The Bicolorist's Tale

Master Hands of Sweyns Ey

Canterbury Tales of Hot QFTs,
Oxenford 13th July 2017

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- Why two colors?
- Bulk thermodynamics for $\mu, T \neq 0$: number/energy densities, pressure, trace anomaly, quark number susceptibility
- Order parameters: superfluidity & deconfinement
- lessons from the attoworld
- Phase diagram
- Spectroscopy, wavefunctions, topology

Why Two Colors?

(PDG only recognises 3)

- Chance to explore systematics of lattice simulations at $\mu \neq 0$

Good news: cutoff fixed as μ varies, no quantum corrections to $n_q = -\partial f/\partial\mu$

Bad news: UV/IR classical artifacts are complicated enough

- Chance to explore "deconfinement" in a new physical régime

- No sign problem stupid!

Why no Sign Problem for QC_2D?

Let K be complex conjugation, and T unitary

If $\exists$ KT s.t. $[KT,M]=0$, then detM is real

ie. $M\psi=\lambda\psi \Rightarrow M\varphi\equiv M(KT\psi)=KT\lambda\psi=\lambda^*\varphi$ so λ,λ^* both in spectrum of M

But is it positive?

Consider real eigenvalues $\lambda\equiv\lambda^*$?

2 cases labelled by **Dyson index**:

$\beta=4$: $(KT)^2 = -1$: $\langle\psi|\varphi\rangle=\langle\psi|KT\psi\rangle=\langle T\psi|TKT\psi\rangle$

$=\langle(KT)^2\psi|KT\psi\rangle=-\langle\psi|\varphi\rangle=0$

$\Rightarrow$ degenerate real eigenvalues $\Rightarrow$ detM > 0

$\beta=1$: $(KT)^2 = +1$: $\langle\psi|\varphi\rangle\neq 0$

$\Rightarrow$ non-degenerate real eigenvalues $\Rightarrow$ Sign Problem!

for N odd

So for QC_2D....

	Continuum/Wilson fermions	Staggered fermions (a>0)
Fundamental (2)	$T = C\gamma_5 \otimes \tau_2$	$T = 1_4 \otimes \tau_2$
$(KT)^2$	+1	-1
χSB	SU(2N)→Sp(2N)	U(2N)→O(2N)
Adjoint (3)	$T = C\gamma_5 \otimes 1_2$	$T = 1_4 \otimes 1_2$
$(KT)^2$	-1	+1
χSB	SU(2N)→O(2N)	U(2N)→Sp(2N)

Staggered fermions away from the weak-coupling continuum limit describe a *different* universality class

See also:
6 of SU(4)
7 of G_2
QCD with $\mu_{isospin} \neq 0$

Note that for $(KT)^2 = +1$ isolated real eigenvalues give a potential ergodicity problem, since only way to change sgn(detM) is to flow through origin

What goes wrong with the usual positive HMC measure?

$$\text{det}M^\dagger M \begin{cases} M & \text{describes} \qquad\qquad\qquad \text{quarks } q \in 3 \\ M^\dagger & \text{describes conjugate quarks } q^c \in \bar{3} \end{cases}$$

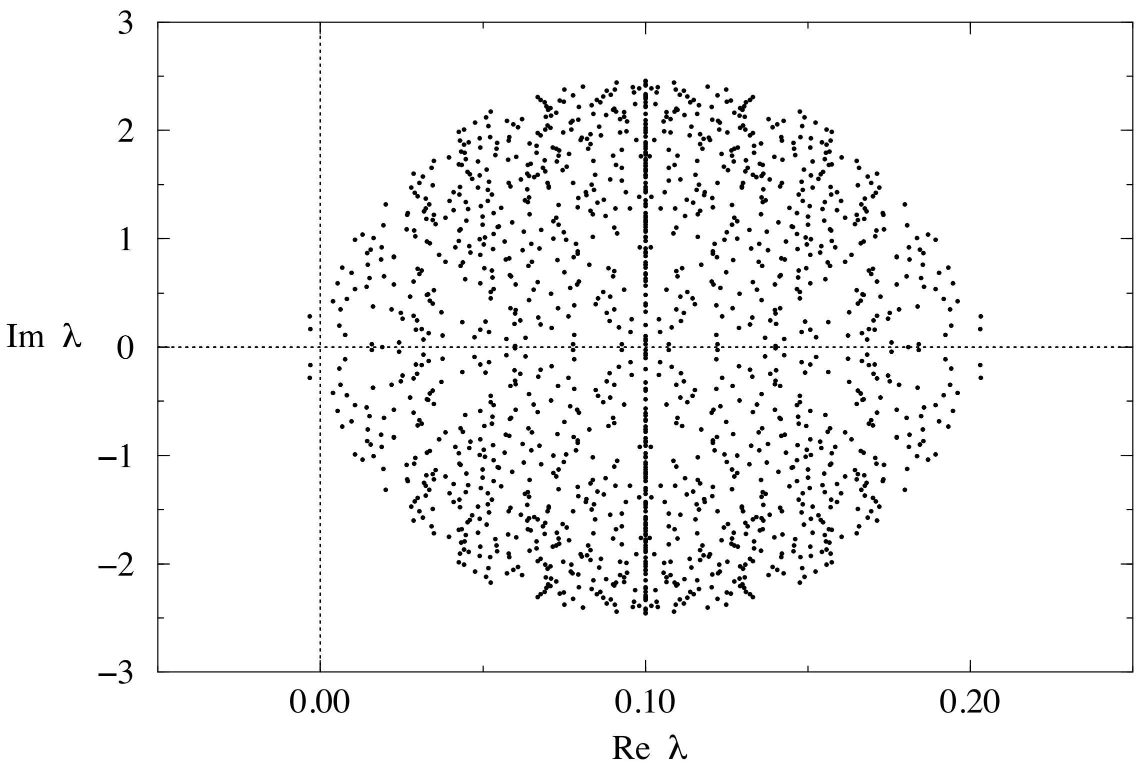
In general $\exists\ qq^c$ gauge singlet bound states with $B > 0$

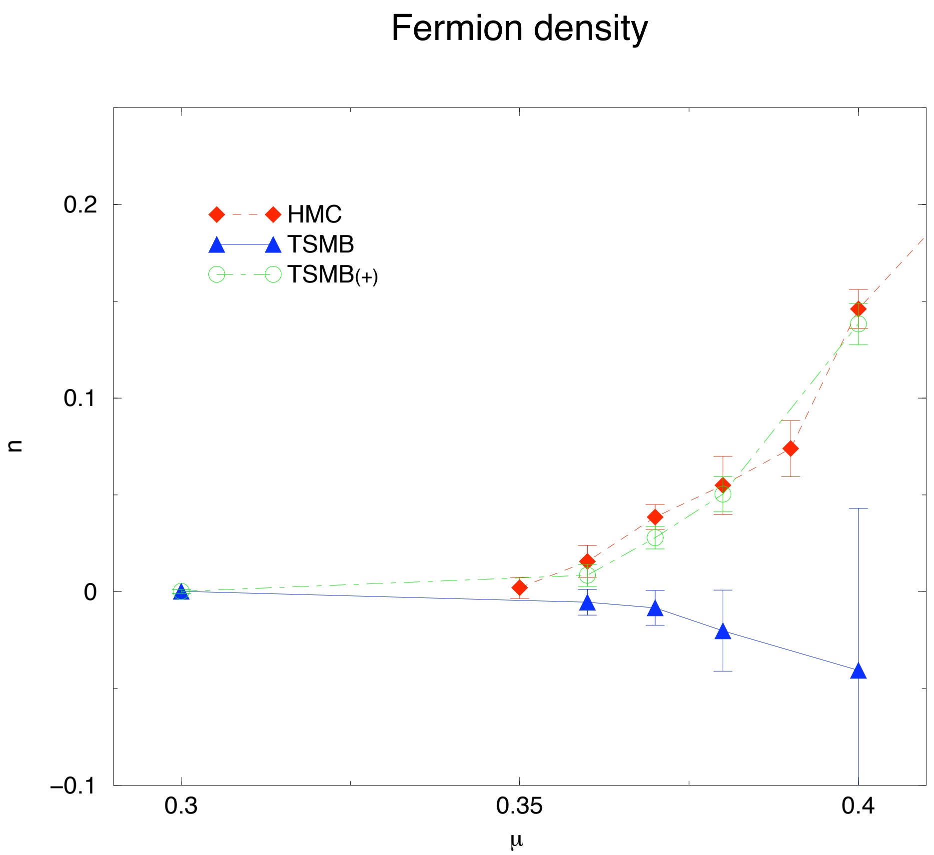
In QCD some qq^c states degenerate with the pion
$\Rightarrow$ unphysical onset of "nuclear matter" at $\mu_o \simeq \frac{1}{2}m_\pi$.

Goldstone baryons: bug for QCD, feature for QC$_2$D...

Calculations with the true complex measure $\text{det}^2 M$ nullify effects of qq^c states for the vacuum with $T = 0$, $\frac{1}{2}m_\pi < \mu \lesssim \frac{1}{3}m_N$ by cancellations among configurations with different signs/phases

The *Silver Blaze* Problem...





This has been numerically verified, eg. in TSMB simulations of Two Color QCD with $N = 1$ adjoint staggered quarks. ie. β=1

SJH,Montvay,Scorzato,Skullerud, EurPJ C22 (2001) 451

The fake transition to a superfluid phase, forbidden by the Pauli Principle, at $\mu_o a \simeq 0.35$ disappears once configurations with det$M < 0$ are included with the correct weight.

QC_2D – the large N_c^{-1} limit

QCD with gauge group SU(2) and an even N_f of fundamental quarks has a real positive functional measure even once $\mu \neq 0$. It is the simplest system of dense matter with long-ranged interactions amenable to LGT simulation.

Hadron multiplets contain both $q\bar{q}$ mesons and $qq, \bar{q}\bar{q}$ (anti-)baryons. For $m_\pi \ll m_\rho$ the μ-dependence can be studied using chiral effective theory.

Key result: for $\mu \geq \mu_o = \frac{1}{2} m_\pi$ a baryon charge density $n_q > 0$ develops, along with a gauge-invariant scalar isoscalar **superfluid** condensate $\langle qq \rangle \neq 0$.

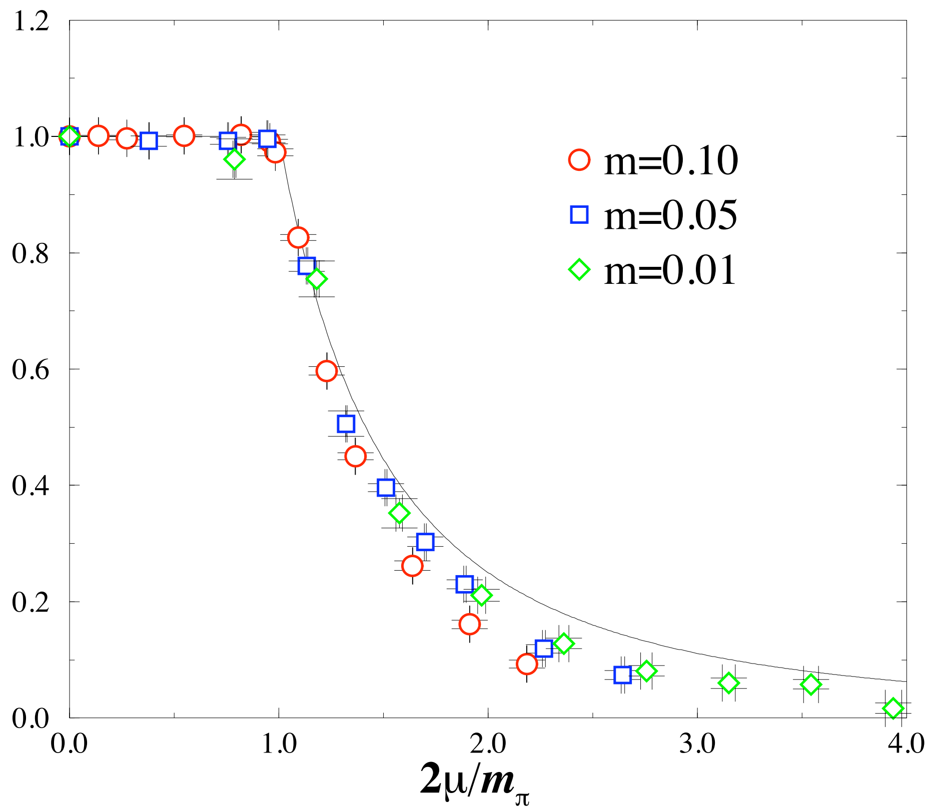
For $\mu \gtrsim \mu_o$ the system is a BEC consisting of dilute weakly-interacting 0^+ qq diquarks.

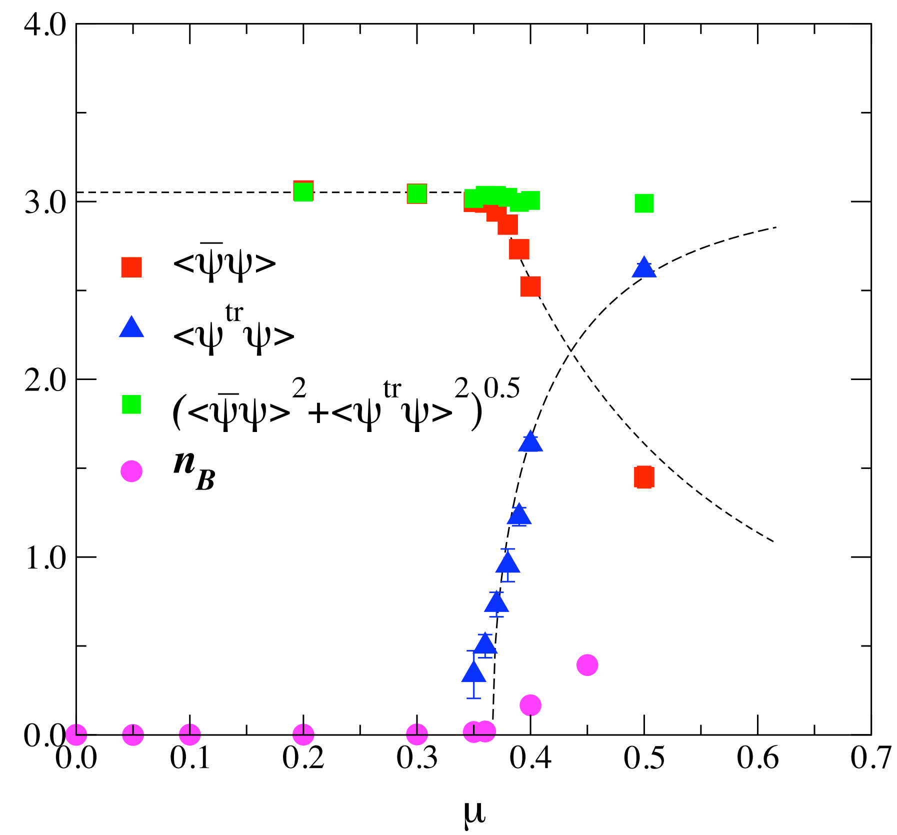
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu\left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

confirmed by QC_2D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

See also Braguta et al PRD94 (2016)205147

Thermodynamics at $T = 0$ from χPT

quark number density $n_{\chi PT} = 8 N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4 N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

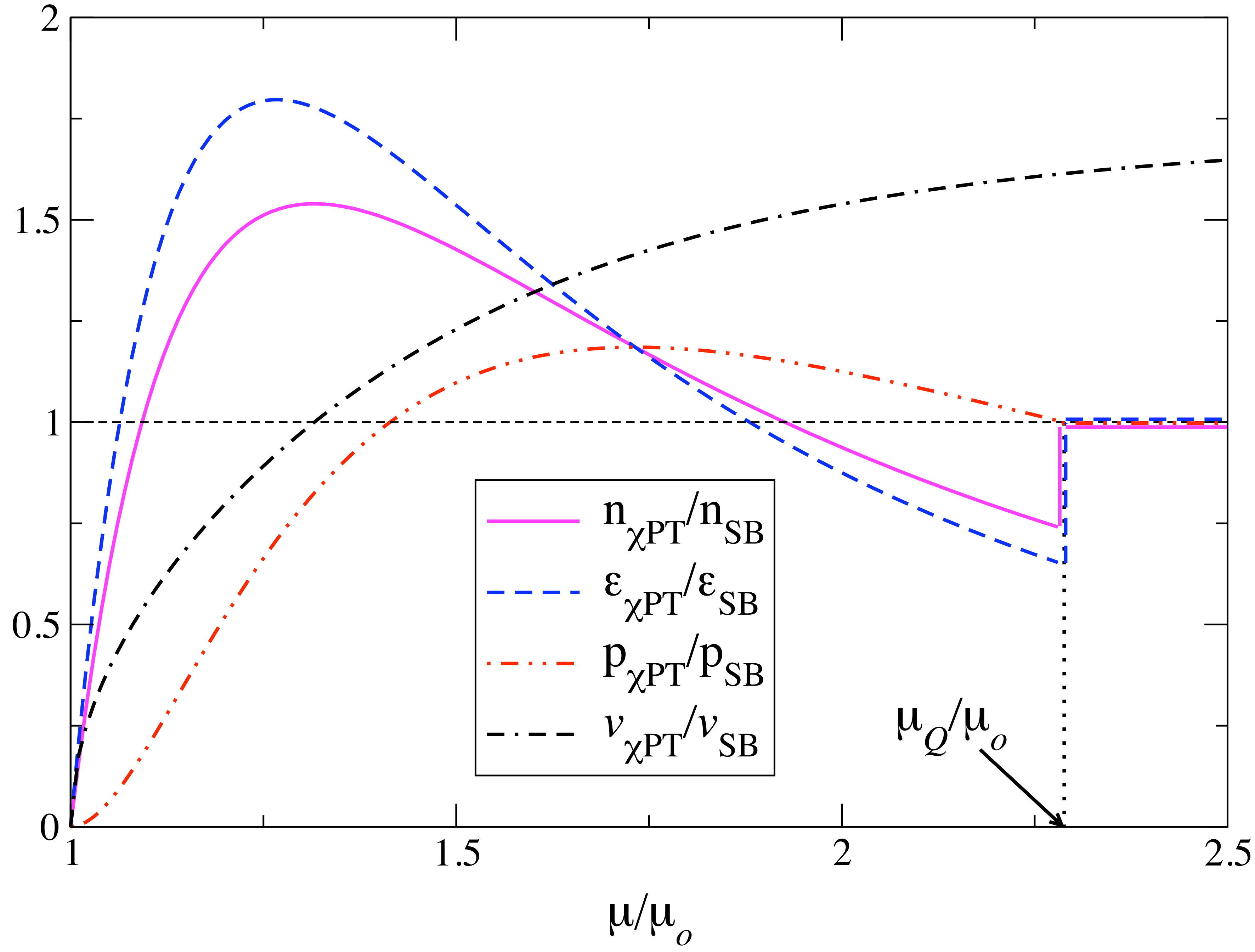
energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4 N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

conformal anomaly

$$(T_{\mu\mu})_{\chi PT} = \varepsilon - 3p = 8 N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1 - \frac{\mu_o^4}{\mu^4}}{1 + 3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter; eg. for $f_\pi^2 = N_c/6\pi^2$, $\quad \mu_d \approx 2.3\mu_o$.

This is to be contrasted with another paradigm for cold dense matter, namely a degenerate system of weakly interacting (deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow \quad n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4;$$

$$\delta_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \quad \langle qq \rangle \propto \Delta\mu^2$$

Simulation Details ($N_f = 2$ Wilson flavors)

SJH, S. Kim & J.I Skullerud, EPJC48 (2006) 193; PRD81 (2010) 091502(R)
S. Cotter, P. Giudice, SJH & J.I Skullerud, PRD87 034507 (2013)
Boz, Cotter, Fister, Mehta & Skullerud, EPJA49 (2013) 87

Machines range from u/g lab PCs to IBM BlueGene

		a(fm)	$m_\pi a$	m_π/m_ρ	T(MeV)
coarse	8^3x16	0.229(3)	0.78(1)	0.804(10)	55(1)
medium	12^3x24	0.178(6)	0.645(8)	0.805(9)	47(2)
fine (new)	16^3x32	0.13	0.45	0.81	49

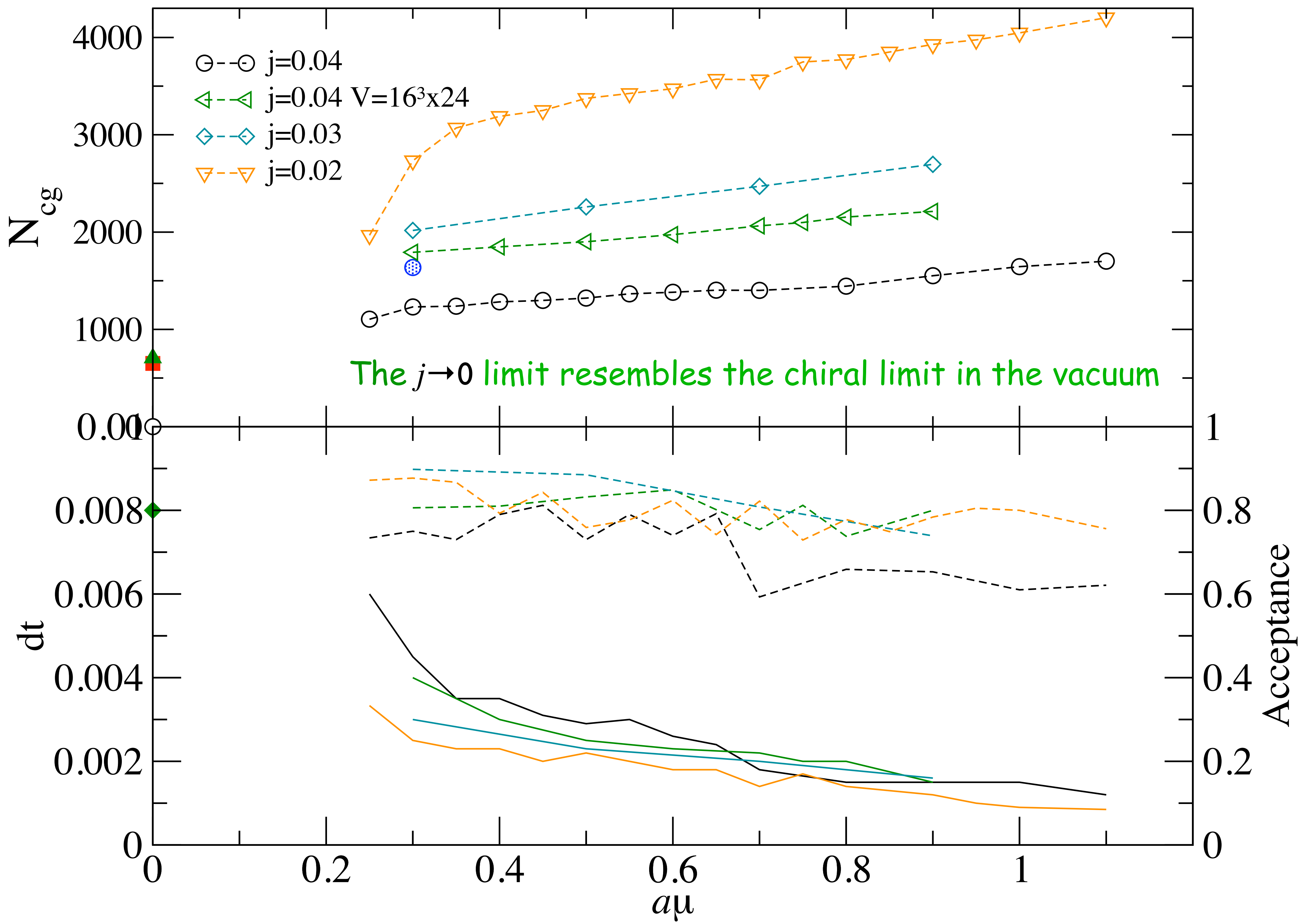
also have μ-scans on 12^3x16, 16^3x20,....,8 ⇒ T = 47,70,94,141 MeV
16^3x32,....,12 ⇒ T = 49,79,99,131 MeV

To counter IR fluctuations and maintain HMC ergodocity, we introduce a diquark source term $j\kappa(\psi_2^{tr} C\gamma_5\tau_2\psi_1 - \bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^{tr})$

Have results for ja=0.04 everywhere

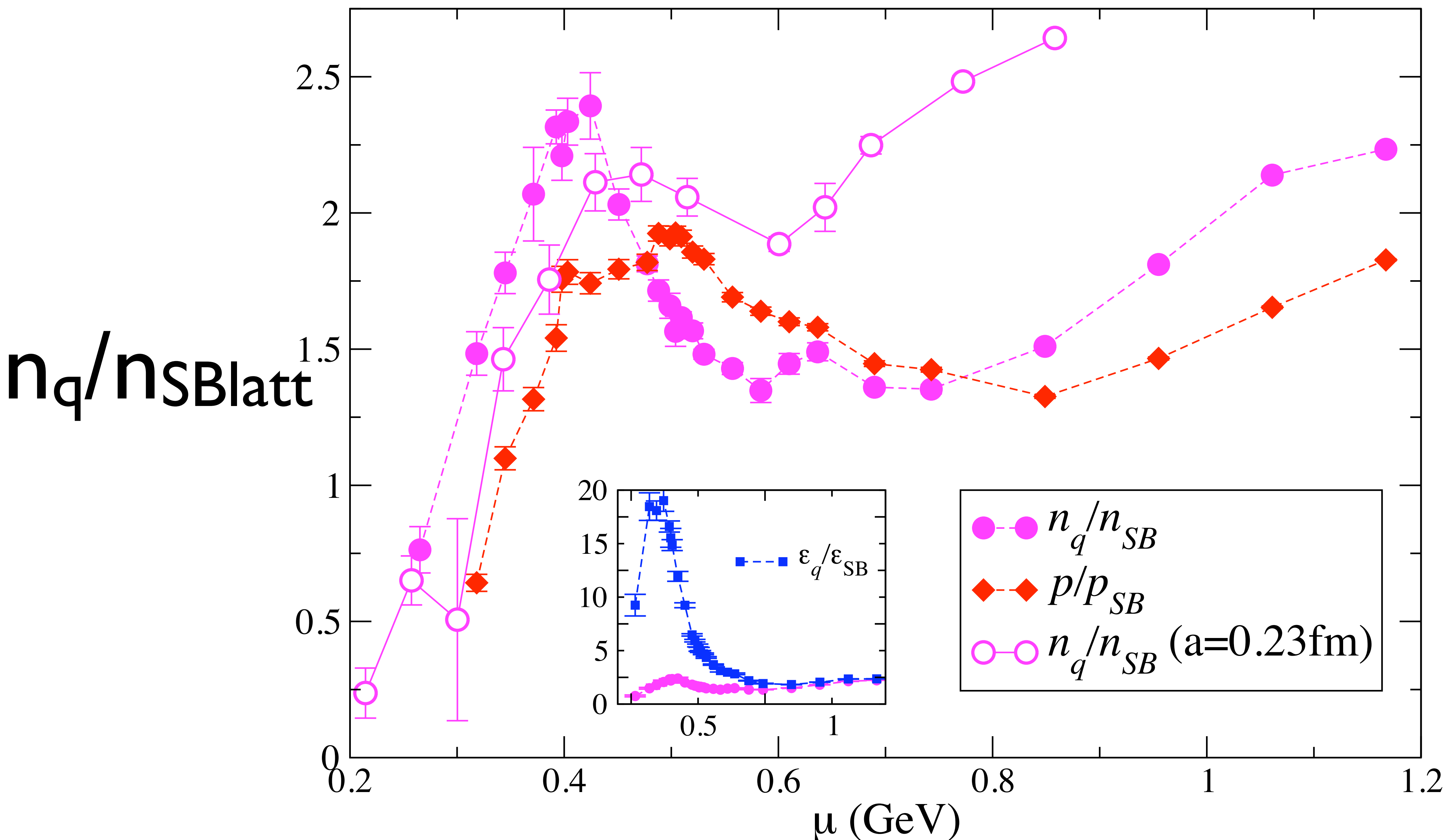
to enable $j \to 0$ have ja=0.02, 0.03 at selected points

Computer Effort (*sans* Sign Problem!)



The number of **`congrad`** iterations required for convergence during HMC guidance rises with μ ⇔ accumulation of small eigenvalues of M?

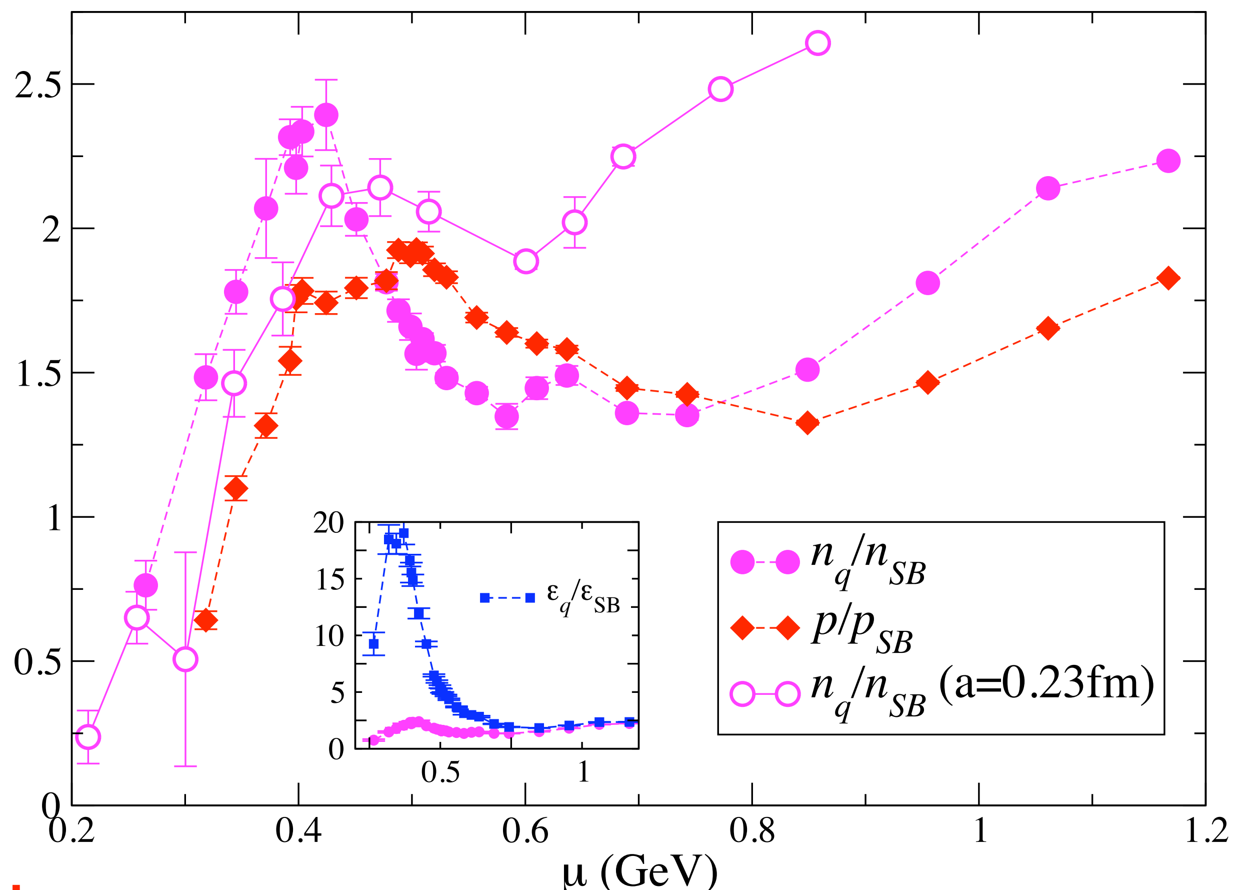
Equation of State on Fine Lattice (12^3x24, ja=0.04)

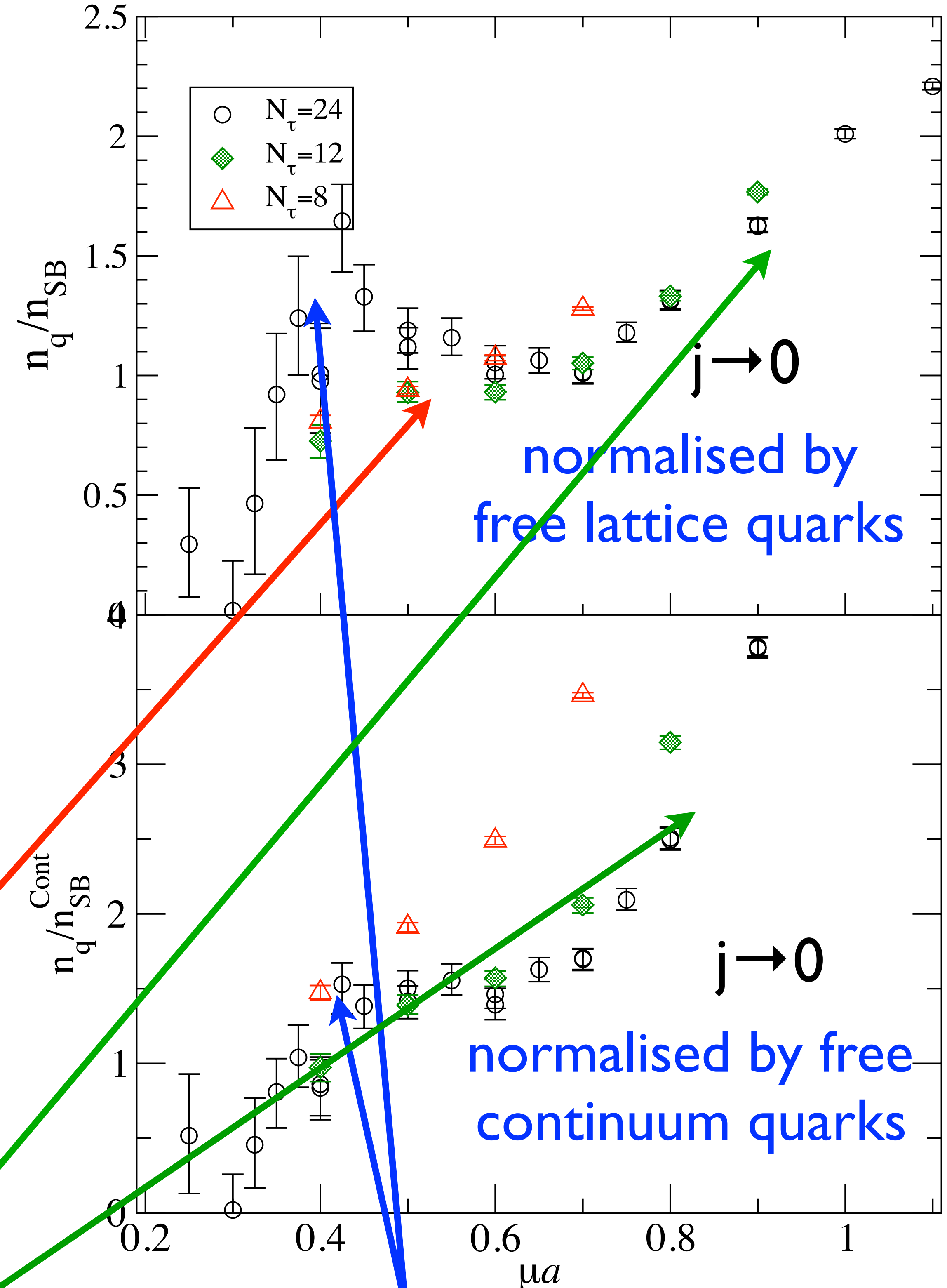


Identify:

onset	$\mu_o \approx$ 360MeV
crossover to "quarkyonic phase"	$\mu_Q \approx$ 530MeV $n_q \approx$ 4 - 5 fm^{-3}
"deconfinement"	$\mu_d \approx$ 850MeV $n_q \approx$ 16 - 32 fm^{-3}

Artifacts





However:

(a) the j→0 extrapolation gives large corrections at small μ, so plateau closer to non-interacting value

j≠0 promotes diquark pairing

significant correction for interacting quarks

(b) the peak above onset at low T is very sensitive to IR artifacts (non-sphericity of Fermi surface)

$T \ll \Delta k = 2\pi/L_s$

significant correction for free *lattice* quarks

(c) UV artifacts are present at larger μ

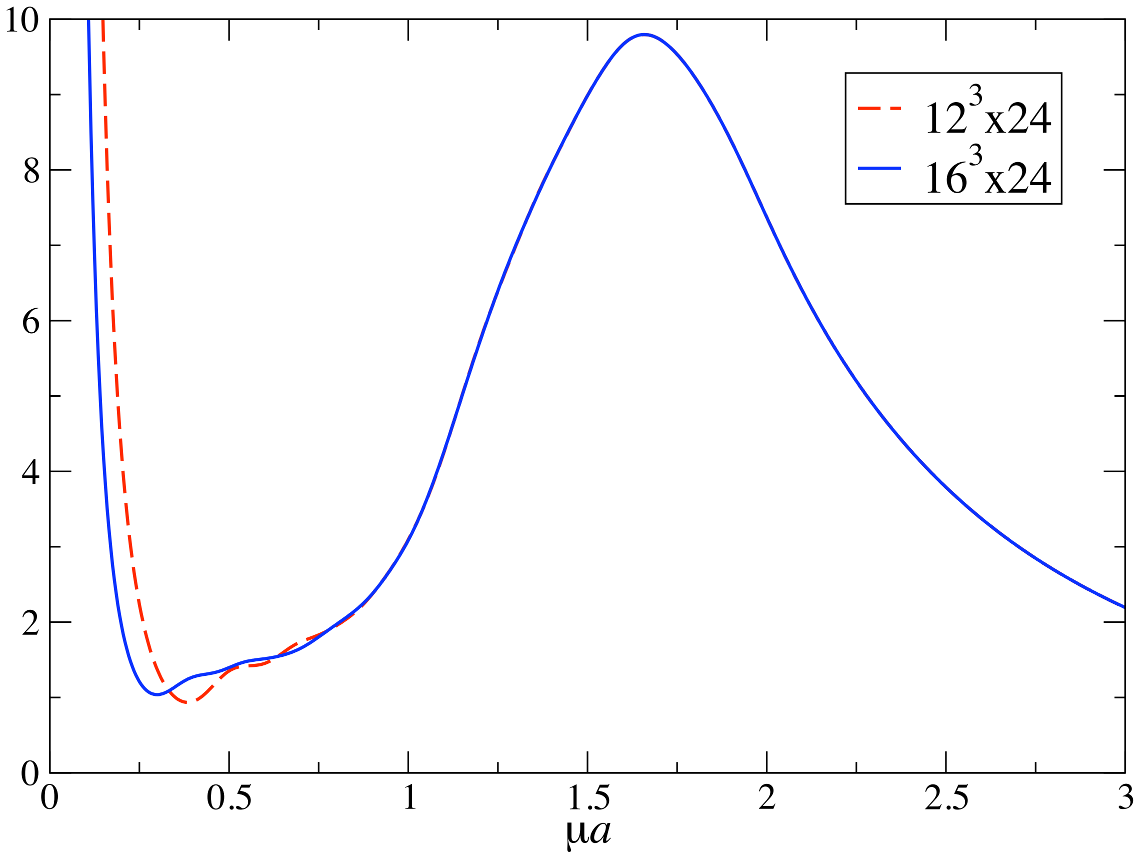
free *lattice* quark correction

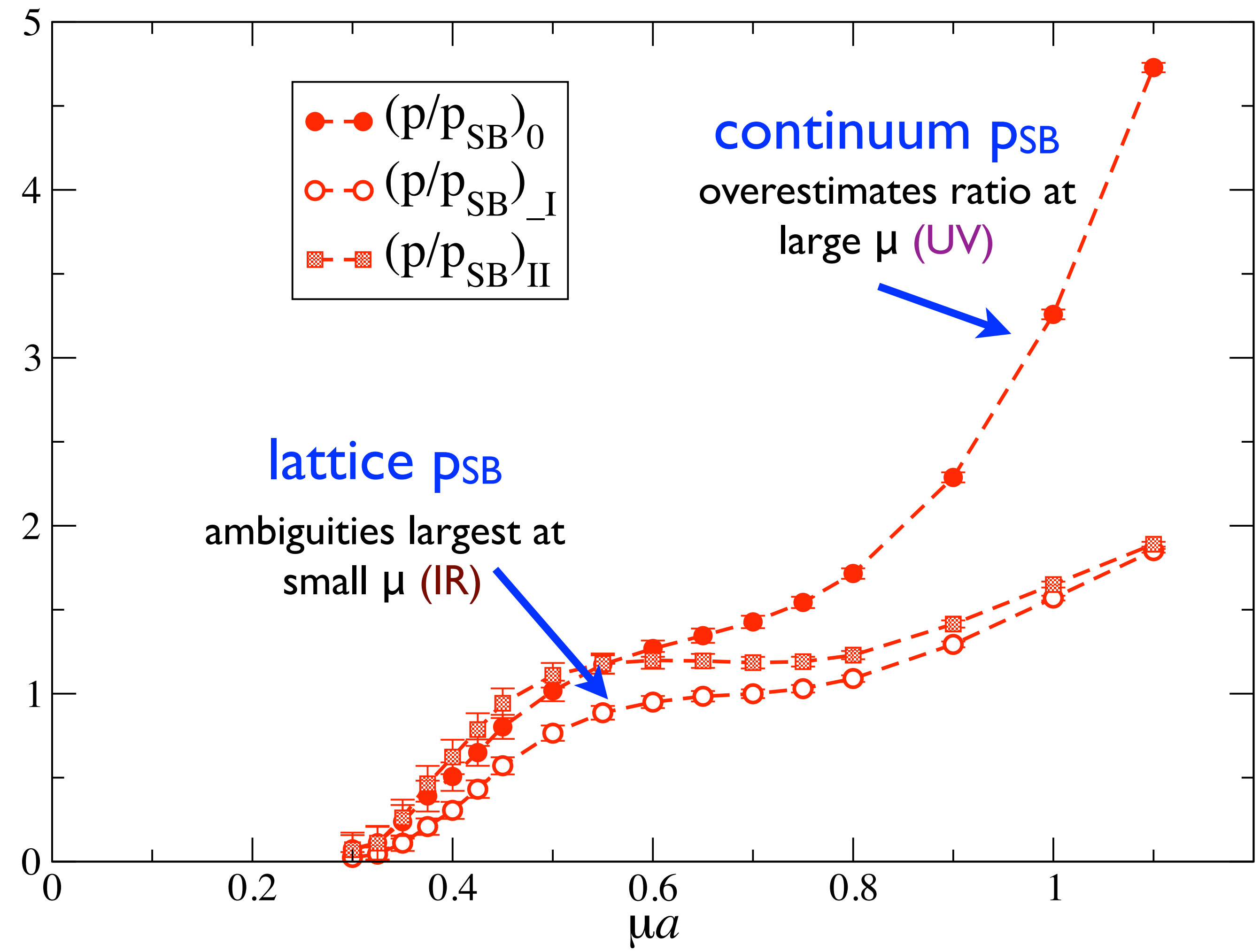
more reliable here

Pressure for $j \to 0$ on $12^3 \times 24$

$$p = \int^{\mu} n_q \, d\mu$$

n_{SB}(latt)/n_{SB}(continuum)





Robust: Still see onset at $\mu_o \approx$ 360MeV

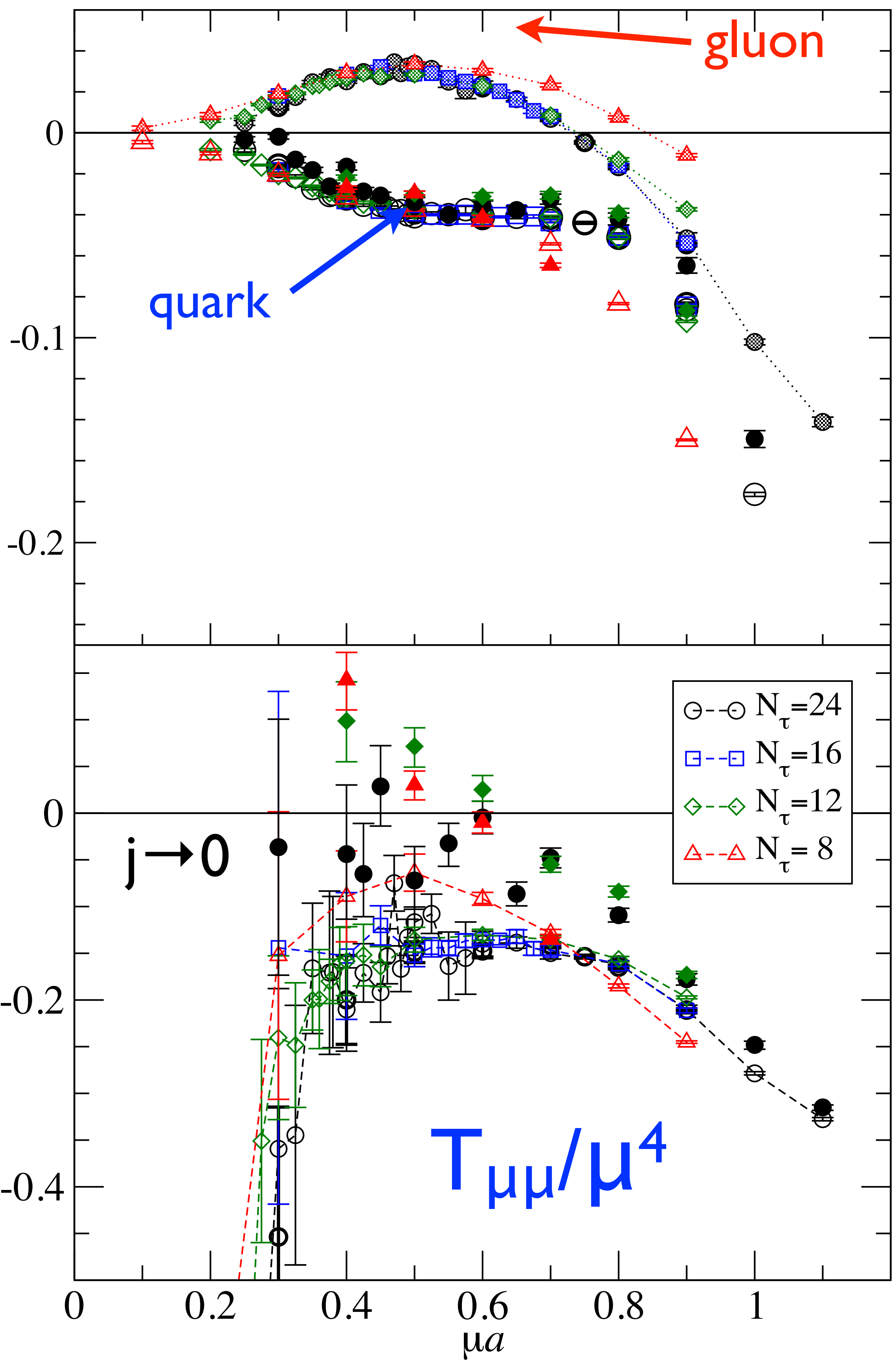
Transition to "quark matter" at $\mu_Q \approx$ 530MeV $E_F \approx k_F$

"Deconfinement" sets in at $\mu_d \approx$ 850MeV $E_F < k_F$

But: no longer any firm evidence for a BEC "peak" just above onset

Conformal Anomaly

$T_{\mu\mu} = \varepsilon\text{-}3p$



$$(T_{\mu\mu})_g = -a\frac{\partial\beta}{\partial a}\bigg|_{LCP} \times \frac{3\beta}{N_c}\mathrm{Tr}\langle\Box_t + \Box_s\rangle;$$
$$(T_{\mu\mu})_q = -a\frac{\partial\kappa}{\partial a}\bigg|_{LCP} \times \kappa^{-1}(4N_fN_c - \langle\bar\psi\psi\rangle)$$

Quark and gluon contributions:

almost cancel for $\mu<\mu_Q$: conformal?

differ for $\mu>\mu_Q$

$T_{\mu\mu} < 0$ for $\mu \gtrsim \mu_Q$

$(T_{\mu\mu})_q$ changes sharply at $\mu_d \approx$ 850MeV

$\Rightarrow \varepsilon<3p$ in limit $\mu\to\infty$

consistent with self-binding?

Calculation of Energy Density

$$\varepsilon = -\frac{1}{V}\frac{\partial Z}{\partial T^{-1}}\Big|_V = -\frac{\xi}{N_s^3 N_\tau a_s^3 a_\tau}\left\langle \frac{\partial S}{\partial \xi}\Big|_{a_s}\right\rangle$$ with $$\xi \equiv \frac{a_s}{a_\tau}$$ physical anisotropy

anisotropic action

$$\mathcal{L} = -\frac{\beta}{N_c}\left[\frac{1}{\gamma_g}\Box_s + \gamma_g \Box_\tau\right] + \bar{\psi}\left[1 + \gamma_q \kappa D_0[\mu] + \kappa \sum_i D_i\right]\psi$$

$$\Rightarrow \quad \frac{\varepsilon_g}{T^4} = \frac{3N_\tau^4}{\xi^2 N_c}\left[\langle \Box_s \rangle \left(\gamma_g^{-1}\frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \gamma_g^{-1}}{\partial \xi}\right) + \langle \Box_\tau \rangle \left(\gamma_g \frac{\partial \beta}{\partial \xi} + \beta \frac{\partial \gamma_g}{\partial \xi}\right)\right]$$

$$\Rightarrow \quad \frac{\varepsilon_q}{T^4} = -\frac{N_\tau^4}{\xi^2}\left[\sum_i \langle \bar{\psi} D_i \psi \rangle \frac{\partial \kappa}{\partial \xi} + \langle \bar{\psi} D_0 \psi \rangle \left(\gamma_q \frac{\partial \kappa}{\partial \xi} + \kappa \frac{\partial \gamma_q}{\partial \xi}\right)\right]$$

Karsch coefficients

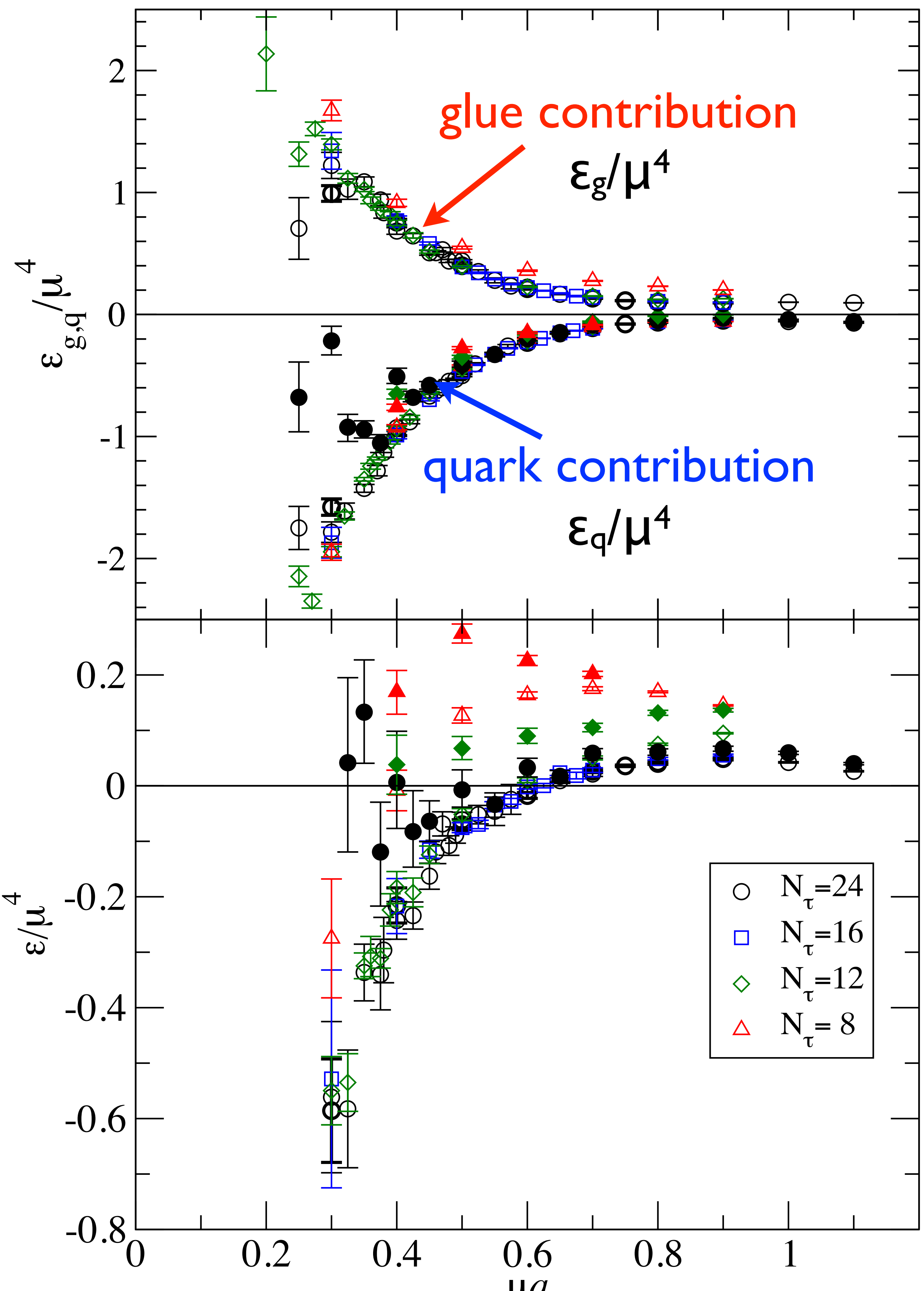
$$\frac{\partial \beta}{\partial \xi}; \quad \frac{\partial \gamma_g}{\partial \xi}; \quad \frac{\partial \kappa}{\partial \xi}; \quad \frac{\partial \gamma_q}{\partial \xi}$$

estimated at $\xi=1$, $\mu=T=0$
by simulating with
$\gamma_g=1\pm\delta\gamma_g$, $\gamma_q=1\pm\delta\gamma_q$
and assuming linear response

ξ_g from "sideways potential", ξ_q from pion dispersion

Levkova, Manke & Mawhinney, PRD73 (2006) 074504; R. Morrin (TCD thesis)

Energy densities



ε_q/μ^4 now negative for all μ -

no more peak!

again, consistent with self-binding.

(indeed ε only barely positive for smaller μ)

Results very sensitive to values of Karsch coefficients (particularly $\frac{\partial\kappa}{\partial\xi}; \frac{\partial\gamma_q}{\partial\xi}$) $\Rightarrow$

systematic error O(100%)?

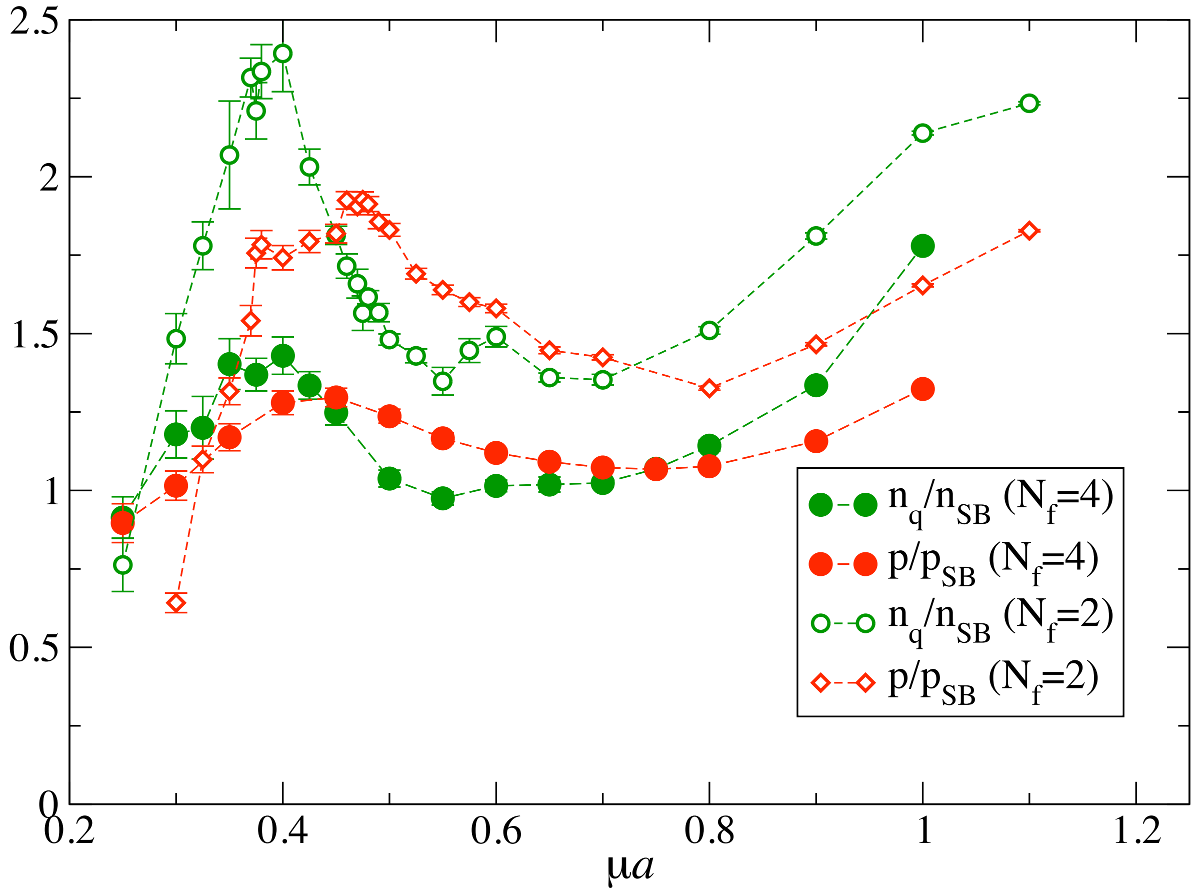
$j \rightarrow 0$ limit is key!

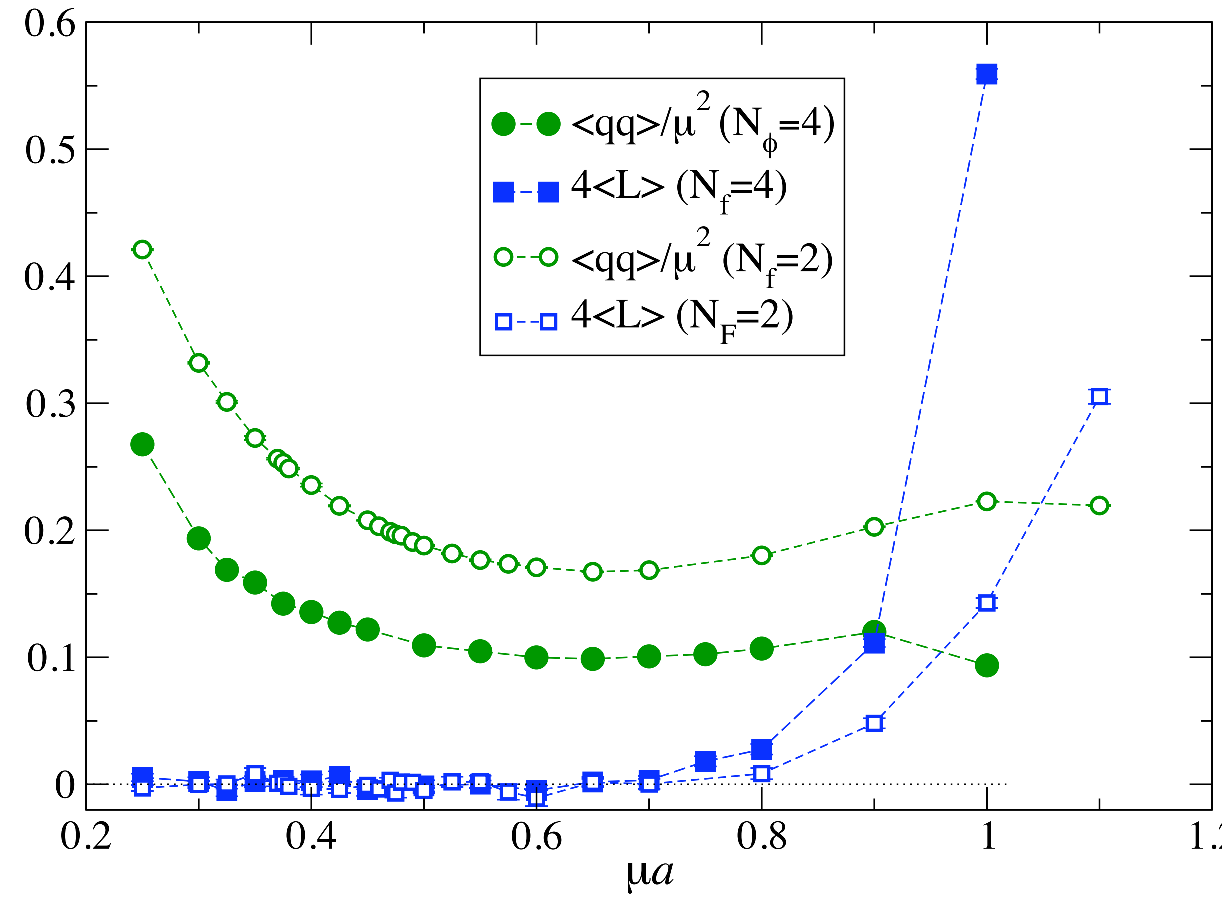
BUT qualitatively similar to bare ε found for $N_f = 4$

Note $a^{N_f=4} \approx ⅓\, a^{N_f=2}$

SJH, P. Kenny and J.I. Skullerud, EPJA 47 (2011) 60

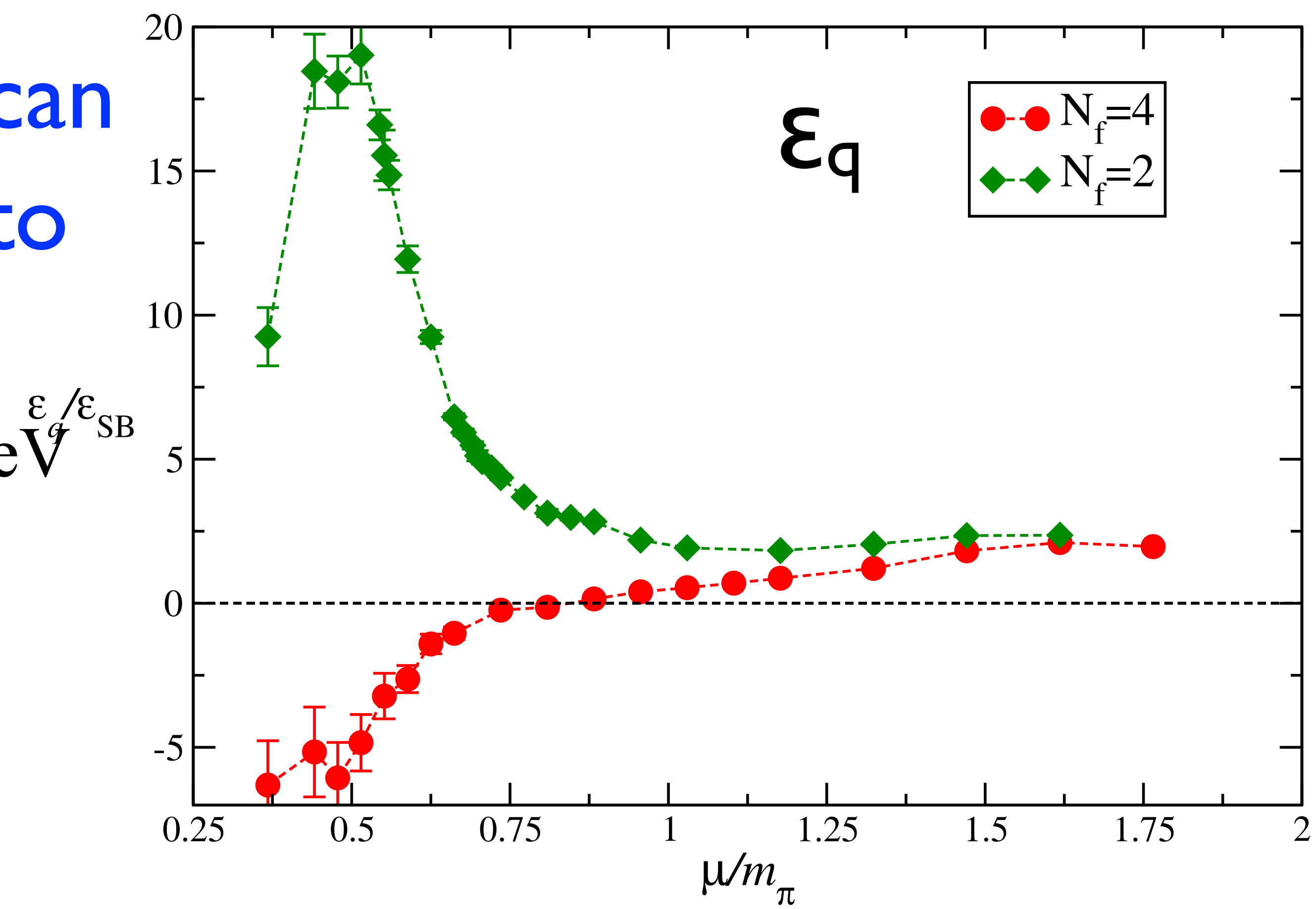
And $N_f=4$? SJH, P. Kenny, S. Kim & J.I. Skullerud, EPJA47 (2011) 60





Same distinct physical regimes can be identified but *much* closer to continuum: a=0.062(2)fm

$\Rightarrow$ μ_Q≈1.5GeV, μ_d≈2.5GeV, T=133(4)MeV

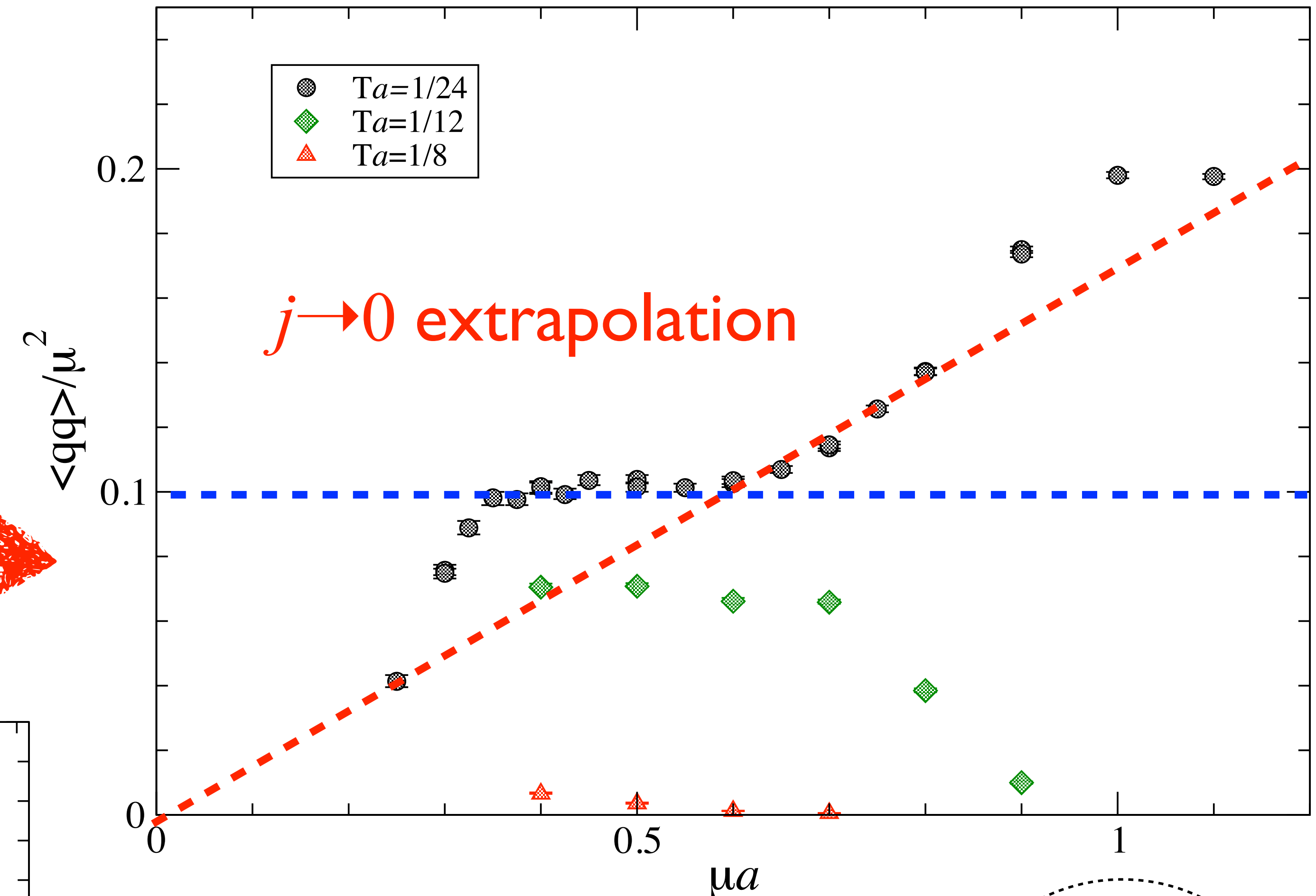


Negative ε_q consistent with renormalised result at $N_f=2$

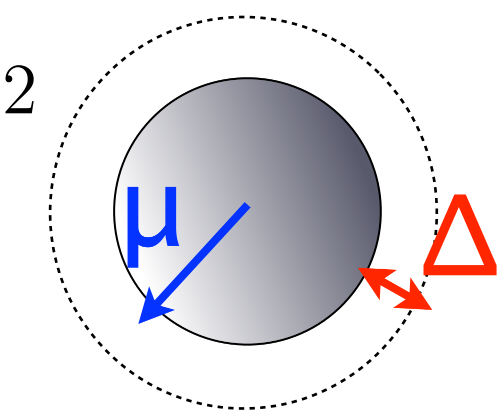
Order parameters

Superfluid condensate $\langle qq \rangle$ scales à la BCS (ie $\propto \mu^2$) for $\mu_Q \leq \mu \leq \mu_d$

Vanishes as T increases

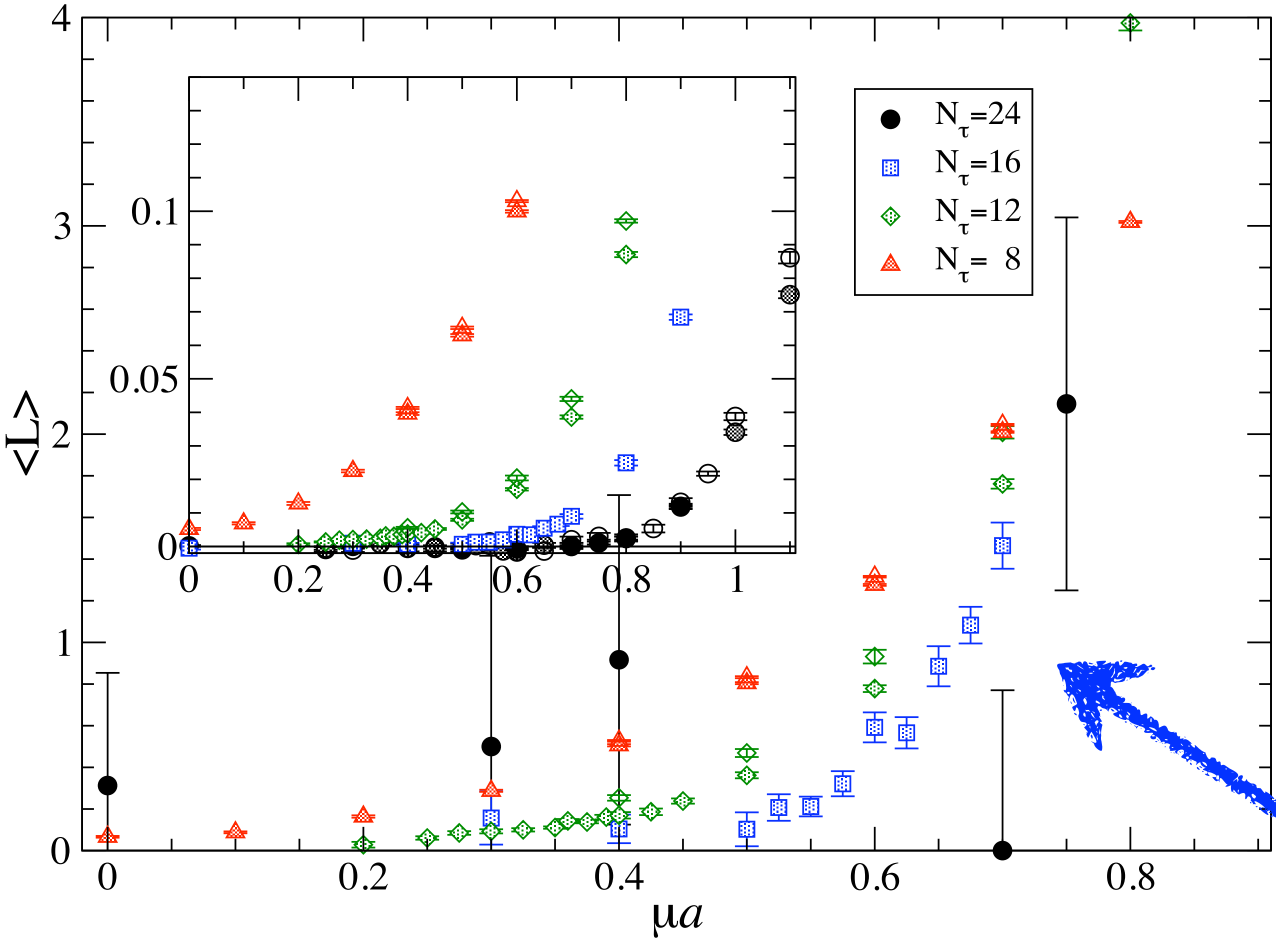


$$\langle qq(\mu) \rangle \propto \Delta(\mu)\mu^2$$



$\mu \lesssim \mu_d \quad \Delta \propto \Lambda_{QCD}$

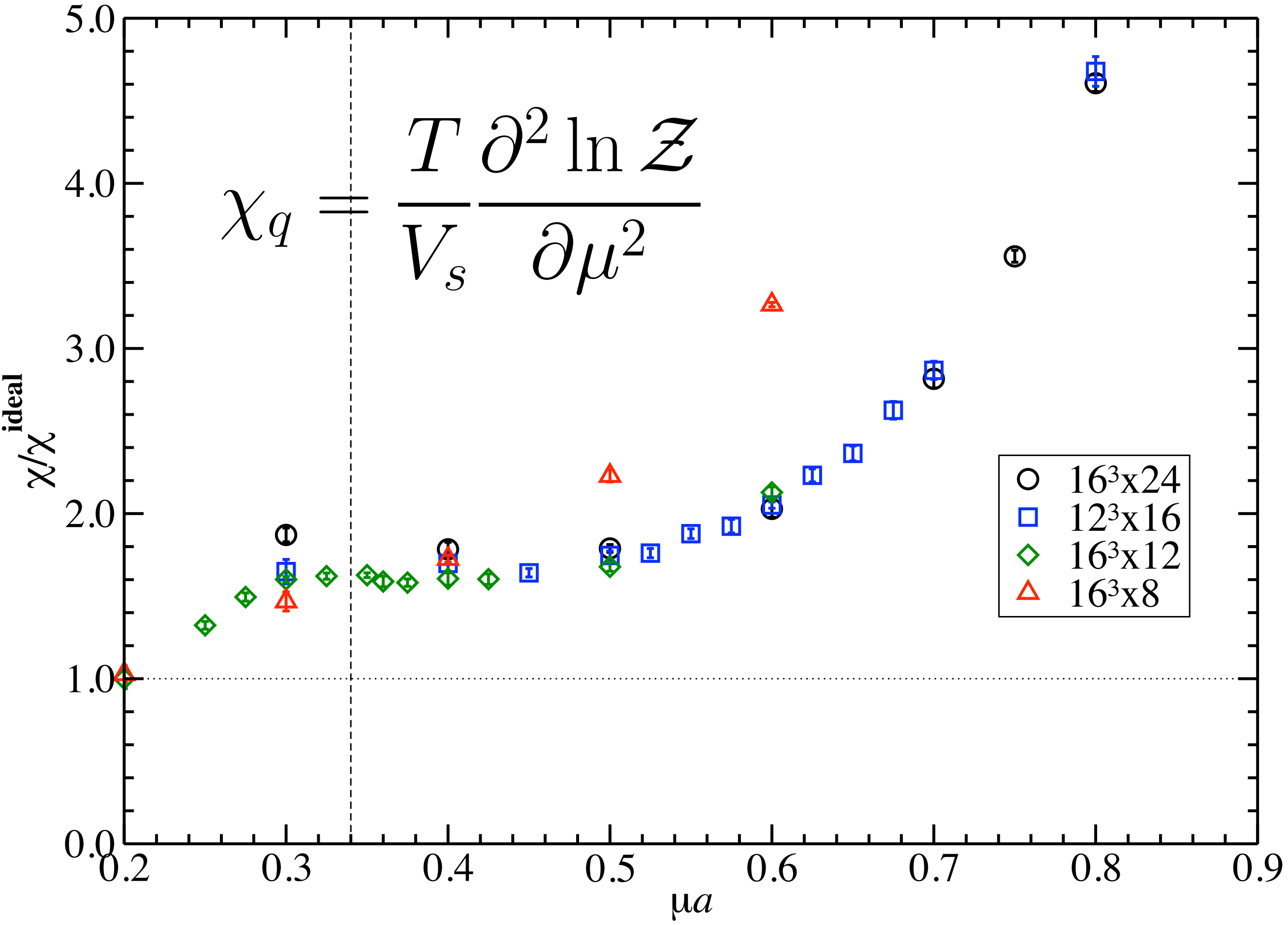
$\mu \gtrsim \mu_d \quad \Delta \propto \mu$

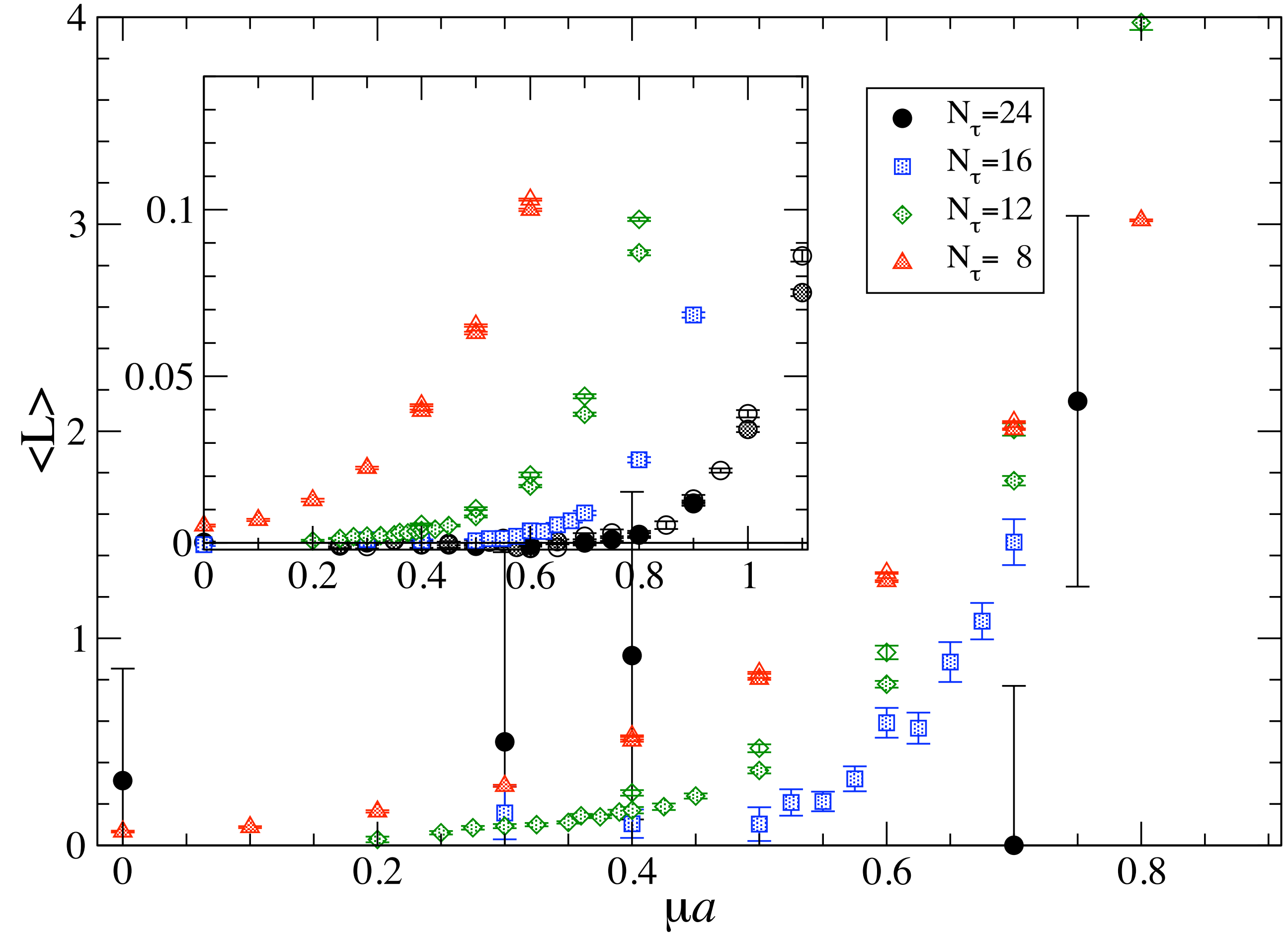


(renormalised) Polyakov line rises from zero at $\mu \approx \mu_d$

⇒ **Deconfinement** at $\mu_d \approx$ 850MeV $n_q \approx$ 16-32 fm^{-3}

Quark Number susceptibility $\chi_q(\mu)$ does not show same T-dependence as the Polyakov loop L





The increase in χ_q is **not** associated with "deconfinement"

So χ_q is ***not*** a proxy for L when $\mu/T >> 1$

Qualitatively different from:

(a) the thermal QCD phase transition

Aoki et al. PLB643 (2006), Borsanyi et al. JHEP 1009 (2010) 073
Bazavov et al. PRD80 (2009) 014504

(b) strong coupling with heavy quarks

Fromm, Langelage, Lottini, Neuman, Philipsen PRL110 (2013) 12200

(c) analytic/numerical studies on small, cold volumes (the "attoworld")

SJH, J. Myers, T.J. Hollowood, JHEP 1007 (2010) 086, 1012 (2010) 057

Two Color Attoworld

w/ Joyce Myers, Tim Hollowood

JHEP 1007 (2010) 086, 1012 (2010) 057

Consider theory on $S^3 x S^1$, with (hyper)sphere radius $R << \Lambda_{QCD}$

$\Rightarrow$ WCPT applicable

At one loop, can consistently retain just the gluon zero mode $A_0^{\text{const}} = \beta^{-1}\text{diag}(\theta_1, \ldots, \theta_{N_c})$

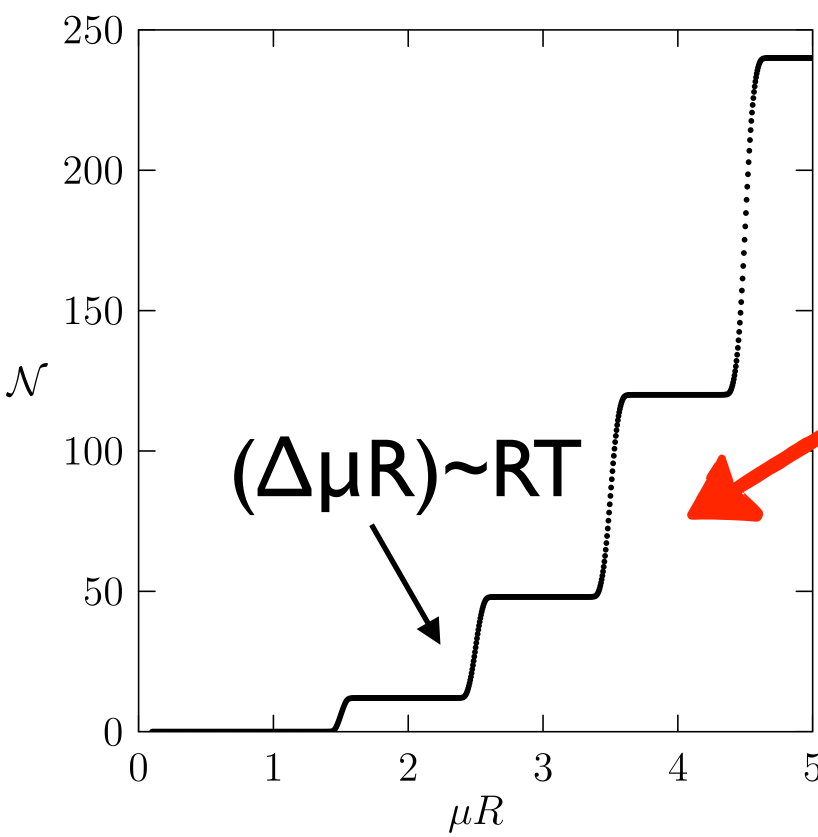
$$\Rightarrow \quad S(\theta) = \sum_{n=1}^{\infty} \frac{1}{n}\left(1 - \sum_{\ell=1}^{\infty} 2\ell(\ell+1)e^{-n\beta(\ell+1)/R}\right) \times \sum_{ij=1}^{N_c} \cos(n(\theta_i - \theta_j))$$

$$+N_f \sum_{n=1}^{\infty} \frac{(-)^n}{n} \sum_{\ell=1}^{\infty} 2\ell(\ell+1)e^{-n\beta\sqrt{(\ell+\frac{1}{2})^2+m^2R^2}/R} \times \sum_{i=1}^{N_c} 2\cosh(n\beta\mu + in\theta_i)$$

gluon bit

quark bit

this term couples chemical potential to Polyakov line (CF. PNJL)

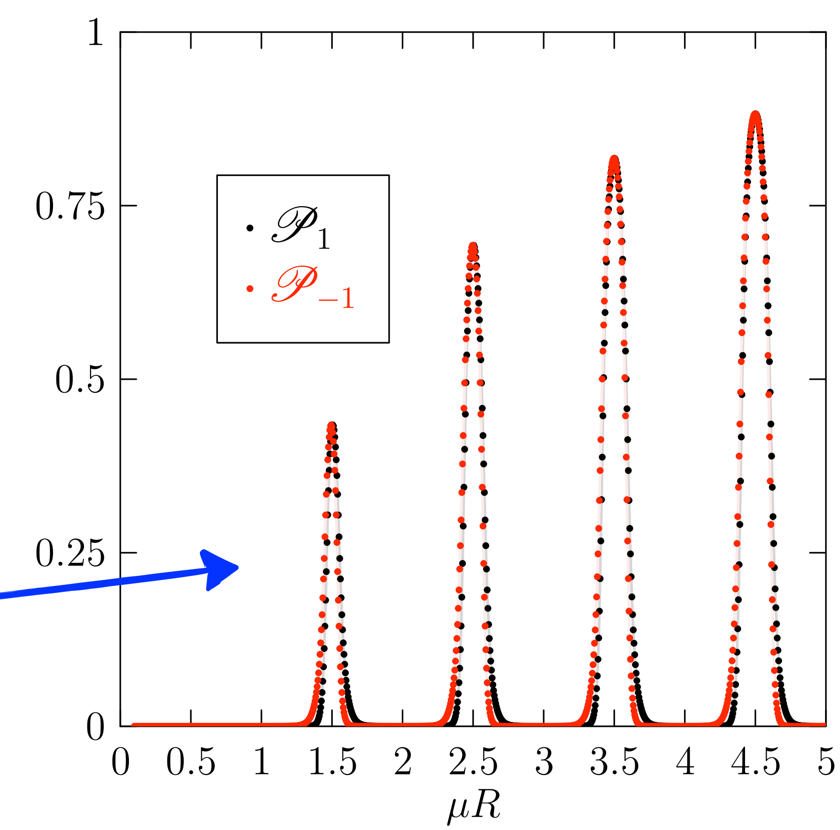


Quark density rises stepwise as finite "shells" are filled with occupancy

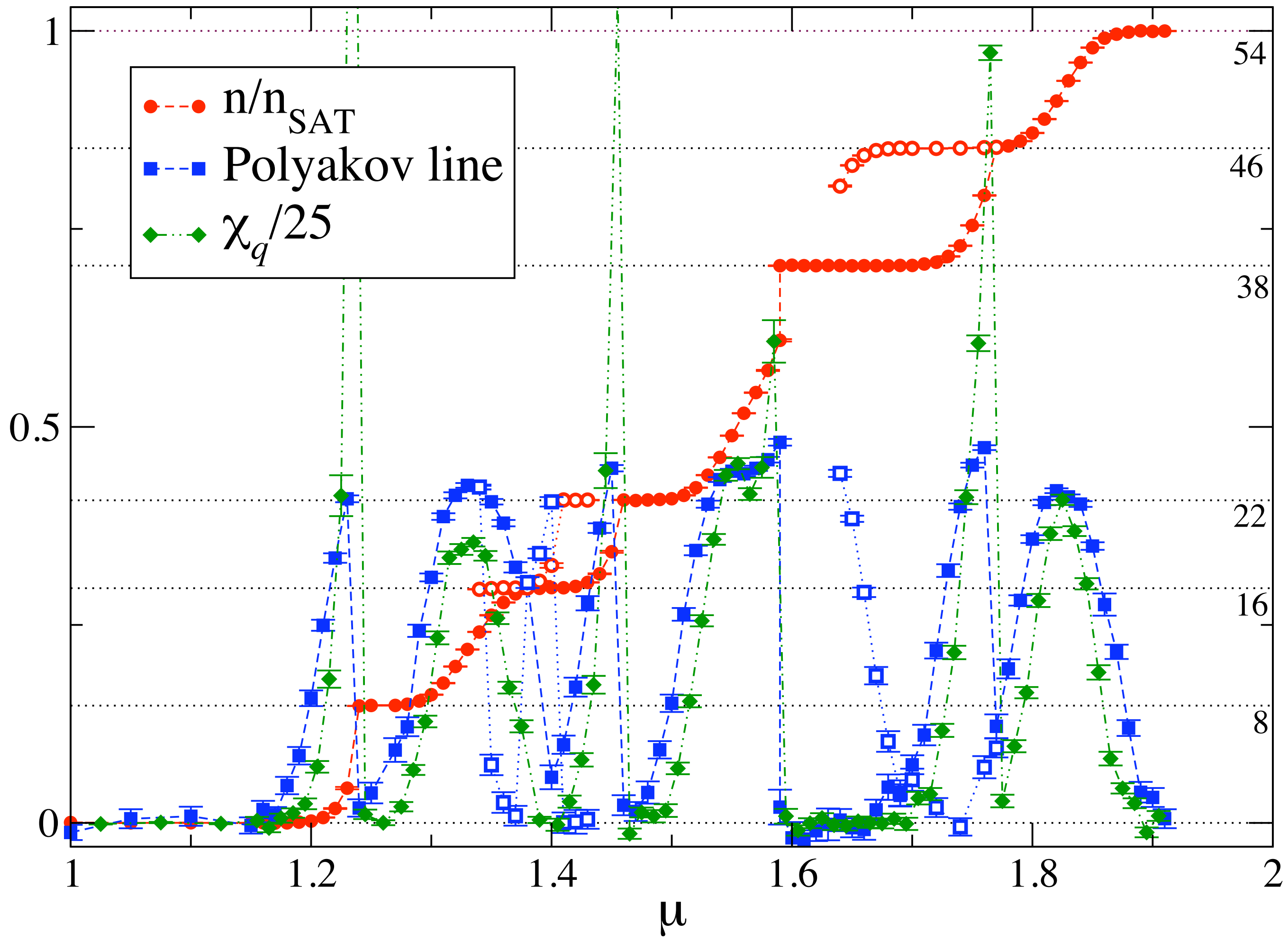
$$\mathcal{N}_L = N_c N_f \sum_{\ell=1}^{L} 2\ell(\ell+1)$$

Polyakov line indicates alternating ranges of confinement and deconfinement

$\Leftrightarrow$ partially-filled shell



Insight to all orders on $(S^1)^4$ from the lattice



$3^3 \times 64$, $\beta=24.0$, $\kappa=0.124$, $j=0$

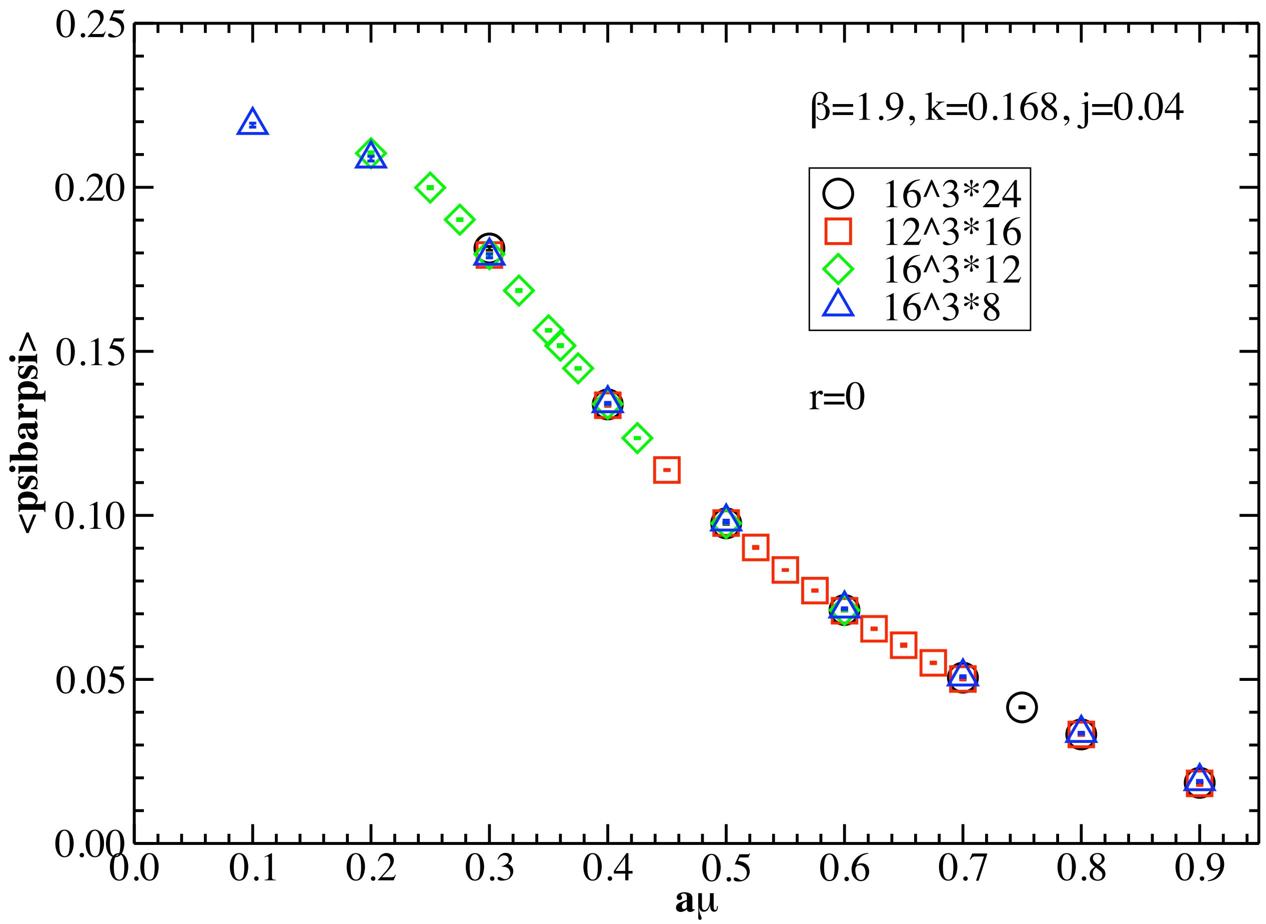
Qualitatively consistent, suggests deconfinement associated with non-vanishing density of states at Fermi energy

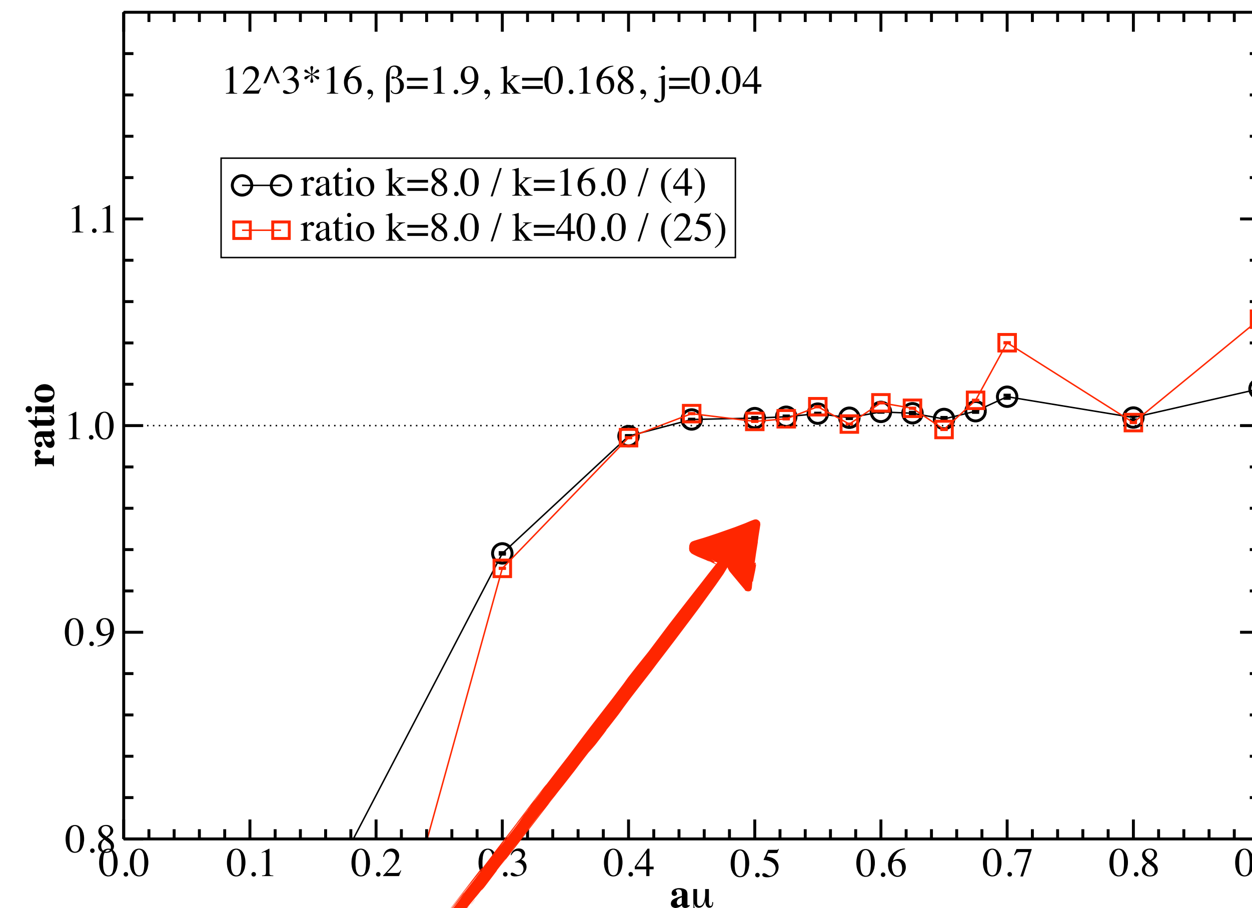
BUT shell degeneracies **not** those of single particle states

So in the attoworld, deconfinement and a rise in χ_q **are** correlated

Conjecture: at low T deconfinement requires massless excitations at the Fermi surface, so is inhibited by a superfluid gap $\Delta > 0$

And chiral symmetry?....





χPT prediction: $\langle\bar\psi\psi\rangle \propto \frac{m}{\mu^2}$

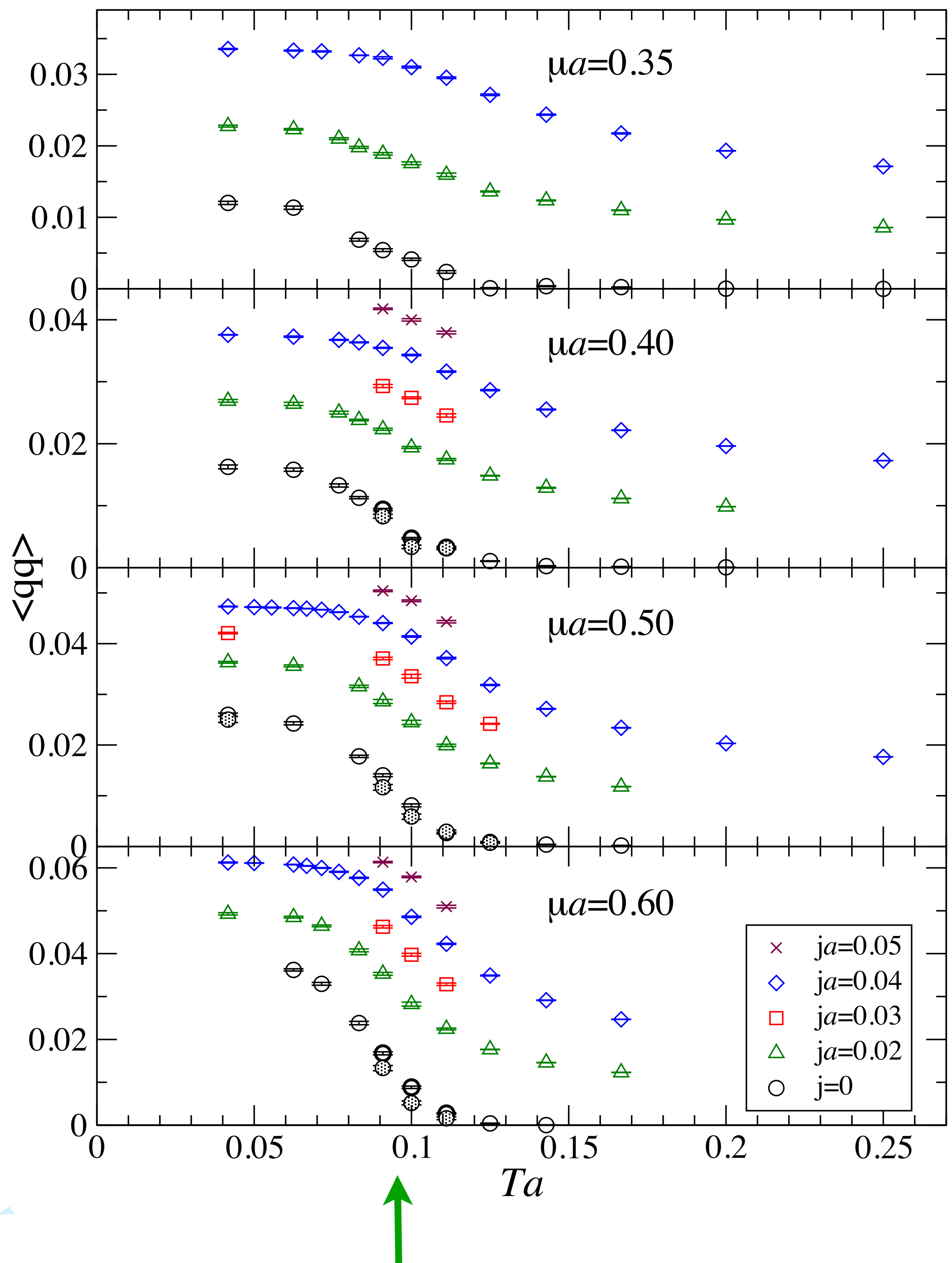
$$\frac{\kappa_1^2\langle\bar\psi\psi\rangle_1}{\kappa_2^2\langle\bar\psi\psi\rangle_2} = \frac{m_2\langle\bar q q\rangle_1}{m_1\langle\bar q q\rangle_2} \begin{cases} = 1 & \text{chirally symmetric;} \\ < 1 & \chi\text{SB with } m_2 < m_1. \end{cases}$$

interrogate configurations using "naive" fermions with $r = 0, ja = 0.04$ and $\kappa = 8.0,\ 16.0,\ 40.0$

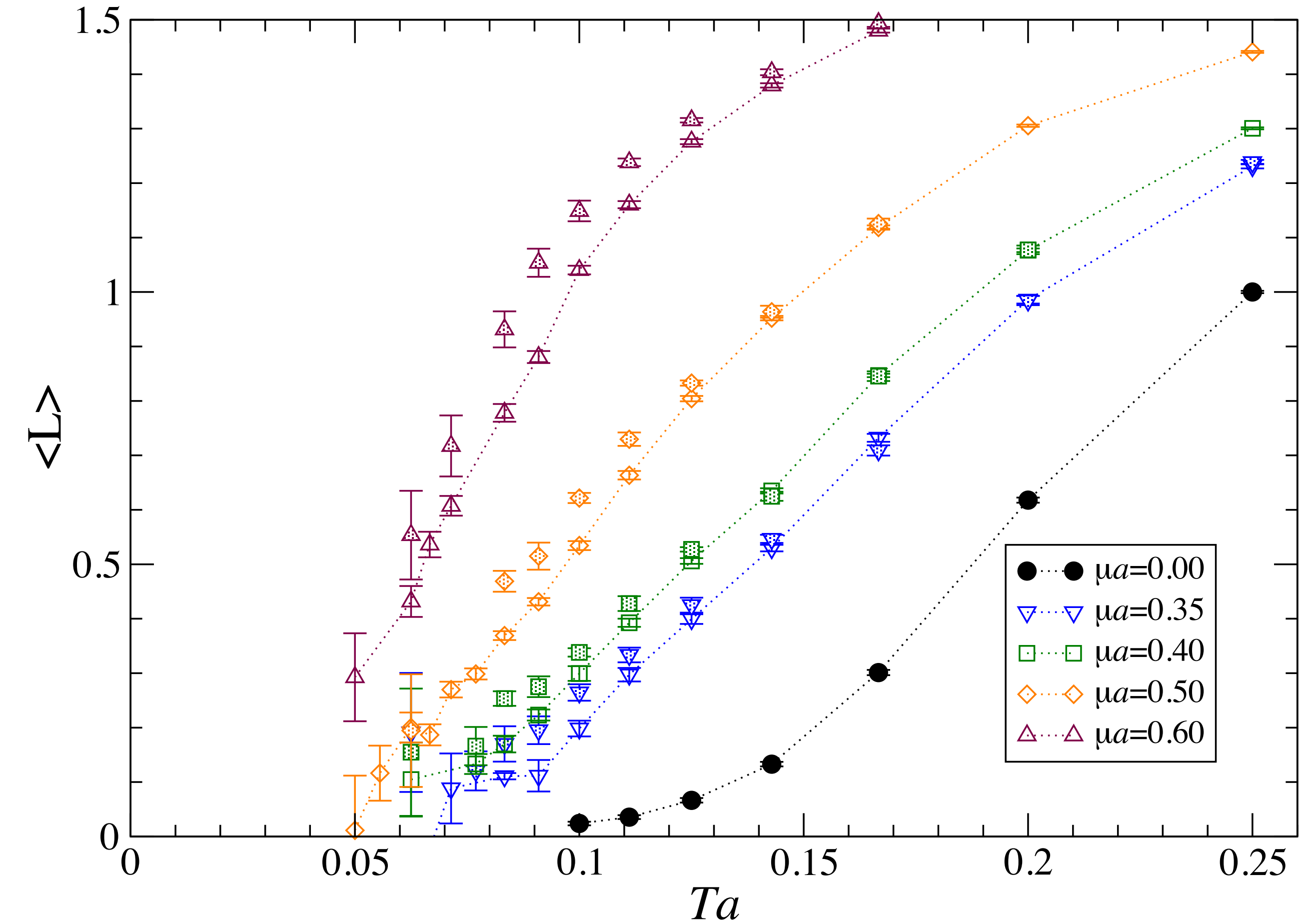
Chiral symmetry restored for μa>0.4 ?

Smulations on $16^3 x N_T=4,...,20$ sketch the picture at higher T, intermediate μ

Boz, Cotter, Fister, Mehta & Skullerud, EPJA49(2013)87



T_s is strikingly μ-independent

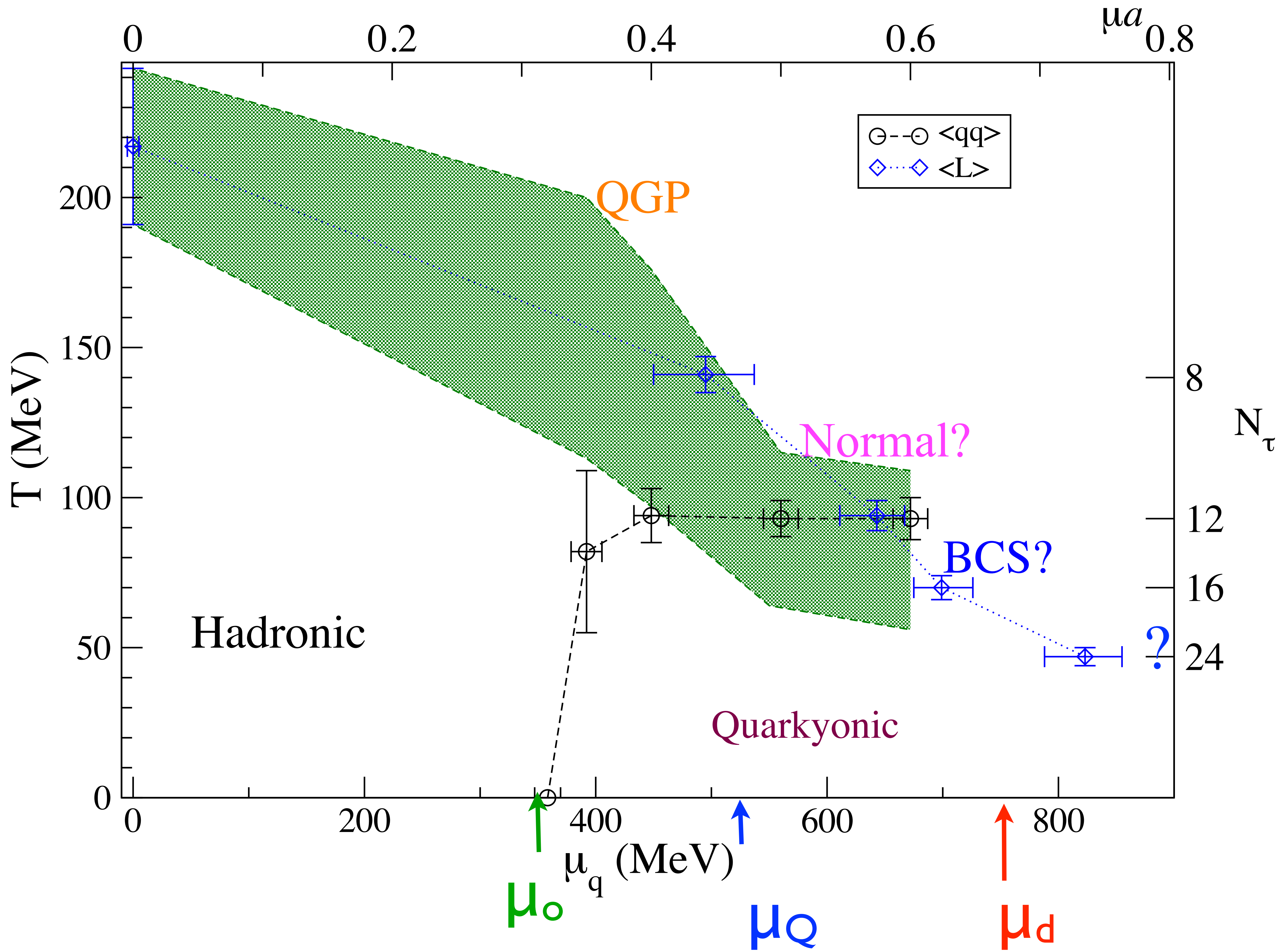


Identify:

superfluid → normal transition via point of inflection of <qq(T)>

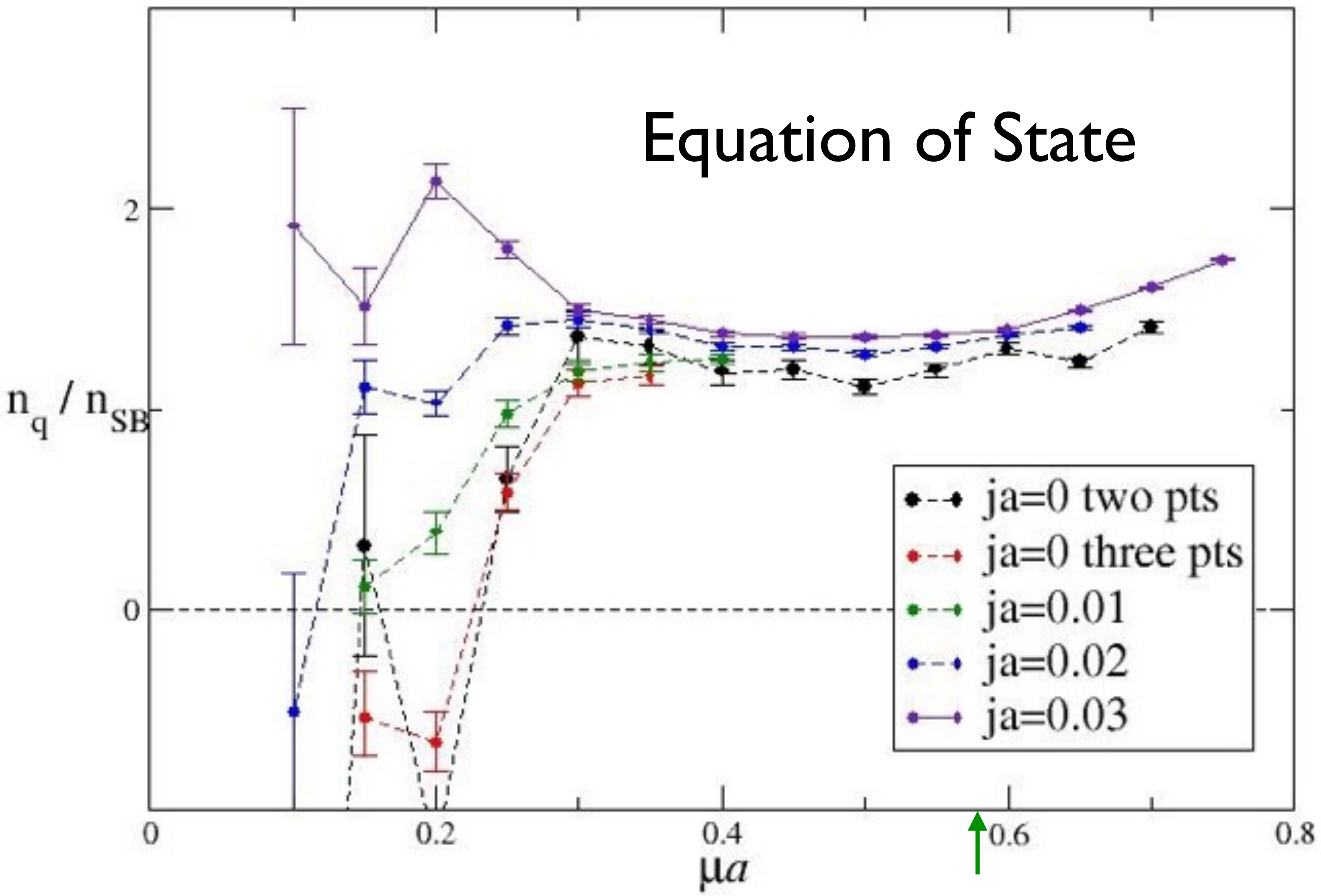
deconfining crossover via linear regime of <L(T)>

Crude map of the T-μ plane...



Preliminary results from fine lattice

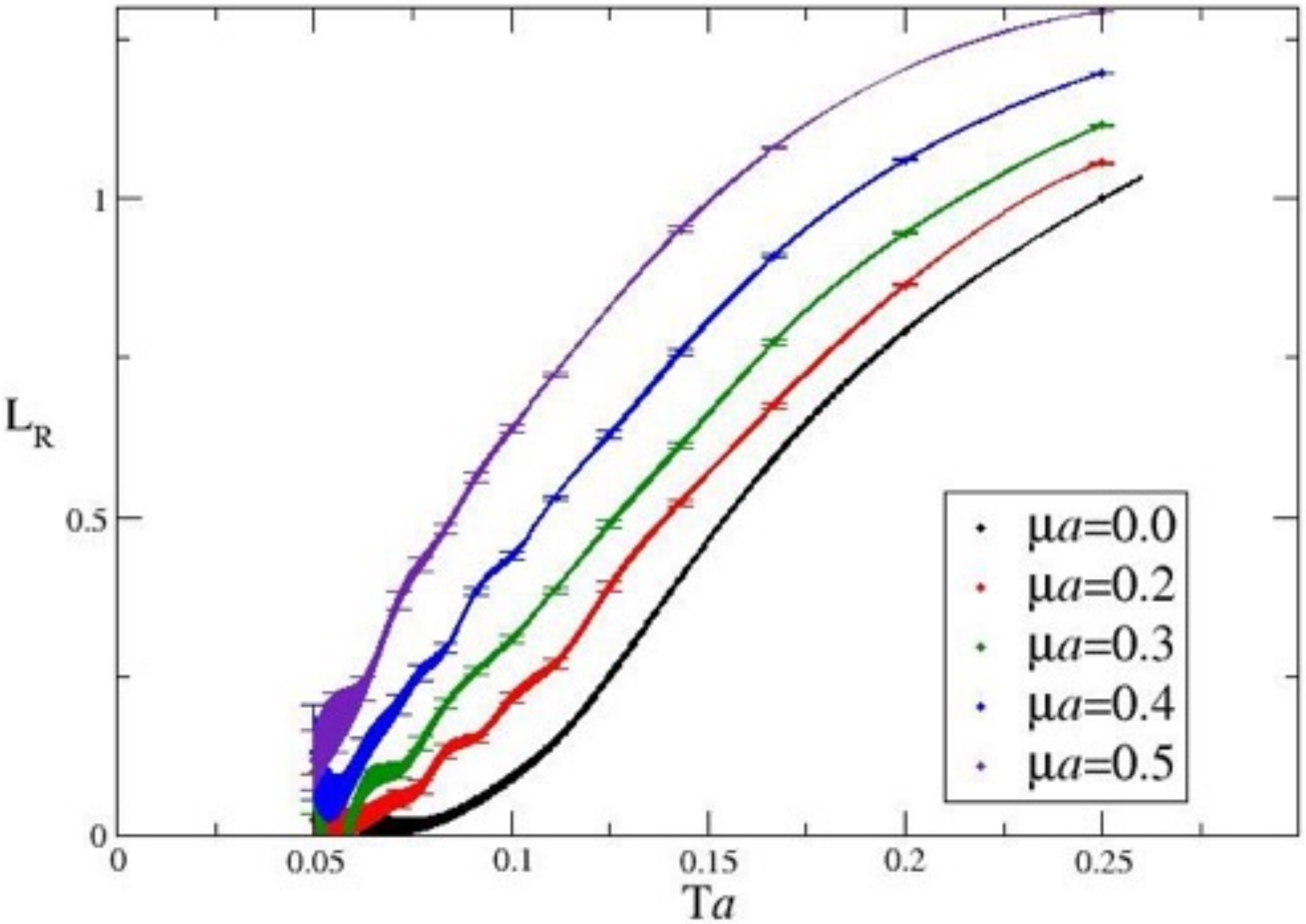
$16^3 \times 32$ a=0.13fm



"μ_d" from medium lattice

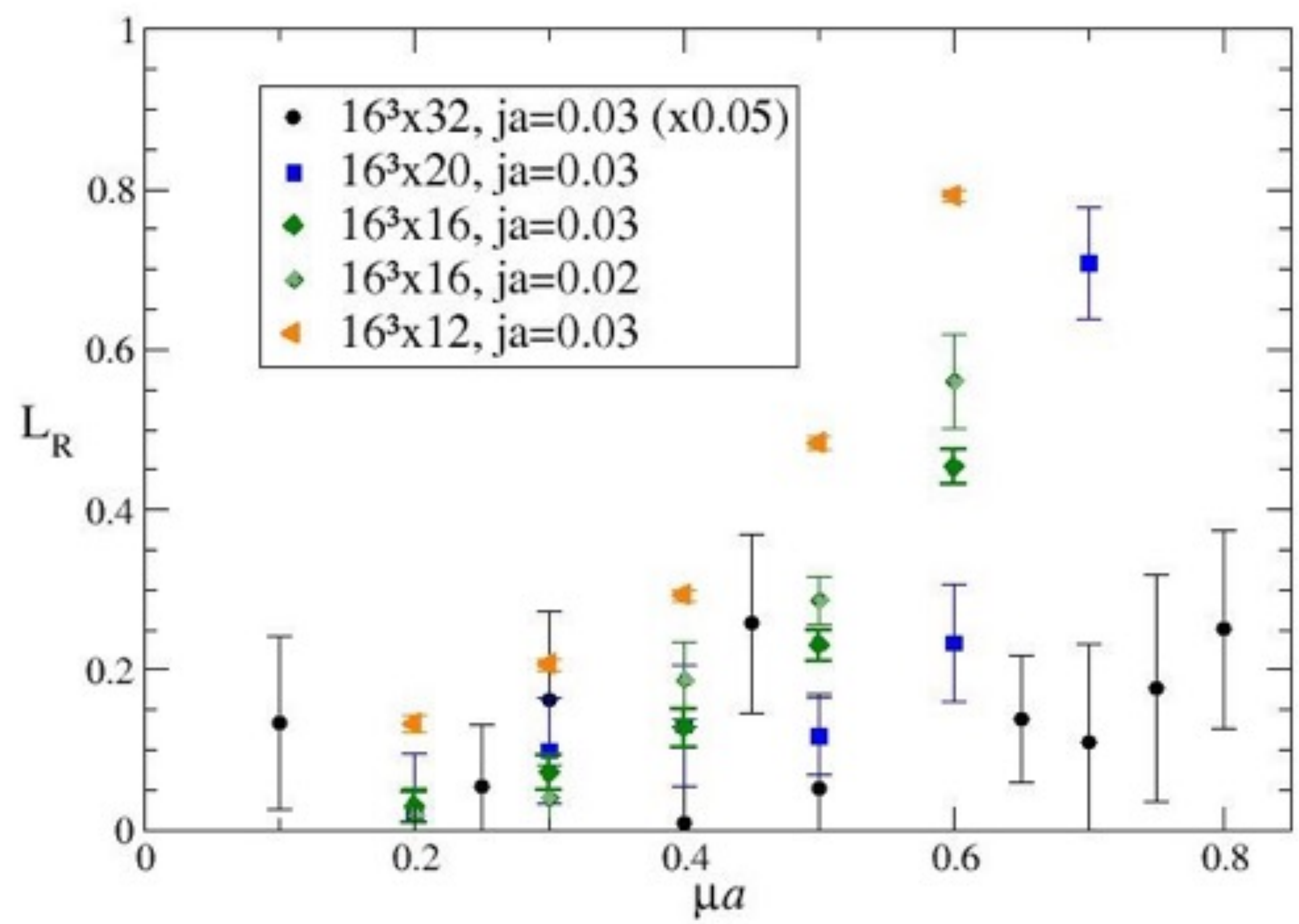
No sign of deconfining transition at $\mu \approx$850MeV

lattice artifact?



T-dependence of Polyakov line consistent with previous, showing deconfinement for Ta>0.05, ie T≳80MeV

No sign of deconfinement for L_t=32, ie. T≈50MeV



Braguta, Ilgenfritz, Kotov, Molochov, Nikolaev, PRD94 (2016)205147
Nikolaev, talk at *Lattice 2017* Granada

News from Russia…

staggered fermions with N_f=2 (rooting!)

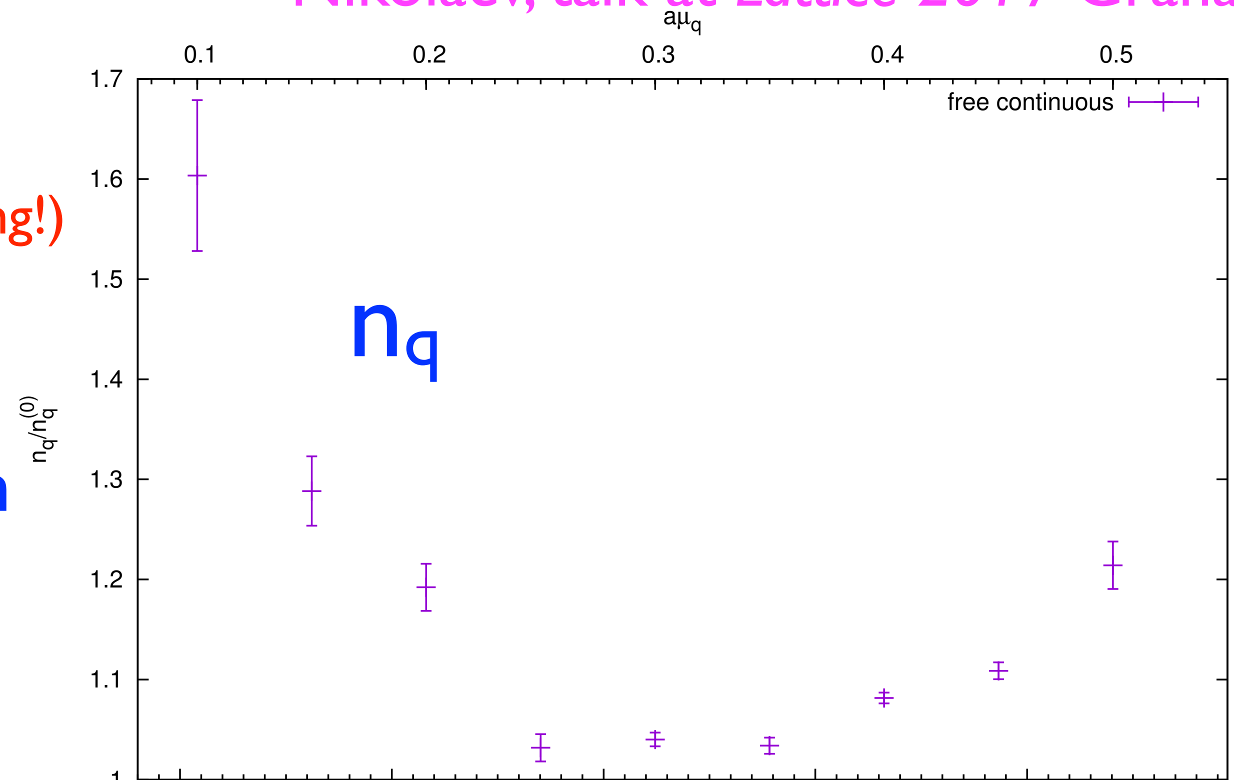
32^3x8,…..32

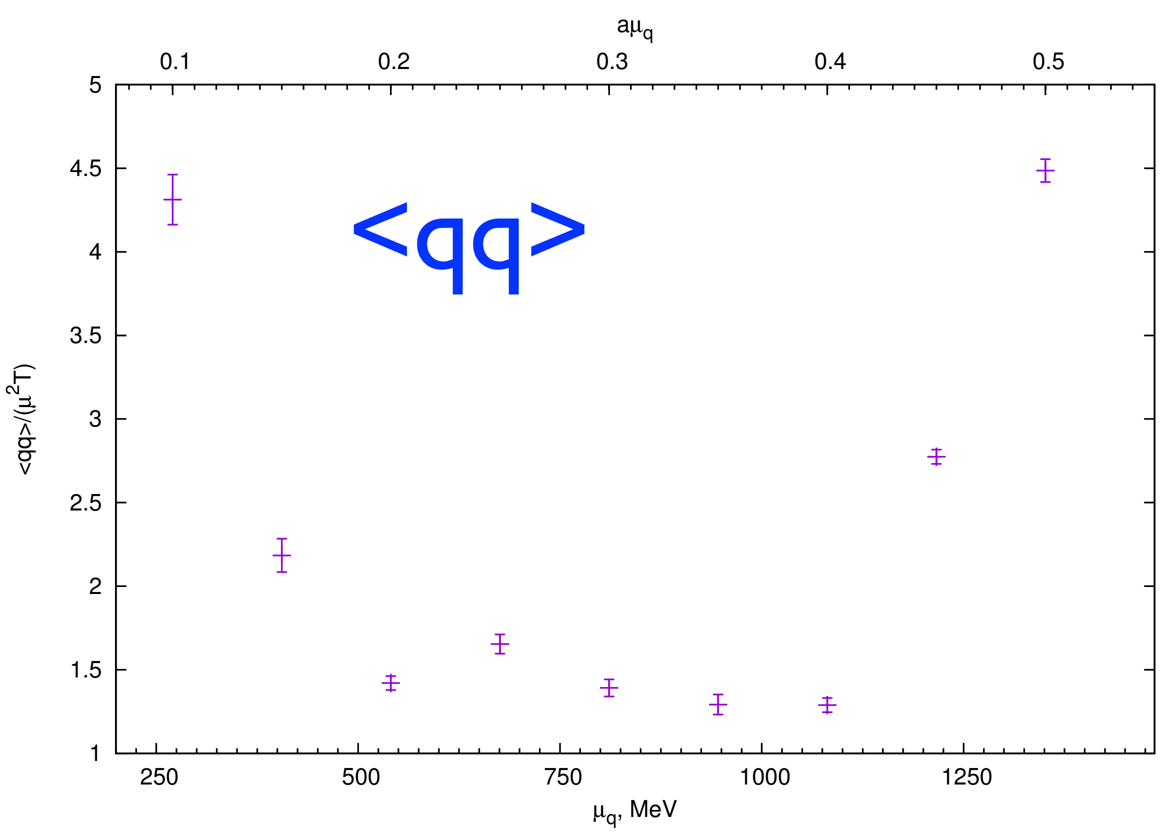
a=0.073fm finer

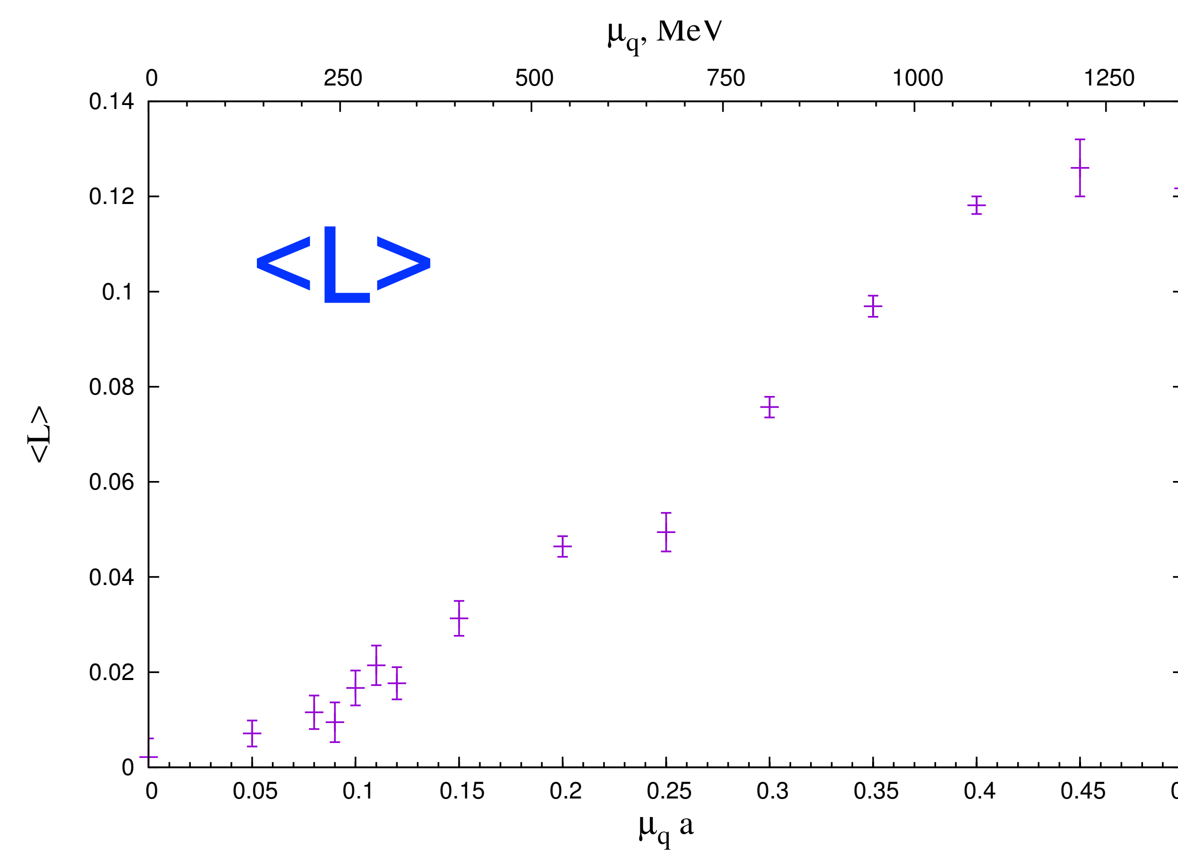
m_π=434MeV lighter ⇔ better separation of scales

T(32)=86MeV hotter

ja=0.00075 closer to "physics"



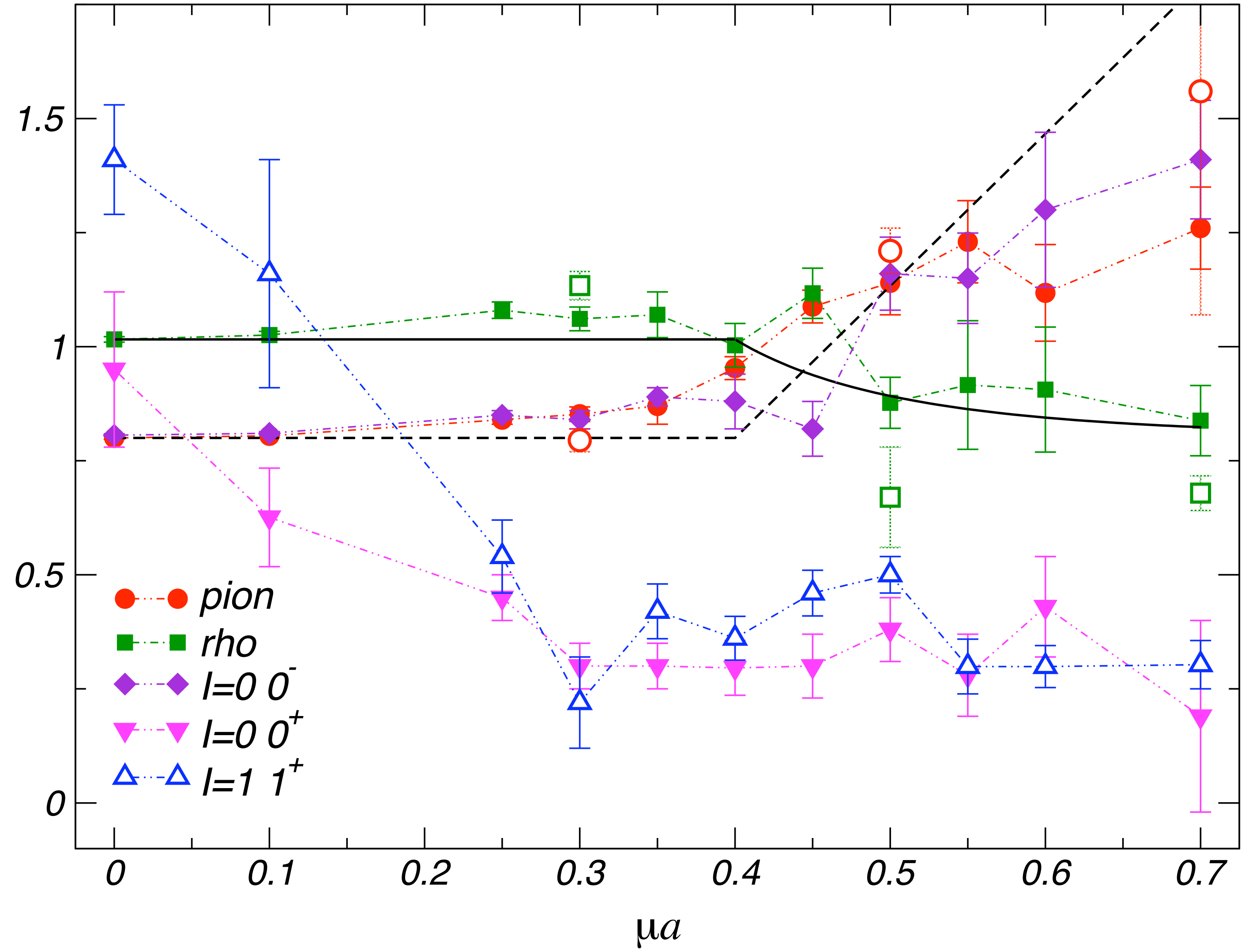




Better contact with BEC regime just above onset?

Clear deconfinement at T=86MeV

Mesons on $8^3 \times 16$ SJH, P. Sitch, J.I. Skullerud PLB662 405 (2008)



Meson spectrum roughly constant up to onset. Then $m_\pi \approx 2\mu$ in accordance with χPT, while m_ρ decreases once $n_q > 0$, in accordance with effective spin-1 action

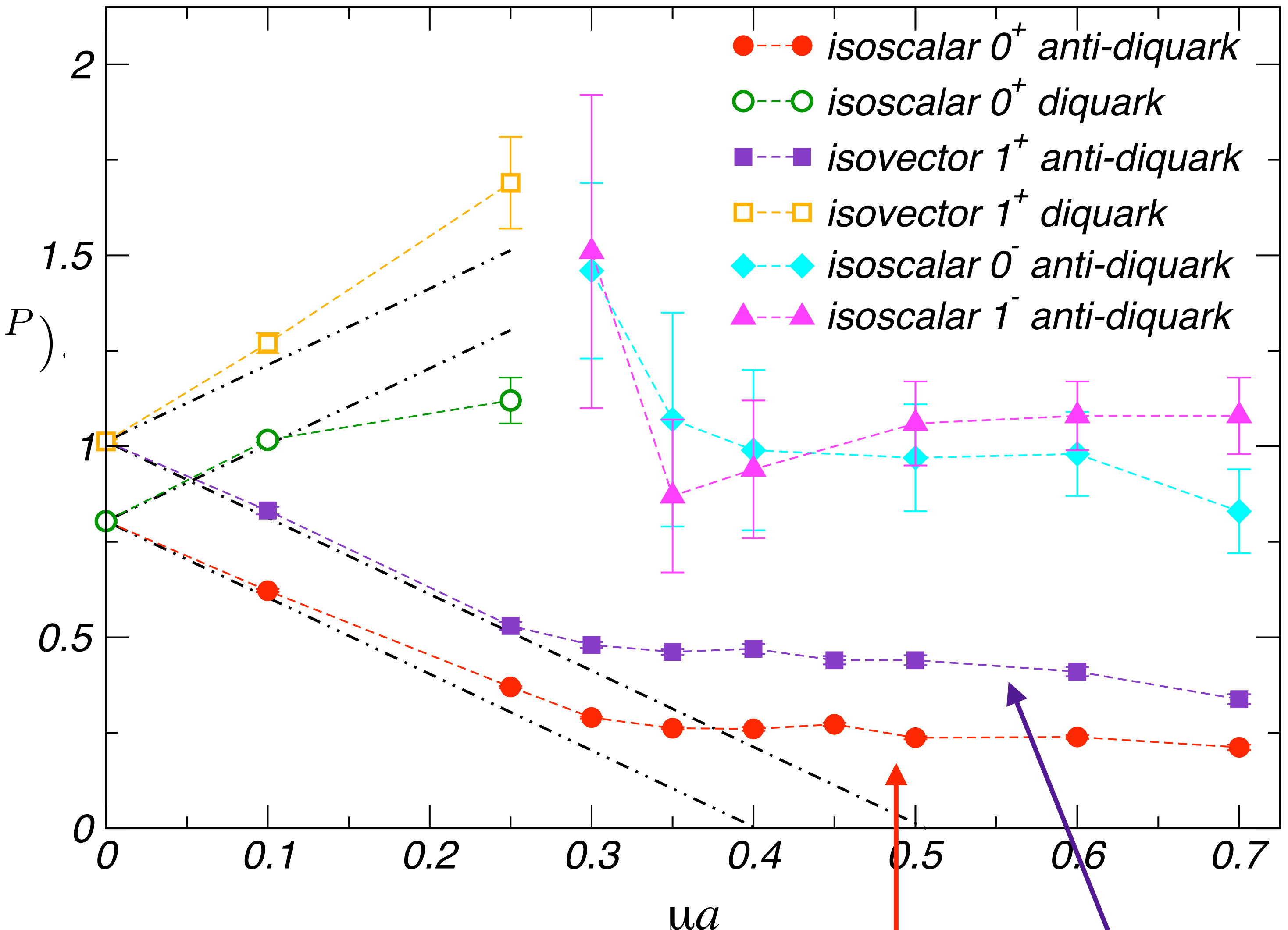
[Lenaghan, Sannino & Splittorff PRD65:054002(2002)]

Cf. Hiroshima group [Muroya, Nakamura & Nonaka PLB551(2003)305]

Diquark Spectrum on $8^3 \times 16$

Note for μ=j=0

$M_D(J^P) = M_M(J^{-P})$.

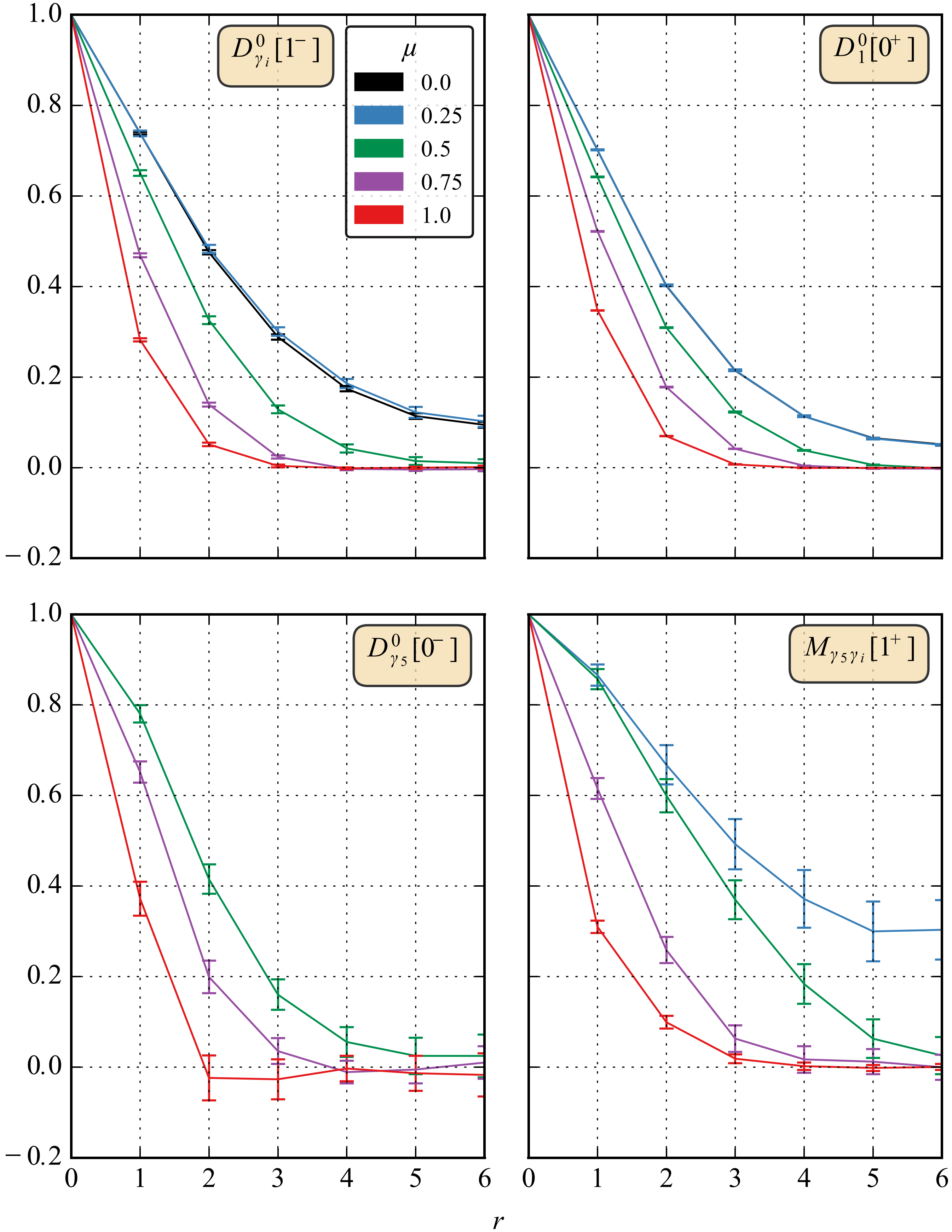


Diquark spectrum modelled by $m_{\pi,\rho} \pm 2\mu$ up to onset, while post-onset:

- Splitting of "Higgs/Goldstone" degeneracy in $I = 0\ 0^+$ channel
- Meson/Baryon degeneracy in $I = 0\ 0^+$ and $I = 1\ 1^+$ channels

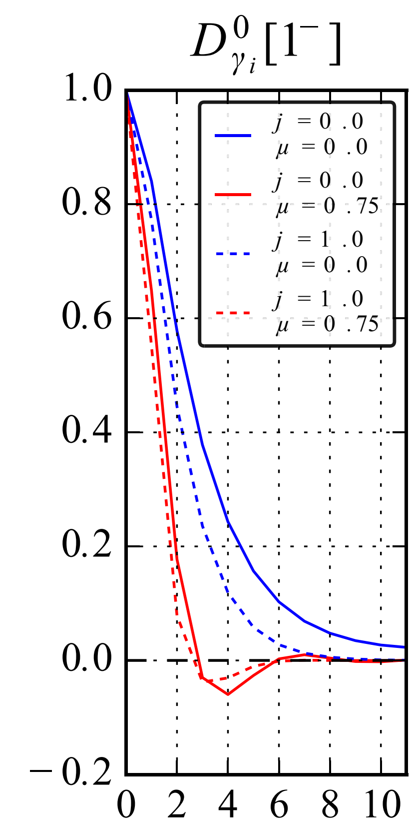
Hadron Wavefunctions

$$\Psi(\vec{r},\tau) = \int d^3\vec{x}\langle 0|\bar{\psi}(\vec{x},\tau)\psi(\vec{x}+\vec{r},\tau)|H\rangle.$$

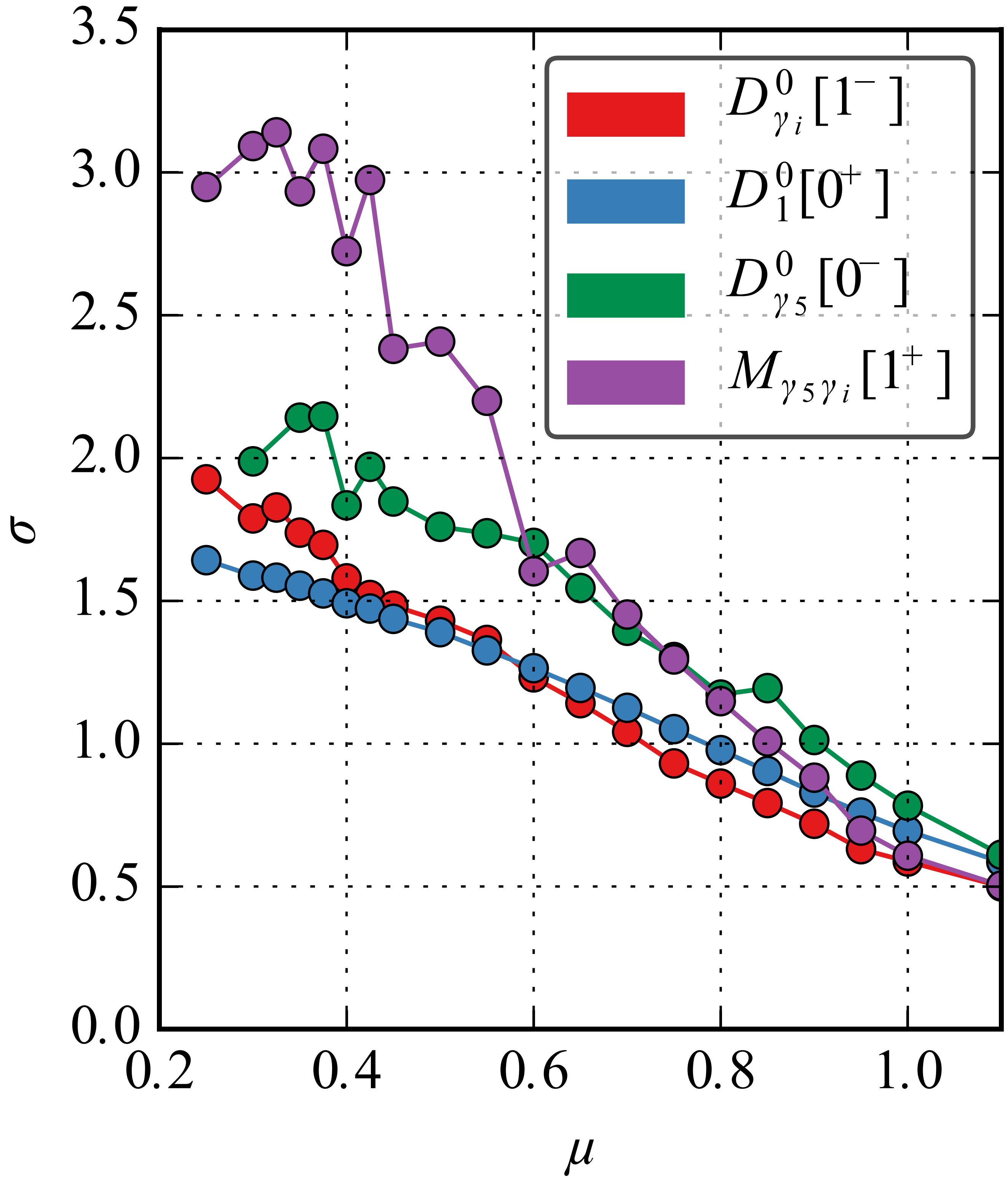


examine both meson and diquark channels using Coulomb gauge-fixing

no Friedel oscillations indicating a sharp Fermi surface $\Leftrightarrow \Delta > 0$?



Amato, Giudice & SJH, EPJA(2015)51, 39



Scale hierarchy in superfluid phase

$$\sigma(0^+) \sim \sigma(1^-) < \sigma(0^-) < \sigma(1^+)$$

Cf. Mass hierarchy

$$m(0^+) < m(1^+) \ll m(1^-) < m(0^-)$$

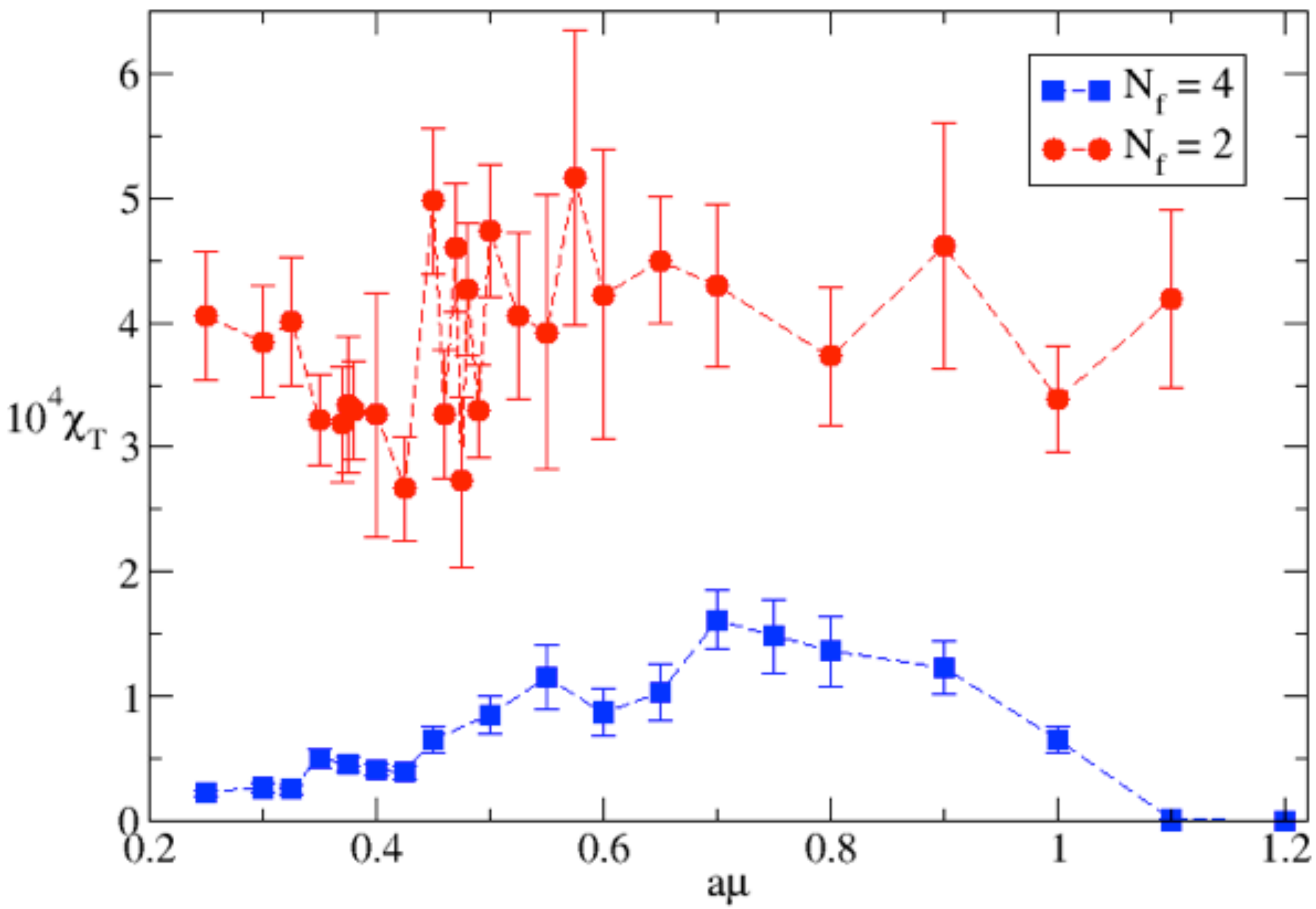
hadron sizes decrease as density rises

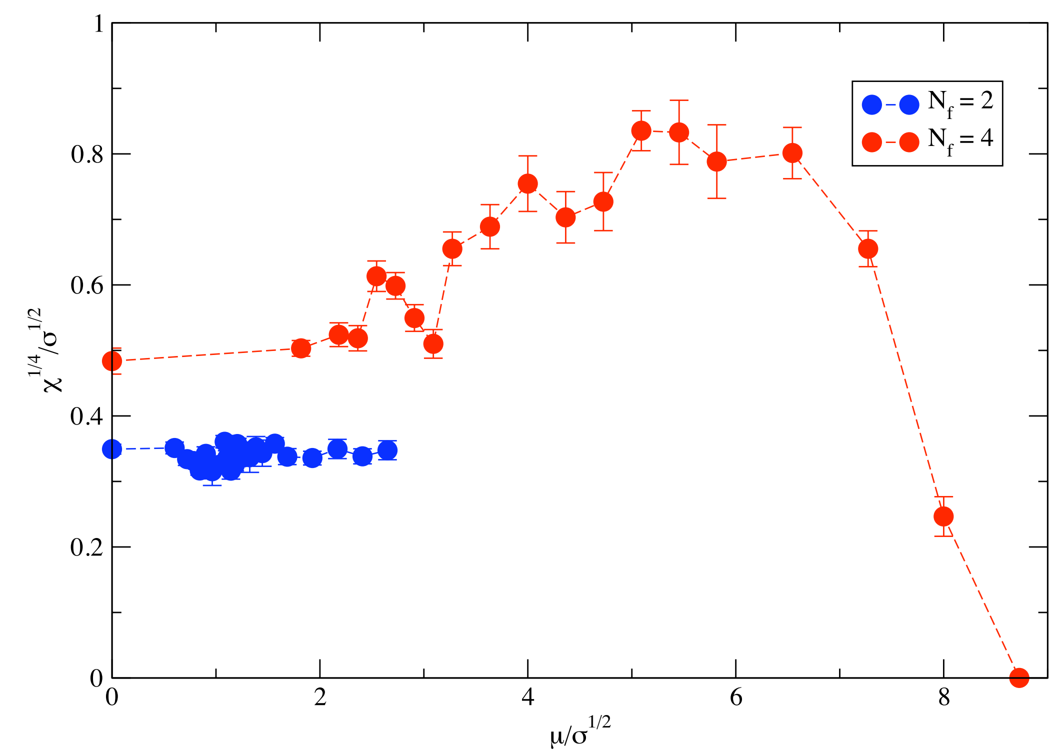
Who knew?

Topological Susceptibility

SJH, P. Kenny, PLB701 (2011) 373

We have investigated instanton distributions and sizes using **cooling**





Topological susceptibility shows no structure for N_f=2

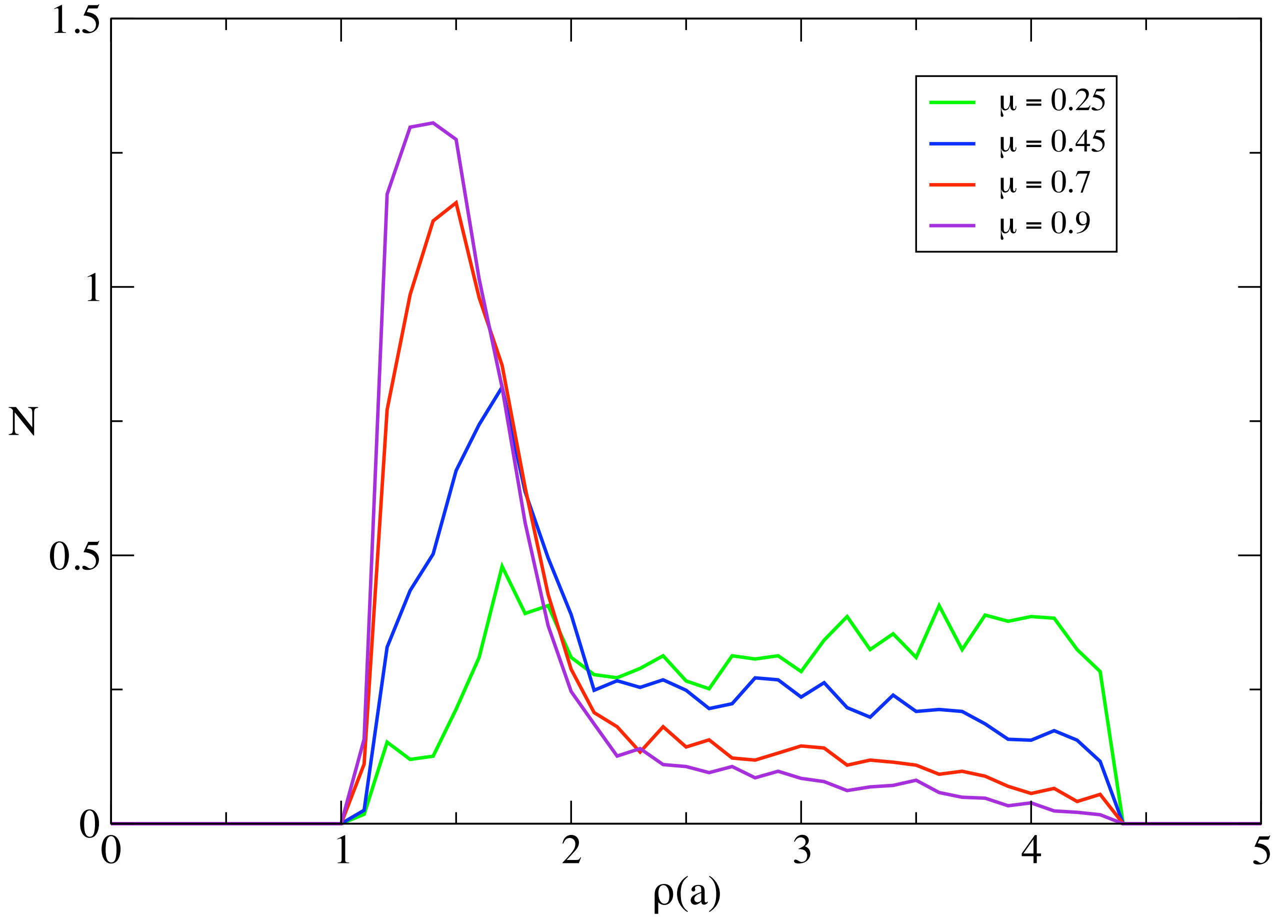
(maybe lattice too coarse?)

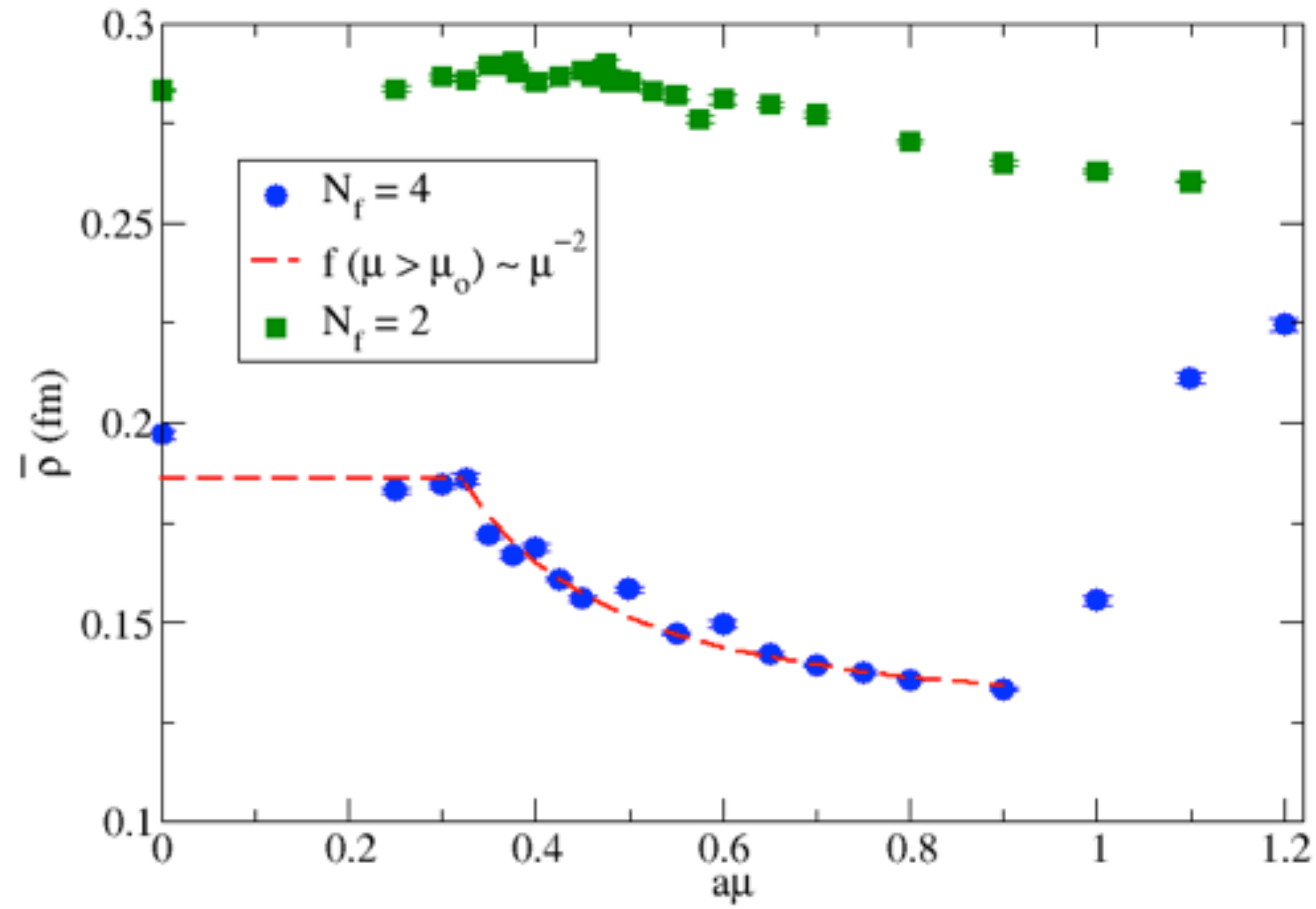
but appears **enhanced** in quarkyonic region for N_f=4

dimensionless plot $\chi^{0.25}/\sigma^{0.5}$ vs. $\mu/\sigma^{0.5}$

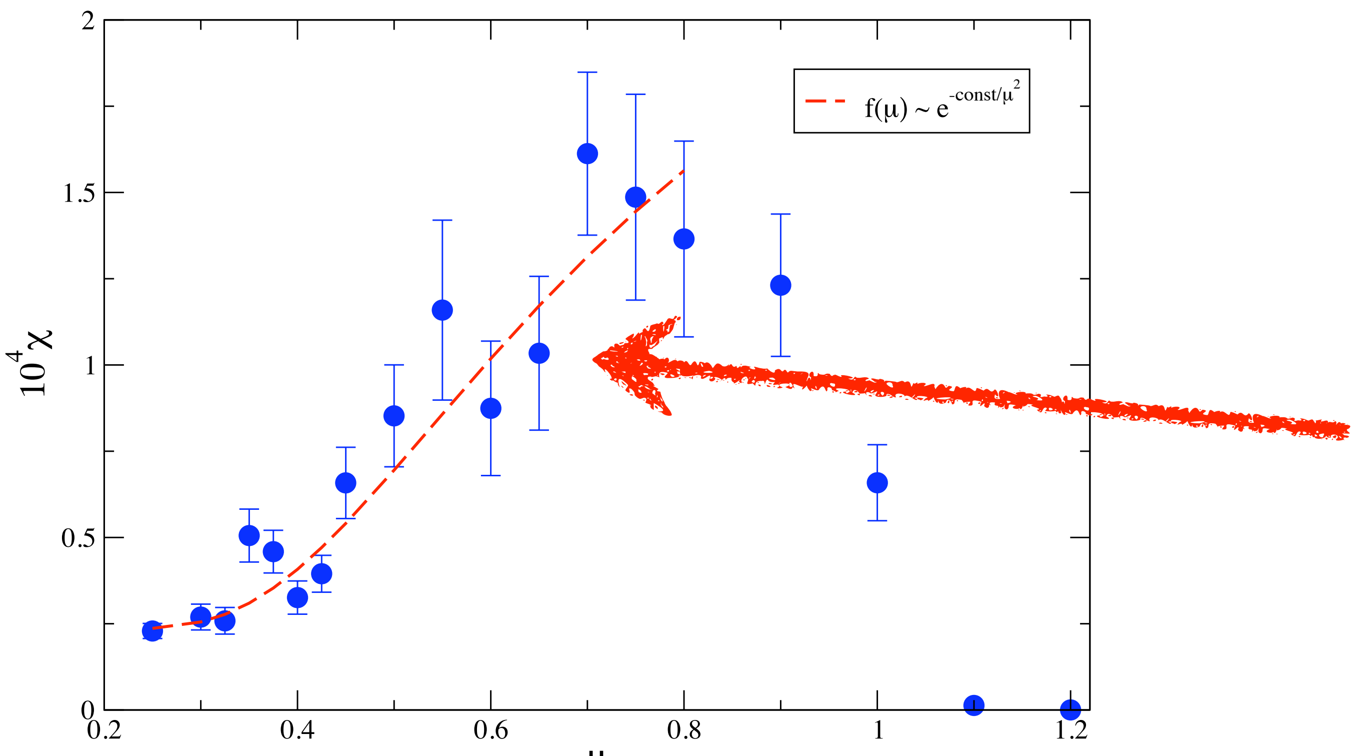
Cf. suppression in superfluid phase for N_f=8

B. Alles, M. D'Elia & M.P. Lombardo, NPB752(2006)124





For $\mu_o < \mu < \mu_d$ the mean instanton size ρ_I decreases



One-loop Debye screening:

Schäfer & Shuryak RMP 70(1998)323

$$n_I(\mu) \;\propto\; \exp\left[-N_f \rho_I^2 \mu^2\right]$$

$$\propto\; \exp\left[-\frac{\text{const}}{\mu^2}\right]$$

Summary

★ QC_2D offers an accessible theoretical laboratory for dense baryonic matter

★ Despite UV & IR artifacts a robust picture is emerging. For low T (at least) 3 distinct regions:

Vacuum for $\mu < \mu_o$

Confined "Quarkyonic" superfluid for $\mu_Q < \mu < \mu_d$

Deconfined phase for $\mu > \mu_d(T)$

★ Deconfinement is delayed by presence of superfluid gap and is plausibly absent as $T \rightarrow 0$

★ Not discussed today: quarkonia, static quark potential, quark and gluon propagators