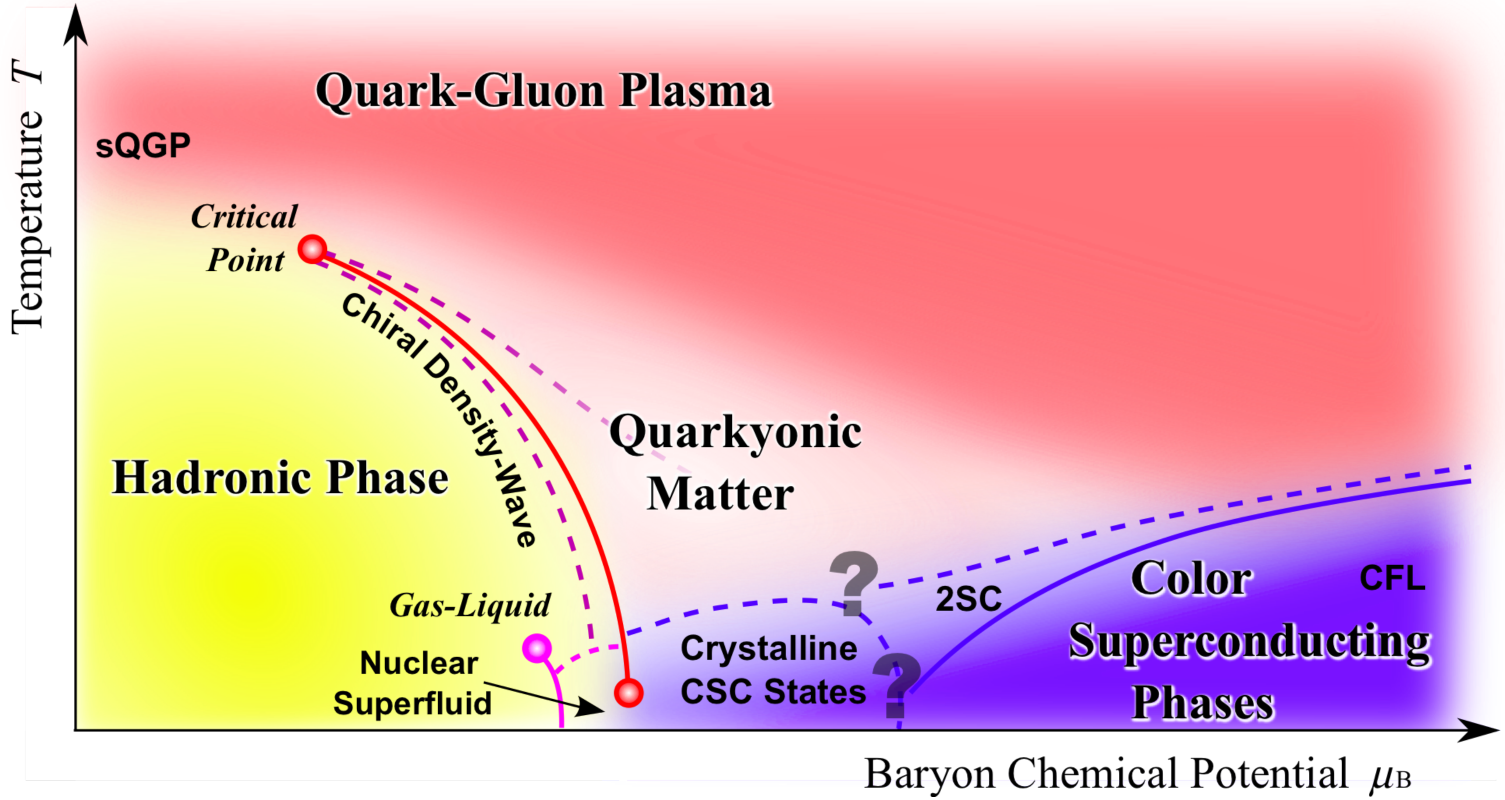
From a Yeoman's Tale in the Canterbury Tales:

Who-so takes up this science, for my part,
If he persists, is done in from the start.
So help me God, there's nothing he can gain
Except an empty purse and addled brain.
And when he by his crazy foolishness
Has lost his goods through all this risky mess,
He then entices others with their pelf
To lose it all as he has done himself.
For rascals find their comfort and delight
In seeing fellow men in pain and blight

Large N and Quarkyonic Matter

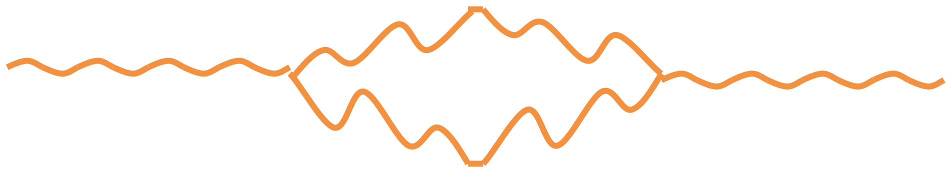
Oxford, July 2017



Hatsuda and Fukushima

QCD Phase Diagram

Confinement at Finite Temperature and Density:



$$g^2 N_c T^2 \sim \alpha_N T^2$$

Generates Debye Screening => Deconfinement at Tc

$$g^2 \mu_Q^2 \sim \alpha_N \mu_Q^2 / N_c$$

$$\mu_Q = \mu_B / N_c$$

Quark loops are always small by 1/N_c

For finite baryon fermi energy, confinement is never affected by the presence of quarks!

T_c does not depend upon baryon density!

Confinement remains to very high baryon number density

Brief Review of Large N

$$N_c \to \infty \qquad g^2 N_c \; finite$$

Mesons: quark-antiquark, noninteracting, masses ~ Λ_{QCD}

Baryons: N quarks, masses M ~$N_c \Lambda_{QCD}$, baryon interactions ~ N_c

Spectrum of Low Energy Baryons:

Multiplets with I = J; I,J = 1/2 -> I,J = N/2

$$M_B(I,J) \sim M_N(1 + O(I^2/N_c^2, J^2/N_c^2, IJ/N_c^2))$$

$$M_\Delta - M_N \sim \Lambda_{QCD}^2/N_c$$

$$e^{(\mu_B - M_B)/T} = 0 \; if \; \mu_B < M_B$$

The confined world has no baryons because baryons are very massive!

Hidaka, Pisarski, Mclerran, Kojo

In large number of colors limit:

Baryons are infinitely massive compared to QCD energy scale
Quarks do not affect the confining potential:
De-confinement temperature independent of density

At least 3 phases of matter:

Confined matter with no baryons
(low temperature and low baryon density)

De-confined matter with light mass quarks carrying baryon number
(high temperature and baryon chemical potential greater than nucleon mass,
Or quark chemical potential greater than QCD scale

In addition there is color superconductivity at very high density
Liquid phase of nuclear matter

Quarkyonic Matter

$$T < T_c \qquad \Lambda_{QCD} << \mu_{quark} << \sqrt{N_c}\Lambda_{QCD}$$

Inside the Fermi sea, expect light mass gas of approximately free quarks

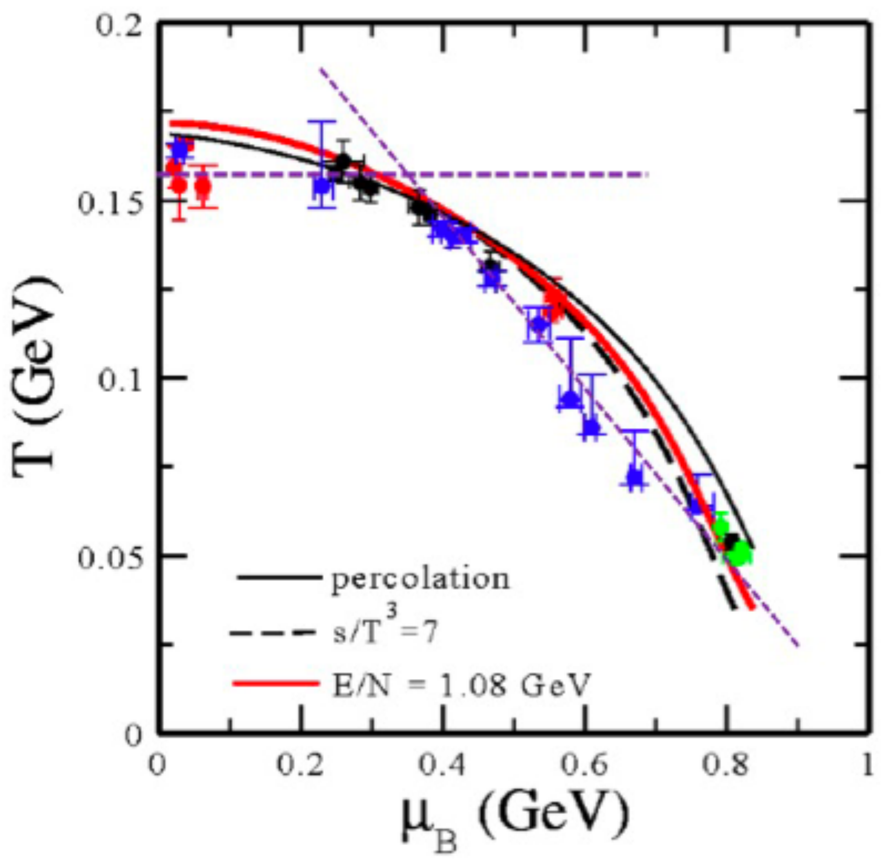
Thermal excitations are mesons, glueballs and baryons, all are confined

Fermi surface composed of mesons and baryons that are confined

Hence the word quarkyonic

The crucial role of the Fermi surface is seen in NJL models

What about chiral symmetry restoration?



Dashed line indicate simple models of deconfinement and quarkyonic transition

$$\mu_B/T - M_B/T = C \qquad T = (\mu_B - M_B)/C$$

A. Andronic, D. Blaschke, P. Braun-Munzinger, J. Cleymans, K. Fukushima, L.D. McLerran, H. Oeschler, R.D. Pisarski, K. Redlich, C. Sasaki, H. Satz, J. Stachel,

Reinhard Stock, Francesco Becattini, Thorsten Kollegger, Michael Mitrovski, Tim Schuster

Measured abundances fall on curve with fixed baryon chemical potential and temperature at each energy: suggests a phase transition with a rapid change in energy density

High density low T points deviate from expectations of deconfinement transition

Fundamental Problem with Naïve Large N_c Limit:

Pion-Nucleon Interactions Strength is of Order N_c
Coupling is spin-isospin dependent
Makes spin-isospin braking crystal with binding energy
of the order of that of nuclear matter with binding
energy of order M_N:

NUCLEAR MATTER DOES NOT EXIST

Can this be cured:

Perhaps by the large N limit being a scale invariant critical point
Skokov and LDM
(More about this later in the talk)
If true can generate a short range repulsive interaction
of strength N_c but long range weak forces associated with pion exchange

Will assume this in this talk.

How Do We Understand Confinement and Chiral Symmetry Breaking in Quarkyonic Matter?

At high baryon density for matter quarks with E << E_f but E >> Λ_{QCD}

Quarks interact weakly. Fermi blocking requires interactions to be at large angles

For quarks near the Fermi surface, small angle scattering can become important, and non-perturbative scales become important. Picture is that of perturbative Fermi sea of quarks, but Fermis surface degrees of freedom are confined: Mesons and glueballs are excitations that feel the effects of confinement, and quarks near the Fermis surface are either in esons or bound into baryons

Chiral condensation: Condensate of quark- quarkhole pairs with energy near the Fermis surface. The (quark—quarkhole) pair has net momentum $2E_{fermi}$

In the confined phase, expect chiral symmetry is broken. Only way to do this with low relative momenta between particle hole pair is from a particle hole pair at the Fermi surface => carge-density waves

Chiral Spiral Formation

$$E = \mu_B + \Delta E, \ particle, \ k_F \sim \mu_B$$
$$E = \mu_B - \Delta E, \ hole, \ k_F \sim -\mu_B$$
$$antiparticle, \ E = \mu_B + \Delta E, k_F \sim \mu_B$$

If form a bound state with negative binding energy =>

Chiral condensate

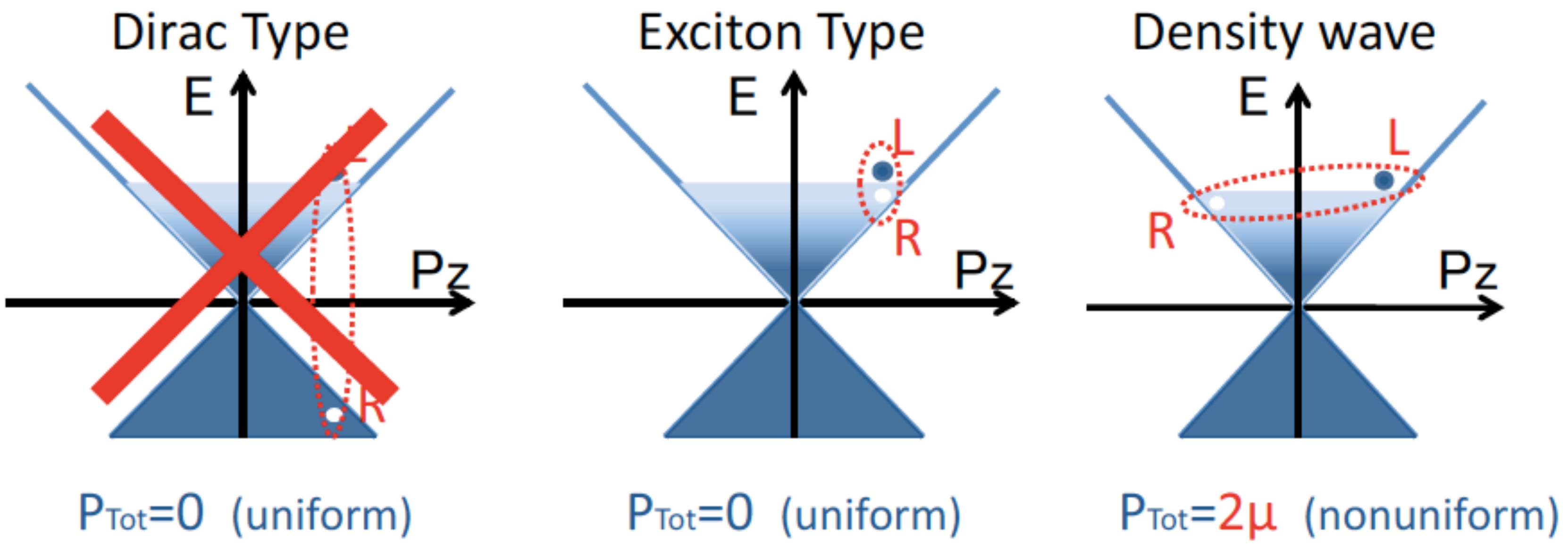
Condensate breaks translational invariance => crystal Chiral symmetry breaking of order

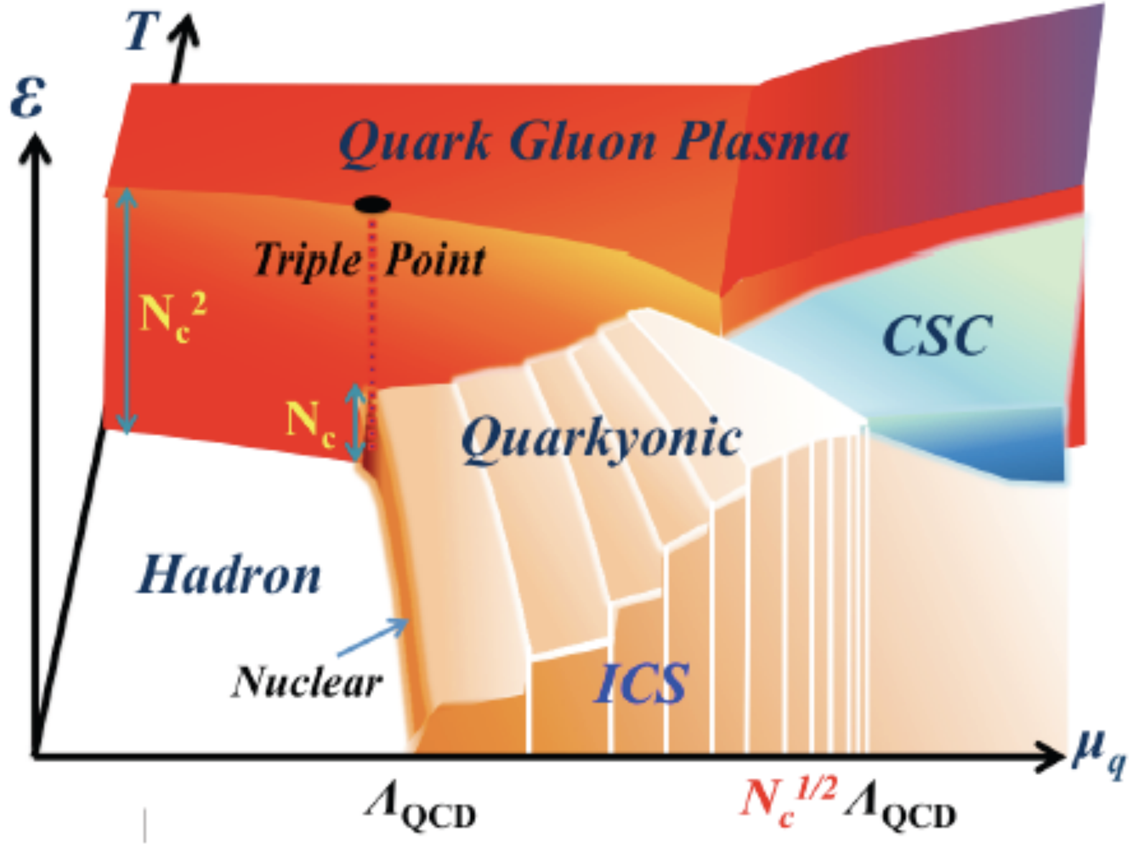
$$\Lambda_{QCD}^2/\mu_Q^2$$

Hidaka, Kojo, McLerran, Pisarski

Quarkyonic phase weakly breaks chiral symmetry

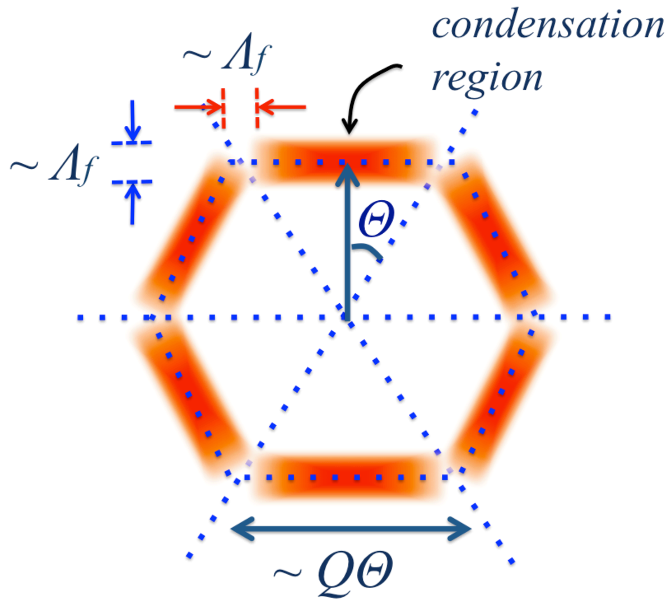
- Candidates which spontaneously break Chiral Symmetry





Breakdown of spherically symmetric Fermi surface

Translationally non-invariant chiral condensate, but baryon number distribution is to good approximation invariant

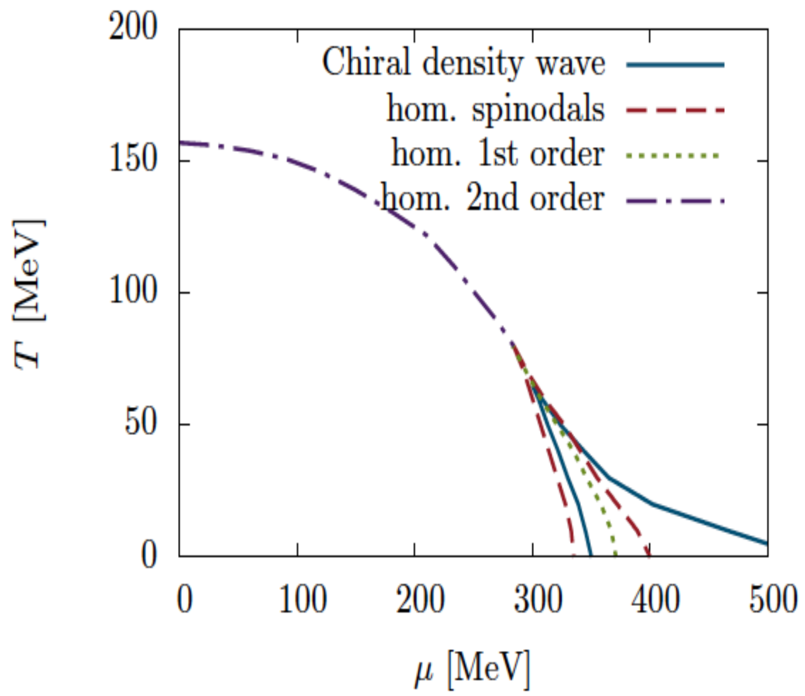


In coordinate space, quasi-crystalline structure for condensate?

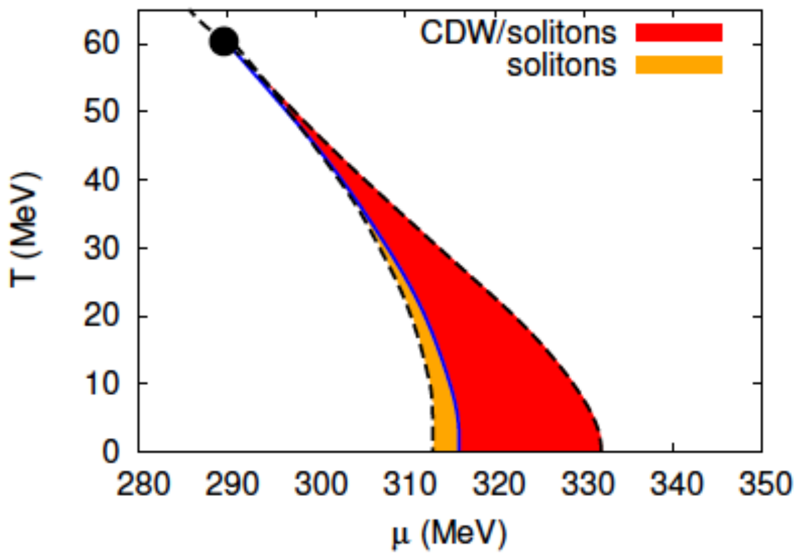
No Consensus about mechanism of breaking of translational invariance

Various model computations:

Muller, Buballa and Wambach



Corignano, Buballa, Schaeffer



What about the baryons?

Two cases:
N_c Even: Baryons are bosons
N_c odd: Baryons are fermions

Bosonic case: When baryon chemical potential is > M_B, Bose condensation is induced. This is true until the hard core of Bose interactions, due to the repulsive interaction of the quark constituents becomes important. This occurs at some momentum scale when quark chemical potential is

$$p_F < \Lambda_{QCD}$$

When $p_F >> \Lambda_{QCD}$, then for p << p_F, the degrees of freedom are quarks, but when p_f is near the Fermis surface the quarks are bound into bosons. The bosonic baryons must sit there because the constituents of the bosons must have energy high enough to avoid the Pauli blocking in the Fermi sea. Therefore the Bosons are acting like Fermis surface excitations

Odd N_c: Fermionic case:

When baryon chemical potential first exceeds the Fermi momenta, Fermions are inserted at the bottom of the Fermis sea. Their energy is

$$p^2/2M \sim 1/N_c$$

So they occupy an effectively zero energy state as was the case for even N_c and the Bose condensed state. This, as was the case for Bosons is true until interactions are important.

At large chemical potential the baryons are bound near the Fermis surface, as was the case for the Bosons

Kinematically, the Fermionic and Bosonic cases are similar implying some sort of duality of description. Is this more general, and do the Bose condensate and "Fermionic Condensate" have distinguishable properties due to the Bose coherence?

Issues with the Nucleon Force and Large N_c

Standard Large N_c description: Baryons are Skyrmions of Large N_c Pion Lagrangian
Multibaryon solutions have binding energy per nucleon of order N_c
Pion nucleon interaction is of order N_c
Pion nucleon force is spin-isospin dependent:

$$\partial_i \pi_a \overline{\psi} \tau^a \sigma^i \psi$$

In chiral limit, pion exchange generate long range strong force
Nuclear Matter is bound into a Skyrme crystal with binding energy of order the nucleon mass

In nature, the binding energy of nuclear matter is 15 MeV << 1 GeV
There is a short range strongly interacting core to the potential at a scale of order 1Fm and a pion tail which is attractive

$$g_A \sim 1 \; not \sim N_c$$

At intermediate range, there is a cancellation between sigma and omega exchange to make for a
Weak intermediate range potential interpolating with the hard core reulstion

One possible way out is BPS Skyrme Model:

C. Adam, J. Sanchez-Guillen and A. Wereszczynski,

Assume interaction is entirely due to topological baryon-baryon interactions and that the ordinary skyrme term and pion kinetic energy interaction term are not present

Leads to BPs soliton solution with no interactions. Skyrmion is stabilized by pion mass so that in chrial limit baryons are of infinite size and zero mass

$$M_N \sim N_c m_\pi$$

$$R^3 \sim 1/m_\pi \Lambda^2_{QCD}$$

This theory has essentially vanishing expectation value for the sigma field or f_π

It has no natural reason why the pion kinetic term should vanish: is there a natural way one can generate a theory with small VEV and that has a more robust treatment of the nucleon?

Pion-Nucleon Sigma Model in Large N_c

$$-\frac{1}{2}\frac{m^2}{N_c}(\phi^2+\pi^2)+\frac{\lambda}{4N_c}(\phi^2+\pi^2)^2 \cdot$$

In strict large N limit, interaction term vanishes and theory is unstable
An example of such a truncation would be quenched QCD

Large N is a free theory but the sigma-pion mode is unstable and generate runaway

What can we do to change this?
If the mass term is positive, it clearly would not generate chiral symmetry breaking so only possibility is zero.
In the 't Hooft model in 1+1 dimension this happens: The Bethe-Salpeter equation for the bound states has a massless scalar in the spectrum for massless quarks

Suppose we assume that the mass squared tem vanishes to leading order in N:
Large N limit is at a critical point?

Naturalness of the massless limit:

If at tree level one assumed a mass of order 1/N, does it stay that way when one has quantum corrections?

Yes

Tadpole diagrams generate a mass squared of order

$$\delta M^2 \sim \frac{\lambda}{N_c} \Lambda^2_{QCD}$$

The VEV is not of order N,

$$f_\pi \sim < \sigma > \sim O(1) \ not \ O(N_c)$$

In nature this is about 100 MeV

This is its value at scale of order the sigma meson mass which is of order 1/N. Presumably at higher momentum scales, sum rules require the axial vector curent coupling return to its value of order N, but nothing obvious prevents its suppression at low momentum scales

If the sigma become massless, then nucleon-nucleon interaction have
A long range scalar attraction in the isosinglet channel

This requires a vector which has a coupling to nuclear greater or equal to the scalar, and in the large N limit, must be less massive or of equal mass to the scalar
This is natural in theories with a hidden local symmetry

So to make a sensible theory one requires also a cancellation of the intermediate range force force

How would nucleons appear?

Assume they are from topological skyrmion like configurations

Kinetic energy is of order 1/N, so kinetic energy will also be of order 1/N

$$R_N \sim \sqrt{N_c/f_\pi} \sim \sqrt{N_c/\Lambda_{QCD}}$$

$$M_N \sim \int d^3x \frac{1}{N_c} \Lambda_{QCD}^4 \sim \sqrt{N_c}\Lambda_{QCD} \sim N_c/R_N$$

Nucleons are large and not so massive

How can this be consistent with what is know from lattice gauge theory?

Lattice gauge theory at large N is done for massive pions.
Need to include "sigma term"

$$S = \frac{1}{2}\left[(\partial\phi)^2 + (\partial\pi)^2\right] - \frac{1}{2}\frac{m^2}{N_c}(\phi^2 + \pi^2) + \frac{\lambda}{4N_c}(\phi^2 + \pi^2)^2 - \sqrt{N_c} m_q \mu^2 \phi.$$

Note that if the quark masses are large, then one can ignore the m^2 term, and naïve large N scaling follows

The transitions between large and small quark mass occurs when

$$m_q \ll \Lambda / N_c^{3/2}$$

For physical pion mass for N =3, we are in the small mass limit

Degrand et al
Bali et al

Lattice data for large N are in the large mass limit.

Can have an acceptable description of the lattice data for variable N with parameters that fit the data for N = 3

Interesting observation on the lattice data for variable quark masses: Indeed the omega, sigma and nucleon mass scale with f_\pi, suggesting there is only one scale in the theory

Pion-Nucleon interaction generated by higher dimensional term

$$\frac{g^2}{\Lambda_{QCD}^2}\left\{\overline{\psi}_L\chi\gamma\cdot\partial\chi^\dagger\psi_L+\overline{\psi}_R\chi^\dagger\gamma\cdot\partial\chi\psi_R\right\}$$

$$\chi=\sigma+i\pi$$

For vev of O(1), pion nucleon coupling is generically of order 1:

$$V\sim 1/\Lambda_{QCD}^2R^3$$

At distances greater than the nucleon size, this is suppressed by powers of N_c since

$$R\sim\sqrt{N}/\Lambda$$

Two pion exchange even more strongly suppressed

Conventional nuclear physics picture arises of a strongly interacting core and a weakly interacting tail