Holographic phase transitions in real time

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Outline

Motivation

The AdS/CFT description of equilibrium phase transitions

How to model nonconformal plasma?

Equilibrium configurations

Linearized dynamics

Why are quasi-normal modes interesting?

Nonlinear evolution

Conclusions and Outlook

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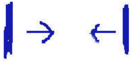

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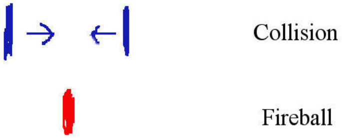

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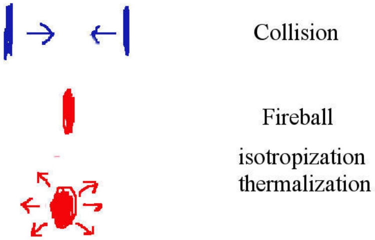

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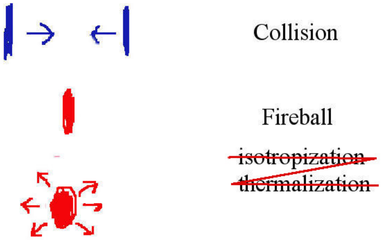

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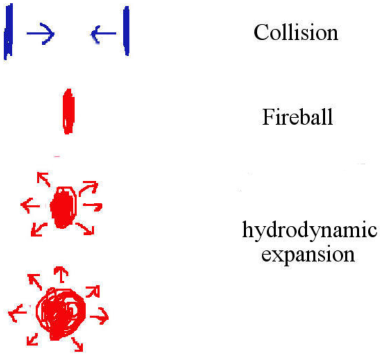

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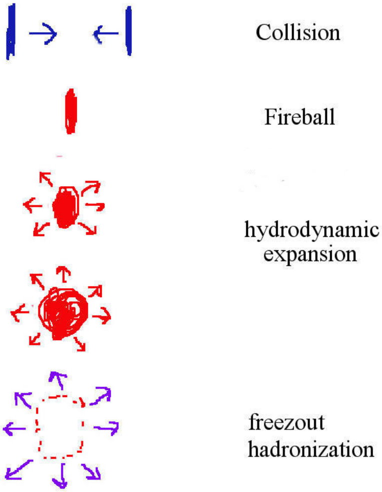

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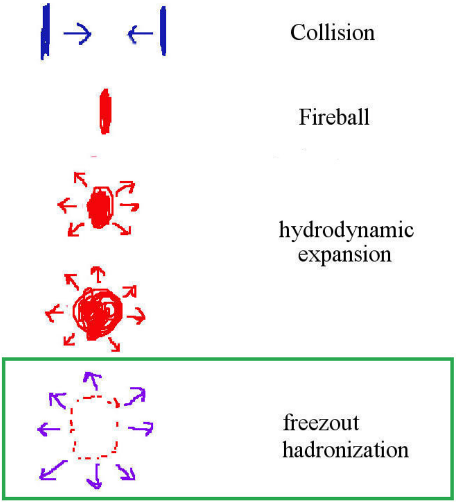

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Understand the AdS/CFT description of a dynamical phase transition... (in Minkowski signature!!)

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The AdS/CFT description of equilibrium phase transitions

- $\mathcal{N} = 4$ SYM on $\mathbb{R}^4$ is a conformal theory — cannot have a phase transition
- $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ can have a phase transition and in fact does have it (TR is a dimensionless quantity).
- In equilibrium we study a field theory at nonzero temperature by compactifying euclidean time on a circle of radius $1/T$
- We thus have to find dual geometries to $\mathcal{N} = 4$ SYM on $S^1 \times S^3$...

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Witten '98

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$$ds^2 = (r^2 + 1)dt^2 + \frac{dr^2}{r^2 + 1} + r^2 d\Omega_3^2$$

2. (Euclidean) AdS black hole

$$ds^2 = \left(r^2 + 1 - \frac{C}{r^2}\right) dt^2 + \frac{dr^2}{r^2 + 1 - \frac{C}{r^2}} + r^2 d\Omega_3^2$$

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- Evaluate the free energies from the gravitational action evaluated on the relevant classical solution
- **Conclusions:** Witten 9803131
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2. The 1^{st} order phase transition occurs when switching between the two saddle points...
3. These are completely distinct 5-dimensional geometries ($\times S^5$)

Question:

What happens in real time??

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Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – some explicitly known (but rather complicated) gravitational backgrounds

2. **Bottom-up approach:** $\leftarrow$ this talk

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to exhibit the physics of interest

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The nonconformal models

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = -12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

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Equilibrium configurations

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- We look for black hole solutions of the form

$$ds^2 = -A(z)dv^2 - \frac{dvdz}{z^2} + S^2(z)dx_i^2 \qquad \phi = \phi(z)$$

- The Eddington-Finkelstein coordinates are very convenient for finding quasinormal modes...
- We can choose the coordinate system so that the horizon is at $z = 0$
- The nonconformality of the theory is ensured by the boundary condition for the scalar field

$$\phi(z) \sim 1 \cdot z^{\#} + \ldots$$

The value 1 defines appropriate units
- We parametrize our solutions by setting the value of the scalar field at the horizon

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Equilibrium configurations

For each value of ϕ_H solve numerically for the black hole geometry

Evaluate observables:

1. Find entropy $S(\phi_H)$ from the area of the horizon
2. Find temperature $T(\phi_H)$ from the euclidean time periodicity
3. Find free energy $F(\phi_H)$ from the on-shell value of the gravitational action (with appropriate counterterms)
4. Find the energy density $E(\phi_H)$ and pressure $p(\phi_H)$ from the near boundary asymptotics of the solution

Scanning through ϕ_H we get various equation of state plots like $S(T)$, $E(T)$ etc.

Equilibrium configurations

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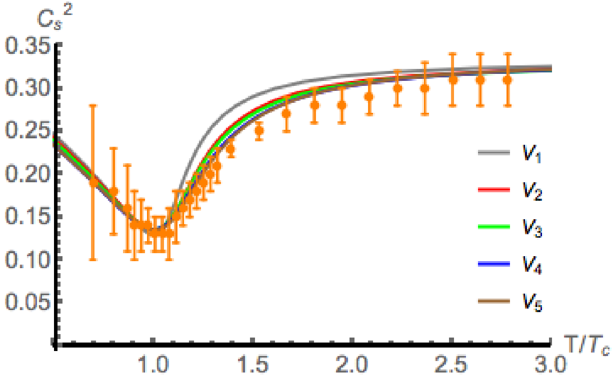

2^{nd} **order phase transition** potential

$$V(\phi) = -12\cosh(0.7071\phi) + 1.958\phi^2$$

This gives the following entropy-temperature $S(T)$ curve with $c_s^2(T_c) = 0$:

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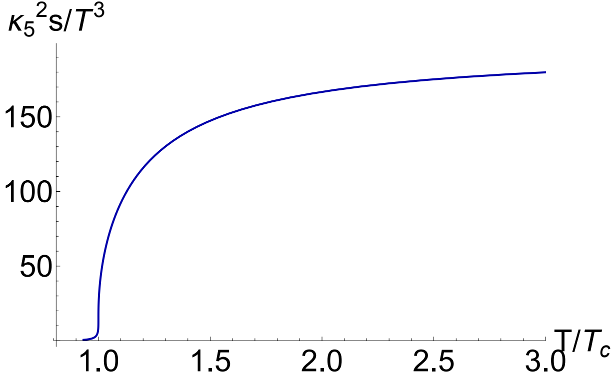

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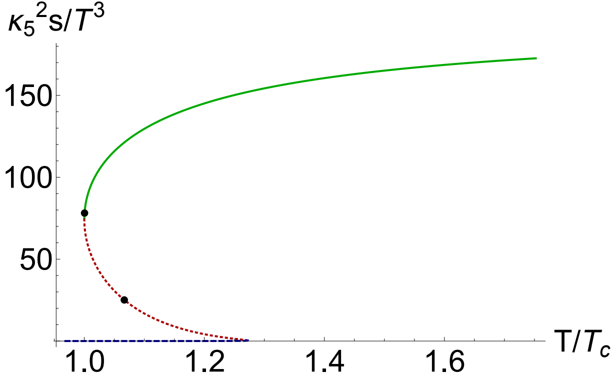

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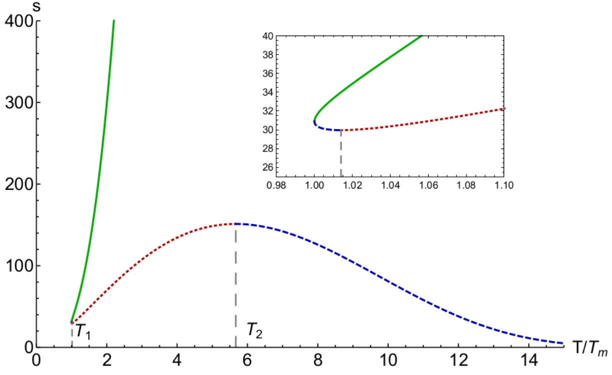

Linearized dynamics

- We are interested in small perturbations of an equilibrium system

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

- We obtain hydrodynamic excitations

$$\omega_{shear} = -i \frac{\eta}{E+p} k^2 + \mathcal{O}\left(k^3\right) \quad \omega_{sound} = c_s k - i \frac{2}{3} \frac{\eta + \frac{3}{4}\zeta}{E+p} k^2 + \mathcal{O}\left(k^3\right)$$

- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$
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- Small disturbances of the uniform static plasma ≡ small perturbations of the black hole metric (≡ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D,black\ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

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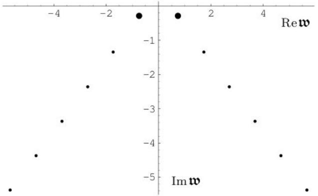

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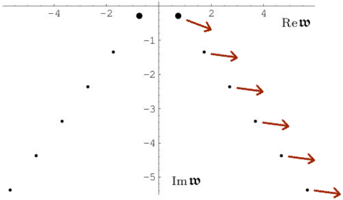

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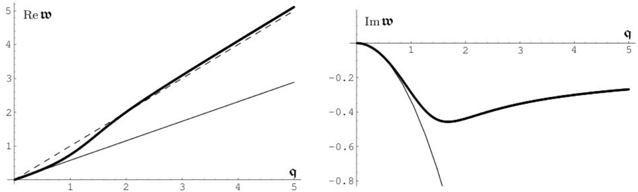

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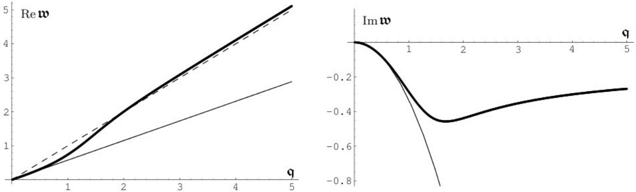

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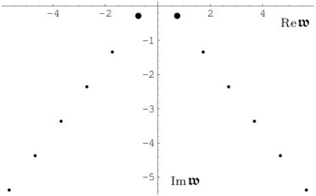

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1. They describe what kind of small excitations propagate on top of a thermal system
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3. ...as well as the **nonhydrodynamic** QNMs whose behaviour is unknown in QCD..

   Lattice QCD??
4. In generic black hole geometries these QNM frequencies (say for $k = 0$) have **comparable real and imaginary parts..** Boltzmann equations/weak coupling lead to purely imaginary frequencies
5. We can study when the hydrodynamic modes are dominant and when the higher modes may be non negligible
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The range of applicability of hydrodynamic excitations

- Once we know the (complex) dispersion relation of all modes we can ask whether for all momenta k, the hydrodynamic modes are less damped than the higher QNM's
- In the **conformal** case in the sound channel this is always the case:

from Kovtun,Starinets hep-th/0506184

- However Amado, Hoyos, Landsteiner, Montero discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$

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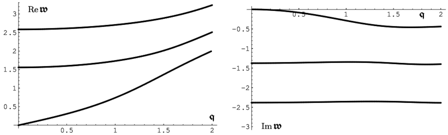

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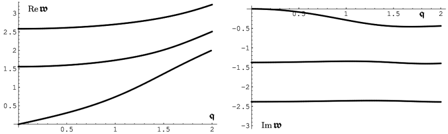

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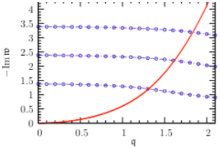
- However Amado, Hoyos, Landsteiner, Montero discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$



from Landsteiner 1202.3550

Selected results

Scalar QNM's – QCD crossover potential

The damping of quasinormal modes decreases by a factor of two around T_c:

- The damping is essentially insensitive to differences in the UV
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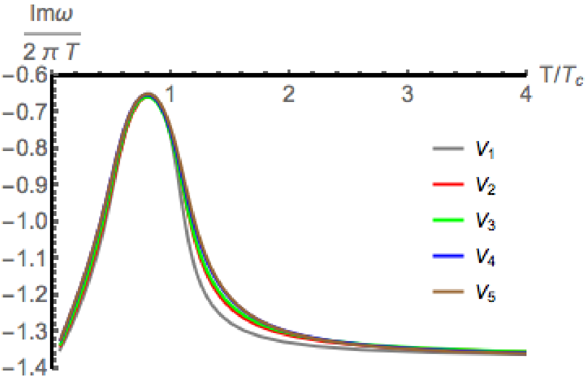

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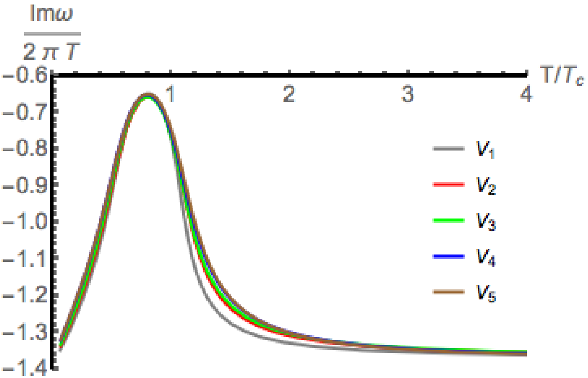

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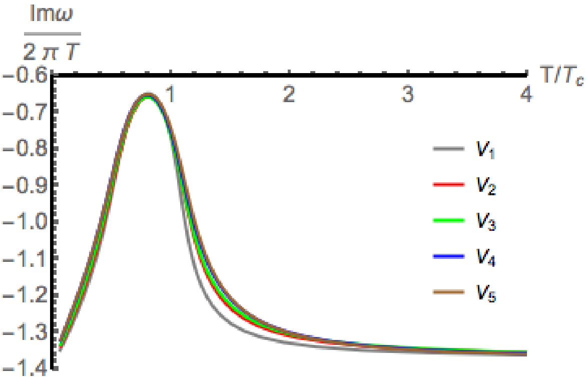

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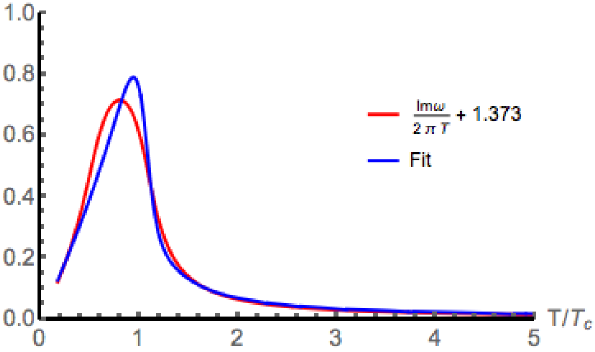

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QCD crossover

2^{nd} **order phase transition**

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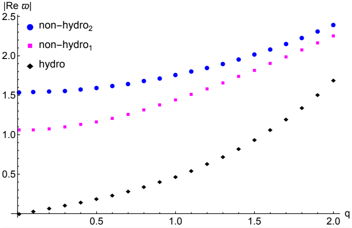

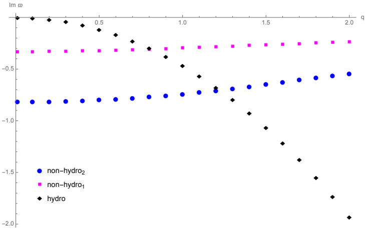

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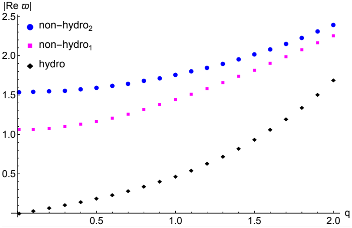

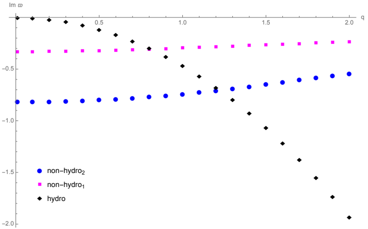

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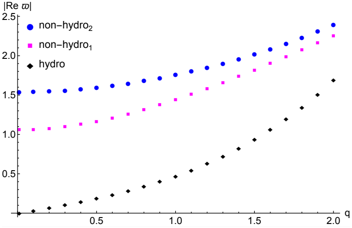

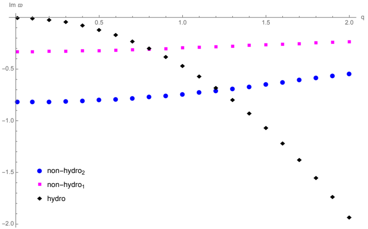

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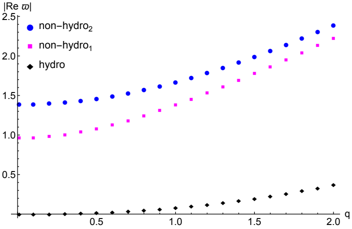

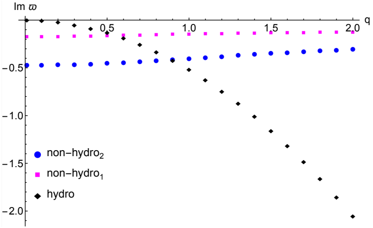

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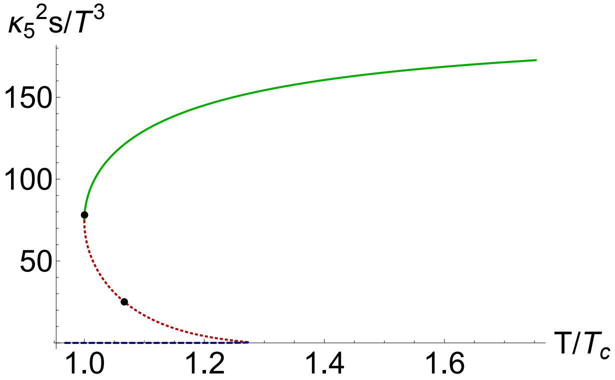

1^{st} **order phase transition** potential

Overcooled branch $T \sim 1.00004\, T_{min}$:

- Speed of sound is very small
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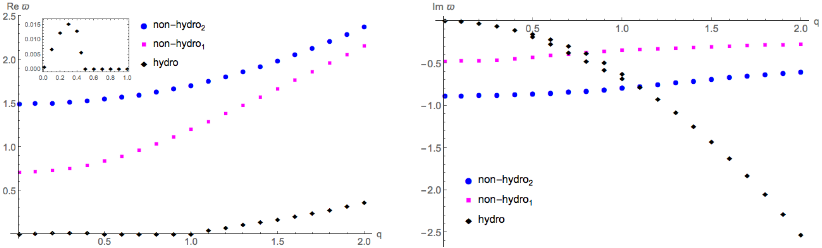

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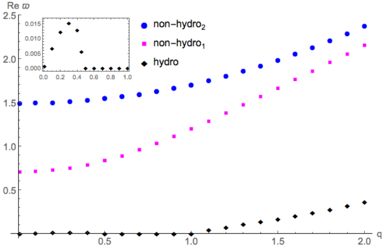

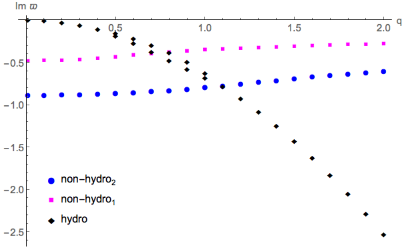

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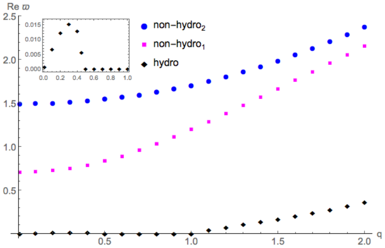

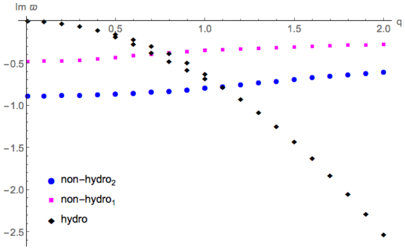

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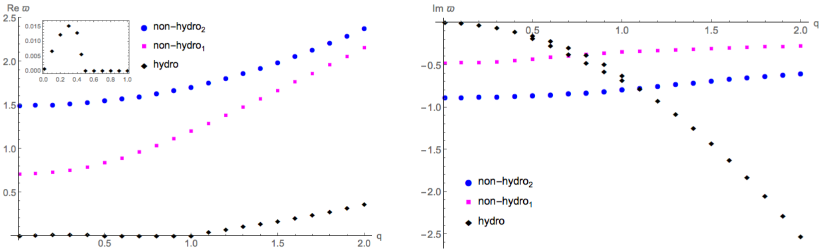

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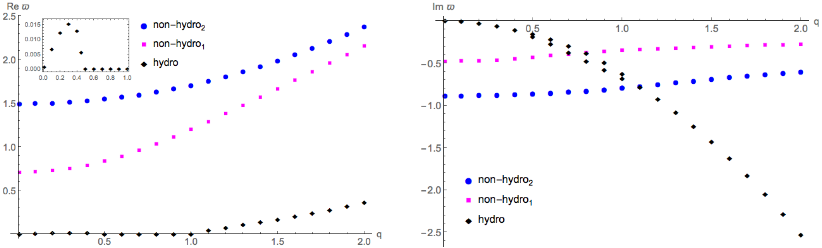

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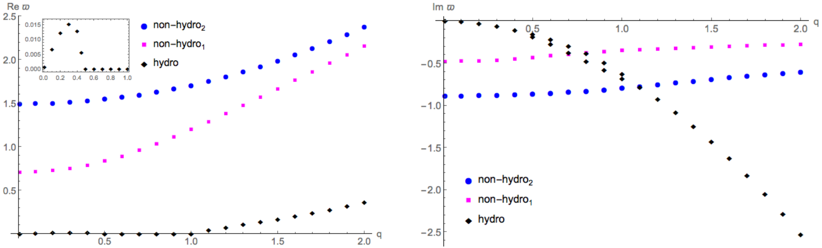

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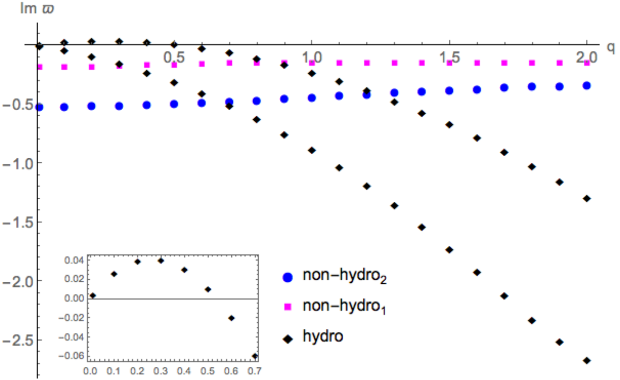

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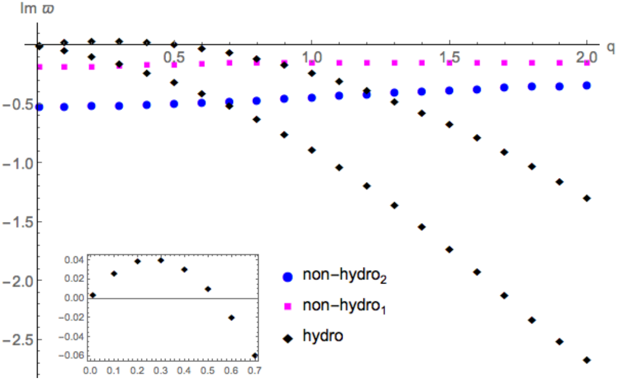

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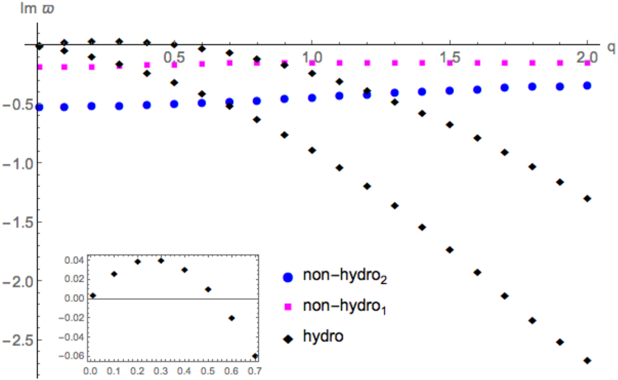

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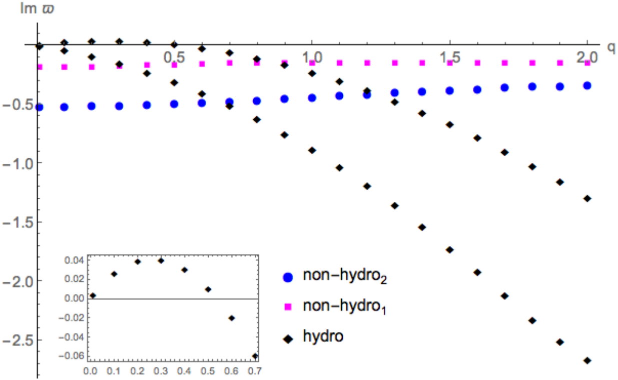

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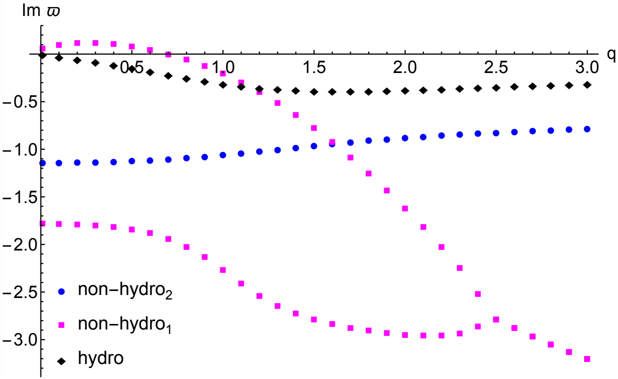

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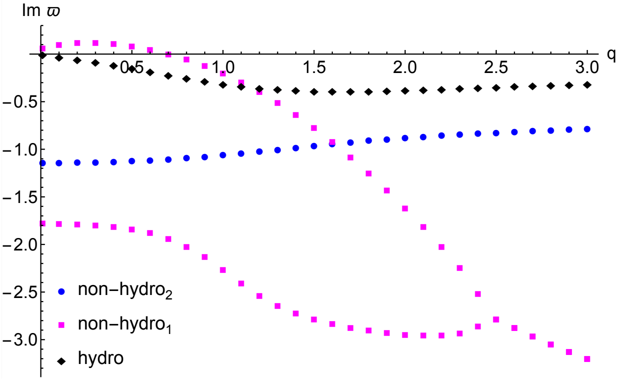

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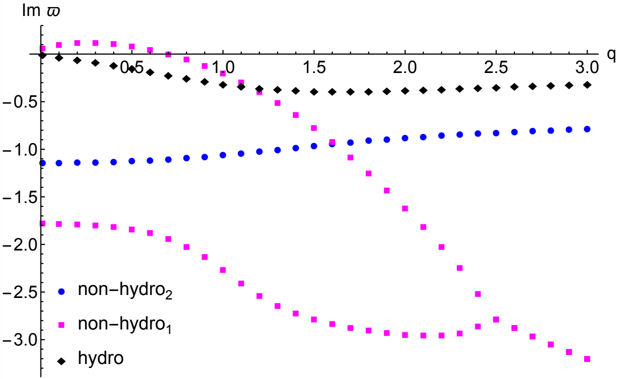

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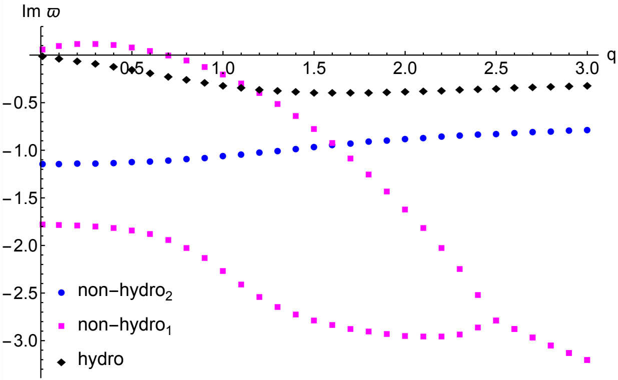

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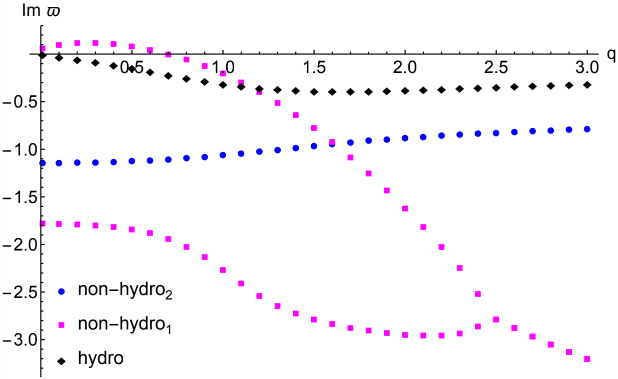

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- Initially we planned looking at a 2D field theory with a 1^{st} order phase transition and 3D dual gravitational description..
- This would have the advantage of not having to make any symmetry assumptions
- However in 3D (and 5D) the metric coefficients develop subleading logarithmic behaviour at the boundary $z = 0$
- This caused huge difficulties with numerical evolution in the inhomogeneous case..
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1^{st} **order phase transition** potential for the 3D case

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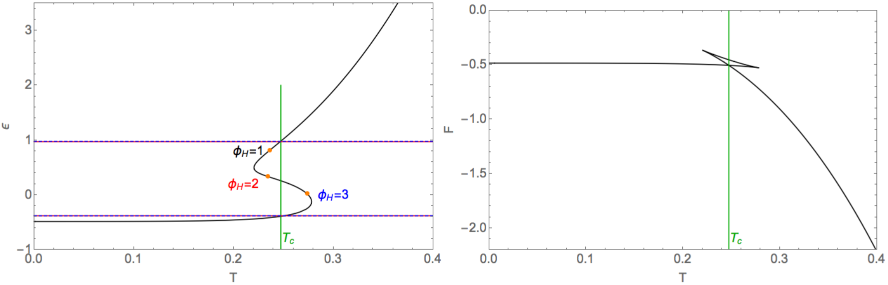

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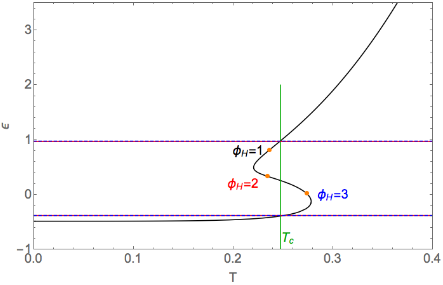

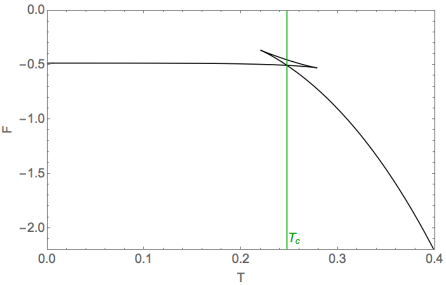

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The functions A, B, S, G and the scalar field ϕ are functions of (v, x, z)

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$\phi_H = 1$ **(overcooled phase)**

- We found no nonlinear instability...
- We tried choosing also other initial overcooled configurations on the line of linear stability with the same conclusions..

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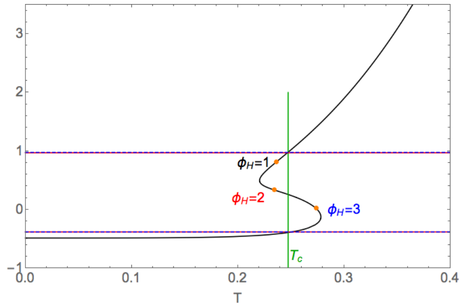

What is the endpoint of spinoidal instability?

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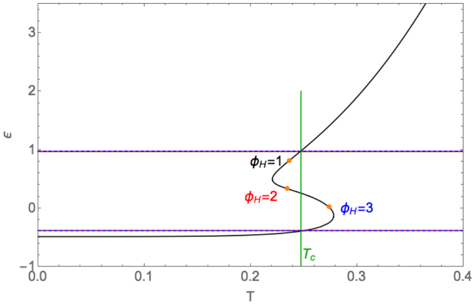

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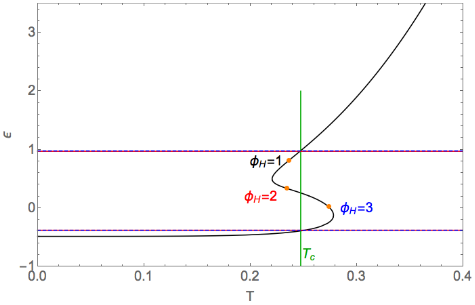

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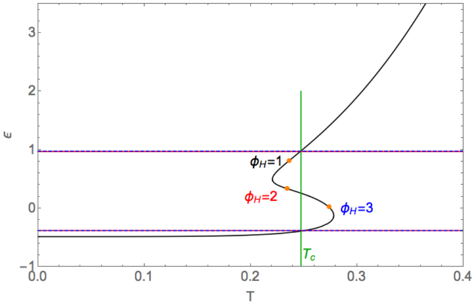

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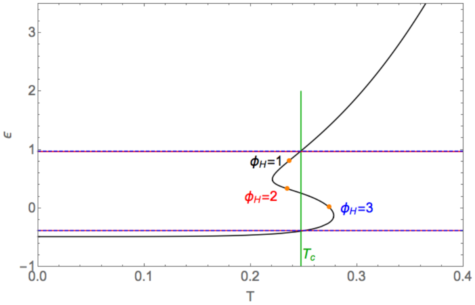

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$\phi_H = 2$ **(spinoidal branch)**

Energy density and $\langle O_\phi \rangle$ as a function of t and x

Clear spatially constant regions separated by domain walls...

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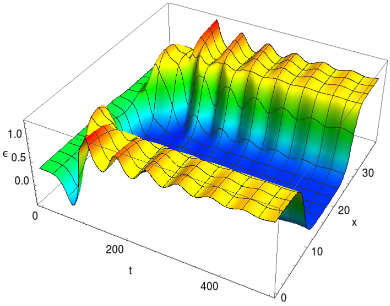

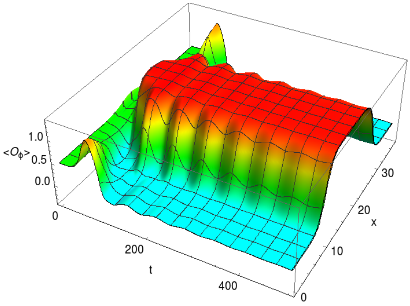

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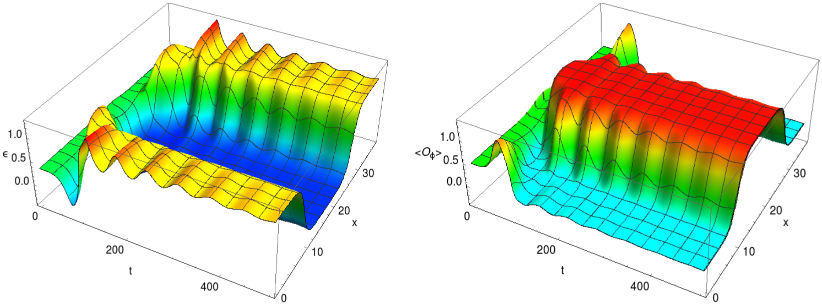

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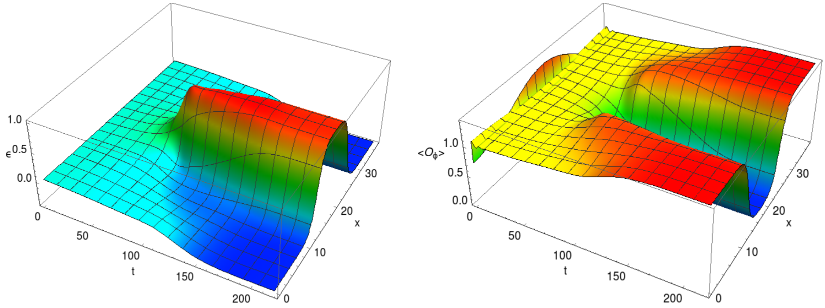

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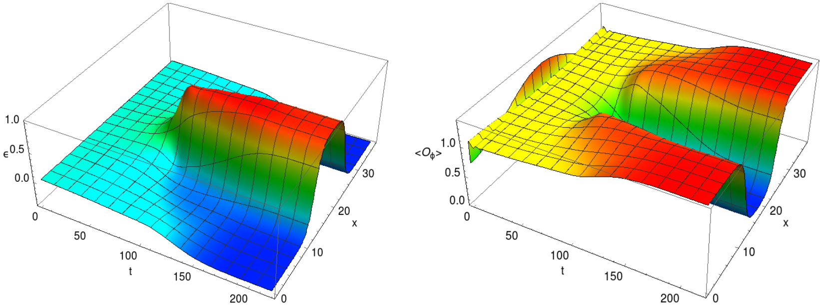

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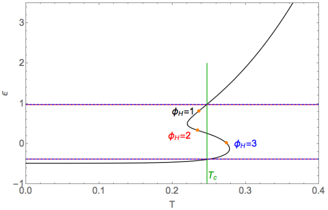

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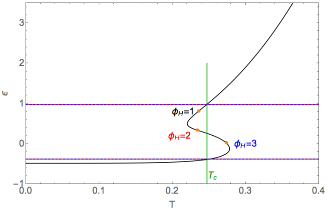

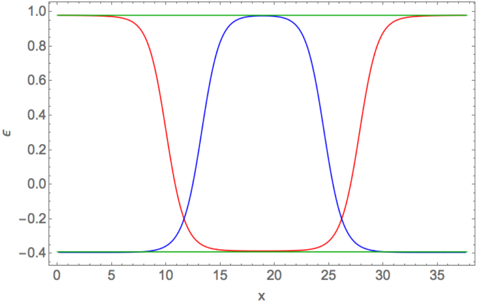

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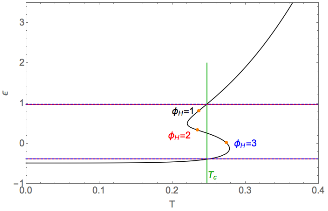

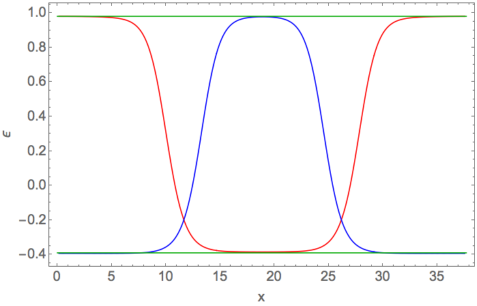

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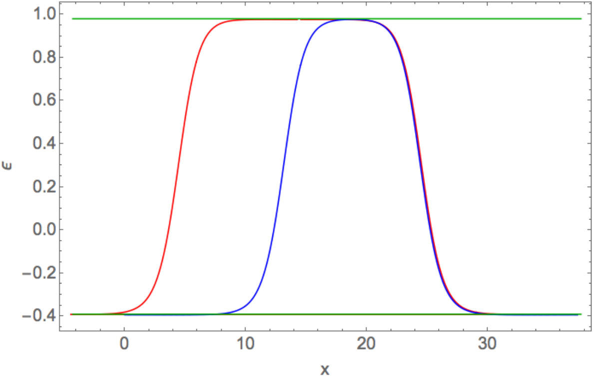

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Directions for future research:

- Boost-invariant setup
- Collisions of domains/bubbles
- Setups with non black hole phases
- Effective description of domain boundaries

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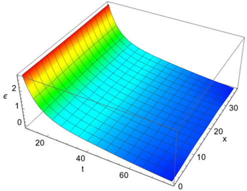

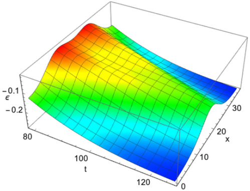

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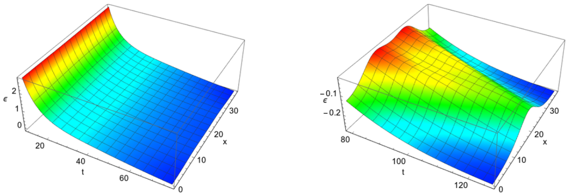

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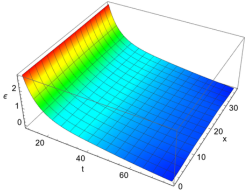

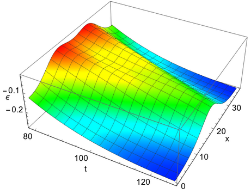

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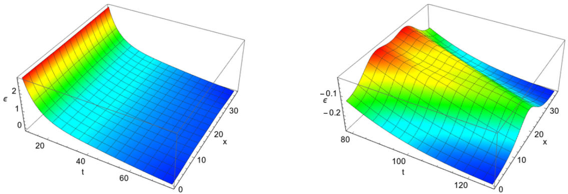

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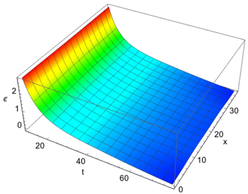

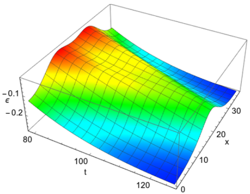

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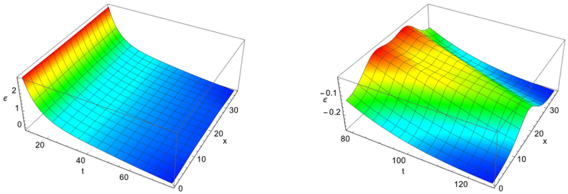
Directions for future research:

- Boost-invariant setup



- Collisions of domains/bubbles
- Setups with non black hole phases
- Effective description of domain boundaries

  c.f. Attems, Bea, Casalderrey-Solana, Mateos, Triana, Zilhao
- Setup with conserved charges
- Less symmetry/higher # of dimensions