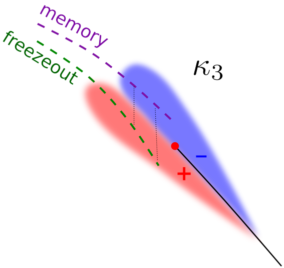
Mukherjee-Venugopalan-Yin

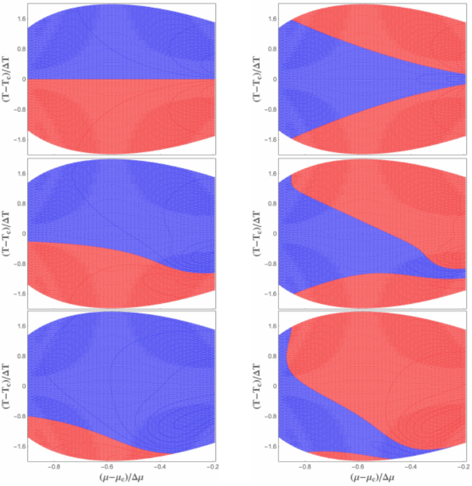
Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





κ_3 κ_4

Signs of cumulants also depend on off-equilibrium dynamics.

Hydrodynamics breaks down at CP

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \tilde{T}^{\mu\nu}_{\text{visc}}$$

Hydrodynamics breaks down at CP

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \tilde{T}^{\mu\nu}_{\rm visc}$$

$$\tilde{T}^{\mu\nu}_{\rm visc} = -\zeta \Delta^{\mu\nu}(\nabla \cdot u) + \ldots$$

Near CP gradient terms are dominated by $\zeta \sim \xi^3 \to \infty$ ($z - \alpha/\nu \approx 3$).

Hydrodynamics breaks down at CP

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \tilde{T}^{\mu\nu}_{\rm visc}$$

$$\tilde{T}^{\mu\nu}_{\rm visc} = -\zeta\Delta^{\mu\nu}(\nabla\cdot u) + \ldots$$

Near CP gradient terms are dominated by $\zeta \sim \xi^3 \to \infty$ $(z - \alpha/\nu \approx 3)$.

When $k \sim \xi^{-3}$ hydrodynamics breaks down, i.e., while $k \ll \xi^{-1}$ still.

(For simplicity, measure dim-ful quantities in units of T, i.e., $k \sim T(T\xi)^{-3}$.)

Why does hydro break at so small k?

Critical slowing down and bulk viscosity

Bulk viscosity is the effect of system taking time to adjust to local equilibrium (Khalatnikov-Landau).

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta\, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$\boldsymbol{\nabla} \cdot \boldsymbol{v}$ – expansion rate

$$\zeta \sim \tau_{\text{relaxation}} \sim \xi^3$$

Critical slowing down and bulk viscosity

Bulk viscosity is the effect of system taking time to adjust to local equilibrium (Khalatnikov-Landau).

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta\, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$\boldsymbol{\nabla} \cdot \boldsymbol{v}$ – expansion rate

$$\zeta \sim \tau_{\text{relaxation}} \sim \xi^3$$

Hydrodynamics breaks down because of large relaxation time (critical slowing down).

Similar to breakdown of an effective theory due to a low-energy mode which should not have been integrated out.

Critical slowing down and Hydro+

- There is a critically slow mode ϕ with relaxation time $\tau_\phi \sim \xi^3$.

Critical slowing down and Hydro+

- There is a critically slow mode ϕ with relaxation time $\tau_\phi \sim \xi^3$.
- To extend the range of hydro – extend hydro by the slow mode.

(MS-Yin 1704.07396, in preparation)

Critical slowing down and Hydro+

- There is a critically slow mode ϕ with relaxation time $\tau_\phi \sim \xi^3$.
- To extend the range of hydro – extend hydro by the slow mode.

(MS-Yin 1704.07396, in preparation)

- "Hydro+" has two competing limits, $k \to 0$ and $\xi \to \infty$;

  or competing rates $\Gamma_\phi \sim \xi^{-3} \to 0$ and $\Gamma_{\text{hydro}} \sim k \to 0$.

Critical slowing down and Hydro+

- There is a critically slow mode ϕ with relaxation time $\tau_\phi \sim \xi^3$.
- To extend the range of hydro – extend hydro by the slow mode.

(MS-Yin 1704.07396, in preparation)

- "Hydro+" has two competing limits, $k \to 0$ and $\xi \to \infty$;

  or competing rates $\Gamma_\phi \sim \xi^{-3} \to 0$ and $\Gamma_{\rm hydro} \sim k \to 0$.
- Regime I: $\Gamma_\phi \gg \Gamma_{\rm hydro}$ – ordinary hydro ($\zeta \sim \xi^3 \to \infty$ at CP).

Critical slowing down and Hydro+

- There is a critically slow mode ϕ with relaxation time $\tau_\phi \sim \xi^3$.
- To extend the range of hydro – extend hydro by the slow mode.

(MS-Yin 1704.07396, in preparation)

- "Hydro+" has two competing limits, $k \to 0$ and $\xi \to \infty$;

  or competing rates $\Gamma_\phi \sim \xi^{-3} \to 0$ and $\Gamma_{\rm hydro} \sim k \to 0$.
- Regime I: $\Gamma_\phi \gg \Gamma_{\rm hydro}$ – ordinary hydro ($\zeta \sim \xi^3 \to \infty$ at CP).

  Crossover occurs when $\Gamma_{\rm hydro} \sim \Gamma_\phi$, or $k \sim \xi^{-3}$.
- Regime II: $k > \xi^{-3}$ – "Hydro+" regime.

Advantages/motivation of Hydro+

- Extends the range of validity of “vanilla” hydro near CP
  to length/time scales shorter than $\mathcal{O}(\xi^3)$.

Advantages/motivation of Hydro+

- Extends the range of validity of “vanilla” hydro near CP
  to length/time scales shorter than $\mathcal{O}(\xi^3)$.

- No kinetic coefficients diverging as ξ^3.
  (Since noise $\sim \zeta$, also the noise is not large.)

Ingredients of “Hydro+”

- Nonequilibrium entropy, or quasistatic EOS:

$$s^*(\varepsilon, n, \phi)$$

Equilibrium entropy is the maximum of s^*:

$$s(\varepsilon, n) = \max_{\phi} s^*(\varepsilon, n, \phi)$$

Ingredients of "Hydro+"

- Nonequilibrium entropy, or quasistatic EOS:

$$s^*(\varepsilon, n, \phi)$$

Equilibrium entropy is the maximum of s^*:

$$s(\varepsilon, n) = \max_{\phi} s^*(\varepsilon, n, \phi)$$

- The 6th equation (constrained by 2nd law):

$$(u \cdot \partial)\phi = -\gamma_\phi \pi - G_\phi(\partial \cdot u), \qquad \text{where } \pi = \frac{\partial s^*}{\partial \phi}$$

- Another example: relaxation of axial charge.

Linearized Hydro+

Linearized Hydro+ has 4 longitudinal modes (sound$\times 2$ + density + ϕ).

In addition to the usual c_s, D, etc. Hydro+ has two more parameters $\Delta c^2 = c_*^2 - c_s^2$ and $\Gamma = \Gamma_\phi$.

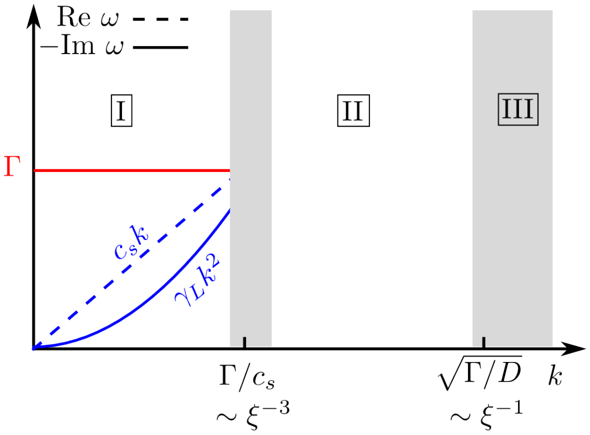
The sound velocities are different in Regime I ($c_s k \ll \Gamma$) and II:

$$c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{s/n,\pi=0} \quad \text{and} \quad c_*^2 = \left(\frac{\partial p^*}{\partial \varepsilon}\right)_{s/n,\phi}$$

The bulk viscosity receives large contribution from the slow mode given by Landau-Khalatnikov formula

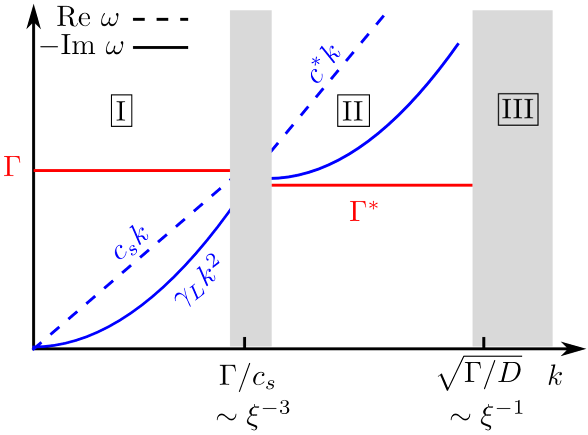
$$\Delta\zeta = w\Delta c^2/\Gamma$$

Modes

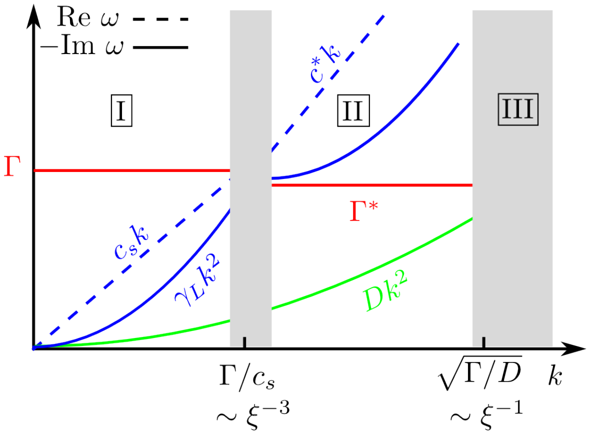


Re ω
$-$Im ω
I
II
III
Γ
$c_s k$
$\gamma_L k^2$
Γ/c_s
$\sim \xi^{-3}$
$\sqrt{\Gamma/D}$
$\sim \xi^{-1}$
k

Modes


Re ω
$-$Im ω
I
II
III
Γ
Γ^*
$c^* k$
$c_s k$
$\gamma_L k^2$
Γ/c_s
$\sim \xi^{-3}$
$\sqrt{\Gamma/D}$
$\sim \xi^{-1}$
k

Modes



Re ω
$-$Im ω
I
II
III
Γ
Γ^*
c^*k
$c_s k$
$\gamma_L k^2$
Dk^2
Γ/c_s
$\sim \xi^{-3}$
$\sqrt{\Gamma/D}$
$\sim \xi^{-1}$
k

Microscopic origins of Hydro+

Understanding the microscopic origin of the slow mode:

The fluctuations around equilibrium are controlled by the entropy functional $P \sim e^S$.

Near the critical point convenient to "rotate" the basis of variables to "Ising"-like critical variables $\mathcal{E}$ and $\mathcal{M}$. $\mathcal{M} \sim s/n - (s/n)_{\rm CP}$.

$$\delta\mathcal{S}[\delta\mathcal{E}, \delta\mathcal{M}] = \left[\frac{1}{2}\, a_{\mathcal{M}}\ (\delta\mathcal{M})^2 + \frac{1}{2}\, a_{\mathcal{E}}\ (\delta\mathcal{E})^2 + b\,\delta\mathcal{E}\,\delta\mathcal{M}^2 + \dots\right].$$

Since $a_{\mathcal{M}} \ll a_{\mathcal{E}}$ fluctuations of $\mathcal{M}$ are large and are slow to equilibrate.

Their magnitude is related to the slow relaxation mode ϕ.

Hydro + mode distribution

Separate "hard" $k > \xi^{-1}$ and "soft" $k \ll \xi^{-1}$ modes.

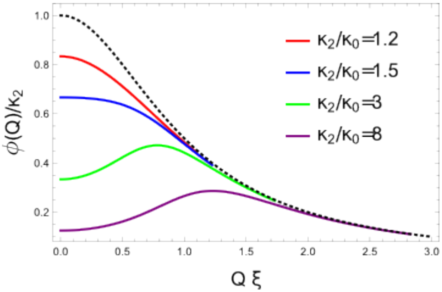
The new variable, "mode distribution function":

$$\phi(t, \boldsymbol{x}, \boldsymbol{Q}) = \int_{\boldsymbol{y}} \langle\, \delta\mathcal{M}(t, \boldsymbol{x} + \boldsymbol{y}/2)\, \delta\mathcal{M}(t, \boldsymbol{x} - \boldsymbol{y}/2) \,\rangle\; e^{-i\boldsymbol{Q}\cdot\boldsymbol{y}}$$

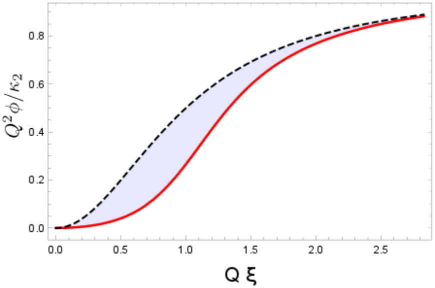
The additional mode distribution function relaxation equation:

$$(u \cdot \partial)\phi(t, \boldsymbol{x}, \boldsymbol{Q}) = 2\Gamma_{\mathcal{M}}(\boldsymbol{Q}) \left[a_{\mathcal{M}}^{-1} - \phi(t, \boldsymbol{x}, \boldsymbol{Q})\right]$$

where $\Gamma_{\mathcal{M}}(\boldsymbol{Q})$ is known from model H (Kawasaki).



Shaded area $\sim(\kappa_{2,eq} - \kappa_{2,ne})$



Summary

- A fundamental question for Heavy-Ion collision experiments:

  Is there a critical point on the boundary between QGP and hadron gas phases?

  Theoretical framework is needed – the goal for BEST COLLABORATION.

- Large (non-gaussian) fluctuations – universal signature of a critical point.

- In H.I.C., the magnitude of the signatures is controlled by dynamical non-equilibrium effects. The physics of the interplay of critical and dynamical phenomena can be captured by hydrodynamics with a critically slow mode(s) – Hydro+.