Interplay between hydrodynamic gradient expansion and transient modes in holography, kinetic theory and relativistic fluid mechanics

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many works, but see

1610.02023 [hep-th] lecture notes

1707.02282 [hep-ph] review with Florkowski and Spalinski

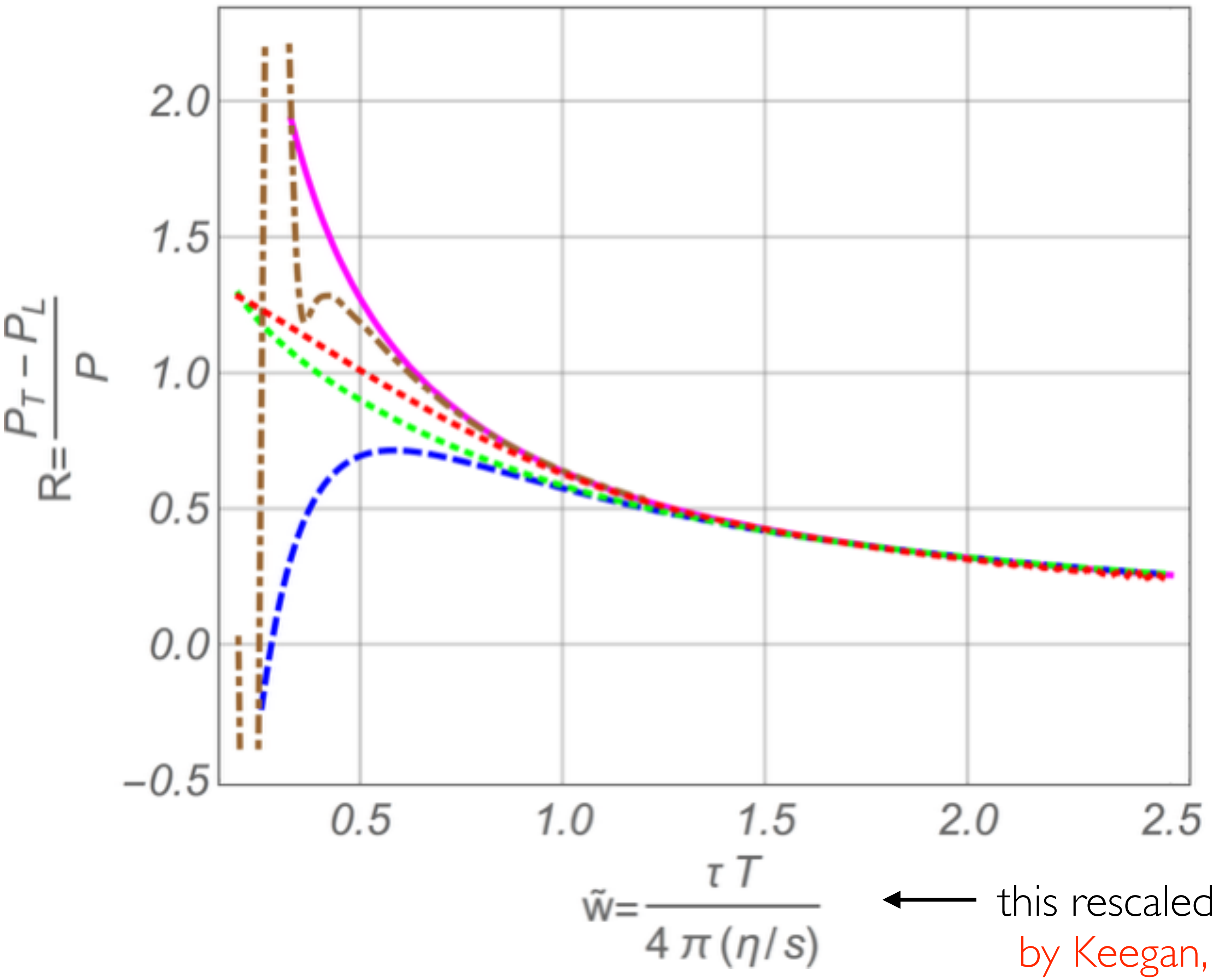
Hydrodynamization

1103.3452 with Janik & Witaszczyk

1609.04803 with Kurkela & Spalinski

Hydrodynamization (across conformal theories)

1609.04803 with Kurkela & Spalinski



N=4 SYM

EKT with $\eta/s = 0.624$

RTA with $\eta/s = 0.624$

RTA with $\eta/s = 1/(4\pi)$

$$\frac{\Delta\mathcal{P}}{\mathcal{E}/3} = \frac{2}{\pi}\tilde{w}^{-1}$$

← this rescaled variable is motivated by **1512.05347** by Keegan, Kurkela, Romatschke, van der Schee

Viscous hydrodynamics works despite huge anisotropy in the system

0906.4426, 1011.3562 by Chesler & Yaffe; **1103.3452** with Janik & Witaszczyk

Relativistic hydrodynamics

hydrodynamics is an EFT of the slow (?) evolution of conserved currents in collective media close to equilibrium (?)

DOFs: always local energy density ϵ and local flow velocity u^μ $(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$ for $\langle T^{\mu\nu} \rangle$ expanded in gradients

$\Pi^{\mu\nu}$

$$\langle T^{\mu\nu} \rangle = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\} - \eta(\epsilon)\, \sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\}(\nabla \cdot u) + \ldots$$

microscopic input:

EoS
$(P(\epsilon) = \frac{1}{3}\epsilon$ for CFTs)

shear viscosity contribution

$$\frac{\Delta \mathcal{P}}{\mathcal{E}/3} = \frac{2}{\pi} \tilde{w}^{-1}$$

bulk viscosity
(vanishes for CFTs)

This talk: behaviour of the gradient expansion at large orders in the number of ∇

Hydrodynamic & transient modes

Theories of (viscous) hydrodynamics

There is a crucial subtlety: $\nabla_\mu\Big(\,\epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\} - \eta(\epsilon)\,\sigma^{\mu\nu} + \ldots\Big) = 0$ does not have a well-posed initial value problem ⟶ hydrodynamic theories

Overall idea (MIS): make $\pi^{\mu\nu}$ obey an independent PDE ensuring its ↘ to $-\eta\,\sigma^{\mu\nu}$

$$(\tau_\pi u^\alpha \mathcal{D}_\alpha + 1)\ [\pi^{\mu\nu} - (-\eta\,\sigma^{\mu\nu})] = 0 \longrightarrow \pi^{\mu\nu} = -\eta\,\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, \pi^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, (\eta\,\sigma^{\mu\nu})$$

decay timescale

Modern incarnation: Baier-Romatschke-Son-Starinets-Stephanov theory 0712.2451

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

BRSSS theory will be treated here on equal footing with holography & kinetic theory

Modes in BRSSS theory

Mode = solution of linearized equations of finite-T theory without any sources

Technical issue: tensor perturbs. ⟶ channels (**here everywhere sound channel**):

Assuming momentum along x^3 direction $e^{-i\,\omega\,x^0 + i\,k\,x^3}$: δT, δu^3 & $\delta \pi^{33}$

conservation

+

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

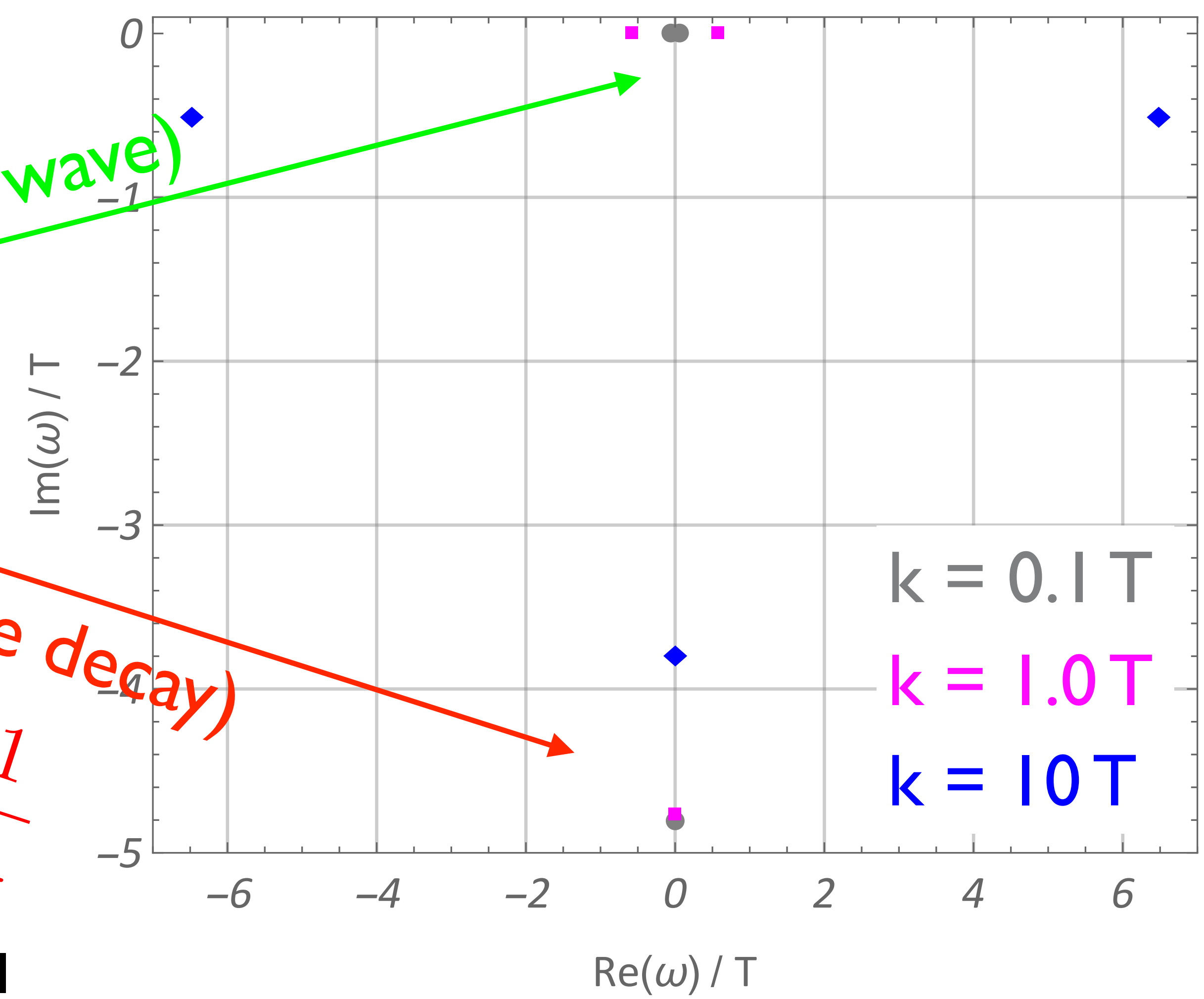
$$\omega^3 + (\ldots)\,\omega^2 + (\ldots)\omega + (\ldots) = 0$$

two modes:

hydro (sound wave)

transient (pure decay)

$$\omega\Big|_{k=0} = \frac{1}{\tau_\pi}$$

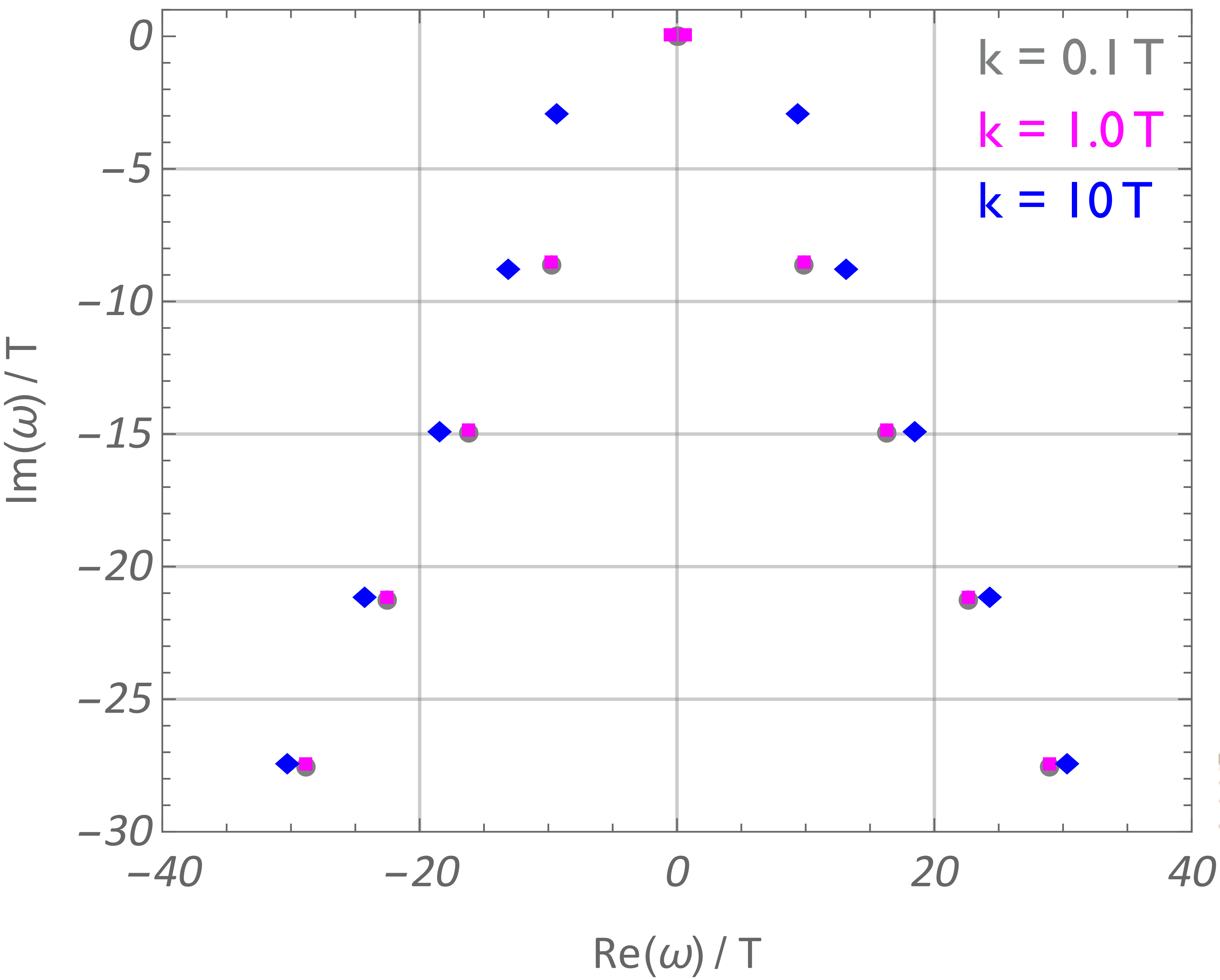


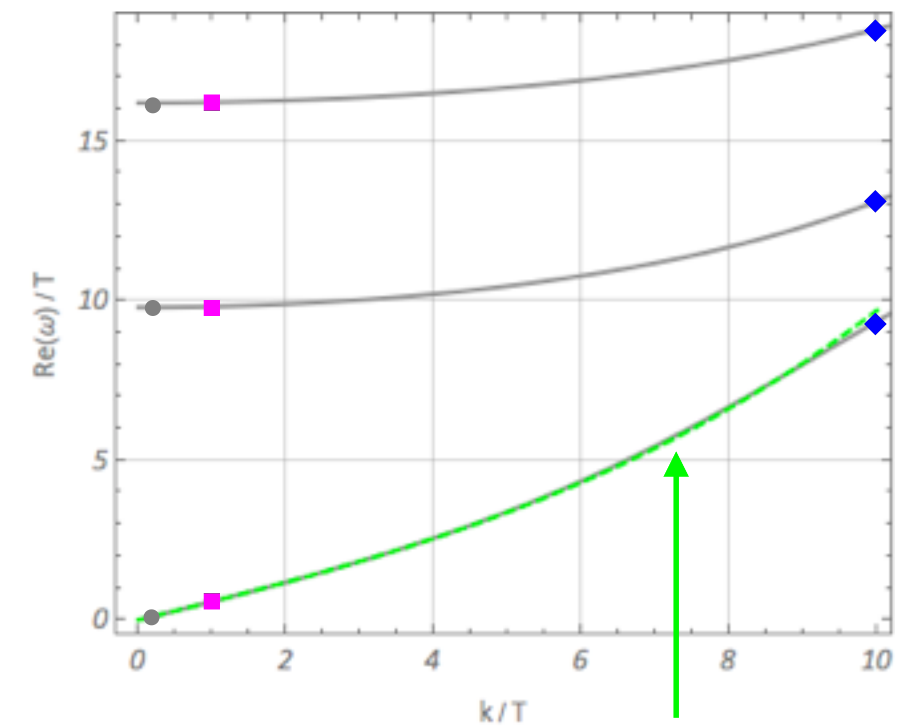
Modes in Einstein-Hilbert holography = QNMs

$$ds^2 = \frac{L^2}{u^2}\left\{-2dx^0du - \left(1-\pi^4T^4u^4\right)\left(x^0\right)^2 + d\vec{x}^2\right\} + \delta g_{ab}(u)\, e^{-i\,\omega\, x^0 + i\, k\, x^3}$$

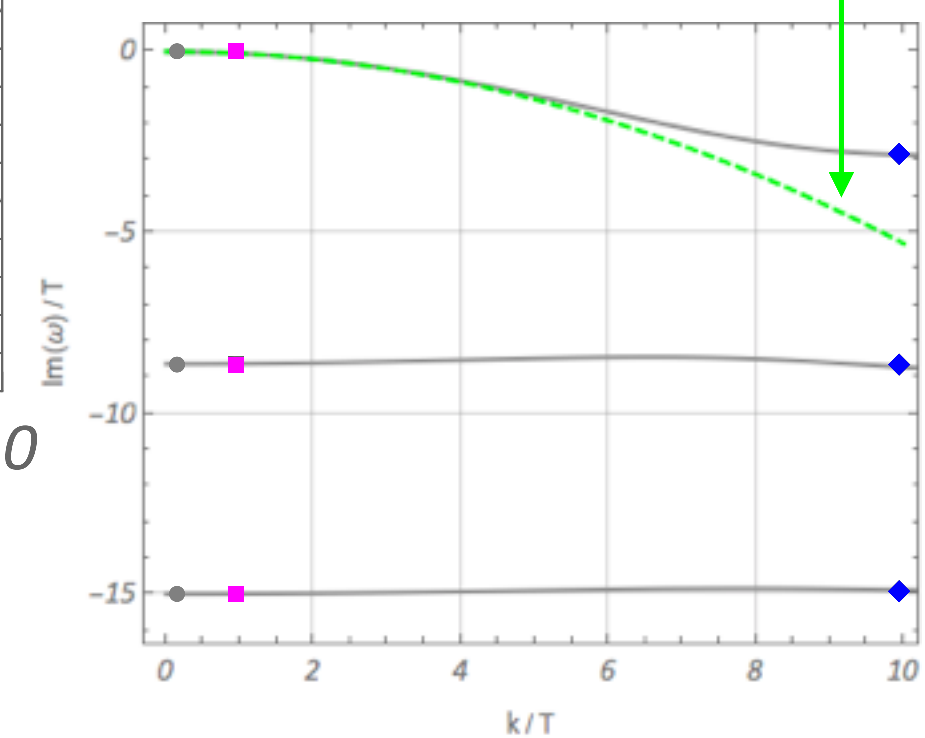
vanishes at the boundary

ingoing (regular) at the horizon





$$\omega/T \approx \pm\frac{1}{\sqrt{3}}\,k/T - i\,\frac{2}{3}\,\frac{1}{4\pi}\,(k/T)^2 \pm \frac{3-2\log 2}{24\sqrt{3}\,\pi^2}\,(k/T)^3$$

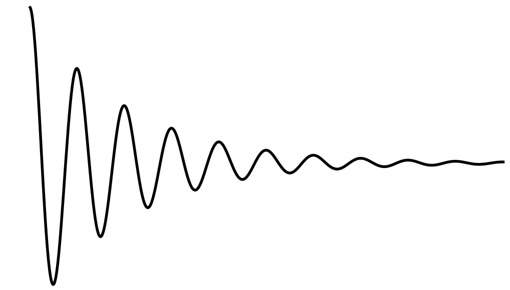


hep-th/0506184 by Kovtun & Starinets

HJSW theory and its modes

1409.5087 with Janik, Spalinski & Witaszczyk (see also 1104.2415 by Noronha & Denicol)

MIS/BRSSS idea: $\pi^{\mu\nu}$ decays exponentially to $-\eta\,\sigma^{\mu\nu}$. In holography:



HJSW: go from relaxation-type eqn. to damped harmonic oscillator-type eqn. for $\pi^{\mu\nu}$:

$$\left\{\left(\frac{1}{T}\mathcal{D}\right)^2 + 2\,\Omega_I\,\frac{1}{T}\mathcal{D} + |\Omega|^2\right\}\pi^{\mu\nu} = \eta\,|\Omega|^2\,\sigma^{\mu\nu} - c_\sigma\,\frac{1}{T}\mathcal{D}\,(\eta\,\sigma^{\mu\nu}) + \ldots \quad \text{with} \quad \frac{1}{T}\,\omega^1_{QNM}\big|_{k=0} = \pm\Omega_R + i\,\Omega_I$$

linearization

$$\omega^4 + (\ldots)\,\omega^3 + (\ldots)\,\omega^2 + (\ldots)\,\omega + (\ldots) = 0$$

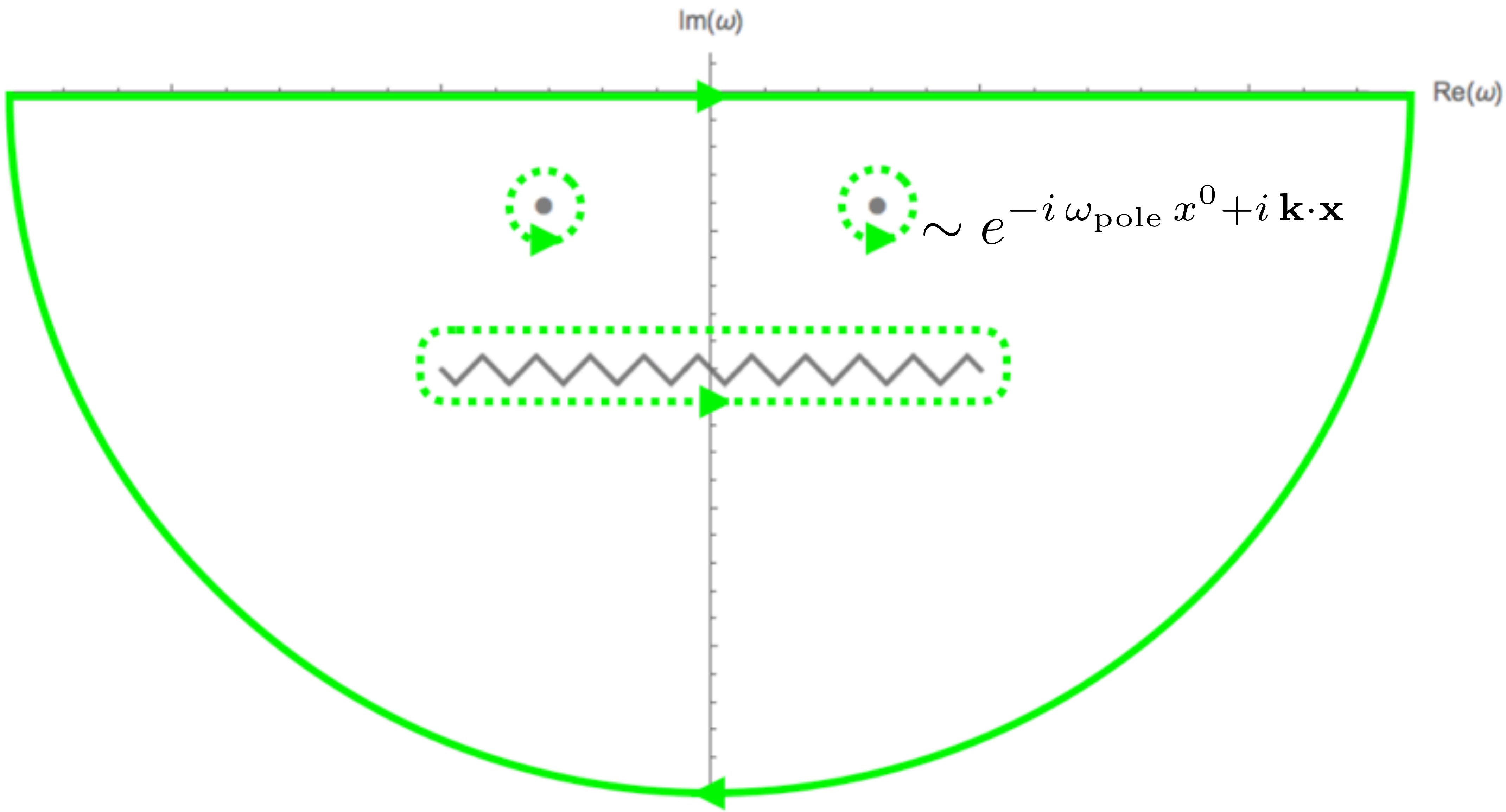
hydrodynamics (sound wave)

transient (decay + oscillation)

Tested using holography V (note initialization requires not only $\pi^{\mu\nu}$ but also $\partial_0\,\pi^{\mu\nu}$)

Modes in linear response theory

$$\delta\langle \hat{T}^{\mu\nu}\rangle(x) = -\frac{1}{2\times(2\pi)^4}\int d^3k \int d\omega\, e^{-i\,\omega\, x^0 + i\,\mathbf{k}\cdot\mathbf{x}}\, G_R^{\mu\nu,\,\alpha\beta}(\omega,\mathbf{k})\,\delta g_{\alpha\beta}(\omega,\mathbf{k})$$



In all hydrodynamic theories and Einstein-Hilbert holography modes are single poles of the thermal retarded two-point function of $T^{\mu\nu}$ in the Fourier space at fixed k

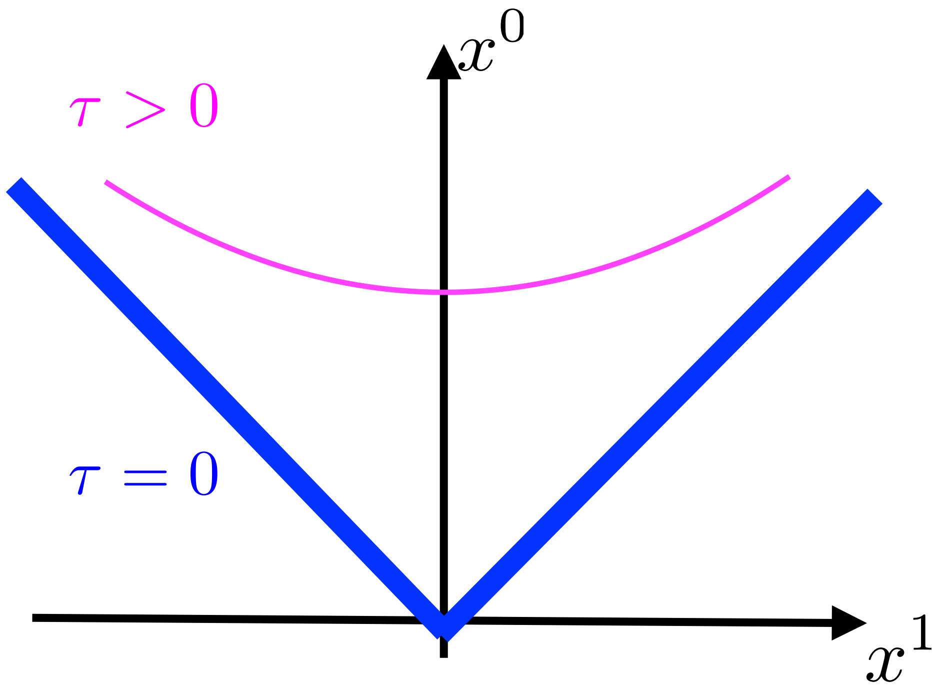
Hydrodynamics & Transient Modes I:
Theories of Hydrodynamics & Holography

1503.07514 with Spalinski

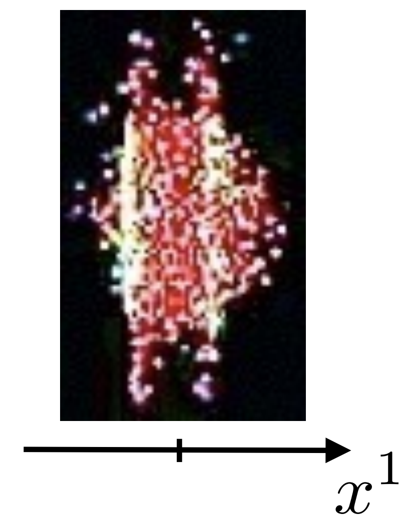
1302.0697 with Janik & Witaszczyk

1603.05344 with Buchel & Noronha

Boost-invariant flow [Bjorken 1982]



const x^0 slice:



Boost-invariance: in $\left(\tau \equiv \sqrt{x_0^2 - x_1^2}, \quad y \equiv \operatorname{arctanh}\frac{x_1}{x_0}, \, x_2, \, x_3\right)$ coords no y-dep

In a CFT: $\langle T^\mu_{\ \nu} \rangle = \operatorname{diag}\left\{-\mathcal{E}(\tau), -\mathcal{E} - \tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}\right\}$

$\langle T^2_{\ 2} \rangle - \langle T^y_{\ y} \rangle \equiv \Delta\mathcal{P}$

and via scale-invariance $\frac{\Delta\mathcal{P}}{\mathcal{E}/3} \equiv R$ is a function of $w \equiv \tau\,T$

$T \equiv \left(\frac{\mathcal{E}(\tau)}{\frac{3}{8}\pi^2 N_c^2}\right)^{1/4}$

Gradient expansion: series in $\frac{1}{w}$.

1103.3452 with Janik & Witaszczyk

Large order gradient expansion: BRSSS 1503.07514 with Spalinski

conservation (always the same) $\longrightarrow$ $\frac{\tau}{w}\frac{dw}{d\tau} = \frac{2}{3} + \frac{1}{18}R$

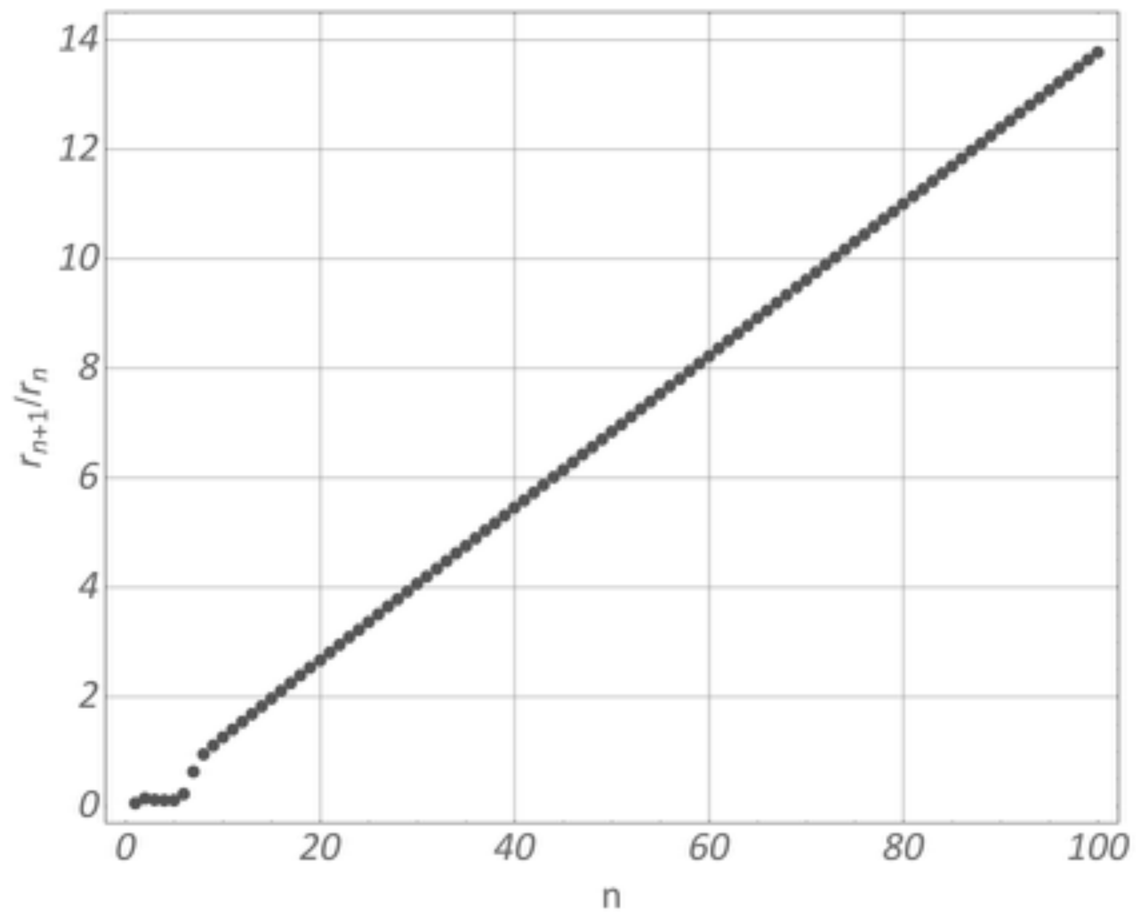
$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} \rightarrow C_{\tau_\pi} w \left(1 + \frac{1}{12}R\right) R' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right) R^2 + \frac{3}{2} w R - 12\, C_\eta = 0$$

$$\left(\eta = C_\eta\, \mathcal{S}, \quad \tau_\pi = \frac{C_{\tau_\pi}}{T}, \quad \lambda_1 = C_{\lambda_1}\frac{\eta}{T}\right)$$

with $C_\eta = \frac{1}{4\pi}$, $C_{\tau_\pi} = \frac{2-\log 2}{2\pi}$, $C_{\lambda_1} = \frac{1}{2\pi}$

$$R(w) \approx \sum_{n=1}^{\infty} \frac{r_n}{w^n} = 8\, C_\eta \frac{1}{w} + \frac{16}{3} C_\eta (C_{\tau_\pi} - C_{\lambda_1}) \frac{1}{w^2} + \ldots \longrightarrow$$

(note that r_n's do not depend on ini. cond.)



Hydrodynamic gradient expansion is a divergent series: $r_n \sim n!$

Hydrodynamics & transient modes: BRSSS

Key observations:

- $\sum_{n=1}^{\infty} \frac{r_n}{w^n}$ does not make sense without a resummation
- there must be sth else that cares about ini. cond.

resurgence

1503.07514 with Spalinski

Linearization of $C_{\tau_\pi} w \,(1 + \frac{1}{12} R)\, R' + \left(\frac{1}{3} C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right) R^2 + \frac{3}{2} w\, R - 12\, C_\eta = 0$ around $\sum_{n=1}^{\infty} \frac{r_n}{w^n}$ gives:

integration const. (ini. cond.)

further hydro dressing (another div. series)

$$\delta R = \sigma\, e^{-\frac{3}{2}\frac{1}{C_{\tau\pi}} w}\, w^{\frac{C_\eta - 2 C_{\lambda_1}}{C_{\tau\pi}}} \left\{ 1 + \sum_{j=1}^{\infty} \frac{r_j^{(1)}}{w^j} \right\}$$

In equilibrium one has $e^{-\frac{1}{C_{\tau\pi}} T\, t}$

It is still true here, but only at a given instance: $e^{-\frac{1}{C_{\tau\pi}} \int_{\tau_i}^{\tau} T(\tau')\, d\tau'}$

Using $T = \frac{\Lambda}{(\Lambda\tau)^{1/3}} \left(1 - C_\eta \frac{1}{(\Lambda\,\tau)^{2/3}} + \ldots\right)$ one gets $e^{-\frac{3}{2}\frac{1}{C_{\tau\pi}} w}\, w^{\frac{C_\eta}{C_{\tau\pi}}} \ldots$

To wrap-up, we have just seen the hydro-dressed transient mode of BRSSS at k =0

see also **hep-th/0606149** by Janik & Peschanski

Transseries and resurgence

1503.07514 with Spalinski

approx. analytic cont.

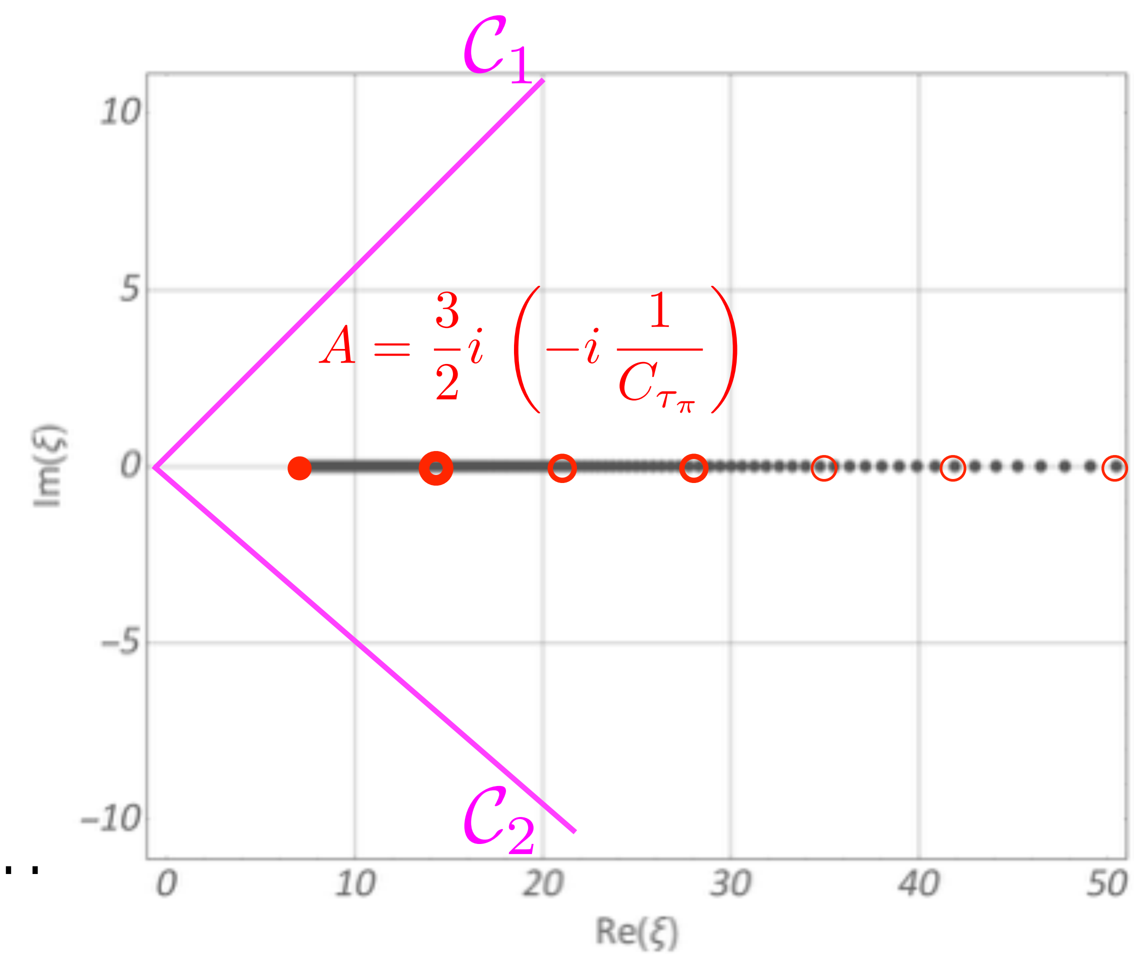
$$R(w) \approx \sum_{n=1}^{\infty} \frac{r_n}{w^n} \xrightarrow{\text{Borel trafo.}} BR(\xi) = \sum_{n=1}^{\infty} \frac{r_n}{n!}\xi^n \approx \frac{a_0 + + \ldots + a_{100}\xi^{100}}{b_0 + + \ldots + b_{100}\xi^{100}}$$

Borel (re)summation

$$\left(\int_{\mathcal{C}_1} d\xi - \int_{\mathcal{C}_2} d\xi\right)\left[w\, e^{-w\,\xi} BR(\xi)\right]$$

$$\sim e^{-\frac{3}{2}\frac{1}{C_{\tau_\pi}} w}\, w^{\frac{C_\eta - 2 C_{\lambda_1}}{C_{\tau_\pi}}} \ldots$$

$\equiv A$, $\equiv \beta$



Ambiguity in resummation

$BR(\xi) = \text{reg.} + (A - \xi)^\beta\, \text{reg.} + \ldots$ $\sim$ transient mode + …

nonlinear effects

Transseries: $R(w) = \sum_{j=0}^{\infty} \sigma^j\, e^{-j\,A\,w}\, w^{j\,\beta}\, \Phi_{(j)}(w)$

~ 1/w expansions

~ resum. ambig. + ini. cond.

Resurgence: transseries yields an unambiguous answer up to 1 real int. const.

Hydrodynamics & transient modes: HJSW

1511.06358

by Aniceto & Spalinski

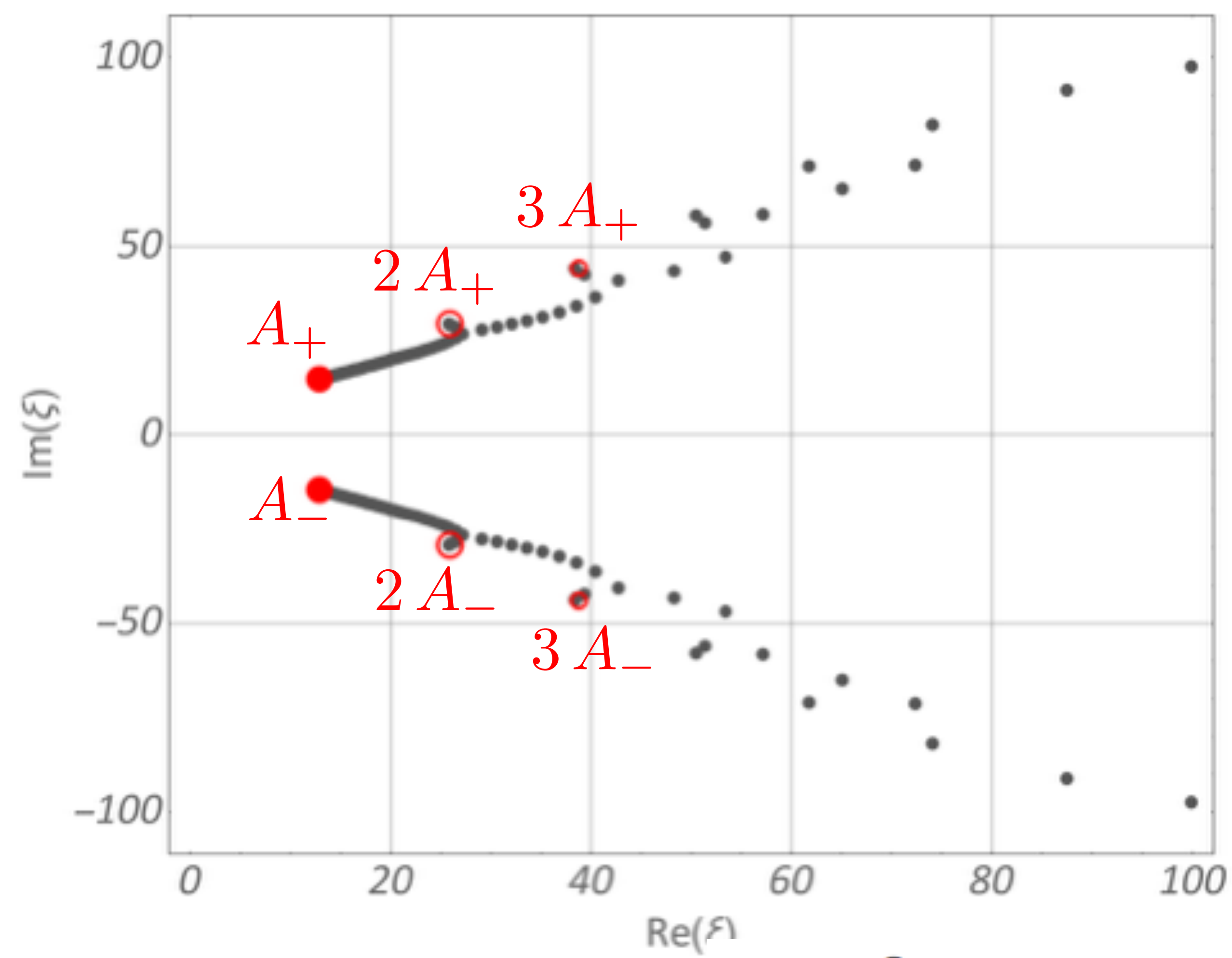
$$a_1\, R'' + a_2\, R'^2 + a_3\, R' + 12\, R^3 + a_4\, R^2 + a_5\, R + a_6 = 0$$

with

$$\begin{aligned} a_1 &= w^2\,(R+12)^2, \\ a_2 &= w^2\,(R+12), \\ a_3 &= 12\,w\,(R+12)\,(R+3\,w\,\Omega_I), \\ a_4 &= 48\,(3\,w\,\Omega_I - 1), \\ a_5 &= 108\,\left(4\,C_\eta\,C_{\tau_\pi} + 3\,w^2\,\Omega^2\right), \\ a_6 &= -864\,C_\eta\,\left(2\,C_{\tau_\pi} + 3\,w\,\Omega^2\right). \end{aligned}$$

$$R(w) = \sum_{n=1}^{\infty} \frac{r_n}{w^n} + \ldots$$

$$BR(\xi) = \sum_{n=1}^{\infty} \frac{r_n}{n!}\,\xi^n \approx \frac{a_0 + + \ldots + a_{300}\xi^{300}}{b_0 + + \ldots + b_{300}\xi^{300}}$$



$$R\,(w) = \sum_{n_\pm=0}^{\infty} \sigma_+^{n_+}\,\sigma_-^{n_-}\,\mathrm{e}^{-(n_+ A_+ + n_- A_-)\,w}\,w^{n_+\beta_+ + n_-\beta_-}\,\Phi_{(n_+|n_-)}\,(w)$$ with $$A_\pm = \frac{3}{2}\,(\Omega_I \pm \mathrm{i}\Omega_R)$$

2nd order EOM ⟶ 2 real int. const. ⟶ 2 parameter ($\sigma_\pm$) transseries

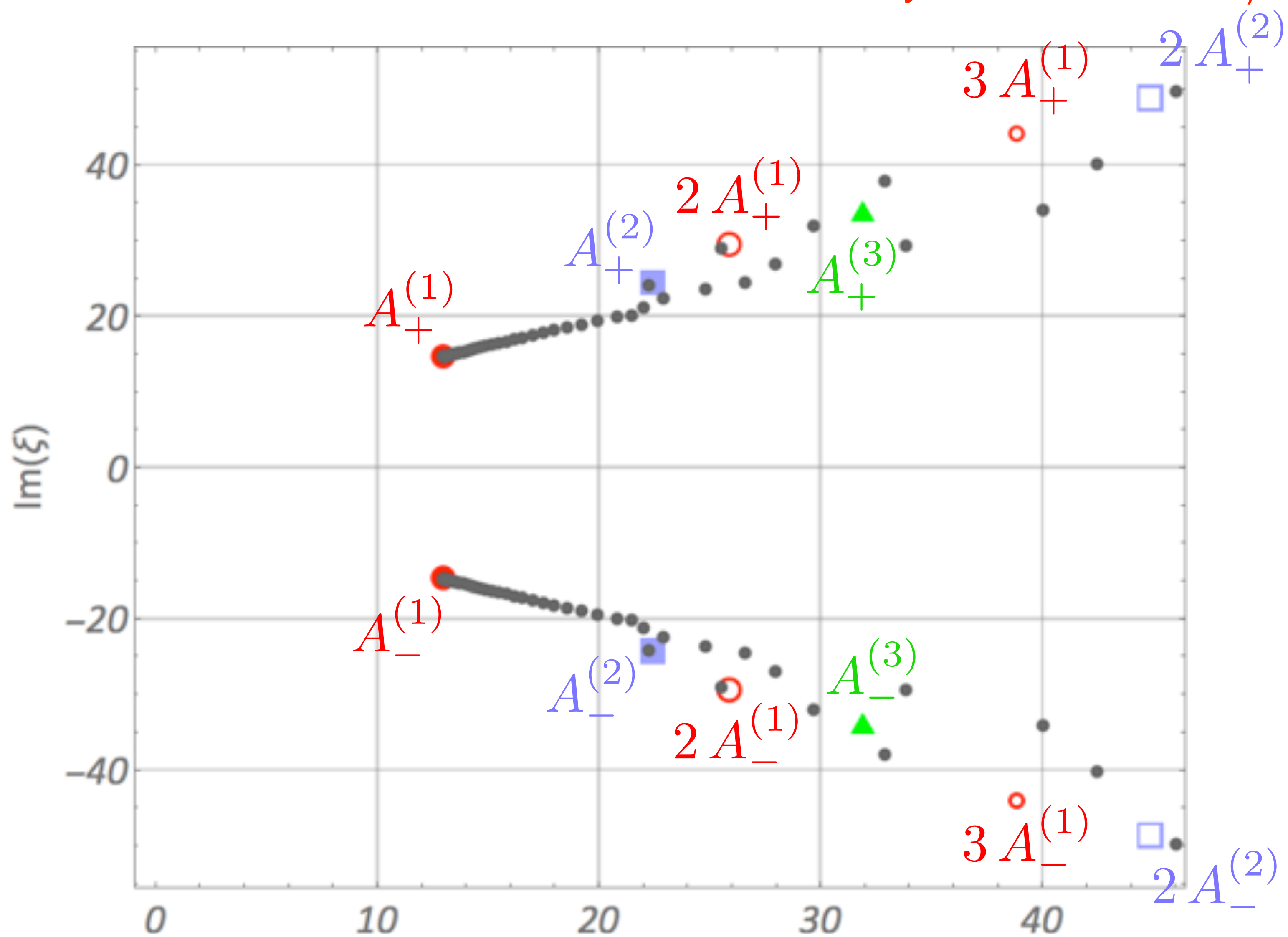
Hydrodynamics & transient modes: holography

1302.0697 with Janik & Witaszczyk

$$R(w) = \sum_{n=1}^{\infty} \frac{r_n}{w^n} + \ldots$$

$$\downarrow$$

$$BR(\xi) = \sum_{n=1}^{\infty} \frac{r_n}{n!}\,\xi^n \approx \frac{a_0 + + \ldots + a_{120}\xi^{120}}{b_0 + + \ldots + b_{120}\xi^{120}}$$



$$R(w) = \sum_{n_\pm^{(1)}, n_\pm^{(2)}, \ldots = 0}^{\infty} \Phi_{\left(n_+^{(1)} | n_-^{(1)} | n_+^{(2)} | n_-^{(2)} | \ldots\right)}(w) \times$$

$$\times \prod_{j=1}^{\infty} \left(\sigma_+^{(j)}\right)^{n_+^{(j)}} \left(\sigma_-^{(j)}\right)^{n_-^{(j)}} \mathrm{e}^{-\left(n_+^{(j)} A_+^{(j)} + n_-^{(j)} A_-^{(j)}\right) w}\, w^{n_+^{(j)} \beta_+^{(j)} + n_-^{(j)} \beta_-^{(j)}}$$

Infinitely many transient QNMs ⟶ infinitely many parameters in the transseries

Lesson from cosmology

1603.05344 with Buchel & Noronha

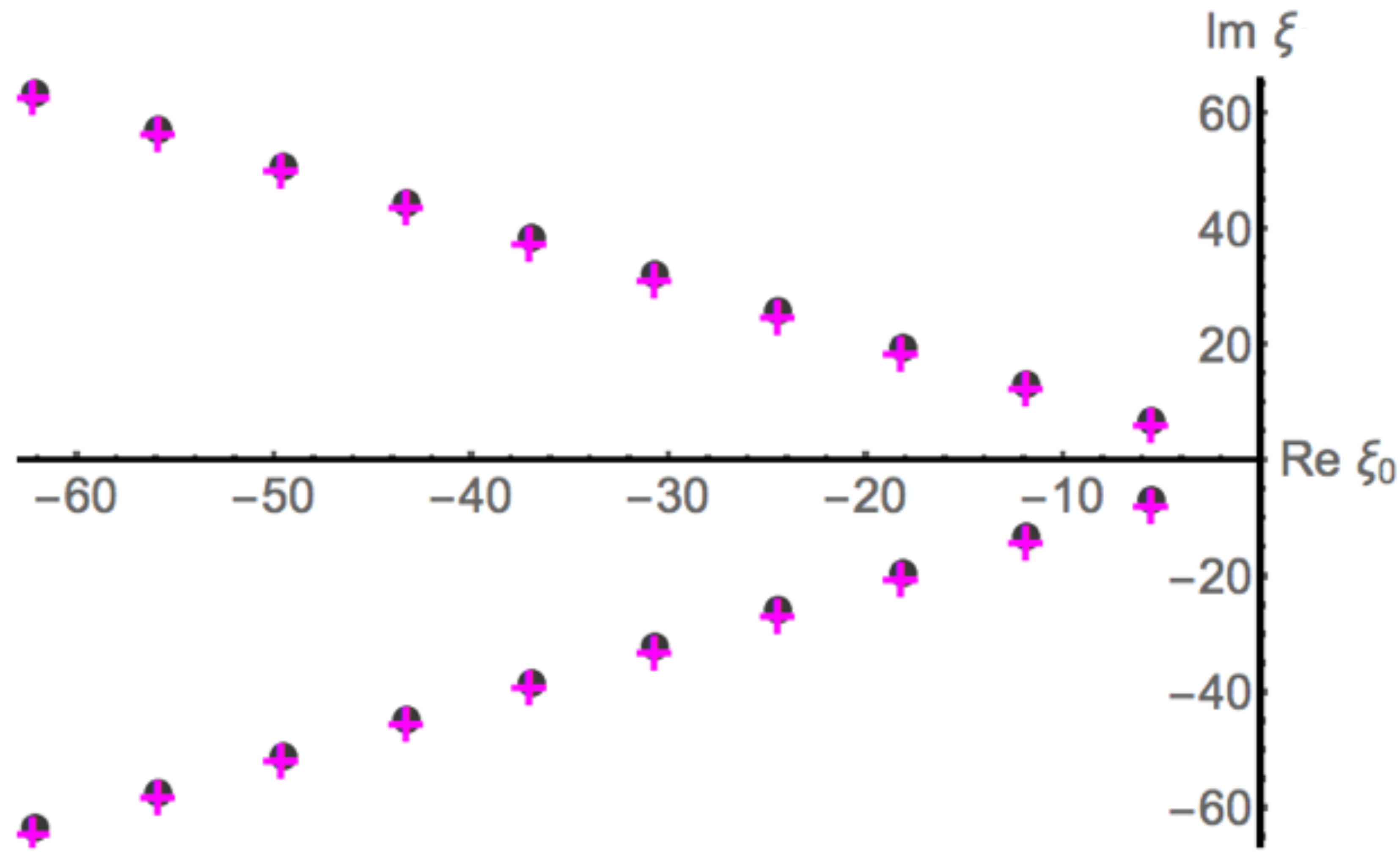
$$\frac{d\,\mathrm{Entropy}}{dt} = V \times \left(\sum_{n=0}^{\infty} c_n \xi^n\right)^2 + \ldots$$ with $\xi = \frac{H}{T}$ for a h~~CFT~~ in $-dt^2 + e^{2Ht}\, d\vec{x}^2$

$$T \sim e^{-H\,t} \longrightarrow e^{-i\,\Omega_\pm \int_{t_i}^{t} T(t')dt'} \sim e^{-i\,\Omega_\pm \cdot \left(-\frac{T(t)}{H}\right)}$$

$$\sum_{n=0}^{300} \frac{c_n}{n!} \xi^n \approx \frac{\sum_{m=0}^{150} d_m\, \xi^m}{\sum_{l=0}^{150} e_l\, \xi^l}$$

● singularities of Borel trafo

\+ 10 lowest transient QNM $\hat{\omega}$'s



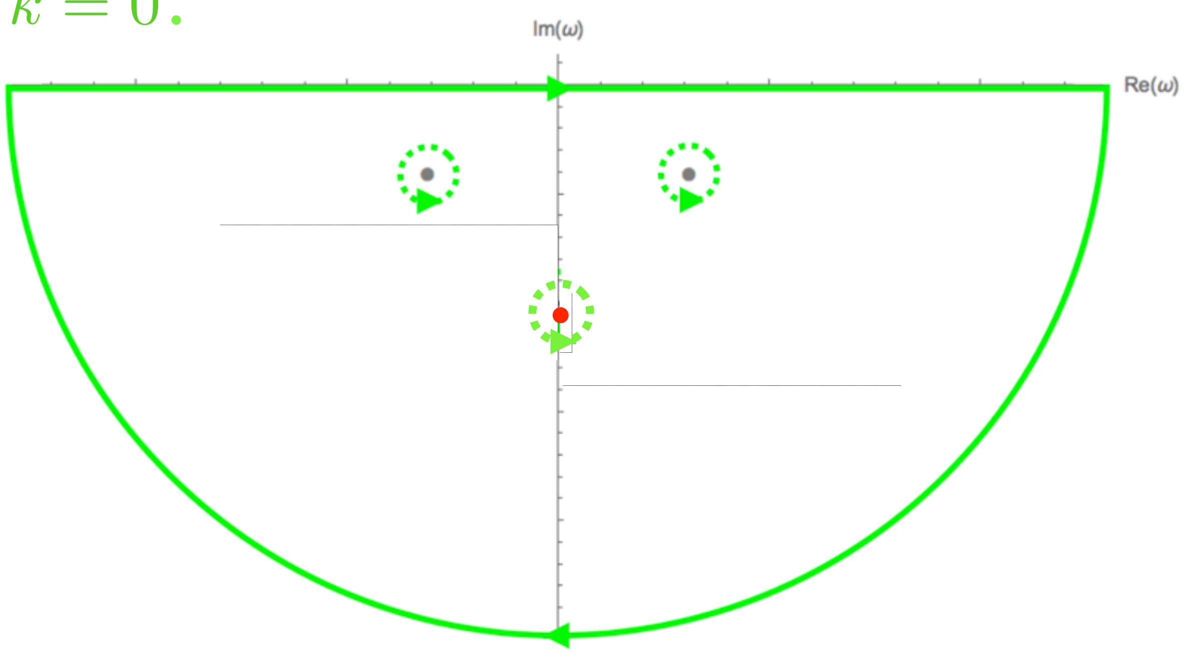
Hydrodynamic gradient expansion knowns about all transient QNMs

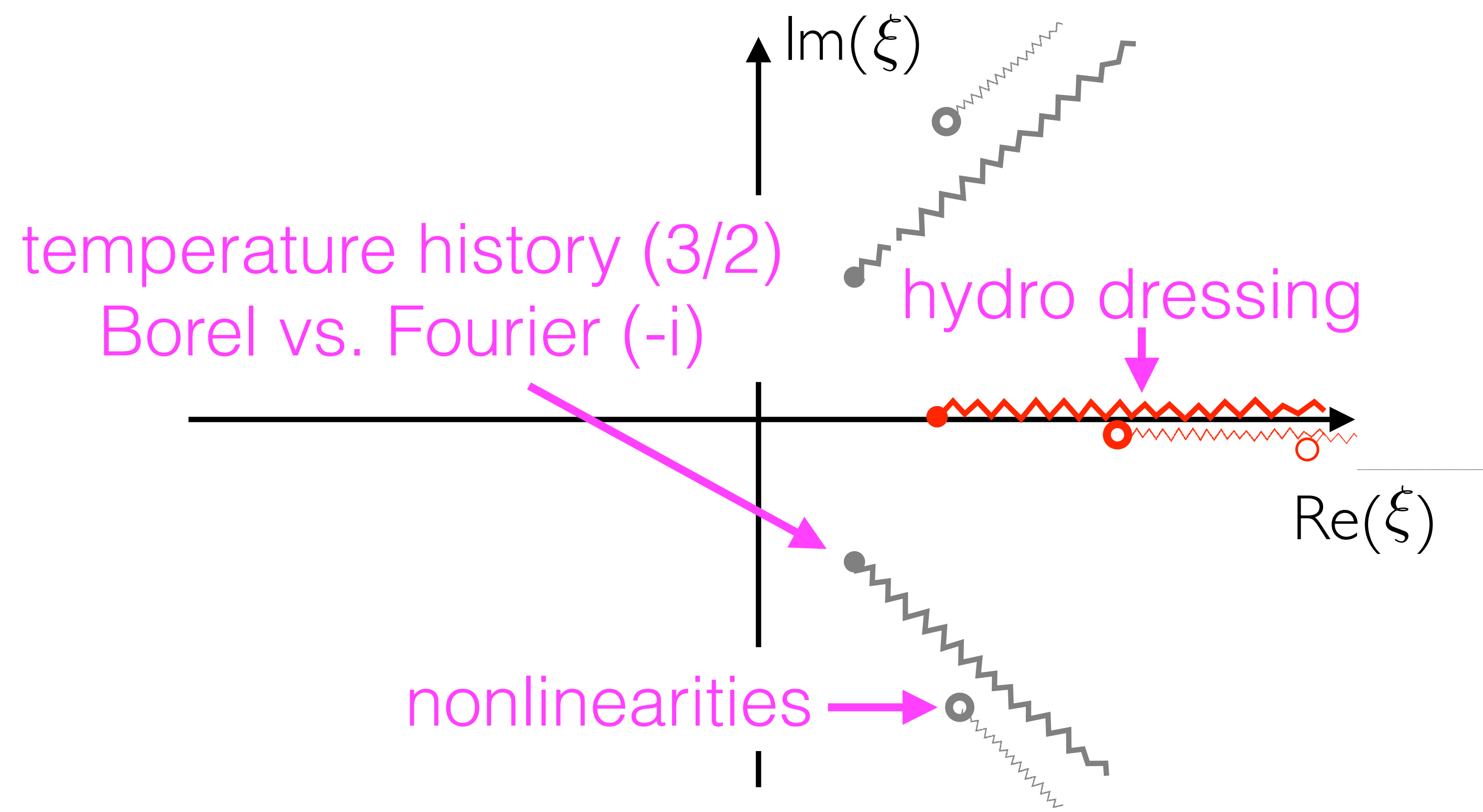
Emerging picture

Hydrodynamic gradient expansion is a divergent series ⟶ hydrodynamization

Transient singularities of $G_R^{T_{\mu\nu}}(\omega, k)$ vs. singularities of Borel transform of hydro

$k = 0$:





Appealing analogy with quantum mechanics:

non-equilibrium physics	QM with $V = -\frac{1}{2}x^2\left(1-\sqrt{g}\,x\right)^2$
gradient expansion in $\frac{1}{w}$	perturbative series in g
transient QNMs $e^{-i\,\frac{3}{2}\,\Omega_{\pm}\,w}(\ldots)$	instanton $e^{-1/(3g)}(\ldots)$

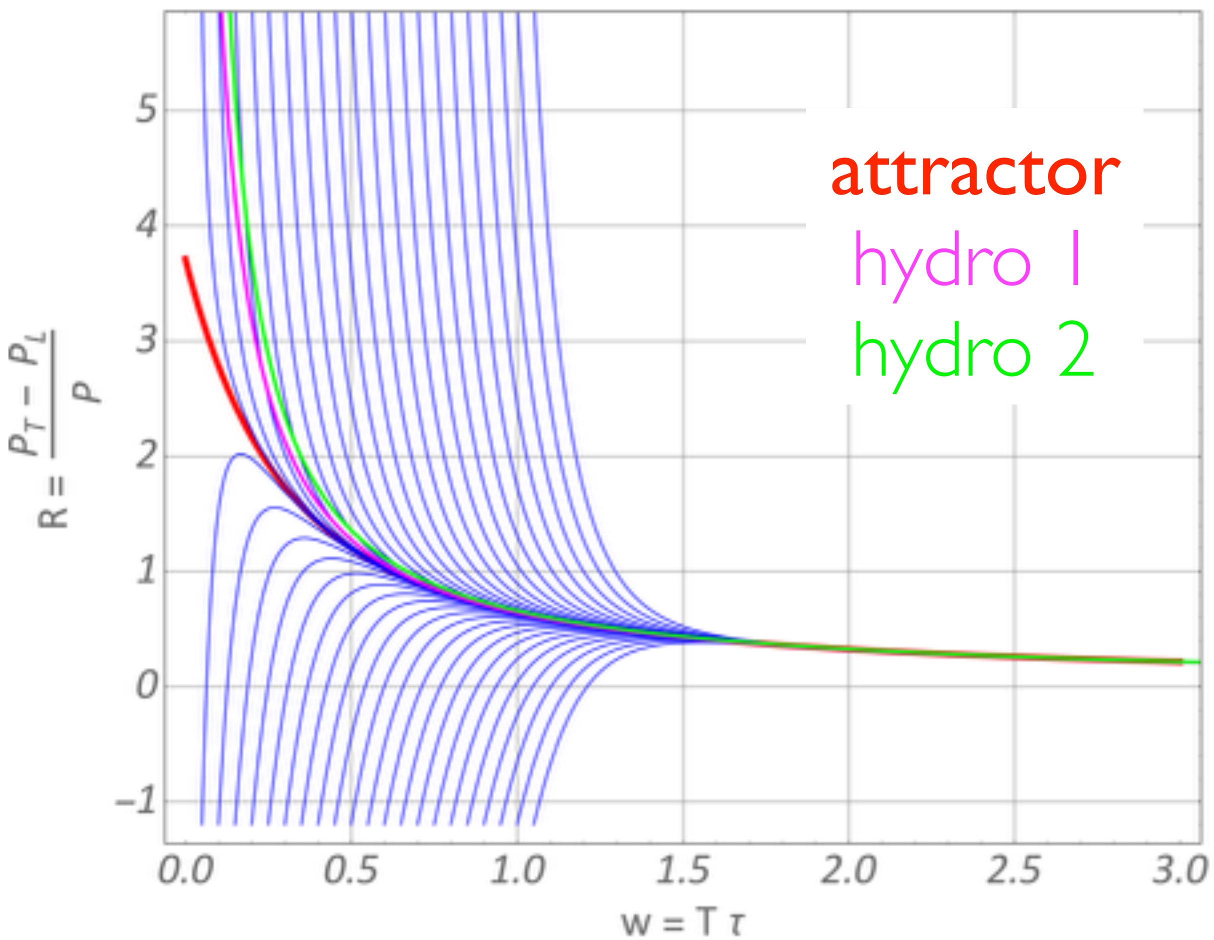
1302.0697 with Janik & Witaszczyk

Resumed hydrodynamics
(Far from equilibrium hydrodynamics)

1503.07514 with Spalinski

(BRSSS) resummed hydrodynamics 1503.07514 with Spalinski

Idea: resummed /all order / far from equilibrium hydrodynamics = attractor solutions

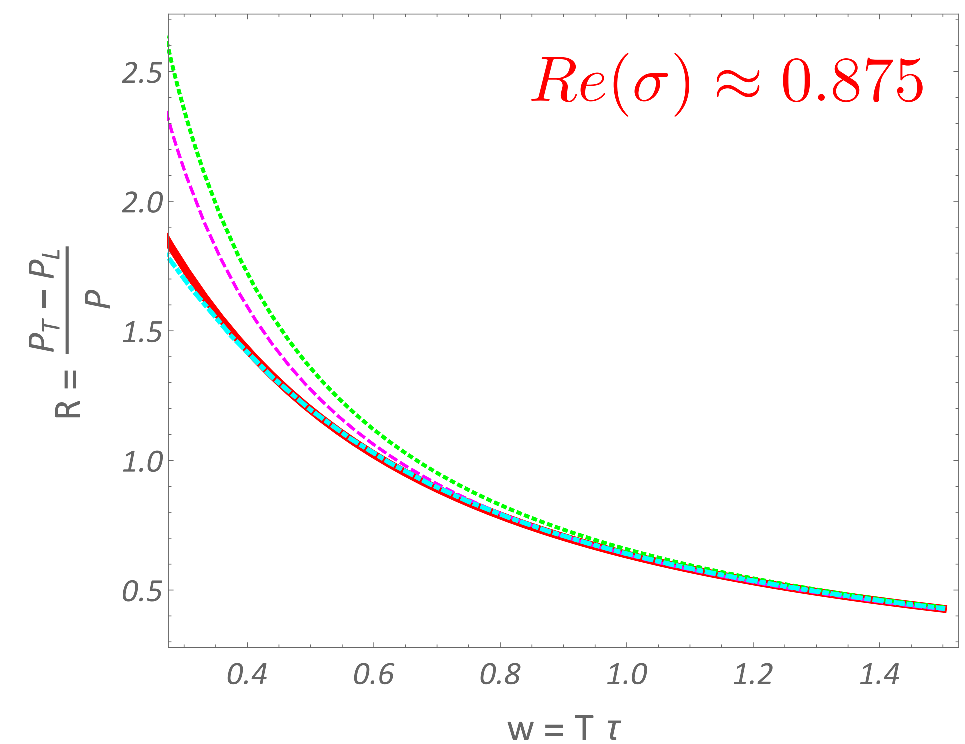


BRSSS:

$$C_{\tau_\pi}\, w\,(1 + \frac{1}{12} R)\, R' + \left(\frac{1}{3} C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right) R^2 + \frac{3}{2}\, w\, R - 12\, C_\eta = 0$$

≈ attractor solution („slow roll'' approximation)

Recently Romatschke in 1704.08699 found such attractors in RTA kinetic theory



One can also approx. resum transseries:

$$R(w) \approx \sum_{j=0}^{2} \sigma^j\, e^{-j\,A\,w}\, w^{j\,\beta}\, \Phi_{(j)}(w)$$

Requires 3 Borel summations

Hydrodynamics & Transient Modes II:
RTA Kinetic Theory

1609.04803 with Kurkela & Spalinski

1707.02282 with Florkowski & Spalinski

work in progress with Svensson

RTA kinetic theory

Natural language to talk about weakly coupled media is the Boltzmann equation:

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \;\text{ with }\; \langle T^{\mu\nu}\rangle(x) = \int_{\text{momenta}} f(x,p)\, p^\mu p^\nu$$

LO $C[\;(x,p)]$ for gauge theories is complicated. We will use instead

$$C[f(x,p)] = -\frac{p^\mu u_\mu}{\tau_{rel}} \Big\{ f(x,p) - f_0(x,p) \Big\} \;\text{ with }\; f_0(x,p) = e^{\frac{u_\mu p^\mu}{T}}$$

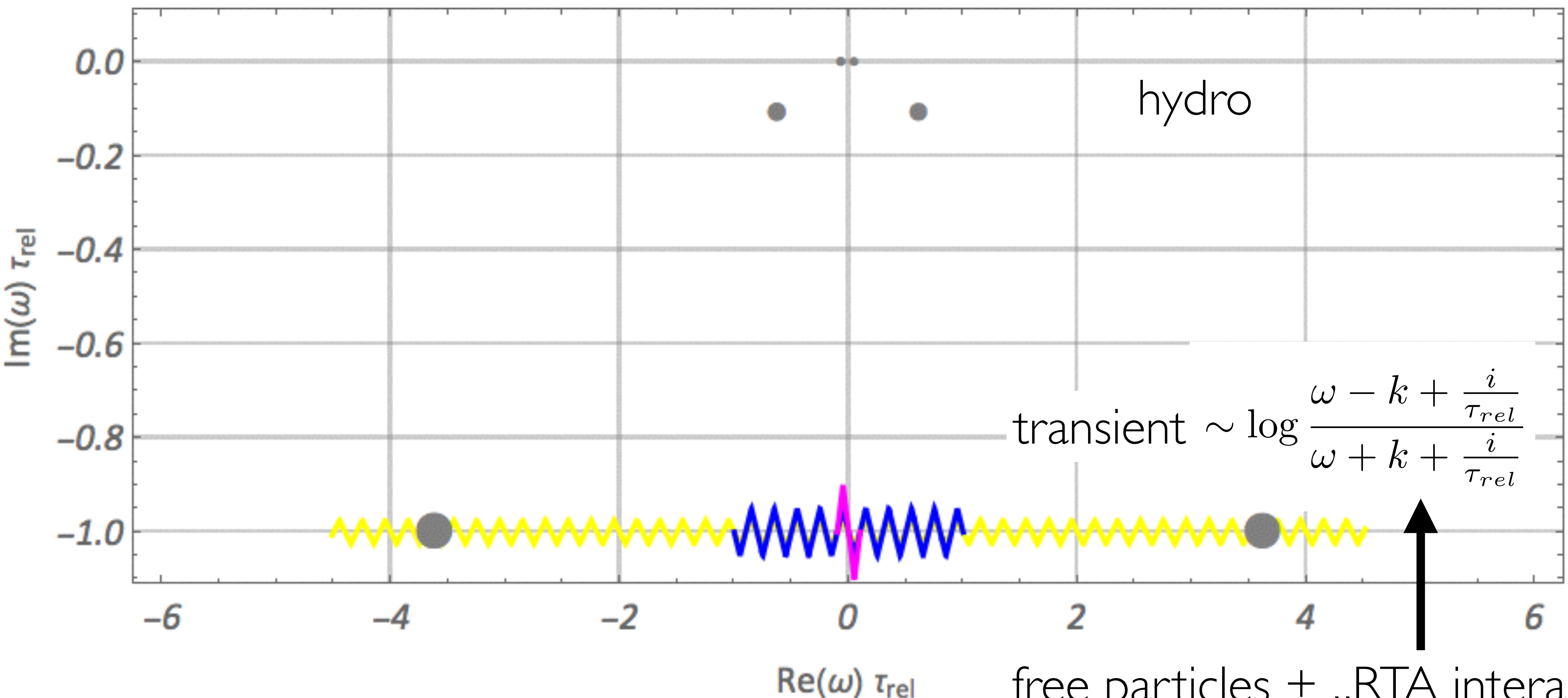
This equation is, typically, highly nonlinear due to $\langle T^{\mu\nu}\rangle\, u_\nu = -\mathcal{E}(T)\, u^\mu$

CFTs: $p^\mu p_\mu = 0$ and $\tau_{rel} = \dfrac{\gamma}{T}$.

Modes in RTA kinetic theory

1512.02641 by Romatschke
1707.02282 with Florkowski & Spalinski

Sound channel at k τ_{rel} = 0.1, 1.0 & 4.531



transient $\sim \log \frac{\omega - k + \frac{i}{\tau_{rel}}}{\omega + k + \frac{i}{\tau_{rel}}}$

free particles + „RTA interactions"

Very different from holography: one hydro mode and one branch-cut at $k \neq 0$

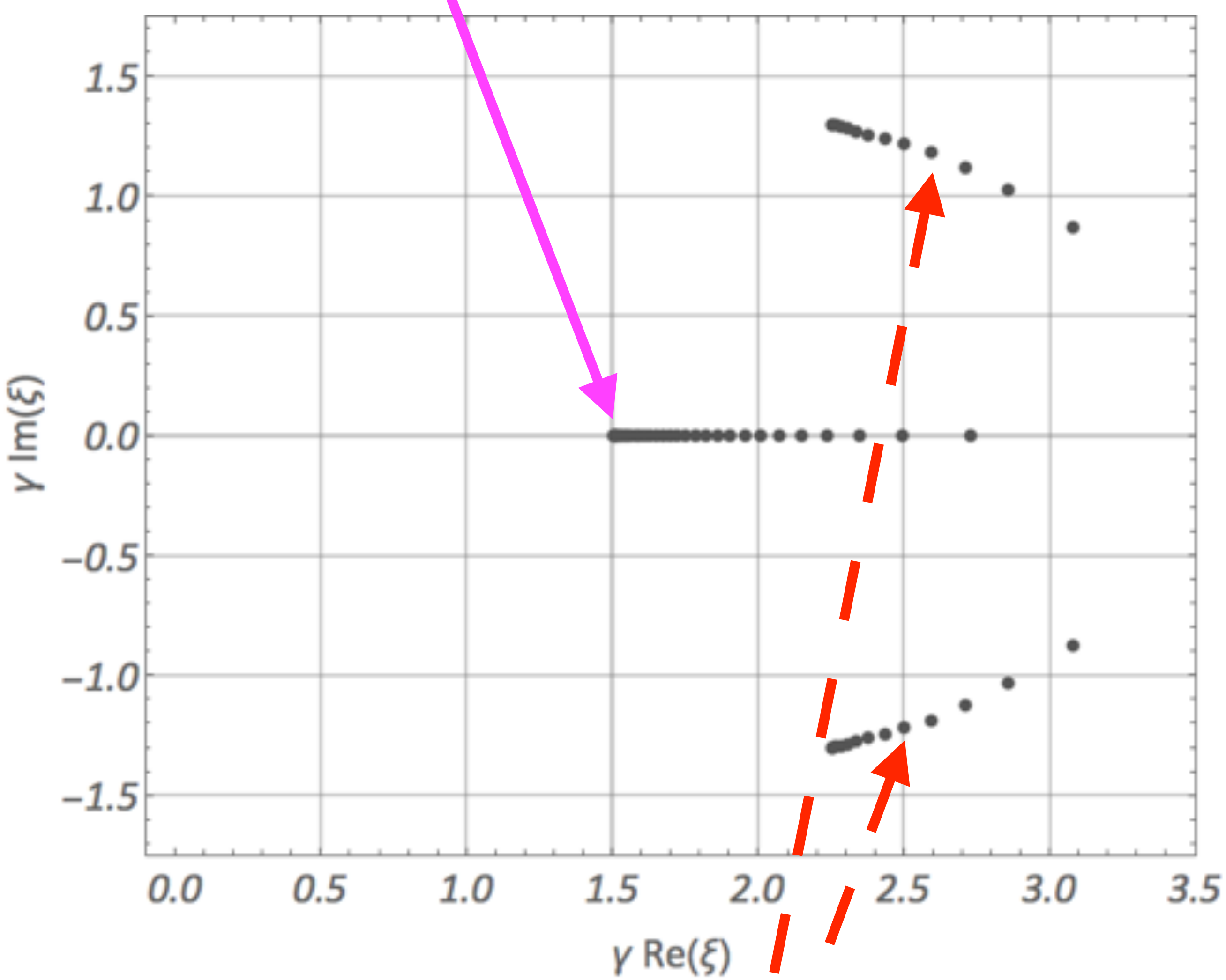
$\downarrow k \to 0$

single pole at $\omega = -i\,\frac{1}{\tau_{rel}}$

QNM in kinetic theory

1609.04803 with Kurkela & Spalinski

$$\xi_{sing} = \frac{3}{2\gamma} \rightarrow \underline{\text{assuming}}\ \text{sing.} \sim \left(\xi - \frac{3}{2\gamma}\right)^{\beta} \rightarrow \delta R \sim \exp\left(-\frac{3}{2\,\gamma}\right) w^{-1.43} (\ldots)$$



$$\delta R \sim \exp\left(-\frac{2.25}{\gamma} \pm \frac{1.3}{\gamma}\, i\right) \ ???$$

Seeing leading transient in dynamics

work in progress with Svensson

$$H(s) = \frac{s^2}{2} + \frac{\arctan\sqrt{\frac{1}{s^2}-1}}{2\sqrt{\frac{1}{s^2}-1}}$$

$$e^{-\frac{1}{\gamma}\int_{\tau_0}^{\tau} T(\tau'')d\tau''}$$

f_{ini}

$$T^4(\tau) = D(\tau,\tau_0)\frac{\pi^2\mathcal{E}^0(\tau)}{6} + \int_{\tau_0}^{\tau} d\tau' \left(\frac{T(\tau')}{\gamma}D(\tau,\tau')\right) \times \left(T^4(\tau')\, H\left(\frac{\tau'}{\tau}\right)\right)$$

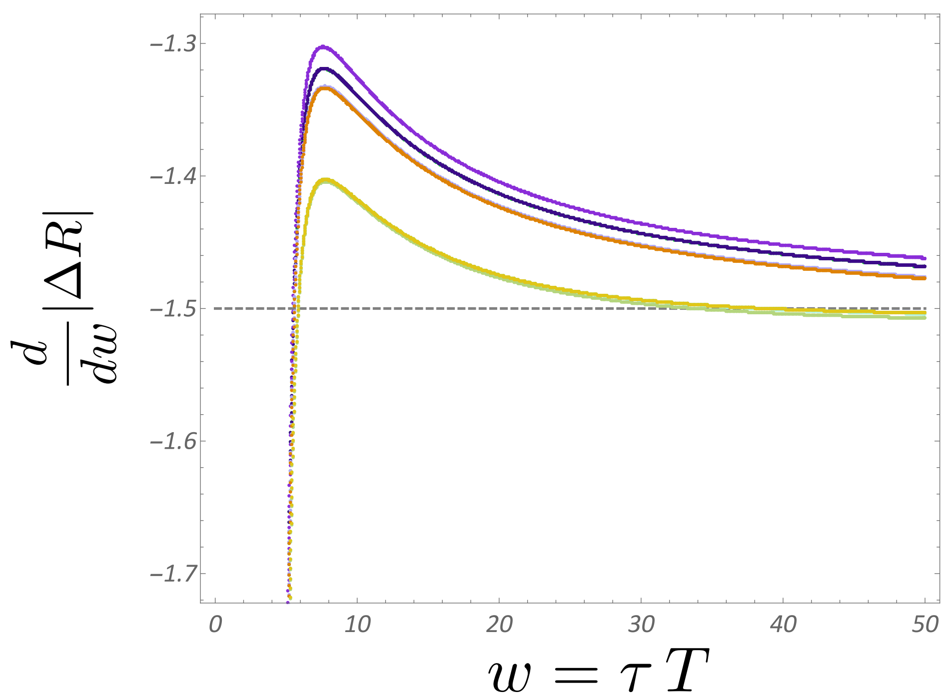
Baym 1984; 1305.7234 by Florkowski, Ryblewski & Strickland

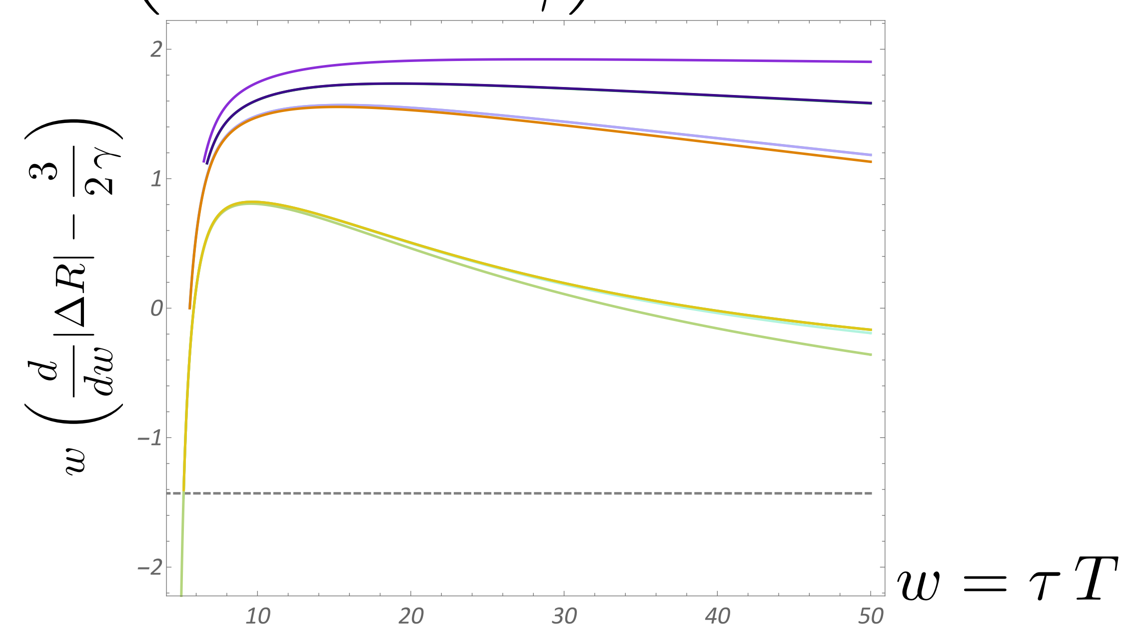
We assume the same as in BRSSS: $\delta R \sim e^{-A\,w}\, w^{\beta}\left(1 + O\left(\frac{1}{w}\right)\right) + \ldots$

Instead of δR we take ΔR and consider:

$$\frac{d}{dw}|\Delta R| \sim A \ ? \quad \sim \checkmark$$

$$w\left(\frac{d}{dw}|\Delta R| - \frac{3}{2\,\gamma}\right) \sim \beta \ ? \quad \times$$





How freedom of choice of f_{ini} maps into R(w)

→ infinite tower of modes in BR(ξ)?

→ many coincident branch pts at $\frac{3}{2\,\gamma}$?

Executive summary

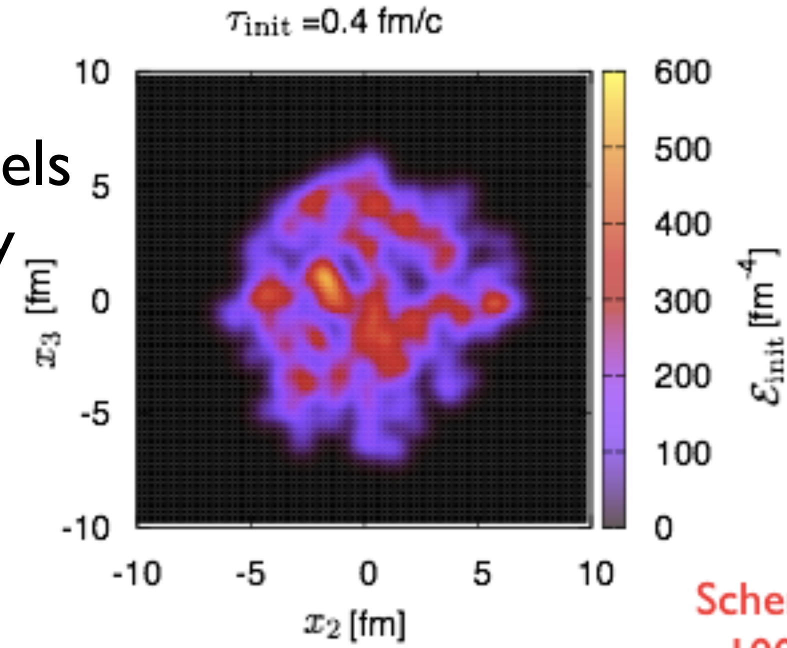
1103.3452 with Janik & Witaszczyk
1302.0697 with Janik & Witaszczyk
1603.05344 with Buchel & Noronha
1503.07514 with Spalinski
1609.04803 with Kurkela & Spalinski

1707.02282 [hep-ph] review with Florkowski and Spalinski

HIC pheno:

hydrodynamization suggests **using simple hydro models under extreme conditions is not completely crazy**

related: ~~**equilibration**~~ **in HICs???**



Schenke et al. 1009.3244

new effects (???):
large contributions from other transport? $(-\zeta \Delta^{\mu\nu} \nabla \cdot u)$

hydrodynamic gradient expansion diverges

1707.02282 [hep-ph] review with Florkowski and Spalinski

towards genericity:
2 flows and
$\infty + \infty$ hQFTs + RTA + MIS + aHYDRO + linear response

new connections:
resurgent series (also in QM & QFT)
gradient expansion as a part of transseries