Jet evolution in a quark-gluon plasma at weak coupling

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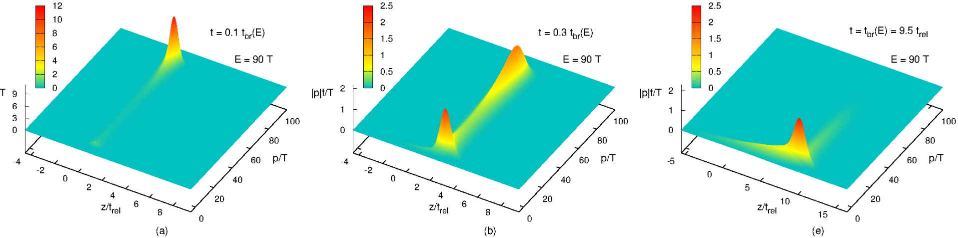
a tale written together with my collaborators (since 2011)

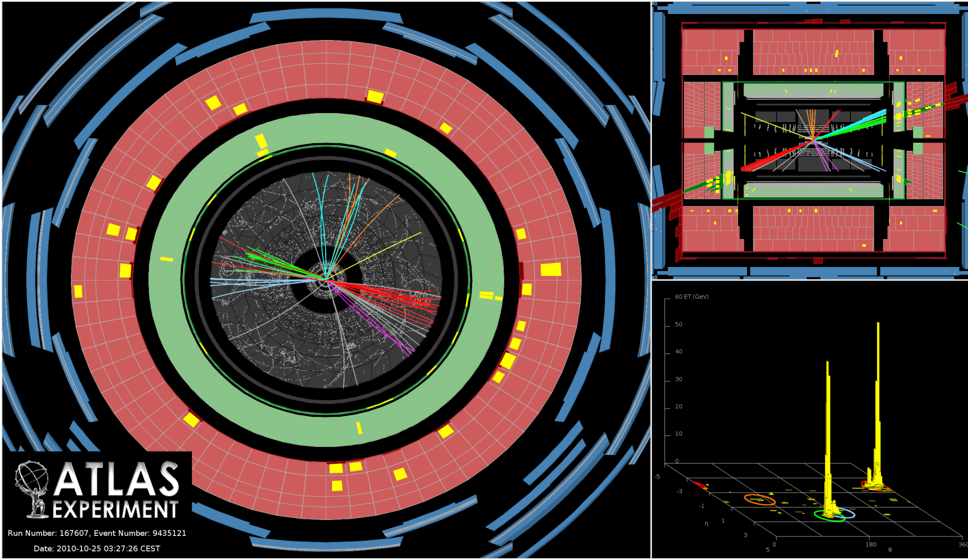
J.-P. Blaizot, F. Dominguez, M. Escobedo, Y. Mehtar-Tani, B. Wu,
and J. Casalderrey-Solana



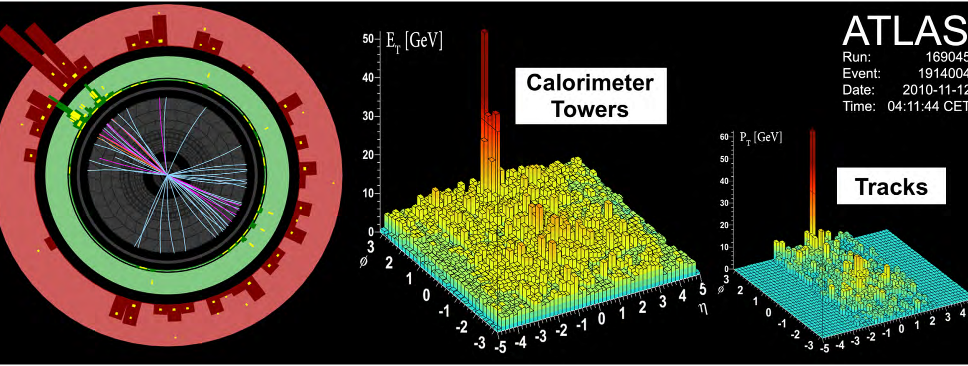
Outline

- Motivation: di-jet asymmetry at the LHC
- Medium-induced radiation: BDMPS-Z
- Multiple branching: physical discussion
- Average gluon distribution & energy loss
- Correlations & fluctuations
- Gluon multiplicities: KNO scaling
- Thermalization of mini-jets

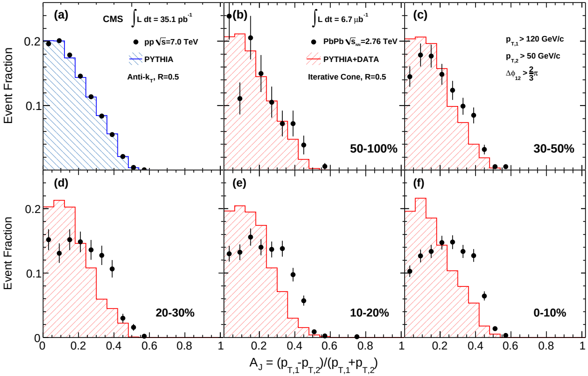
From di–jets in p+p collisions ...



… to mono-jets in Pb+Pb collisions

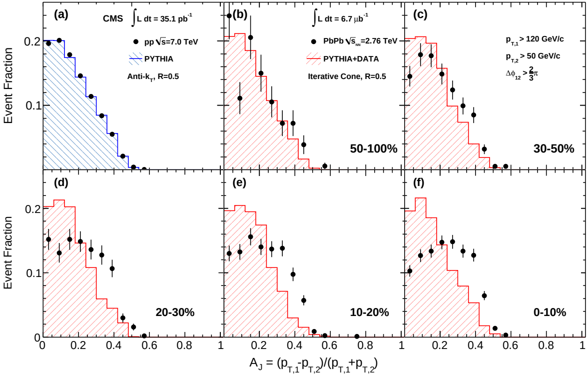


- Central Pb+Pb: 'mono–jet' events
- The secondary jet can barely be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

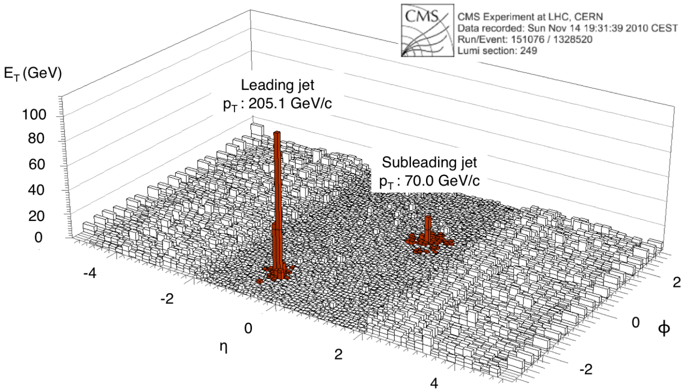


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

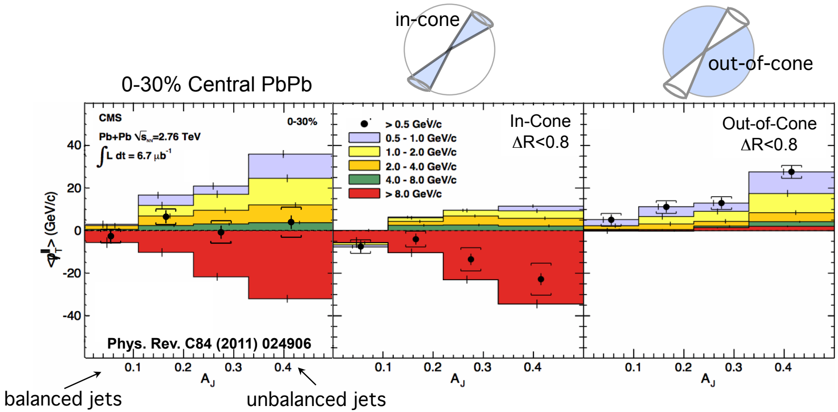
$$A_{\mathrm{J}} = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$



- N.B. A pronounced asymmetry already in p+p collisions !
  - 3-jets events, fluctuations in the branching process
- Central Pb+Pb : the asymmetric events occur more often



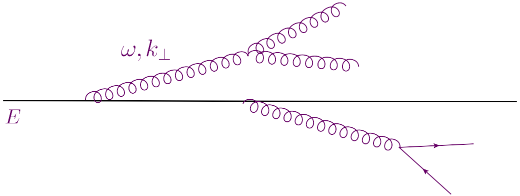
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- The 'missing energy' is actually found in the underlying event:
  - many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles



- No missing energy : $E_{\rm Lead}^{\rm in+out} = E_{\rm SubLead}^{\rm in+out}$
- In-Cone : $E_{\rm Lead}^{\rm in} > E_{\rm SubLead}^{\rm in}$: di-jet asymmetry, hard particles
- Out-of-Cone : $E_{\rm Lead}^{\rm out} < E_{\rm SubLead}^{\rm out}$: soft hadrons @ large angles

A challenge for the theorists

- Very different from the usual jet fragmentation pattern in the vacuum
  - bremsstrahlung favors collinear splittings ⇒ jets are collimated
  - many soft gluons ... but energy remains in the few partons at large x



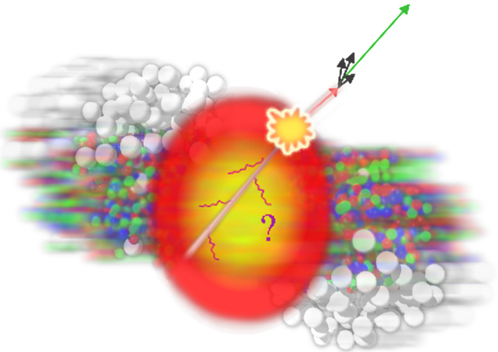
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}x}{x} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

- Soft hadrons can be easily deviated towards large angles
  - elastic scatterings with the medium constituents

- The main question: how is that possible that a significant fraction of the jet energy be carried by its soft constituents ?

The generally expected picture

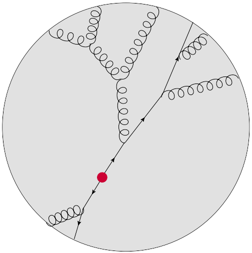
- "One jet crosses the medium along a distance longer than the other"



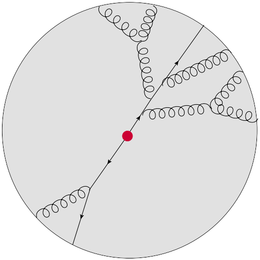
- Implicit assumption: fluctuations in energy loss are small
  - "the energy loss is always the same for a fixed medium size"
- Fluctuations are known to be important for a branching process

The role of fluctuations

- Different path lengths



- Fluctuations in the branching pattern



- Fluctuations in the energy loss are as large as the average value

  (M. Escobedo and E.I., arXiv:1601.03629 & 1609.06104)

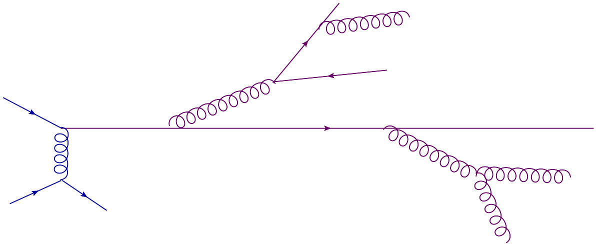
- Similar conclusion independently reached by a Monte-Carlo study

  (Milhano and Zapp, arXiv:1512.08107, "JEWEL")

- One needs a better understanding of the in-medium jet dynamics

Medium-induced jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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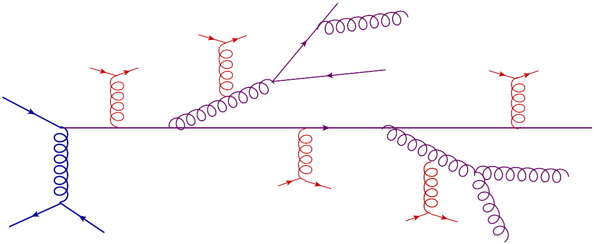

- ... and via collisions off the medium constituents

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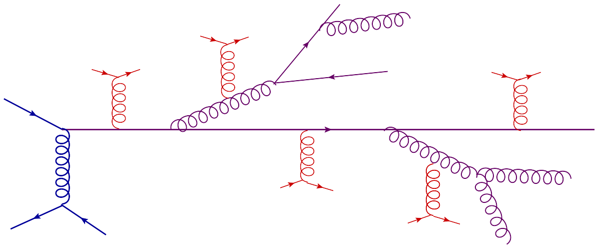

- ... and via collisions off the medium constituents
- Collisions can have several effects
  - broaden the p_T–distribution of the jet constituents
  - trigger additional radiation ('medium-induced branching')
  - thermalize the (soft) products of this radiation

Medium-induced jet evolution

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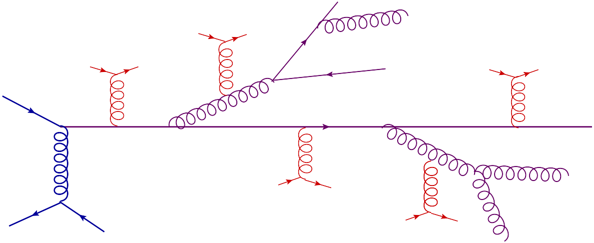


- ... and via collisions off the medium constituents
- BDMPS–Z mechanism for medium-induced radiation in pQCD
  Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov (1996-97)
  Wiedemann (2000), "Bottom-up" (2001), Arnold, Moore, Yaffe (2002-03) ...
  - gluon emission is linked to transverse momentum broadening

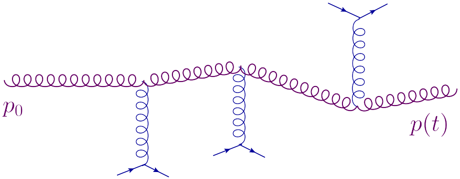
Transverse momentum broadening

- In a weakly-coupled QGP, mean free path $\gg$ Debye screening length

$$\lambda_{\rm mfp} \sim \frac{1}{\alpha_s T \ln(1/\alpha_s)} \gg \lambda_D \sim \frac{1}{\sqrt{\alpha_s}\, T}$$

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$

- $\langle p_\perp^2 \rangle \simeq \hat{q}\, \Delta t$



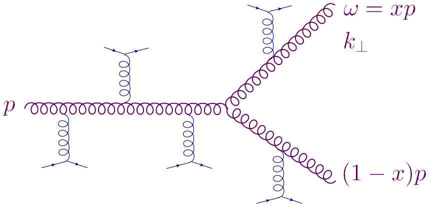
- Jet quenching parameter $\hat{q}$: transport coefficient for p_T–diffusion

$$\hat{q} \simeq \frac{m_D^2}{\lambda_{\rm mfp}} \sim \alpha_s^2 T^3 \ln\frac{1}{\alpha_s}$$

- An average value (theory and data): $\hat{q} \simeq 1 \div 2\,{\rm GeV}^2/{\rm fm}$

Formation time

- Collisions destroy quantum coherence and thus trigger emissions



transverse separation larger than transverse wavelength

$$\frac{k_\perp}{\omega}\,\Delta t \;\gtrsim\; \lambda_\perp \simeq \frac{1}{k_\perp}$$

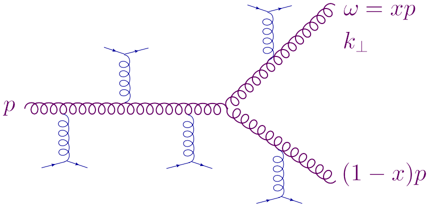
- The transverse momentum can be given by the medium: $k_\perp^2 \sim \hat{q} t_{\rm f}$

$$t_{\rm f} \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q} t_{\rm f} \quad \Longrightarrow \quad t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Implicit assumptions: $\lambda_{\rm mfp} \ll t_{\rm f}(\omega) < L$

$$T \lesssim \omega \leq \omega_c \equiv \hat{q} L^2$$

- Collisions destroy quantum coherence and thus trigger emissions



formation time

$$t_{\rm f} \simeq \frac{1}{\Delta E} \simeq \frac{\omega}{k_\perp^2}$$

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$$t_{\rm f} \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q} t_{\rm f} \quad \Longrightarrow \quad t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Implicit assumptions: $\lambda_{\rm mfp} \ll t_{\rm f}(\omega) < L$

$$T \lesssim \omega \leq \omega_c \equiv \hat{q} L^2$$

Emission angles

$$k_\perp^2 \simeq \hat{q} t_{\rm f} \simeq \sqrt{\hat{q}\omega} \Longrightarrow \theta_{\rm f}(\omega) \simeq \frac{k_\perp}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

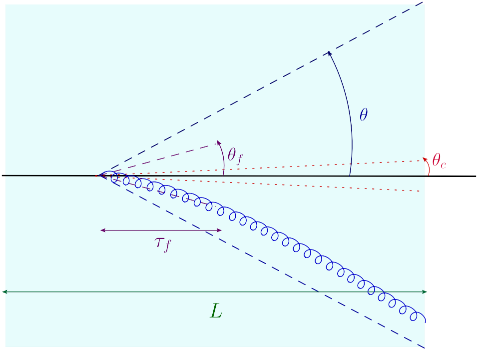
- $k_\perp$ can further increase after the emission, up to a final value $k_\perp^2 \sim \hat{q}L$
- Hard gluons $(\omega \sim \omega_c)$ dominate the energy loss by the leading particle

- large formation times:

$$t_{\rm f}(\omega_c) \simeq L$$

- rare events: prob $\sim \alpha_s$
- propagate at small angles

$$\theta(\omega_c) \simeq \frac{1}{\sqrt{\hat{q}L^3}} \lesssim 0.1$$

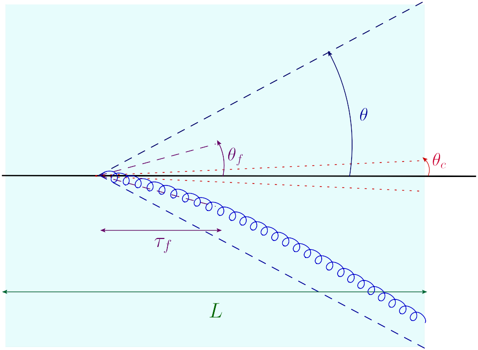


- Irrelevant for the energy loss by the jet

Emission angles

$$k_\perp^2 \simeq \hat{q} t_{\rm f} \simeq \sqrt{\hat{q}\omega} \Longrightarrow \theta_{\rm f}(\omega) \simeq \frac{k_\perp}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- $k_\perp$ can further increase after the emission, up to a final value $k_\perp^2 \sim \hat{q}L$
- Soft gluons $(\omega \ll \omega_c)$ have ...



- small formation times:

$$t_{\rm f}(\omega) \ll L$$

- ... and large production angles:

$$\theta(\omega) \simeq \frac{k_\perp}{\omega} \simeq \frac{\sqrt{\hat{q}L}}{\omega}$$

- promising for dijet asymmetry

- For them, multiple branching becomes important

Multiple branchings

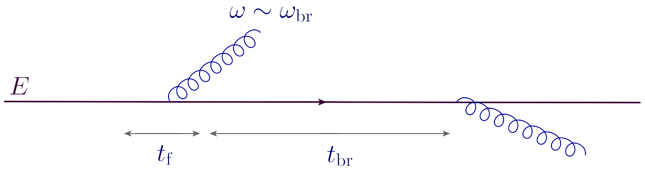
- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

- When $\mathcal{P}(\omega, L) \sim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br}(L) \equiv \alpha_s^2 \hat{q} L^2 \quad \Longleftrightarrow \quad L \gtrsim t_{\rm br}(\omega) \equiv \frac{1}{\alpha_s} t_{\rm f}(\omega)$$

- LHC: the leading particle has $E \sim 100\,{\rm GeV} \gg \omega_{\rm br} \sim 5 \div 10\,{\rm GeV}$



- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$

Multiple branchings

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$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

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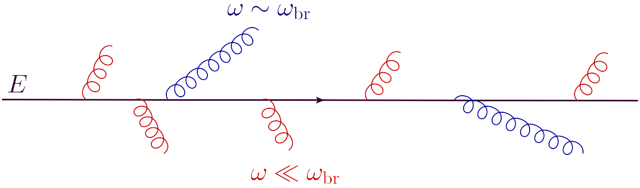

- In a typical event, the LP emits ...
  - a large number of softer gluons with $\omega \ll \omega_{\rm br}$

Multiple branchings

- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

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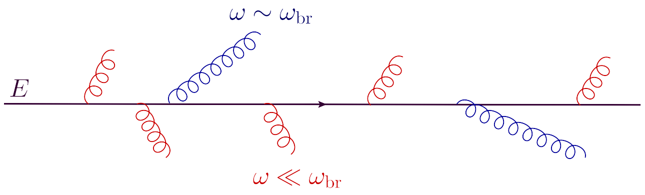
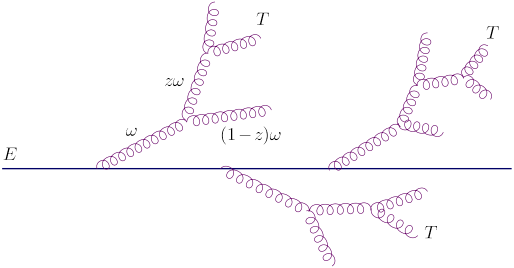


- The energy loss is controlled by the hardest primary emissions

Democratic branchings

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- The primary gluons generate 'mini-jets' via democratic branchings
  - daughter gluons carry comparable energy fractions: $z \sim 1 - z \sim 1/2$

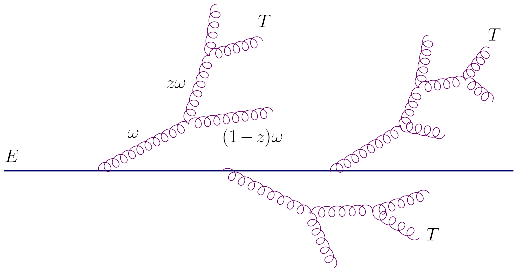


$$\mathcal{P}(z\omega, L) \simeq \frac{L}{t_{\rm br}(z\omega)} \simeq \alpha_s\, L \sqrt{\frac{\hat{q}}{z\omega}}$$

- when $\omega \sim \omega_{\rm br}$, $\mathcal{P}(z\omega, L) \sim 1$ independently of the value of z
- democratic branchings are most efficient in redistributing the energy

Energy loss via democratic cascades

- A mini-jet with $\omega \lesssim \omega_{\rm br}$ decays over a time $t_{\rm br}(\omega) \lesssim L$
- Via democratic branchings, the energy is successively transmitted to softer and softer gluons, down to $\omega \sim T$
- The soft gluons thermalize via elastic collisions
- The energy appears in many soft quanta propagating at large angles



- What is the average energy loss and its fluctuations ?

Probabilistic picture

- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - successive branchings are non-overlapping: $t_{\rm br} \sim \frac{1}{\alpha_s}\, t_{\rm f}$
  - interference phenomena could complicate the picture ...

    (in the vacuum, they lead to angular ordering)
  - ... but they are suppressed by rescattering in the medium

    Casalderrey-Solana, E.I. (2011);
    Blaizot, Dominguez, E.I., Mehtar-Tani (2012)
    Arnold, Iqbal (since 2015); cf. Peter's earlier talk today

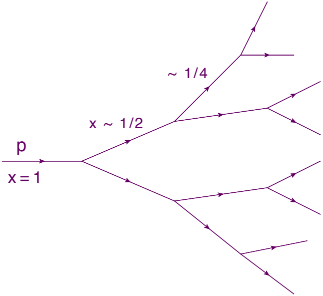
- Hierarchy of equations for n-point correlation functions $(x \equiv \omega/E)$

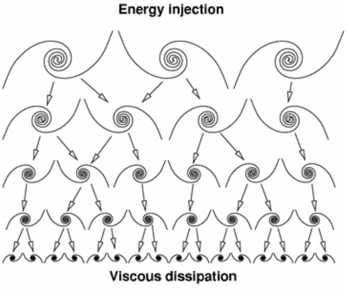
$$D(x,t) \equiv x \left\langle \frac{{\rm d}N}{{\rm d}x}(t) \right\rangle, \qquad D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{{\rm d}N_{\rm pair}}{{\rm d}x\,{\rm d}x'}(t) \right\rangle$$

- Analytic solutions *(Blaizot, E. I., Mehtar-Tani, '13; Escobedo, E.I., '16)*

- New phenomena: wave turbulence, KNO scaling, large fluctuations

- Democratic branchings lead to wave turbulence
  - energy flows from one parton generation to the next one, at a rate which is independent of the generation
  - it eventually dissipates into the medium, via thermalization
  - mathematically: a fixed point $D(x) = \frac{1}{\sqrt{x}}$ (Kolmogorov spectrum)

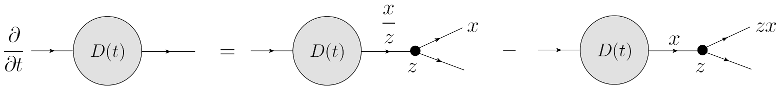




Gluon spectrum: the average energy loss

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- Kinetic equation for $D(x,t) = x(\mathrm{d}N/\mathrm{d}x)$: 'gain' - 'loss'



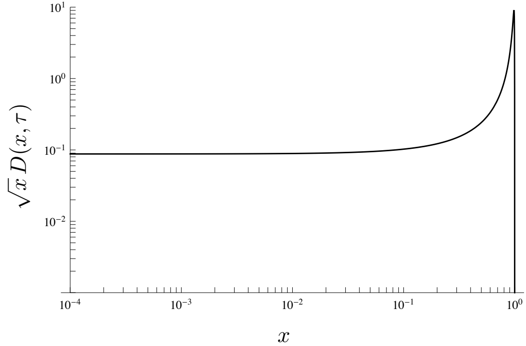
- Simplified version of the AMY and "Bottom-up" kinetic equations
  - no elastic collisions, just branchings
  - simplified branching kernels: poles at $z = 0$ & $z = 1$
- Exact solution with initial condition $D(x, t = 0) = \delta(x - 1)$

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi\frac{\tau^2}{1-x}}, \qquad x \equiv \frac{\omega}{E}, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- $t_{\rm br}(E)$: the lifetime of the LP until its first democratic branching

Gluon spectrum

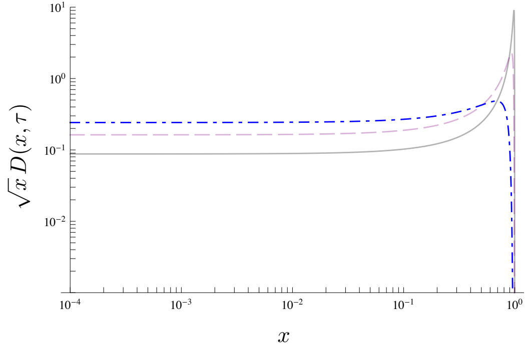
$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\; \mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$



$\tau = 0.1$

- Pronounced LP peak at small times

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\,\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

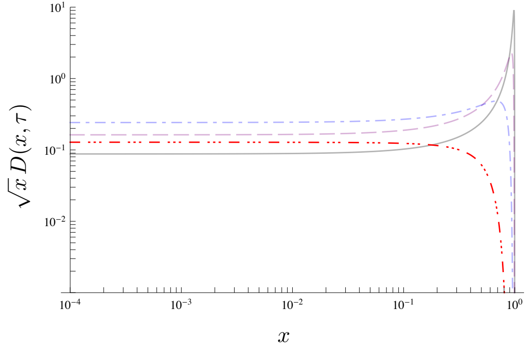


$\tau = 0.1,\ 0.2,\ 0.4$

- Increasing t: the LP peaks decreases, broadens, and moves to the left

Gluon spectrum

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,,\quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

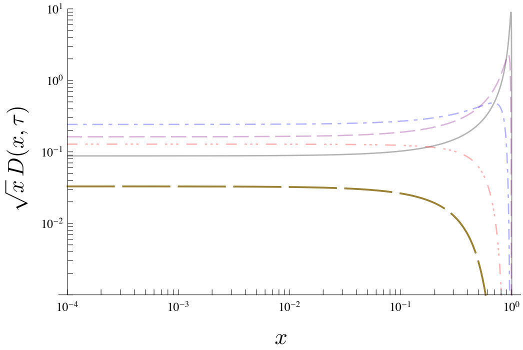


$\tau = 0.1,$ 0.2, 0.4, 0.8

- $\tau \sim 1$: the LP disappears via a democratic branching

Gluon spectrum

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x\equiv\frac{\omega}{E}\,,\quad \tau\equiv\frac{t}{t_{\rm br}(E)}$$

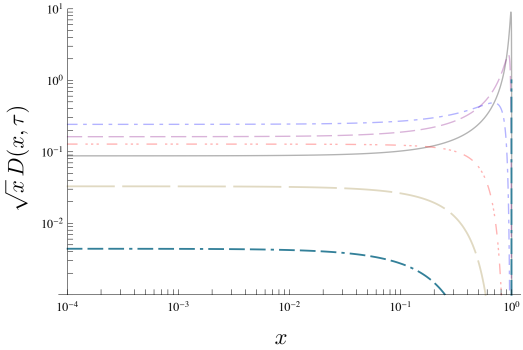


$\tau = 0.1$, 0.2, 0.4, 0.8, 1

- The shape at small x is not changing: genuine fixed point

Gluon spectrum

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\,\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$



$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8,\ 1,\ 1.2$

- The energy flows out of the spectrum: $\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2}$

The average energy loss

- Formally, it accumulates into a condensate at $x = 0$
- Physically, it is transmitted to the medium, via thermalization

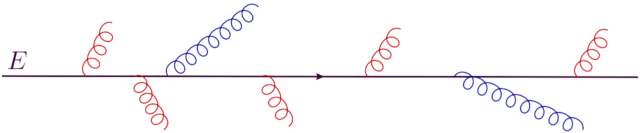
$$\langle \Delta E \rangle = E\big(1 - \mathrm{e}^{-\pi\tau^2}\big) = E\left[1 - \mathrm{e}^{-\pi\frac{\omega_{\rm br}}{E}}\right]$$

- LHC: $E \sim 100\,\mathrm{GeV} \gg \omega_{\rm br} \sim 5 \div 10\,\mathrm{GeV}$

$$\langle \Delta E \rangle \simeq \pi\omega_{\rm br} = \pi\alpha_s^2 \hat{q} L^2$$

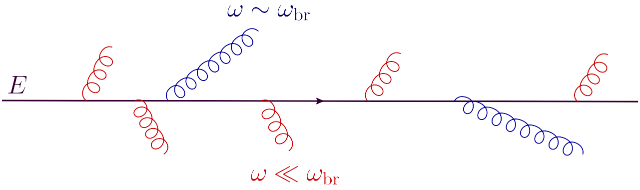
- Consistent with our general physical picture:

  ▷ energy loss is controlled by the primary emissions with $\omega \sim \omega_{\rm br}$



Fluctuations in the energy loss

- Recall: the probability for a primary emission with $\omega \sim \omega_{\rm br}$ is of $\mathcal{O}(1)$



  - the average number of such emissions is of $\mathcal{O}(1)$ (indeed, it is π)
  - successive such emissions are quasi-independent ($E \gg \omega_{\rm br}$)

- Fluctuations in the number of such emissions must be of $\mathcal{O}(1)$ as well
- The fluctuations in the energy loss are expected to be comparable with the average value: $\sigma \sim \langle \Delta E \rangle \sim \omega_{\rm br}$
- Confirmed by exact calculations *(M. Escobedo and E. I., 2016)*

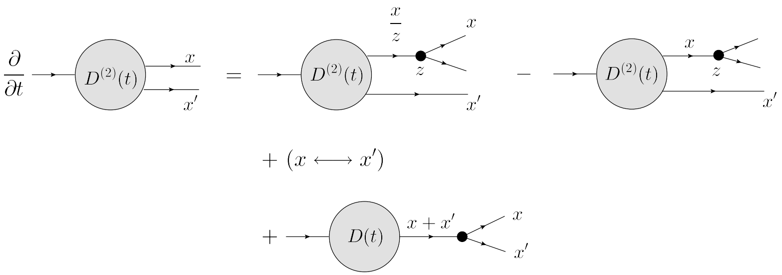
Correlations & fluctuations

M.A. Escobedo and E. I., arXiv:1601.03629, arXiv:1609.06104

- The variance is related to the density $D^{(2)}(x,x',t)$ of gluon pairs:

$$D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}x\,\mathrm{d}x'}(t) \right\rangle$$

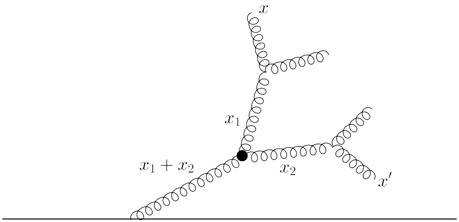
- Kinetic equation for $D^{(2)}(x,x',t)$: correlations due to common ancestors



- The 1-body density $D(x+x',t)$ acts as a source for the 2-body density

The gluon pair density

- The 2 measured gluons x and x' have a last common ancestor (LCA) $x_1 + x_2$



$$D^{(2)}(x, x', \tau) = \frac{1}{2\pi} \frac{1}{\sqrt{xx'(1-x-x')}} \left[\mathrm{e}^{-\frac{\pi\tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi\tau^2}{1-x-x'}} \right]$$

  - 1st term: the splitting of the LCA occurs at late times $\tau' \sim \tau$
  - 2nd term: the splitting of the LCA occurs at early times $\tau' \sim 0$
  - first term dominates at large measurement time $\tau \gtrsim 1$

- All the n-body correlations $D^{(n)}$ have been similarly computed

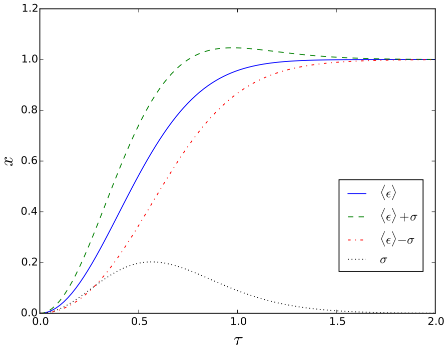
Dispersion in the energy loss

$$\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2 \ : \text{ involves } \int \mathrm{d}x \int \mathrm{d}x' D^{(2)}(x, x', \tau)$$

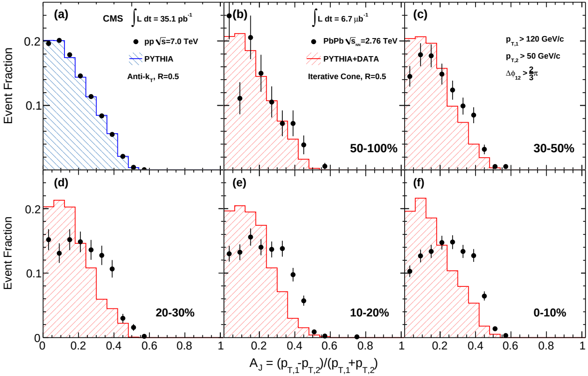
- Small time/high energy $E \gg \omega_{\rm br}$ (LHC) : large fluctuations

$$\sigma^2 \simeq \frac{\pi^2}{3}\,\omega_{\rm br}^2 = \frac{1}{3}\langle \Delta E \rangle^2$$

- Fluctuations die away at large times when $\langle \Delta E \rangle \simeq E$.



Di–jet asymmetry from fluctuations

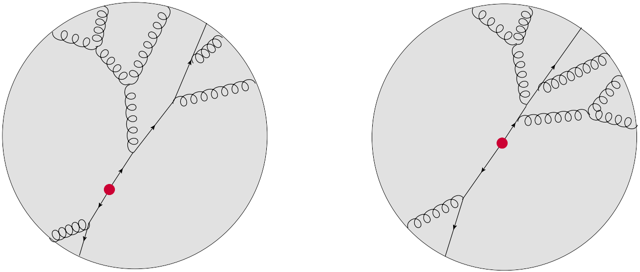


- Event fraction as a function of the di–jet energy imbalance

$$A_{\rm J} = \frac{|E_1 - E_2|}{E_1 + E_2}$$

- Fluctuations cannot cancel since $A_{\rm J}$ is positive-definite, by construction

Di-jet asymmetry from fluctuations



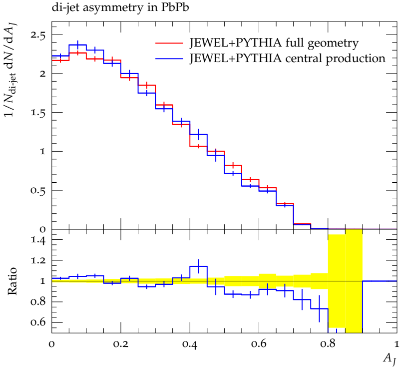
- A relatively large value A_{J} can either correspond to a peripheral di-jet, or (more often) to a large fluctuation in the branching pattern

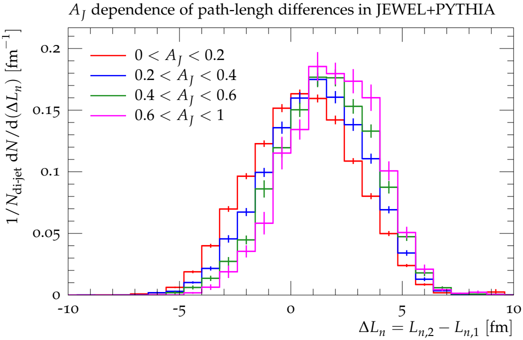
$$\langle (E_1 - E_2)^2 \rangle - \langle E_1 - E_2 \rangle^2 = \sigma_1^2 + \sigma_2^2 \propto \langle L_1^4 + L_2^4 \rangle$$

- Fluctuations dominate whenever $L_1 \sim L_2$ (the typical situation)

- Difficult to check: no direct experimental control of L_1 and L_2

(Milhano and Zapp, arXiv:1512.08107)





- Left: Central production $(L_1 = L_2)$ vs. randomly distributed production points ("full geometry")
- Right: Distribution of $\Delta L \equiv L_1 - L_2$ for different classes of A_J
  - the width of the distribution is a measure of fluctuations

Particle multiplicities

- The average multiplicities and their fluctuations are dominated by very soft gluons :

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{1}{\omega} D(\omega) \propto \frac{1}{\omega^{3/2}}$$

- Number of gluons with $\omega \geq \omega_0$, where $\omega_0 \ll E$:

$$\langle N(\omega_0)\rangle = \int_{\omega_0}^{E} \mathrm{d}\omega \, \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq 1 + 2\left[\frac{\omega_{\rm br}}{\omega_0}\right]^{1/2} \quad \text{(LP + radiation)}$$

  - $\langle N(\omega_0)\rangle \simeq 1$ when $\omega_0 \gg \omega_{\rm br}$: just the LP
  - $\langle N(\omega_0)\rangle \gg 1$ when $\omega_0 \ll \omega_{\rm br}$: multiple branching

- All the higher moments $\langle N^p\rangle$ have been similarly computed

Koba-Nielsen-Olesen scaling

- All the higher moments $\langle N^p \rangle$ have been similarly computed

$$\frac{\langle N^2 \rangle}{\langle N \rangle^2} \simeq \frac{3}{2}\,, \qquad \frac{\langle N^p \rangle}{\langle N \rangle^p} \simeq \frac{(p+1)!}{2^p}$$

- KNO scaling : the reduced moments are pure numbers
- A special negative binomial distribution (parameter $r = 2$)
  - huge fluctuations (say, as compared to a Poissonian distribution)

$$\frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{\langle N \rangle}}$$

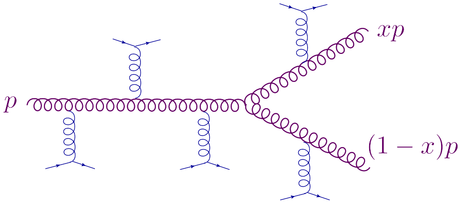
  - fluctuations are stronger than for jets in the vacuum ($r = 3$)

$$\frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{3}}$$

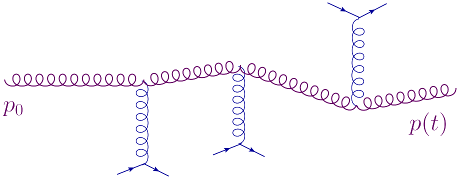
- Difficult to check against the data: huge backgrounds at soft energies

Thermalization of the mini-jets

- So far, elastic collisions used only to trigger emissions



- When $\omega \sim T$, they also matter for the energy-momentum redistribution (between the jet and the medium) : 'drag' & 'diffusion'



$$\langle p_\perp^2(t)\rangle \simeq \hat{q}\,t$$

$$\langle p_z(t)\rangle \simeq p_0 - \eta t$$

$$\langle \Delta p_z^2(t)\rangle \simeq \hat{q}_\ell t$$

Adding elastic collisions

- When $\omega \sim T$, branchings and elastic collisions compete with each other
  - the democratic branching time becomes comparable with the relaxation time towards thermal equilibrium

$$t_{\rm br}(\omega) \simeq \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}} \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)} \simeq t_{\rm rel} \quad \text{when } \omega \sim T$$

  - $t_{\rm rel}$: mean free path for large angle scattering

- When $\omega \sim T$, occupation numbers are of $\mathcal{O}(1)$ (due to plasma constituents)
  - recombination effects become important (including for branchings)
  - detailed balance in thermal equilibrium

- One expects the branching process to be stopped by thermalization

- At weak coupling, all that can be studied within kinetic theory

Kinetic theory for jet evolution

- A Boltzmann equation with both elastic collisions and branchings

  Baier, Mueller, Schiff, Son '01 ('bottom-up'); Arnold, Moore, Yaffe, '03

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\right) f(t,\boldsymbol{x},\boldsymbol{p}) \;=\; \mathcal{C}_{\rm el}[f] + \mathcal{C}_{\rm br}[f]$$

- Complicated in general, yet numerically tractable:
  studies of QGP thermalization *(see talk by Aleksi Kurkela)*

- The jet problem is further complicated by its strong inhomogeneity

$$f(t=0,\boldsymbol{x},\boldsymbol{p}) \;=\; \delta^{(3)}(\boldsymbol{x})\,\delta^{(2)}(\boldsymbol{p}_\perp)\,\delta(p_z - E)$$

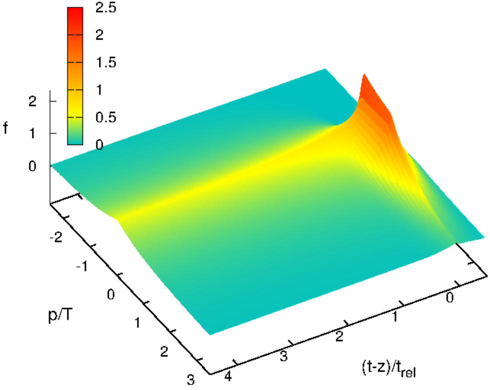
- Simplification: consider only the dynamics along the longitudinal (jet) axis
  (E.I. and Bin Wu, 2015)
  - a correct approximation so long as $\omega \simeq p_z \gg T$
  - parametrically correct down to $\omega \sim T$ (exploratory study)

The source approximation

- The branching process $\approx$ a source of gluons with $p \sim T$

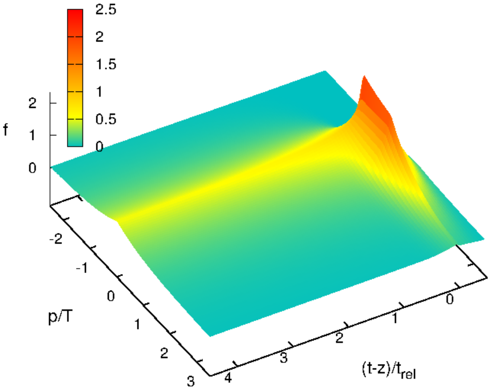
$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right) f = \frac{\hat{q}}{4}\frac{\partial}{\partial p}\left[\left(\frac{\partial}{\partial p} + \frac{v}{T}\right) f\right] + T\delta(t-z)\delta(p-T)$$

- Relativistic Fokker-Planck equation in D=0+1+1: exact solution

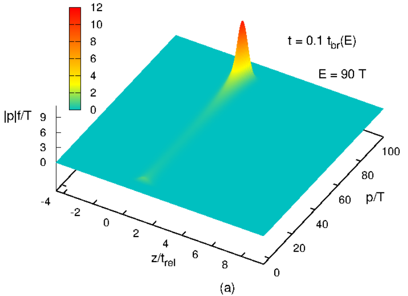


The source approximation

- A front $\propto \delta(t-z)$: gluons with $0 < p \leq T$
  - gluons recently injected that had no time to thermalize
- A thermalized tail at $z \lesssim t - t_{\rm rel}$: $f_p \propto \mathrm{e}^{-|p|/T}$
  - gluons in thermal equilibrium with the medium

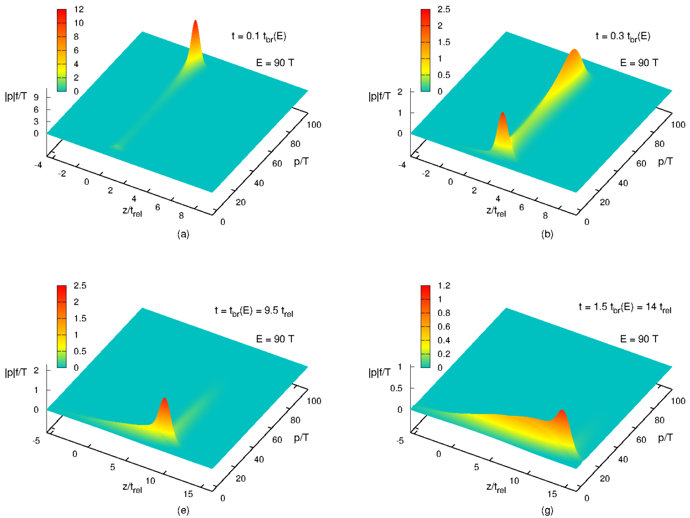


- Initial condition at $t = z = 0$: $E = 90\,T$
  - $t_{\rm br}(E)$: the democratic branching time for the leading particle



- $t = 0.1\,t_{\rm br}(E)$: the broadening in p is already visible (branchings)
- With increasing time, the jet substructure is softening (mostly via branchings) and broadening (via drag and diffusion)

Jet thermalization



- $t = t_{\rm br}(E)$: the leading particle disappears; second peak near $p = T$.
- $t = 1.5\, t_{\rm br}(E)$: the jet is fully quenched

Conclusions

- Effective theory and physical picture for jet quenching from pQCD
  - democratic branchings leading to wave turbulence
  - thermalization of the soft branching products with $p \sim T$
  - efficient transmission of energy to large angles
  - wide probability distribution, strong fluctuations, KNO scaling
- Di-jet asymmetry : geometry (path length difference) competes with fluctuations
- Qualitative and semi-quantitative agreement with the phenomenology of di-jet asymmetry at the LHC
- Important dynamical information still missing: vacuum-like radiation (parton virtualities), medium expansion ...