Hydrodynamization at weak coupling

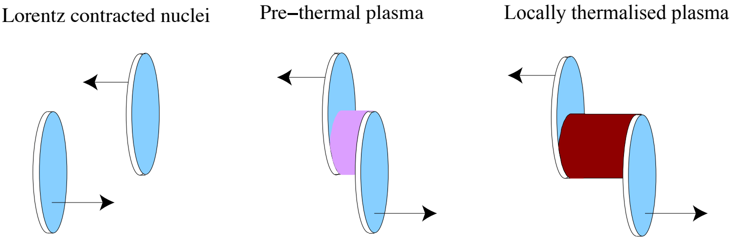
Aleksi Kurkela

AK, Wiedemann in progress
AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress
Keegan, AK, Mazeliauskas, Teaney JHEP 1608 (2016) 171
AK, Zhu PRL 115 (2015) 18, 182301
AK, Lu PRL 113 (2014) 18, 182301
AK, Moore JHEP 1111 (2011) 120
AK, Moore JHEP 1112 (2011) 044

Universitetet
i Stavanger

Oxford, July 2017

Motivation?



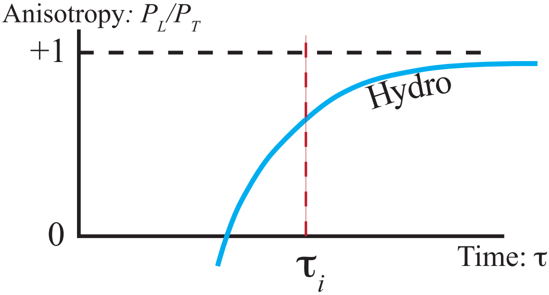
- Soft physics of HIC described by relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- Gradient expansion around local thermal equilibrium

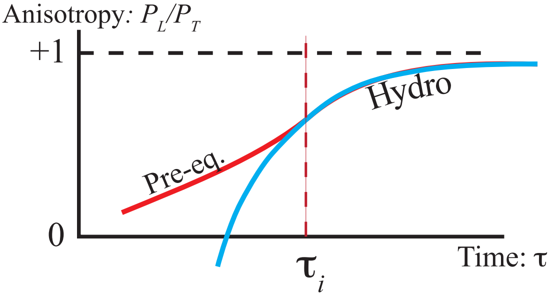
$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} - \eta 2 \nabla^{<\mu} u^{\nu>} + \ldots$$

Motivation?



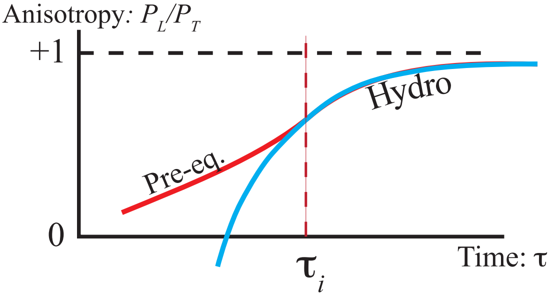
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *intialization time* τ_i

Motivation:



- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i

Motivation:



- If prethermal evolution converges smoothly to hydro, independence of unphysical τ_i
- In most current pheno: either free streaming, or nothing at all

Motivation:

- In AA collisions: pre-equilibrium evolution $\sim 10\%$ of the evolution
  - Pre-equilibrium evolution major uncertainty affects η/s, etc

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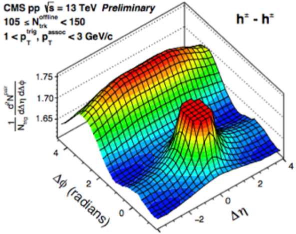
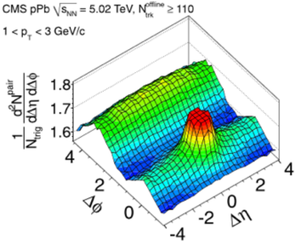
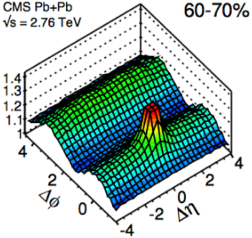


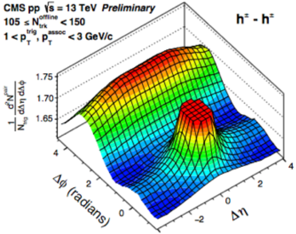


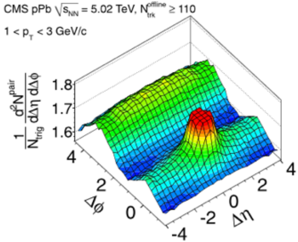


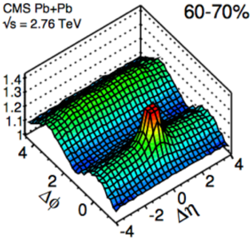
Motivation:

- In AA collisions: pre-equilibrium evolution $\sim 10\%$ of the evolution
  - Pre-equilibrium evolution major uncertainty affects η/s, etc

- In pA collisions: currently no quantitative description
  - even if the system becomes hydrodynamical, "pre-equilibrium" evolution $\mathcal{O}(1)$ of the evolution







Motivation:

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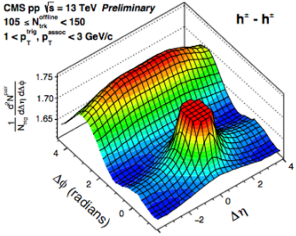
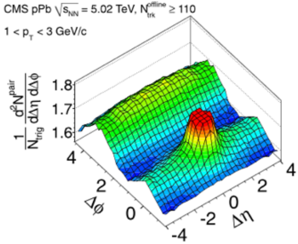
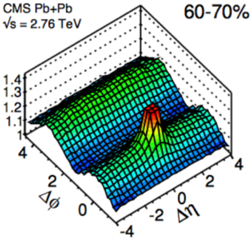
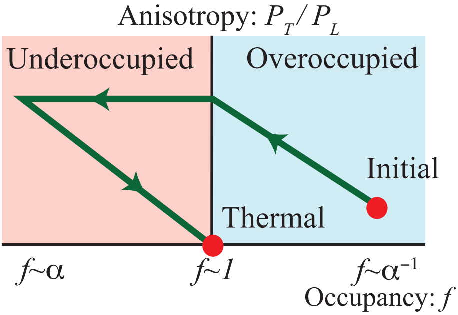
- pp collisions: ?????







Hydrodynamization in weak coupling



- Color Glass Condensate: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) , PRD49 (1994); Gelis et. al Int.J.Mod.Phys. E16 (2007), Ann.Rev.Nucl.Part.Sci. 60 (2010)

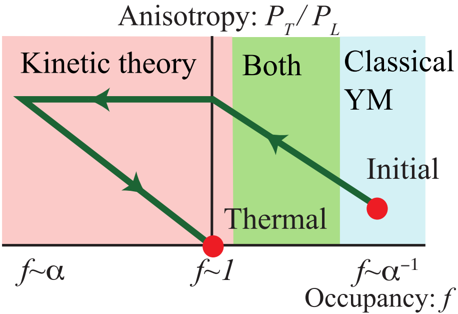
$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

Baier et al PLB502 (2001)

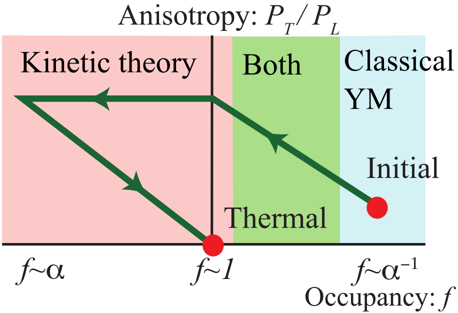
$$f(Q_s) \ll 1$$

Hydrodynamization in weak coupling



- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

Hydrodynamization in weak coupling

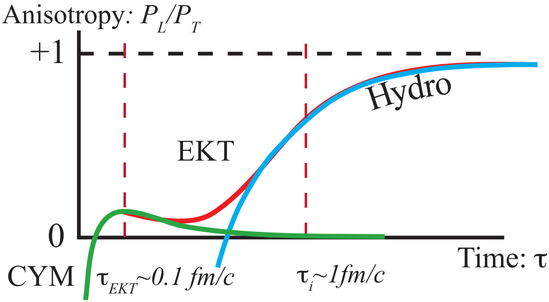


- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873; AK et al PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

- "Bottom-up thermalization" of underoccupied system

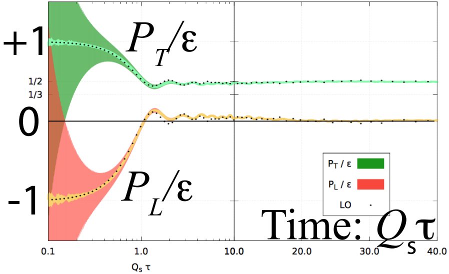
Strategy at weak coupling



Strategy: Switch from CYM to EKT at τ_{EKT}, $1 \ll f \ll 1/\alpha_s$

From EKT to hydro at τ_i, $P_L/P_T \sim 1$

Early times $0 < Q_s\tau \lesssim 1$: classical evolution



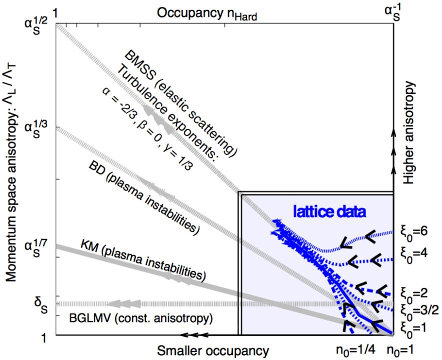
Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields

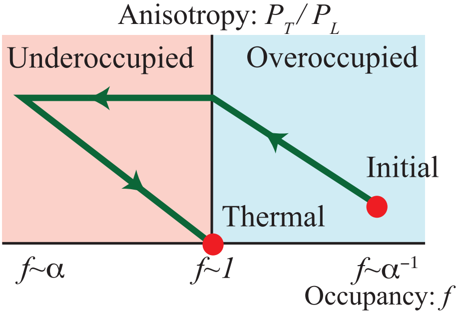
Initial condition from CGC: MV-model, JIMWLK

  - After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$

Later times $Q_s\tau > 1$: classical evolution



Berges et al. Phys.Rev. D89 (2014) 7, 074011

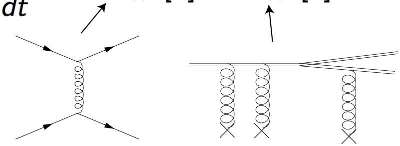


- Numerical demonstration of overoccupied part of the diagram
- Classical theory never thermalizes or isotropizes

Effective kinetic theory of Arnold, Moore, Yaffe

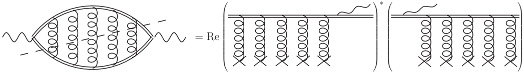
JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$



- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum



- No free parameters; LO accurate in the $\alpha_s \to 0$, $\alpha_s f \to 0$ limit, for

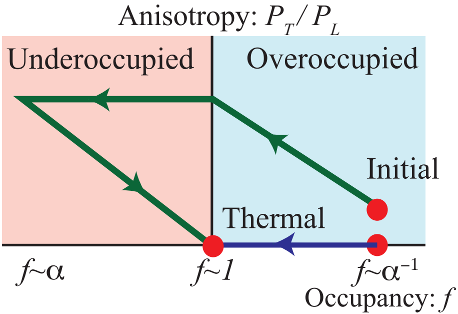
$$\Delta t \sim \omega^{-1} > \text{Typical scattering time } \sim 1/(\alpha^2 T)$$

- Caveat: in anistropic systems screening complicated. Here with isotropic screening. Also no fermions here, yet

plasma instabilities, ...

Outline

- Hydrodynamization and thermalization of homogenous systems
- Hydrodynamization with spatial inhomogenities
- Hydrodynamization as decay of non-hydrodynamic modes and analytic structure of Green functions

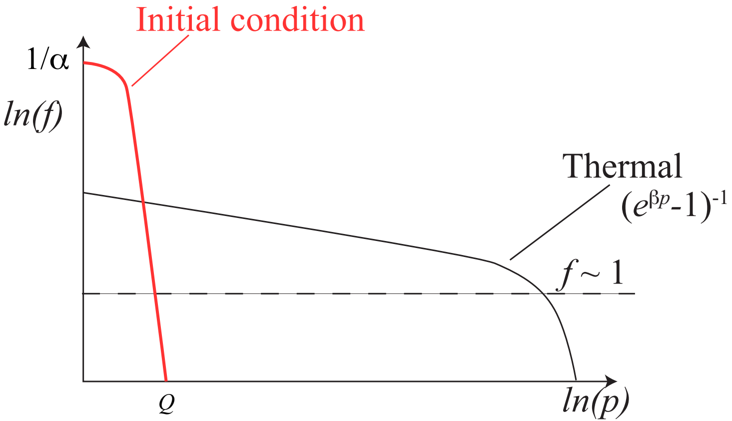


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Radiative break-up
- Effect of longitudinal expansion: Hydrodynamization

Overoccupied cascade

AK, Moore JHEP 1112 (2011) 044

What happens if you have too many soft gluons, $f \sim 1/\alpha_s$.



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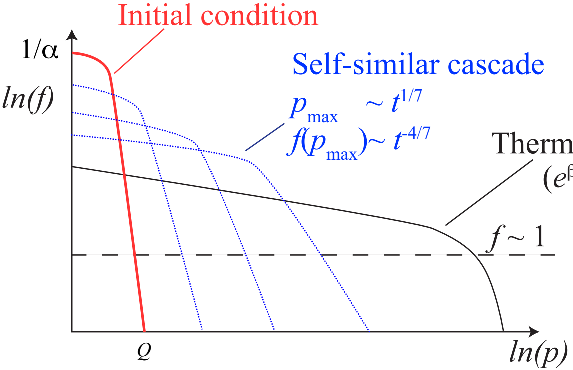
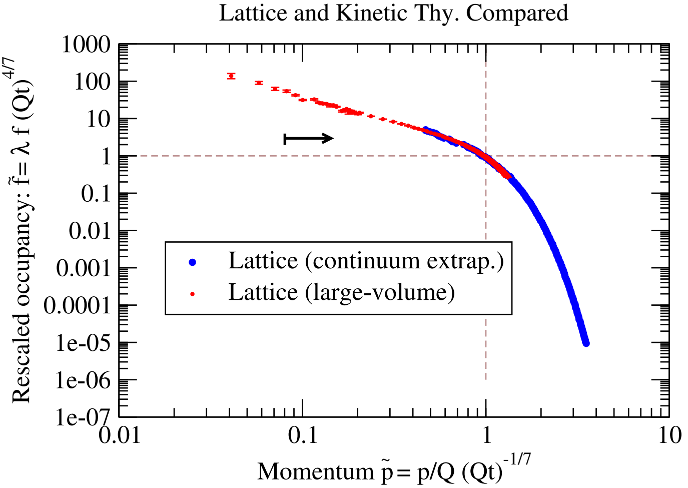


$$\tau_{\text{init}} \sim [\sigma n(1+f)]^{-1} \sim \left(\frac{Q}{T}\right)^7 \frac{1}{\alpha_s^2 T} \ll \frac{1}{\alpha_s^2 T} \sim \tau_{\text{them}}.$$

c.f. Bokuslawski's talk

Overoccupied cascade

AK, Lu, Moore, PRD89 (2014) 7, 074036



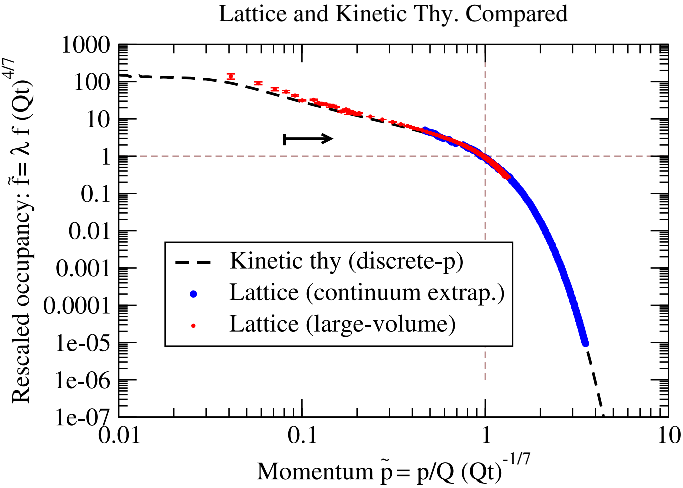
Form of cascade from classical lattice simulation,

$$1 \ll f \lesssim 1/\alpha_s$$

Large-volume: (Qa)=0.2, (QL)=51.2, Cont. extr.: down to (Qa)=0.1, (QL)=25.6, Qt=2000, $\tilde{m} = 0.08$

Overoccupied cascade

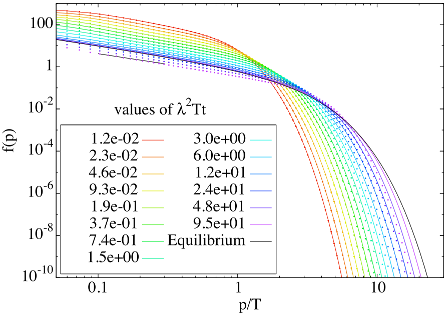
AK, Lu, Moore, PRD89 (2014) 7, 074036

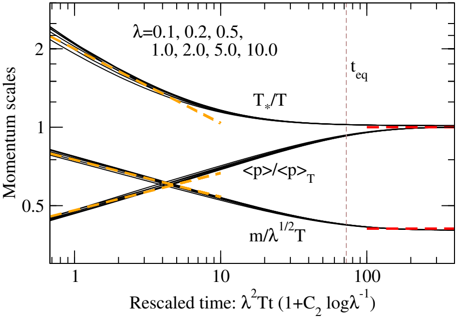


Same system, very different degrees of freedom

$$1 \lesssim f \ll 1/\alpha_s$$

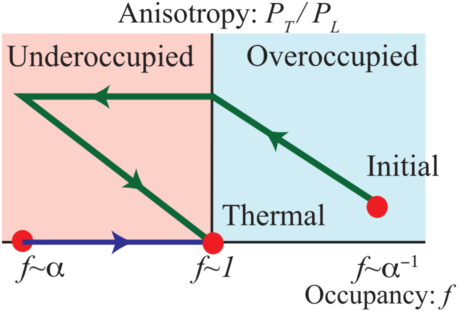
Ending of the overoccupied cascade AK, Lu PRL 113 (2014) 18, 182301





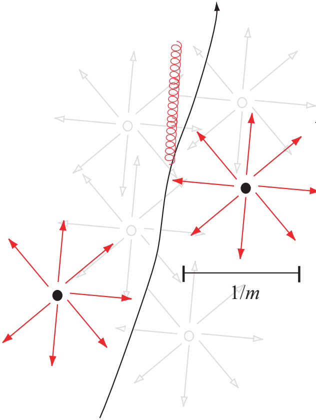
- Thermal equilibrium reached within the kinetic theory

$$t_{\text{eq}} \approx \frac{72.}{1 + 0.12 \log \lambda^{-1}} \frac{1}{\lambda^2 T}$$

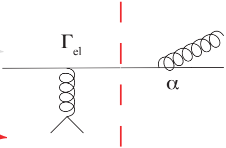


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Bottom-up thermalization
- Effect of longitudinal expansion: Hydrodynamization

Underoccupied cascade: Formation of thermal bath



- Soft modes quick to emit



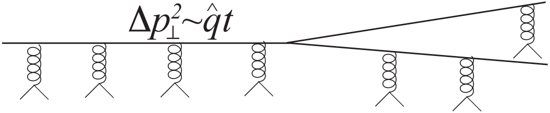
$$\Gamma_{\mathrm{el}} \sim \alpha_s^2 \frac{n}{m_D^2} \sim \alpha_s^2 \frac{\int_{\mathbf{p}} f}{\alpha_s \int_{\mathbf{p}} f/p}$$

$$n_{\mathrm{soft}} \sim \alpha_s \, \Gamma_{\mathrm{el}} \, t$$

- Low-p: easy to thermalize
- Can dominate the dynamics
  scattering, screening, . . .

⇒ Few energetic “jets” propagating in thermal bath

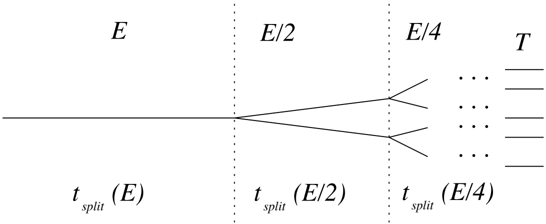
Underoccupied cascade: Radiational breakup



- In vacuum: on-shell splitting kin. disallowed
- In medium:
    - frequent soft scatterings with medium, mom. diffusion: $\Delta p^2 \sim \hat{q}t$
    - Scatterings lead to virtuality: $P^2 \sim \hat{q}t$
    - Now offshell particle may split collinearly: $t_f \sim Q/P^2 \sim \sqrt{Q/\hat{q}}$
    - Splitting time (per particle) $t_{\text{split}}(Q) \sim \frac{1}{\alpha_s} t_f \sim \frac{1}{\alpha_s}\sqrt{\frac{Q}{\hat{q}}}$

QED: Landau, Pomeranchuk, Migdal 1953.
QCD: Baier Dokshitzer Mueller Peigne Schiff hep-ph/9607355

Underoccupied cascade: Radiational breakup



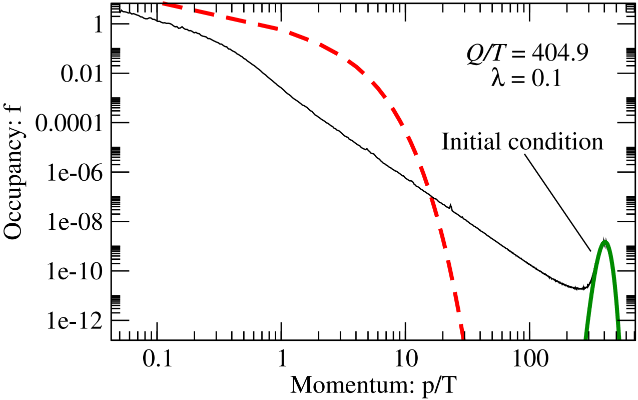
- Successive splittings happen in faster times scales:

$$t_{\text{quench}}(Q) \sim t_{\text{split}}(Q) + t_{\text{split}}(Q/2) + t_{\text{split}}(Q/4) + \ldots \sim t_{\text{split}}(Q)$$

- Once the parton has had time to split it cascades its energy to IR, T increases.

Bottom-up thermalization

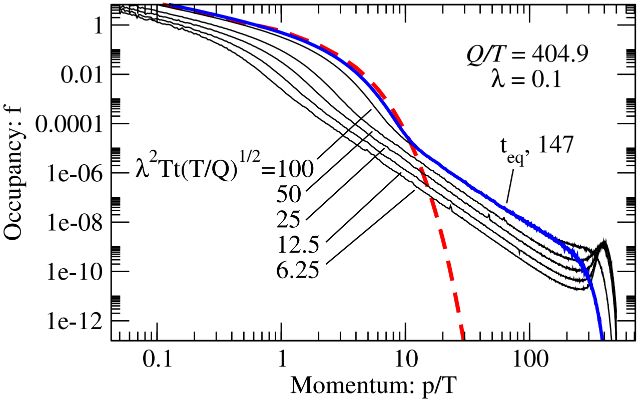
AK, Lu, PRL 113 (2014) 18, 182301



- Start with an underoccupied initial condition $p \sim Q$
- after a very short time, an IR bath is created (1↔ 2–processes)

Bottom-up thermalization

AK, Lu, PRL 113 (2014) 18, 182301



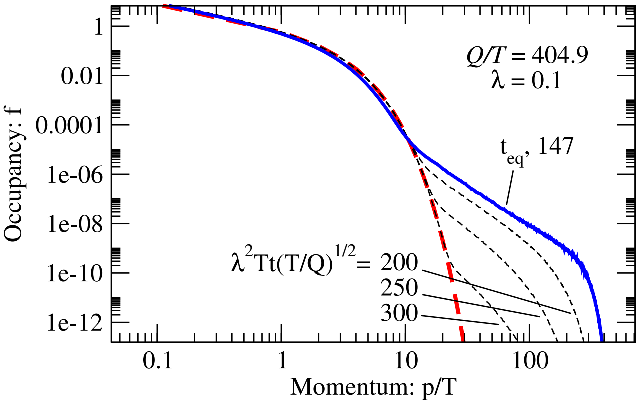
- More energy flows to the IR, temperature increases, “Bottom-up”
- When “bottom” reaches final T, “up” is quenched

AK, Moore JHEP 1112 (2011) 044

$$t_{\mathrm{eq}} \sim (Q/T)^{1/2} \frac{1}{\alpha_s^2 T}$$

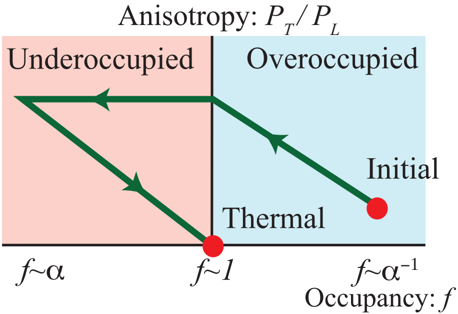
Bottom-up thermalization

AK, Lu, PRL 113 (2014) 18, 182301



- Hardest scales reach equilibrium last.

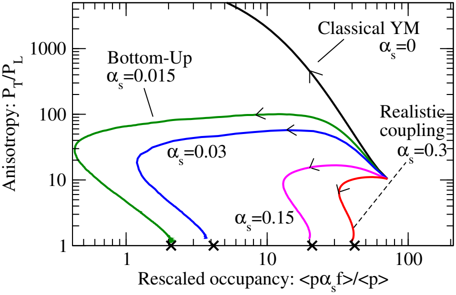
Close resemblance to Blaizot, Iancu, Mehtar-tani for jets PRL 111 (2013) 052001

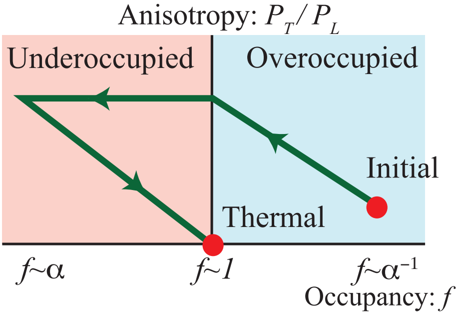


- Isotropic overoccupied: Transmutation of d.o.f's
- Isotropic underoccupied: Radiative break-up
- Application to HIC: effect of longitudinal expansion

Route to equilibrium in EKT

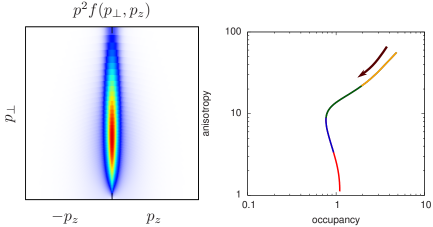
AK, Zhu, PRL 115 (2015) 18, 182301

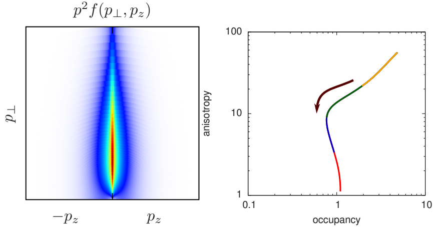


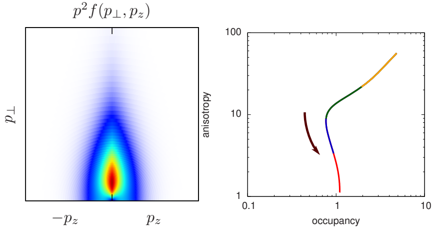


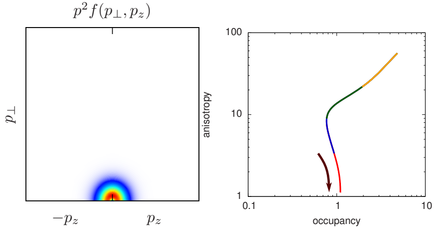
- Initial condition ($f \sim 1/\alpha_s$) from classical field thy calculation
  Lappi PLB703 (2011) 325-330
- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

Route to equilibrium in EKT



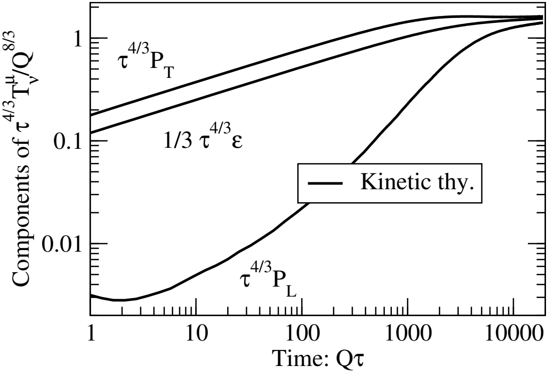






Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha_s = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

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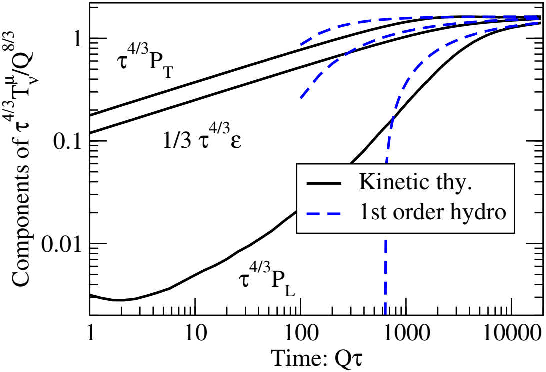


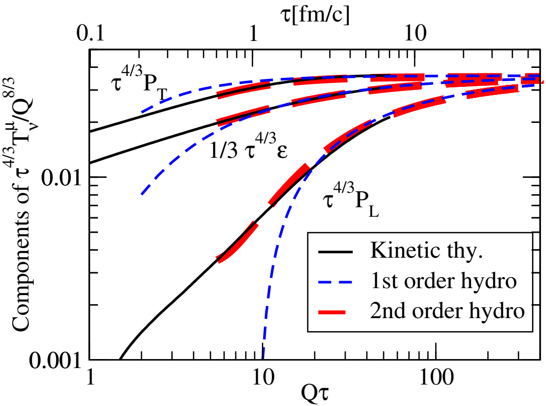
- Kinetic theory converges to hydro smoothly and automatically
- Approach to hydro fixed by perturbative η/s

Arnold et al. JHEP 0305 (2003) 051

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2}, \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha_s = 0.3$$



- For realistic couplings, hydrodynamics reached around $\lesssim 1\text{fm/c}$.
- Hydro gives a good description even when $P_L/P_T \sim 1/5$

Outline

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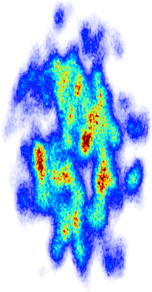
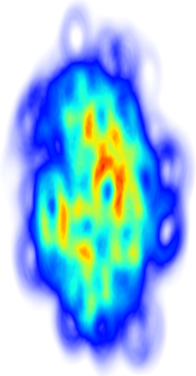
- Hydrodynamization with spatial inhomogenities
- Hydrodynamization as decay of non-hydrodynamic modes and analytic structure of Green functions

Transverse dynamics and preflow



$t = 0$

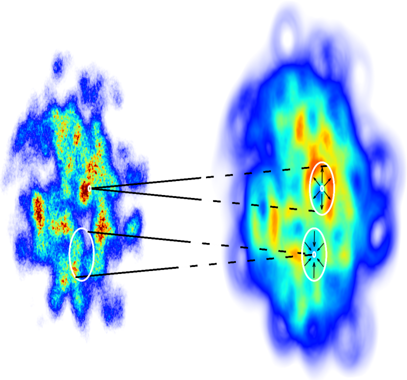


$t = 1 fm/c$

Pre-equilibrium evolution leads to:

- *smearing* of the nuclear geometry
- Generation of *preflow* due to gradients

Transverse dynamics and preflow



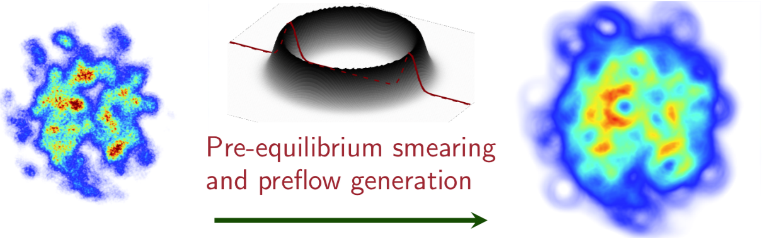
$t = 0$ $\quad$ $t = 1 fm/c$

Nuclear radius $R \ll c\tau_i \sim$ Nucleon radius R_p

- Transverse structure small perturbation within the causal horizon
- Linear response theory for the transverse structures

Transverse dynamics and preflow

$$\int d^2\boldsymbol{x}' \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}} \quad \times \quad \underbrace{E(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_0)} = \underbrace{\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)}, \vec{v}(\tau,\boldsymbol{x})}$$



Pre-equilibrium smearing and preflow generation

- Non-equilibrium Green functions on top of non-equilibrium background E computed in kinetic theory Keegan et al.JHEP 1608 (2016) 171

Linearized perturbations in EKT

Keegan et al. 1605.04287

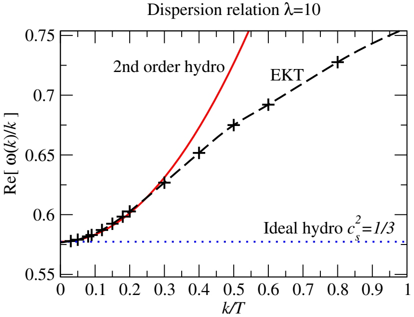
Transverse perturbations characterized by wavenumber **k**

$$f(\mathbf{x}_\perp, \mathbf{p}) = \bar{f}(\mathbf{p}) + \exp(i\mathbf{x}\cdot\mathbf{k})\delta f(\mathbf{p})$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f = C[f]$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + i\mathbf{k}\cdot\mathbf{p}\right) f = C[\bar{f}, f]$$

- For thermal $\bar{f}$: large wavelenght pert. described by hydro



$$\frac{\omega(k)}{k} = c_s^2 + \frac{4}{3}\frac{\eta}{e+p}\left(c_s\tau_\pi - \frac{2}{3c_s}\frac{\eta}{e+p}\right)k^2$$

- For larger k, $c_s^2 \to 1$, with polynomial decay

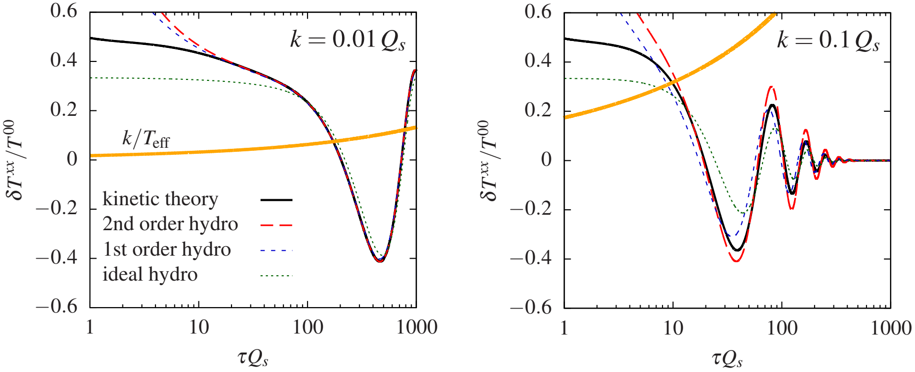
no plot unfortunately...

Hydrodynamization of perturbations

Keegan et al.JHEP 1608 (2016)

$$\delta T^{xx} = \frac{\delta e}{e}\left[\frac{1}{3}e + \frac{1}{3}\eta\tau_\pi k^2 + \frac{\eta}{2\tau} - \frac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \frac{ik\delta T^{0x}}{e}\left[\eta - \frac{1}{\tau}\left(\frac{\eta^2}{2e} + \frac{\eta\tau_\pi}{2} - \frac{2}{3}\lambda_1\right)\right]$$

$k \sim 1/R_{\text{nucleus}}$ $k \sim 1/R_{\text{proton}}$



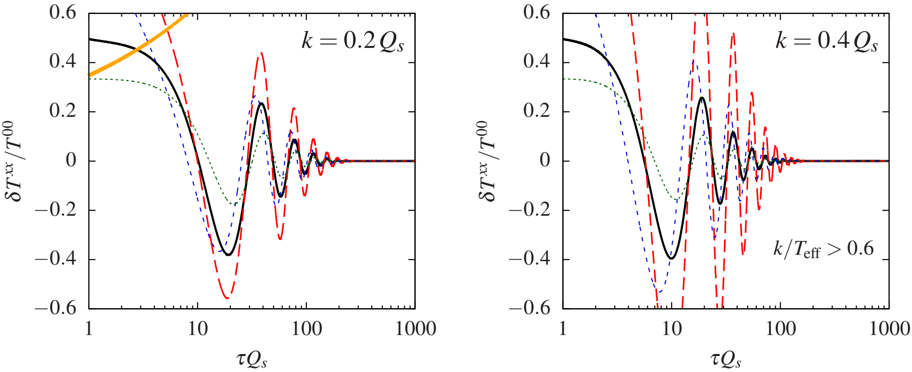
- Perturbations hydrodynamize also at $Q\tau \sim \{10, 20\}$.

Hydrodynamization of perturbations

Keegan et al.JHEP 1608 (2016)

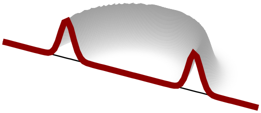
$$\delta T^{xx} = \frac{\delta e}{e}\left[\frac{1}{3}e + \frac{1}{3}\eta\tau_\pi k^2 + \frac{\eta}{2\tau} - \frac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \frac{ik\delta T^{0x}}{e}\left[\eta - \frac{1}{\tau}\left(\frac{\eta^2}{2e} + \frac{\eta\tau_\pi}{2} - \frac{2}{3}\lambda_1\right)\right]$$

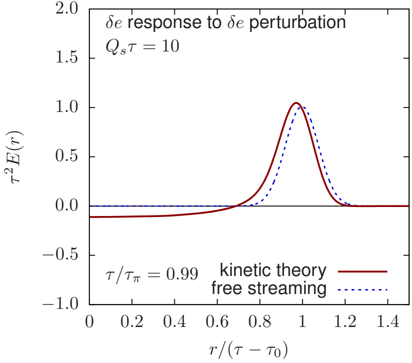
$k \sim 1/0.5R_{\mathrm{nucleus}}$ $\quad$ $k \sim 1/0.25R_{\mathrm{nucleus}}$

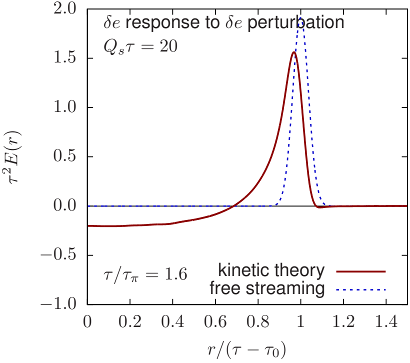


- No hydrodynamics for the large-k modes

Green function in coordinate space

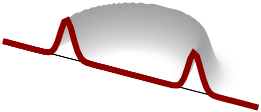


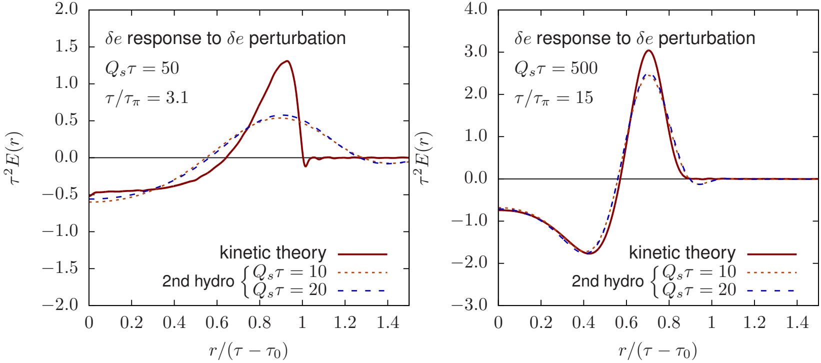




- Nanscent formation of dip in the origin hall mark of hydro

Green function in coordinate space

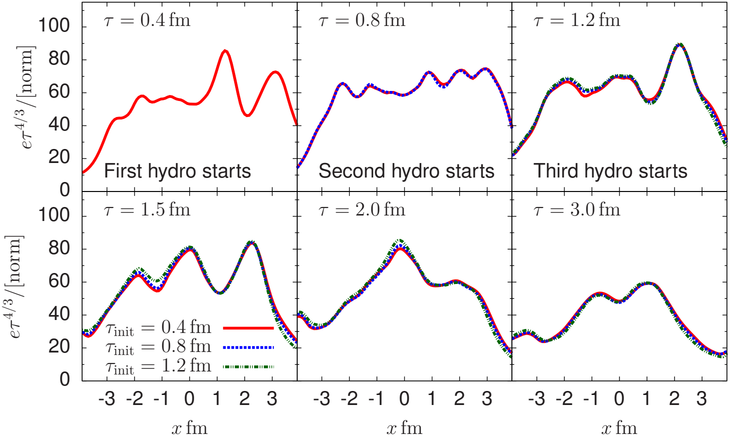




- Evolution after $Q\tau_i > \{10, 20\}$, evolution described by hydro

Transverse dynamics and preflow

- IP-glasma + EKT + Hydro:

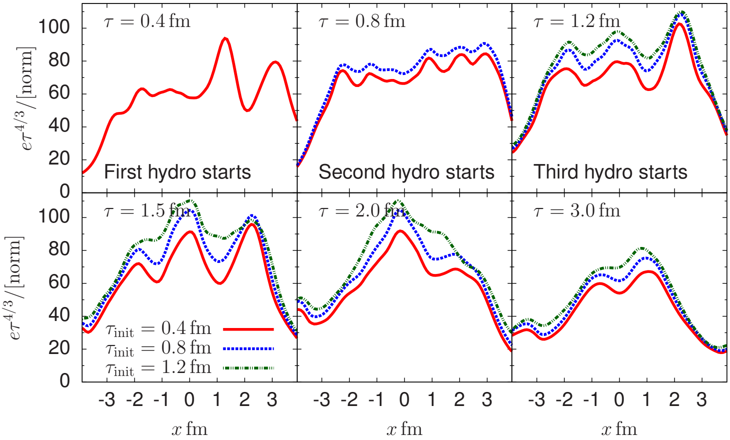


AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress

- Initialization time dependence removed

Transverse dynamics and preflow

- Without EKT:



AK, Mazeliauskas, Paquet, Schlichting, Teaney, in progress

- Strong dependence on initialization time!

What's missing?

- Public code to put this to use, (*KoMPoST*?)

  Soon: aK, Mazeliauskas, Paquet, Schlichting, Teaney

- Role of fermions
  - production of quarks, chemical equilibration Berges et al. PRC 95 (2017)

- Role of far-from-equilibrium plasma instabilities

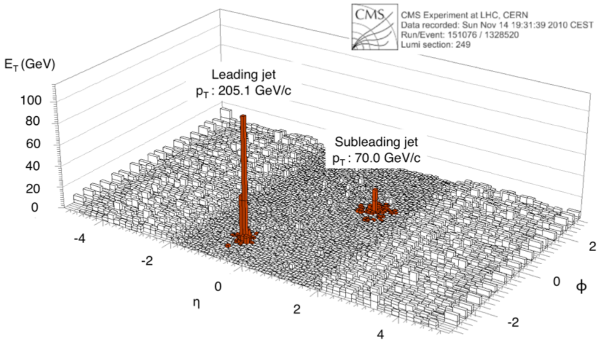
- Extensions to small systems
  - What is the smallest droplet of (weakly coupled) liquid that can be?

    Strong coupling: Chesler JHEP 1603 (2016) 146
  - Can flow-like behaviour arise from only few collisions? Flow without hydro?

- Non-equilibrium correlation functions in strong coupling?

  Casalderrey-Solana, Meiring, van der Schee

What's missing?



- Jet loses energy to the medium: Jet thermalization
- Same physics governs thermalization of the bulk and the medium
- Experimental characterization of jet thermalization will teach about hydrodynamization at intermediate coupling