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# GENERALISED GLOBAL SYMMETRIES AND DISSIPATIVE MAGNETOHYDRODYNAMICS

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# OUTLINE

- hydrodynamics
- generalised global symmetries and magnetohydrodynamics
- plasma and holography
- outlook

# FRAGMENT I

with

D. Hofman and N. Iqbal

# FRAGMENT II

with

N. Poovuttikul

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## Generalized global symmetries and dissipative magnetohydrodynamics

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The conserved magnetic flux of $U(1)$ electrodynamics coupled to matter in four dimensions is associated with a generalized global symmetry. We study the realization of such a symmetry at finite temperature and develop the hydrodynamic theory describing fluctuations of a conserved 2-form current around thermal equilibrium. This can be thought of as a systematic derivation of relativistic magnetohydrodynamics, constrained only by symmetries and effective field theory. We construct the entropy current and show that at first order in derivatives, there are seven dissipative transport coefficients. We present a universal definition of resistivity in a theory of dynamical electromagnetism and derive a direct Kubo formula for the resistivity in terms of correlation functions of the electric field operator. We also study fluctuations and collective modes, deriving novel expressions for the dissipative widths of magnetosonic and Alfvén modes. Finally, we demonstrate that a nontrivial truncation of the theory can be performed at low temperatures compared to the magnetic field: this theory has an emergent Lorentz invariance along magnetic field lines, and hydrodynamic fluctuations are now parametrized by a fluid tensor rather than a fluid velocity. Throughout, no assumption is made of weak electromagnetic coupling. Thus, our theory may have phenomenological relevance for dense electromagnetic plasmas.

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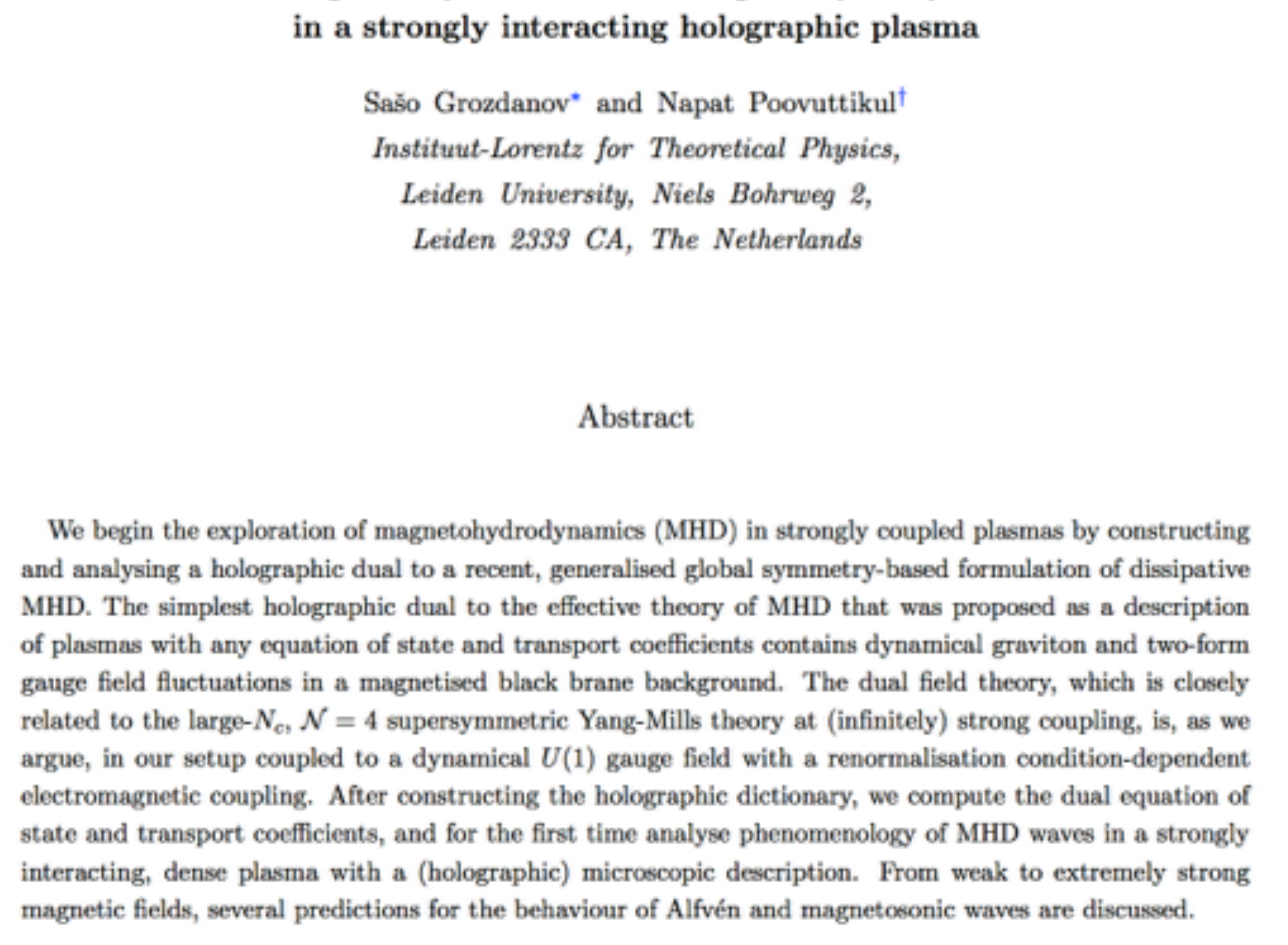

## Generalised global symmetries and magnetohydrodynamic waves in a strongly interacting holographic plasma

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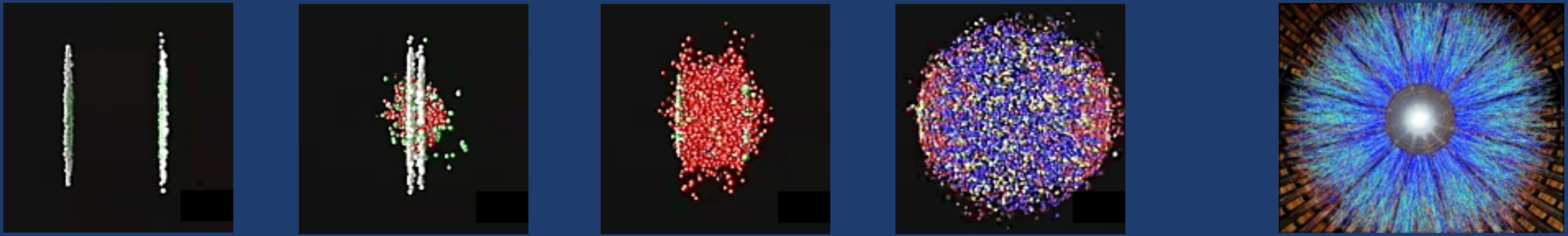
### Abstract

We begin the exploration of magnetohydrodynamics (MHD) in strongly coupled plasmas by constructing and analysing a holographic dual to a recent, generalised global symmetry-based formulation of dissipative MHD. The simplest holographic dual to the effective theory of MHD that was proposed as a description of plasmas with any equation of state and transport coefficients contains dynamical graviton and two-form gauge field fluctuations in a magnetised black brane background. The dual field theory, which is closely related to the large-$N_c$, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at (infinitely) strong coupling, is, as we argue, in our setup coupled to a dynamical $U(1)$ gauge field with a renormalisation condition-dependent electromagnetic coupling. After constructing the holographic dictionary, we compute the dual equation of state and transport coefficients, and for the first time analyse phenomenology of MHD waves in a strongly interacting, dense plasma with a (holographic) microscopic description. From weak to extremely strong magnetic fields, several predictions for the behaviour of Alfvén and magnetosonic waves are discussed.

# HYDRODYNAMICS

# HYDRODYNAMICS

- QCD and quark-gluon plasma



- low-energy limit of QFTs (effective field theory)

$$T^{\mu\nu}\left(u^{\lambda},T,\mu\right)=\left(\varepsilon+P\right)u^{\mu}u^{\nu}+Pg^{\mu\nu}-\eta\sigma^{\mu\nu}-\zeta\nabla\cdot u\Delta^{\mu\nu}+\dots$$

$$J^{\mu}(u^{\lambda},T,\mu)=nu^{\mu}-\sigma T\Delta^{\mu\nu}\nabla_{\nu}(\mu/T)+\dots$$

$$\boxed{\nabla_{\mu}T^{\mu\nu}=0 \qquad\qquad \nabla_{\mu}J^{\mu}=0}$$

- tensor structures (phenomenological gradient expansions) with transport coefficients (microscopic)

# HYDRODYNAMICS

- conformal (Weyl-covariant) hydrodynamics $T^{\mu}{}_{\mu} = 0$

$$g_{\mu\nu} \to e^{-2\omega(x)} g_{\mu\nu} \qquad T^{\mu\nu} \to e^{6\omega(x)} T^{\mu\nu}$$

- infinite-order asymptotic expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} T^{\mu\nu}_{(n)} \longrightarrow \omega = \sum_{n=0}^{\mathcal{O}_H} \alpha_n k^{n+1}$$

- classification of tensors beyond Navier-Stokes

first order: 2 (1 in CFT) - shear and bulk viscosities

second order: 15 (5 in CFT) - relaxation time, … [Israel-Stewart and extensions]

third order: 68 (20 in CFT) - [S. G., Kaplis, PRD 93 (2016) 6, 066012, arXiv:1507.02461]

# HYDRODYNAMICS

- diffusion and sound dispersion relations in CFT

$$\text{shear:} \quad \omega = -i\frac{\eta}{\varepsilon+P}k^2 - i\left[\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{2}\frac{\theta_1}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

$$\text{sound:} \quad \omega = \pm c_s k - i\Gamma_c k^2 \mp \frac{\Gamma_c}{2c_s}\left(\Gamma_c - 2c_s^2\tau_\Pi\right)k^3 - i\left[\frac{8}{9}\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{1}{3}\frac{\theta_1+\theta_2}{\varepsilon+P}\right]k^4 + \mathcal{O}\left(k^5\right)$$

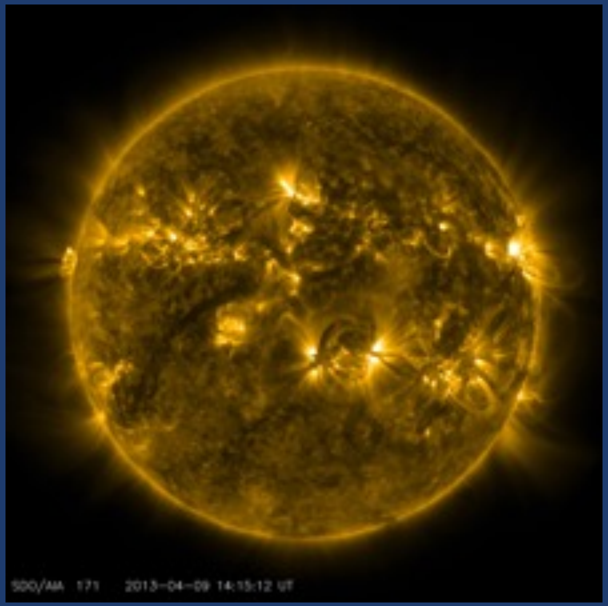
- loop corrections break analyticity of the gradient expansion (long-time tails), but are *1/N* suppressed [Kovtun, Yaffe (2003)]
- entropy current, constraints on transport and new transport coefficients (anomalies, broken parity)
- non-relativistic hydrodynamics
- hydrodynamics from effective Schwinger-Keldysh field theory with dissipation

[Nicolis, et. al.; S. G., Polonyi; Haehl, Loganayagam, Rangamani; de Boer, Heller, Pinzani-Fokeeva; Crossley, Glorioso, Liu]

# MAGNETOHYDRODYNAMICS

# STATES OF MATTER

- solids
- liquids
- gases
- plasma



# PLASMA AND MHD

- ionised gas, uncharged at large distances (screening)
- the theory: magnetohydrodynamics (MHD)

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\epsilon \vec{v}) = 0$$

$$\epsilon \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{v} = -\nabla p + \vec{J} \times \vec{B}$$

continuity + Euler (or Navier-Stokes)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\xcancel{\nabla \cdot \vec{E} = 0}$$

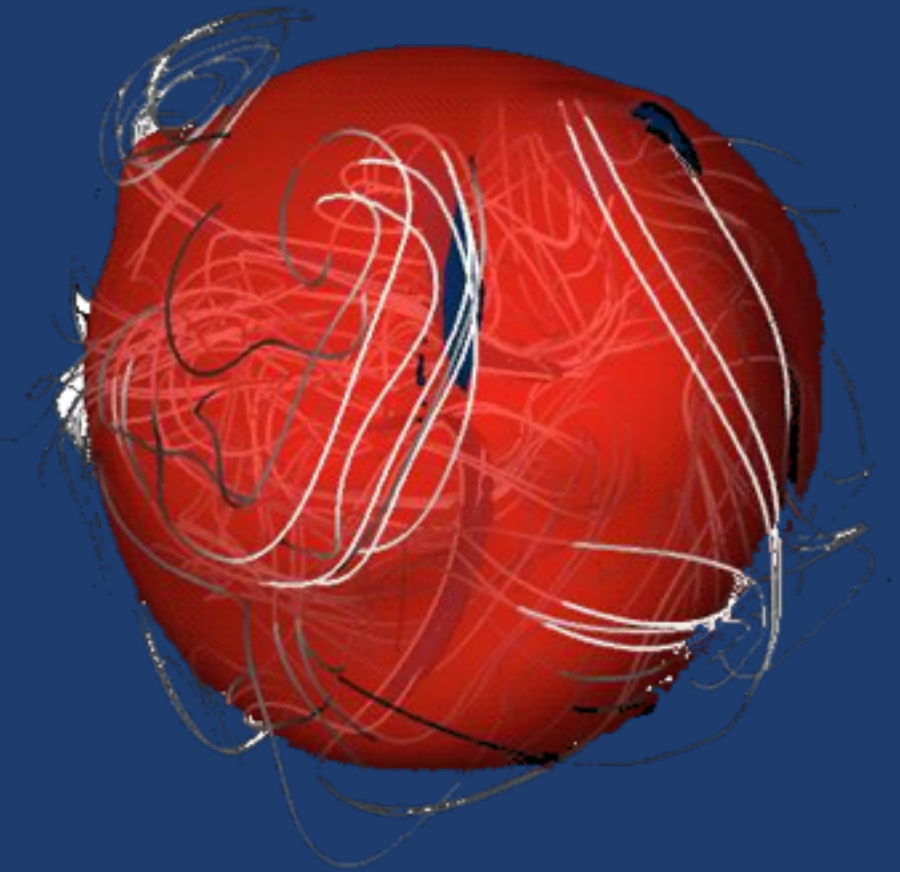
Maxwell

$$\vec{E} + \vec{v} \times \vec{B} = 0\,, \quad (\sigma \to \infty)$$

Ohm's law (ideal)

$$\left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \left( \frac{p}{\epsilon^{\gamma}} \right) = 0$$

Equation of state



# PLAN AND WISHES

- generalise MHD to bring it to the effective field theory level of (charged) relativistic hydrodynamics: symmetries and field content
- is there a way to avoid assumptions of the matter content?
- what if a plasma is strongly coupled, contains strong magnetic fields,

  $$B/T^2 \gg 1$$

  its equation of state depends on magnetic properties and the matter sector is strongly coupled to the charged sector, $p(T, B)$
- quantum effects: pair-creation (Euler-Heisenberg), Landau levels, …?
- systematic derivative expansions: transport coefficients
- look at an example of such a plasma and learn something new about real plasmas (fusion, magnetars, quark-gluon plasma, …)?

# GENERALISED GLOBAL SYMMETRIES

- symmetry based formulation of MHD

  [S. G., Hofman, Iqbal, PRD (2017) 9, 096003, arXiv:1610.07392]

- gauge vs. global *U(1)* symmetry in electromagnetism

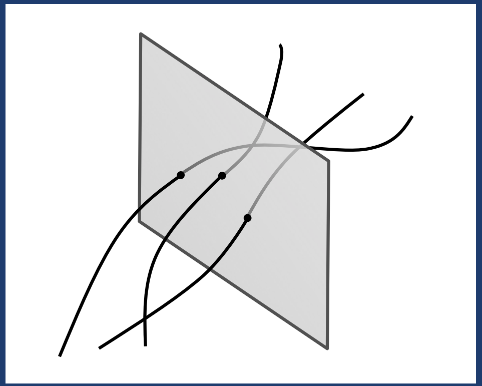
$$\frac{1}{g^2}\nabla_\mu F^{\mu\nu} = j^\nu_{el} \qquad\qquad J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

- 0-form symmetry (1-form conserved current) $\mathcal{O}(x) \to e^{iq\Lambda}\mathcal{O}(x)$

- 1-form symmetry (2-form conserved current)

  [Gaiotto, Kapustin, Seiberg, Willet]

$$W(C) \to \exp\left(iq\int_C \Lambda_\mu dx^\nu\right)W(C)$$



- matter-less electromagnetism has two *U(1)* 1-form symmetries

- spontaneous breaking of the symmetry gives a photon

# GENERALISED GLOBAL SYMMETRIES

- such a symmetry can be source or gauged like any other symmetry

$$S[b] \equiv S_0 + \Delta S[b], \qquad \Delta S[b] \equiv \int d^4x \sqrt{-g}\, b_{\mu\nu} J^{\mu\nu}$$

- related to one-form current by a dualisation

$$\Delta S[b] = \int d^4x \sqrt{-g} A_\sigma j^\sigma_{\text{ext}}, \qquad j^\sigma_{\text{ext}} \equiv \epsilon^{\sigma\rho\mu\nu} \partial_\rho b_{\mu\nu}$$

- our theory has the following conserved global symmetries: energy-momentum and a *U(1)* number of magnetic flux lines (in the presence of charged matter)

$$\begin{tikzcd} \nabla_\mu T^{\mu\nu} = 0 \arrow[d, "\text{external source}"'] \\ \nabla_\mu T^{\mu\nu} = H^{\nu}{}_{\rho\sigma} J^{\rho\sigma} \end{tikzcd}$$

$$\nabla_\mu J^{\mu\nu} = \frac{1}{2} \nabla_\mu \left( \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \right) = 0$$

# IDEAL MHD AND WAVES

- construct a general hydrodynamic gradient expansion with fields:

$$u^\mu,\, T,\, h^\mu,\, \mu \qquad\qquad u^2 = -1,\, h^2 = 1,\, h_\mu u^\mu = 0$$

- ideal MHD

$$T^{\mu\nu}_{(0)} = (\varepsilon + p)\, u^\mu u^\nu + p\, g^{\mu\nu} - \mu\rho\, h^\mu h^\nu$$

(string hydro)

$$J^{\mu\nu}_{(0)} = 2\rho\, u^{[\mu} h^{\nu]}$$

$$\varepsilon + p = Ts + \mu\rho \qquad\qquad dp = sdT + \rho d\mu$$

- thermodynamics

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu J^{\mu\nu} = 0$$

- Alfvén and (slow and fast) magnetosonic waves from

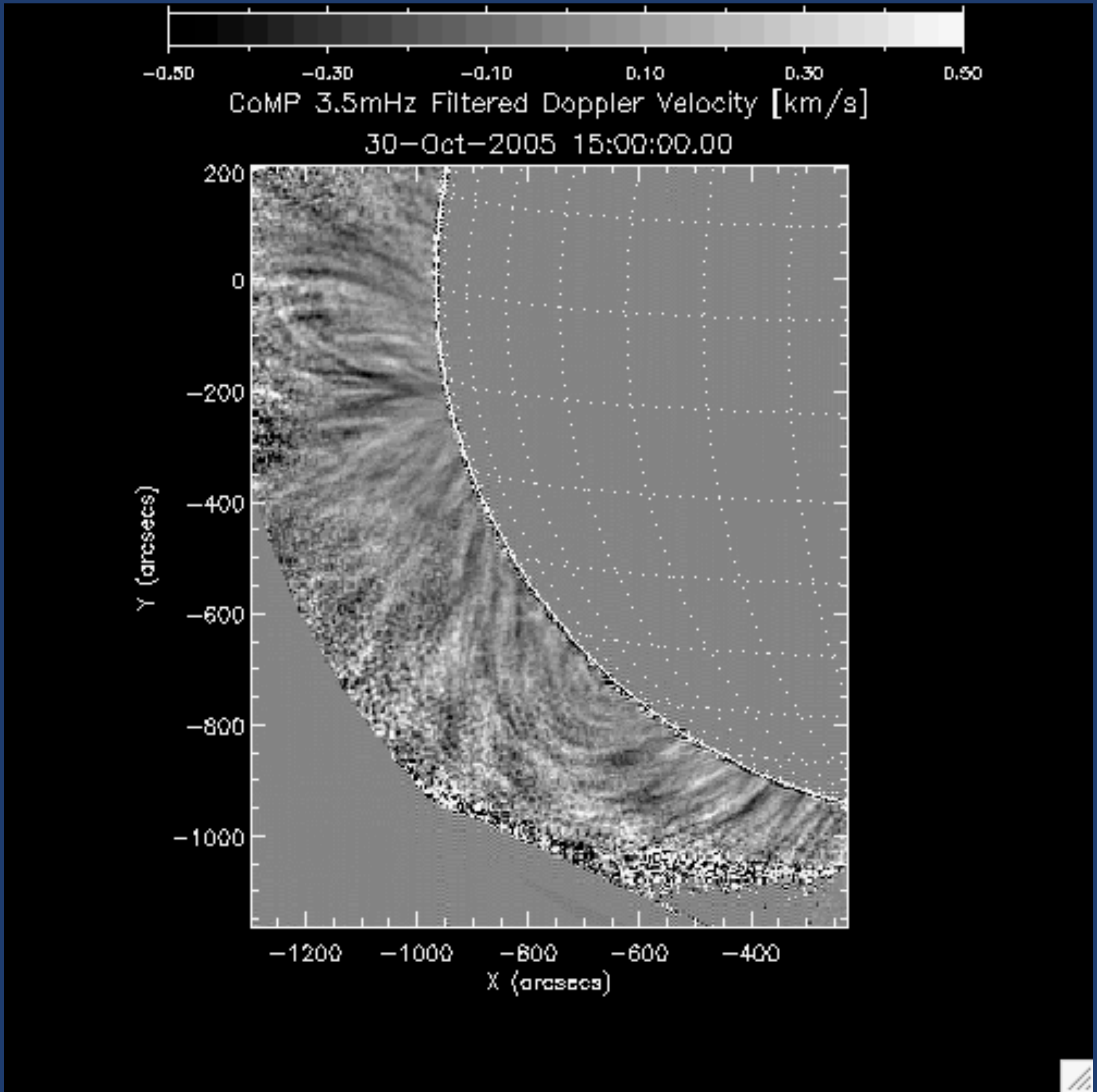
$$\omega = \pm v_A k \qquad\qquad \omega = \pm v_M k$$

$$v_A^2 = \mathcal{V}_A^2 \cos^2\theta$$

$$v_M^2 = \frac{1}{2}\left\{ (\mathcal{V}_A^2 + \mathcal{V}_0^2)\cos^2\theta + \mathcal{V}_S^2 \sin^2\theta \pm \sqrt{\left[(\mathcal{V}_A^2 - \mathcal{V}_0^2)\cos^2\theta + \mathcal{V}_S^2\sin^2\theta\right]^2 + 4\mathcal{V}^4\cos^2\theta\sin^2\theta} \right\}$$

# IDEAL MHD AND WAVES

- Alfvén waves have been observed in solar corona [McIntosh, et. al (2011)]



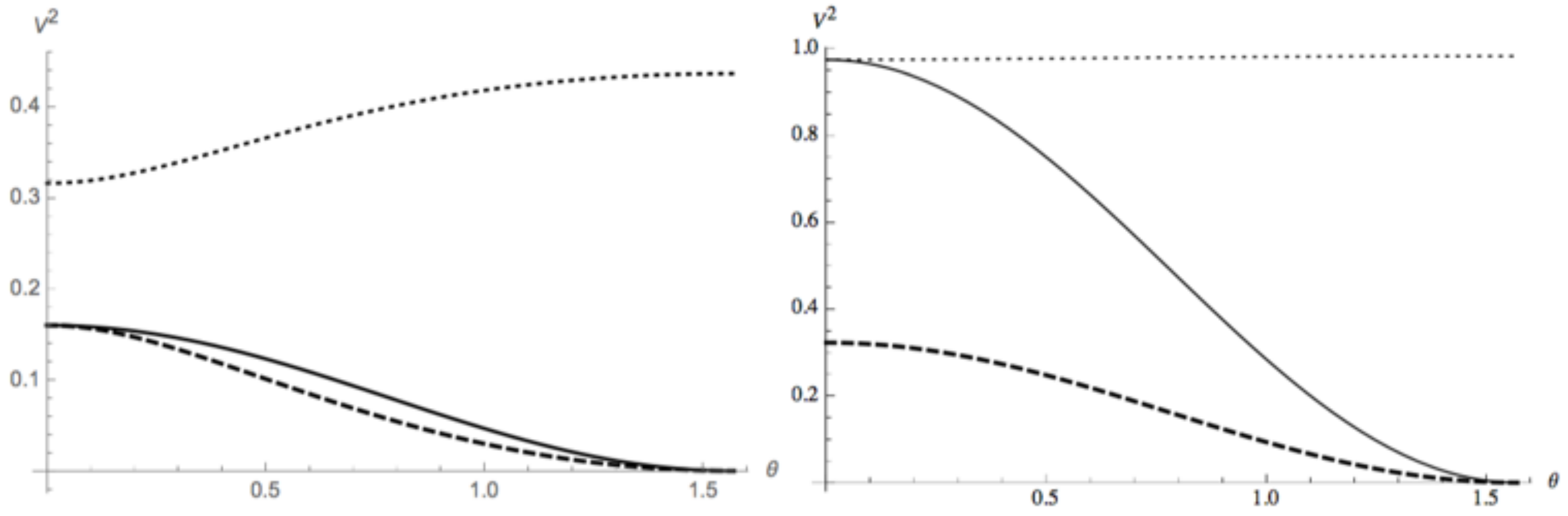
# SPEEDS OF MHD WAVES

- assume weak field limit of the equation of state to reproduce all of standard MHD results (Alfvén and magnetosonic waves) [Hernandez, Kovtun]

$$\mu \ll T^2 : \qquad p_{\text{weak}}(\mu, T) = \frac{a}{4}T^4 + \frac{g^2}{2}\mu^2 + \frac{\beta}{4}\frac{\mu^4}{T^4} + \cdots$$

- strong field limit

$$T^2 \ll \mu : \qquad p_{\text{strong}}(\mu, T) = \frac{g'^2}{2}\mu^2 + \frac{a'}{4}T^4 + \frac{\beta'}{8}\frac{T^8}{\mu^2} + \cdots$$



a) Velocities from high temperature equation of state (4.26) with $\mu/T^2 = 0.4$, $a = g = \beta = 1$.

b) Velocities from high field equation of state (4.31) with $T^2/\mu = 0.4$, $a' = g' = \beta' = 1$.

# DISSIPATION

- construct first-order (anisotropic) dissipative corrections
- 7 terms consistent with positive entropy production and charge-conjugation invariant MHD (2 shear, 3 bulk viscosities, 2 resistivities); 11 in general MHD [Hernandez, Kovtun]

$$S^\mu = s\, u^\mu - \frac{1}{T} T^{\mu\nu}_{(1)} u_\nu - \frac{\mu}{T} J^{\mu\nu}_{(1)} h_\nu , \qquad \nabla_\mu S^\mu \geq 0$$

$$\begin{aligned} T^{\mu\nu}_{(1)} &= \delta f\, \Delta^{\mu\nu} + \delta\tau\, h^\mu h^\nu + 2\, \ell^{(\mu} h^{\nu)} + t^{\mu\nu} \\ J^{\mu\nu}_{(1)} &= 2\, m^{[\mu} h^{\nu]} + s^{\mu\nu} \end{aligned}$$

$$\begin{aligned} \delta f &= -\zeta_\perp \Delta^{\mu\nu} \nabla_\mu u_\nu - \zeta_\times^{(1)} h^\mu h^\nu \nabla_\mu u_\nu \\ \delta\tau &= -\zeta_\times^{(2)} \Delta^{\mu\nu} \nabla_\mu u_\nu - 2\zeta_{||} h^\mu h^\nu \nabla_\mu u_\nu \\ \ell^\mu &= -2\eta_{||} \Delta^{\mu\sigma} h^\nu \nabla_{(\sigma} u_{\nu)} \\ t^{\mu\nu} &= -2\eta_\perp \left( \Delta^{\mu\rho} \Delta^{\nu\sigma} - \frac{1}{2} \Delta^{\mu\nu} \Delta^{\rho\sigma} \right) \nabla_{(\rho} u_{\sigma)} \\ m^\mu &= -2r_\perp T \Delta^{\mu\beta} h^\nu \nabla_{[\beta} \left( \frac{h_{\nu]} \mu}{T} \right) \\ s^{\mu\nu} &= -2r_{||} \mu \Delta^{\mu\rho} \Delta^{\nu\sigma} \nabla_{[\rho} h_{\sigma]} \end{aligned}$$

$$\begin{aligned} \eta_\perp &\geq 0 & \eta_{||} &\geq 0 \\ r_\perp &\geq 0 & r_{||} &\geq 0 \\ \zeta_\perp &\geq 0 & \zeta_\perp \zeta_{||} &\geq \zeta_\times^2 \end{aligned}$$

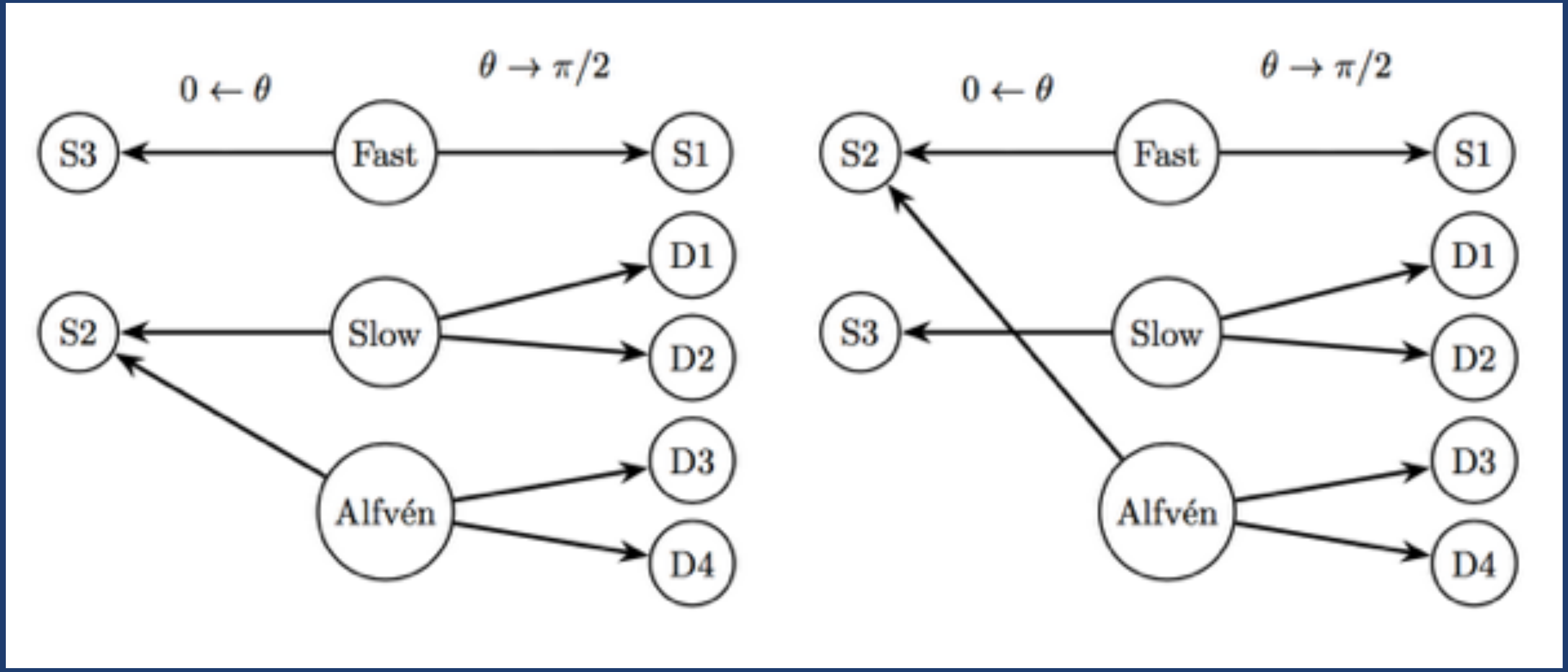
# DISSIPATION

- corrections to the dispersion relations of MHD waves
- limits of small momentum and large angle do not commute
- expand in small *k*, e.g. the Alfvén wave is

$$\omega = \pm k \cos\theta \sqrt{\frac{1}{1+\frac{Ts}{\mu\rho}}} - \frac{1}{2} i \left( \frac{\sin^2\theta}{Ts+\mu\rho}\eta_\perp + \frac{\cos^2\theta}{Ts+\mu\rho}\eta_\parallel + \frac{\mu\cos^2\theta}{\rho} r_\perp + \frac{\mu\sin^2\theta}{\rho} r_\parallel \right) k^2$$

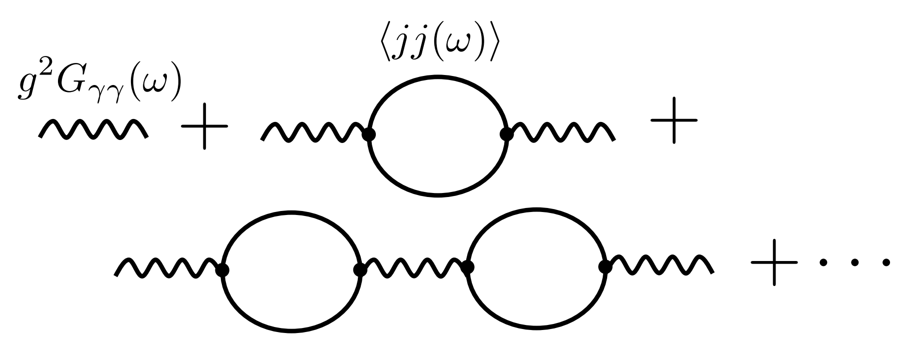
- for non-infinitesimal *k*, there is a critical angle $\theta_c(k)$ at which propagating modes (slow-MS and Alfvén) cease to exist and become diffusive

  (weak/strong *B*)

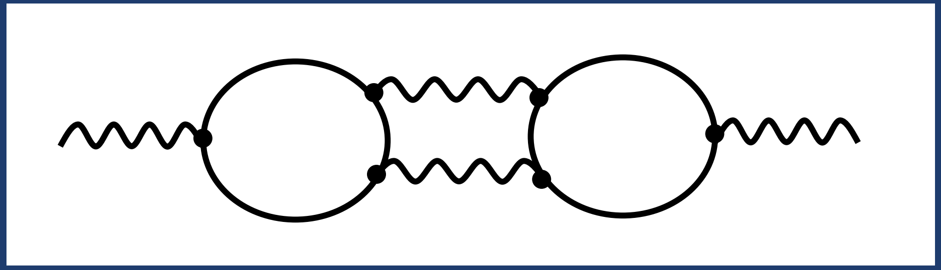


# DISSIPATION

- in the regime of a strong magnetic field, when anisotropic corrections become large, critical angle decreases from
- Kubo formulae, e.g. resistivities $r_{\parallel} = \lim_{\omega \to 0} \frac{G^{xy,xy}_{JJ}(\omega)}{-i\omega}$ $\quad$ $r_{\perp} = \lim_{\omega \to 0} \frac{G^{xz,xz}_{JJ}(\omega)}{-i\omega}$
- normally $\langle EE(\omega) \rangle \sim -(-i\omega)^2 \frac{g^2 G_{\gamma\gamma}(\omega)}{1 - \langle j_{el} j_{el}(\omega) \rangle \, g^2 G_{\gamma\gamma}(\omega)} \longrightarrow r \sim (-i\omega) \frac{1}{\langle j_{el} j_{el}(\omega) \rangle} \sim \frac{1}{\sigma}$



- now, new diagrams: e.g.



$$\longrightarrow \qquad r \neq \frac{1}{\sigma}$$

# MHD AT ZERO TEMPERATURE

- take the limit of zero temperature
- a sector of the theory without entropy production, and $\delta T = 0$
- symmetry breaking patterns

$$\begin{aligned} \text{finite T}: \quad & SO(3,1) \to SO(2) \\ \text{zero T}: \quad & SO(3,1) \to SO(1,1) \times SO(2) \end{aligned}$$

- degrees of freedom $\mu,\ u_{\mu\nu}$ s.t. $u_{\mu\nu} u^{\mu\nu} = -2, \quad u^{\mu\nu} u_{\nu\rho} u^{\rho\sigma} = u^{\mu\sigma}$
- ideal hydrodynamics $T^{\mu\nu}_{(0)} = -\varepsilon\, \Omega^{\mu\nu} + p\, \Pi^{\mu\nu}$ $\quad \Omega^{\mu\nu} \equiv u^{\mu\lambda} u_\lambda{}^\nu$

  (no 1st order correction) $J^{\mu\nu}_{(0)} = \rho\, u^{\mu\nu}$ $\quad \Pi^{\mu\nu} \equiv g^{\mu\nu} - \Omega^{\mu\nu}$
- entropy conservation vs. closure of equations (reduces 8 (7) to 5)

$$(\nabla_\mu T^{\mu\nu})\, \Omega_{\nu\lambda} + \mu\, (\nabla_\mu J^{\mu\nu})\, u_{\nu\lambda} = 0$$

# MHD AT ZERO TEMPERATURE

- this theory has no slow MS waves due to vanishing fluctuation of the temperature
- universal dispersion relation for Alfvén waves (to all orders)

$$\omega = \pm k \cos\theta$$

- fast MS waves receive corrections from 2nd order hydrodynamics and travel at the

$$\omega_M = \pm\left(v_M k + \alpha_M k^3 + \cdots\right)$$

$$v_M = \left(\cos^2\theta + \frac{\rho}{\mu\chi}\sin^2\theta\right)^{1/2} \qquad\qquad \chi = \frac{\partial\rho}{\partial\mu}$$

- in absence of any scales beyond *B*, speed of fast MS waves is the speed of light

# HOLOGRAPHIC PLASMA

# HOLOGRAPHIC MHD

- example of a strongly coupled plasma [S. G., Poovuttikul, arXiv:1707.04182]
- fields at the boundary source dual conserved operators
- generating functional

$$W\left[g_{\mu\nu}, b_{\mu\nu}\right] = \left\langle \exp\left[i\int d^4x\sqrt{-g}\left(\frac{1}{2}T^{\mu\nu}g_{\mu\nu} + J^{\mu\nu}b_{\mu\nu}\right)\right]\right\rangle$$

- construct a theory of a metric and a two-form gauge field in 5D

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(R + \frac{12}{L^2} - \frac{1}{3e_H^2}H_{abc}H^{abc}\right)$$

$$H = dB$$

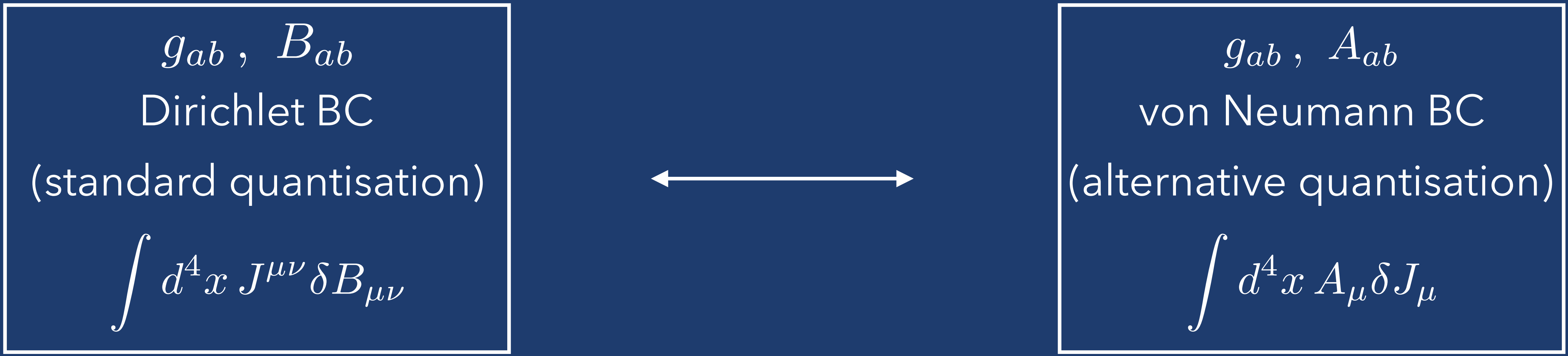
- naively, it is dual to the bulk Einstein-Maxwell theory with $F_{ab}F^{ab}$
  if we write $H = \star F$ : $F \wedge \star F \quad \rightarrow \quad H \wedge \star H$

# HOLOGRAPHIC MHD

- the connection to Einstein-Maxwell theory is more subtle
- dualise by writing the path integral over *F*

$$Z \supset \int \mathcal{D}F_{ab}\,\mathcal{D}B_{ab}\,\exp\left\{ i\,\frac{N_c^2}{8\pi^2}\int d^5x\sqrt{-G}\,\left(F_{ab}F^{ab} + e_H^{-1}B_{ab}\epsilon^{abcde}\nabla_c F_{de}\right)\right\}$$

$$\langle J_R^\mu\rangle = -\frac{N_c^2}{2\pi^2}\lim_{u\to 0} F^{u\mu} = -\frac{N_c^2}{2\pi^2 e_H}\lim_{u\to 0}\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma}$$



$g_{ab}\,,\quad B_{ab}$

Dirichlet BC

(standard quantisation)

$$\int d^4x\, J^{\mu\nu}\delta B_{\mu\nu}$$

$g_{ab}\,,\quad A_{ab}$

von Neumann BC

(alternative quantisation)

$$\int d^4x\, A_\mu \delta J_\mu$$

- the gauge field on the boundary is dynamical (photons) [Hofman, Iqbal]

# HOLOGRAPHIC MHD

- full holographic action

$$S = \frac{N_c^2}{8\pi^2}\left[\int d^5x\sqrt{-G}\left(R+12-\frac{1}{3e_H^2}H_{abc}H^{abc}\right)+\int_{\partial M} d^4x\sqrt{-g}\left(2\,\mathrm{tr}K-6+\frac{1}{e_H^2}\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu}\ln\mathcal{C}\right)\right]$$

$$\mathcal{H}_{\mu\nu}=n^a H_{a\mu\nu}$$

- D'Hoker-Kraus black brane

$$ds^2 = G_{ab}dx^a dx^b = r_h^2\left(-F(u)dt^2+\frac{e^{2\mathcal{V}(u)}}{v}(dx^2+dy^2)+\frac{e^{2\mathcal{W}(u)}}{w}dz^2\right)+\frac{du^2}{4u^3F(u)}$$

$$H = \frac{Br_h^2 e^{-2\mathcal{V}+\mathcal{W}}}{2u^{3/2}\sqrt{w}}\,dt\wedge dz\wedge du$$

- on-shell action at a cut-off (FG coordinates)

$$\delta S_{on-shell} = -\frac{N_c^2}{4\pi^2}\int d^4x\sqrt{-g}\,\mathcal{H}^{\mu\nu}\left(\delta B_{\mu\nu}^{(0)}+\delta B_{\mu\nu}^{(1)}\ln\mathcal{C}^2\rho_\Lambda\right)$$

- source

$$B_{\mu\nu}^{(0)}+B_{\mu\nu}^{(1)}\ln\mathcal{C}^2\rho_\Lambda = \frac{4\pi^2}{N_c^2}b_{\mu\nu}$$

# HOLOGRAPHIC MHD

- expectation values of dual conserved operators

$$\begin{aligned}\langle T_{\mu\nu}\rangle = -\frac{N_c^2}{4\pi^2}\lim_{\epsilon\to 0}\frac{r_h^2}{\epsilon}\Bigg(&K_{\mu\nu}-\gamma_{\mu\nu}K-3\gamma_{\mu\nu}+\frac{1}{2}R[\gamma]_{\mu\nu}-\frac{1}{4}\gamma_{\mu\nu}R[\gamma] &&\longleftarrow \quad \text{matter}\\ &-\Bigg(\mathcal{H}_{\mu\lambda}\mathcal{H}_{\nu}{}^{\lambda}-\frac{1}{4}\gamma_{\mu\nu}\mathcal{H}_{\alpha\beta}\mathcal{H}^{\alpha\beta}\Bigg)\ln(\mathcal{C}\rho)\Bigg)\Bigg|_{\rho=\epsilon} &&\longleftarrow \quad \text{Maxwell}\end{aligned}$$

$$\langle J_{\mu\nu}\rangle = -\lim_{\epsilon\to 0}\mathcal{H}_{\mu\nu}\Big|_{\rho=\epsilon}$$

- stress-energy tensor has two contributions, more precisely [Fuini, Yaffe]

$$\langle T_{\mu\nu}\rangle = \lim_{\epsilon\to 1/\Lambda^2}\frac{N_c^2}{2\pi^2}\Bigg(g^{(2)}_{\mu\nu}-g^{(0)}_{\mu\nu}(g^{(2)})^{\lambda}{}_{\lambda}+\frac{1}{2}\tilde{h}_{\mu\nu}+\tilde{h}_{\mu\nu}\ln\left(\mathcal{C}^2\rho\right)+\mathcal{O}(\rho,\partial^2)\Bigg)\Bigg|_{\rho=\epsilon}$$

$$\frac{N_c^2}{\pi^2}\tilde{h}_{\mu\nu}\ln(\Lambda/\mathcal{C}) = \frac{N_c^2}{\pi^2}\tilde{h}_{\mu\nu}\ln(\Lambda/M)+\tilde{h}_{\mu\nu}\left(\frac{2}{e_r^2}-\frac{N_c^2}{\pi^2}\ln(\Lambda/M)\right)$$

# HOLOGRAPHIC MHD

$$\langle T_{\mu\nu}\rangle = \langle T^{matter}_{\mu\nu}\rangle + \langle T^{EM}_{\mu\nu}\rangle$$

$$\langle T^{matter}_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\left(g^{(2)}_{\mu\nu} - g^{(0)}_{\mu\nu}(g^{(2)})^{\lambda}{}_{\lambda} + \frac{1}{2}\tilde{h}_{\mu\nu}\right) - \frac{N_c^2}{\pi^2}\tilde{h}_{\mu\nu}\ln(\Lambda/M)$$

$$\langle T^{EM}_{\mu\nu}\rangle = -\left(\frac{2}{e_r^2} - \frac{N_c^2}{\pi^2}\ln\left(\Lambda/M\right)\right)\tilde{h}_{\mu\nu}$$

- electromagnetic term comes from the Maxwell action

$$S_{EM} = -\frac{1}{4e(\Lambda/M)^2}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

$$\langle T^{EM}_{\mu\nu}\rangle = \frac{1}{e(\Lambda/M)^2}\left(\langle F_{\mu\alpha}F_{\nu}{}^{\alpha}\rangle - \frac{1}{4}\eta_{\mu\nu}\langle F_{\alpha\beta}F^{\alpha\beta}\rangle\right) = \frac{1}{e(\Lambda/M)^2}\left(\langle F_{\mu\alpha}\rangle\langle F_{\nu}{}^{\alpha}\rangle - \frac{1}{4}\eta_{\mu\nu}\langle F_{\alpha\beta}\rangle\langle F^{\alpha\beta}\rangle\right)$$

- trace anomaly and *N=4 NSVZ* beta function of $U(1)_R$ (from $SU(4)_R$)

$$\langle T^{\mu}{}_{\mu}\rangle = -\frac{\beta(e)}{e^3}\mathcal{B}^2 = -\frac{N_c^2}{4\pi^2}\mathcal{B}^2$$

$$\beta\left(1/e^2\right) = \mu\frac{de^{-2}}{d\mu} = -\frac{N_c^2}{2\pi^2}\left[\frac{1}{6}\sum_{\alpha=1}^{4}\left(q^{\alpha}_{\mathrm{f}}\right)^2 + \frac{1}{12}\sum_{a=1}^{3}\left(q^{a}_{\mathrm{s}}\right)^2\right] = -\frac{N_c^2}{2\pi^2}$$

# HOLOGRAPHIC MHD

- final renormalised expectation values

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\left(g^{(2)}_{\mu\nu} - g^{(0)}_{\mu\nu}(g^{(2)})^\lambda{}_\lambda + \frac{1}{2}\tilde{h}_{\mu\nu}\right) - \frac{2}{e_r^2}\tilde{h}_{\mu\nu}$$
$$\langle J_{\mu\nu}\rangle = 2B^{(1)}_{\mu\nu}$$

- RG-condition-dependent electromagnetic coupling

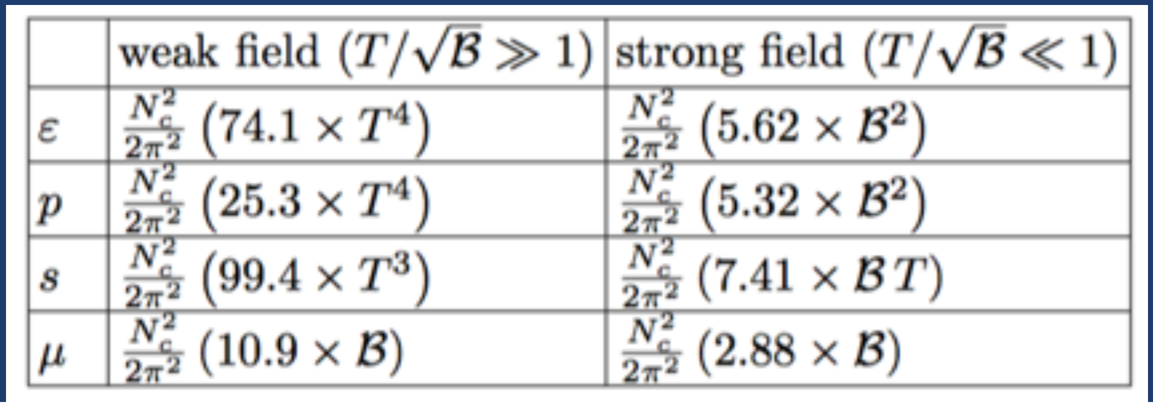
$$\bar{\alpha} = \frac{N_c^2}{2\pi^2}\alpha = \frac{N_c^2}{2\pi^2}\frac{e_r^2}{4\pi} = \frac{1}{137}$$

- Ward identities from MHD are satisfied

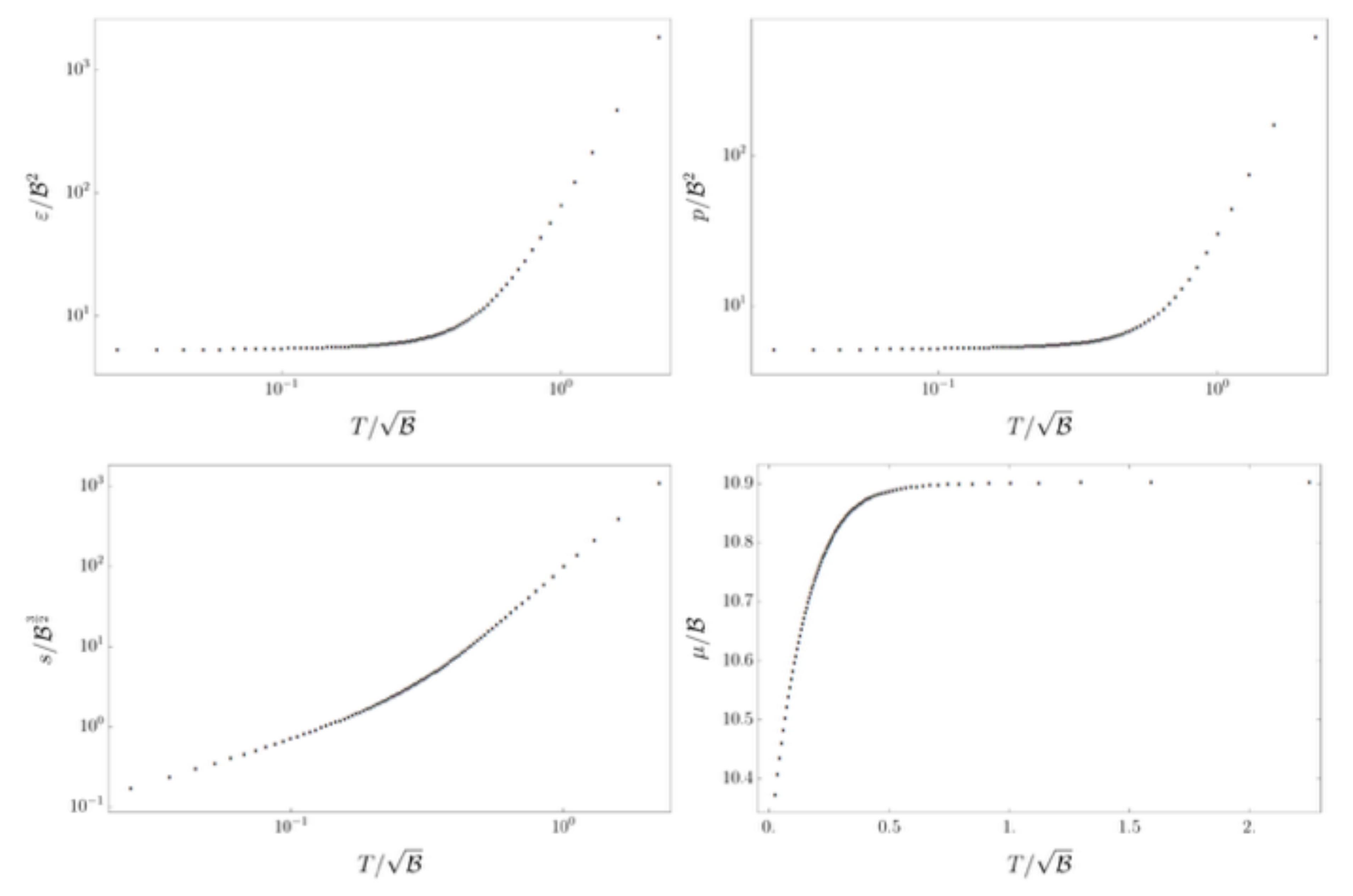
$$\nabla_\mu T^{\mu\nu} = H^\nu{}_{\rho\sigma}J^{\rho\sigma} \qquad \nabla_\mu J^{\mu\nu} = 0$$

- interpretation: not quite gauged *N=4* SYM (gauge anomaly of $U(1)_R$, no Chern-Simons term, *B*-term not in IIB supergravity)

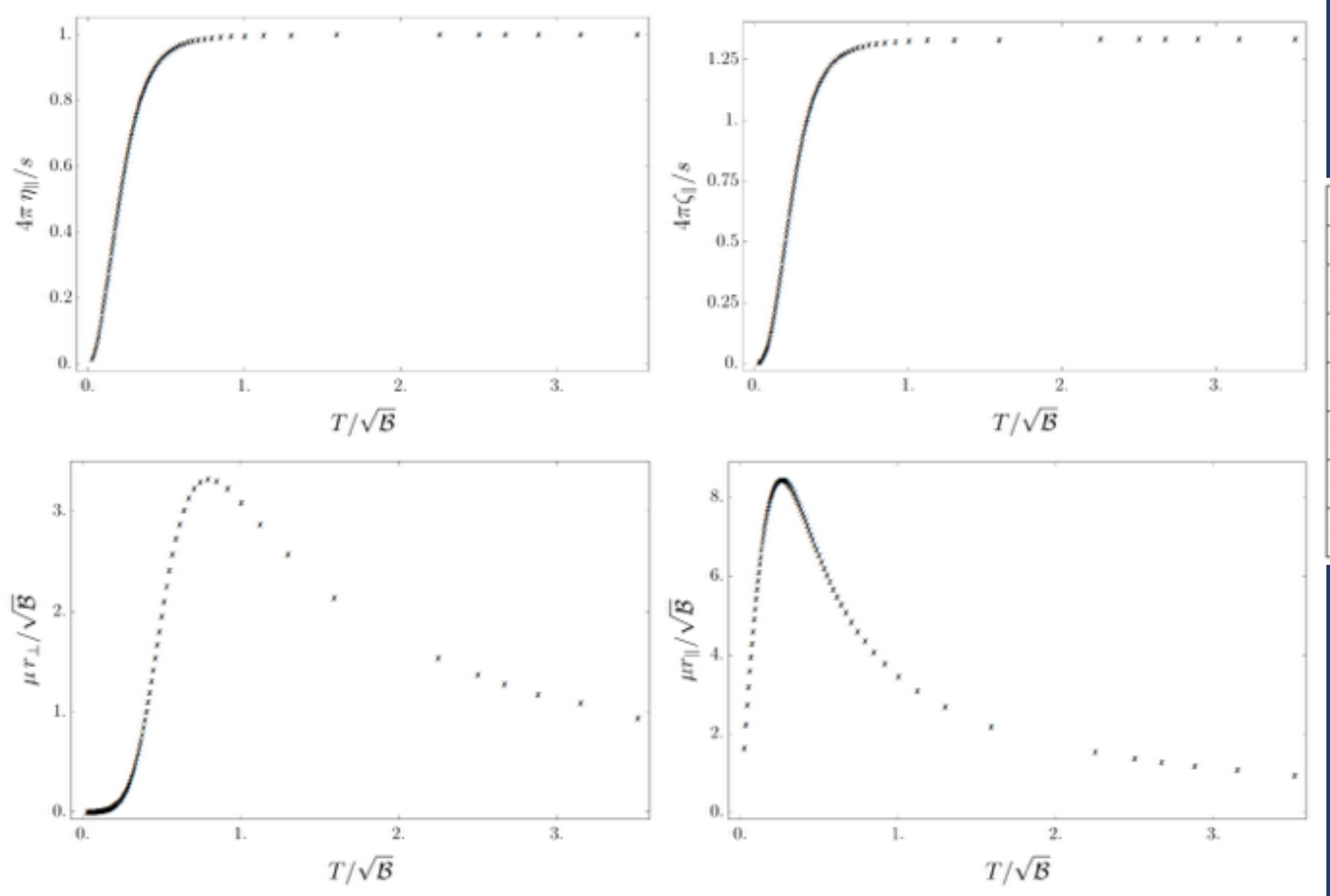
# THERMODYNAMICS

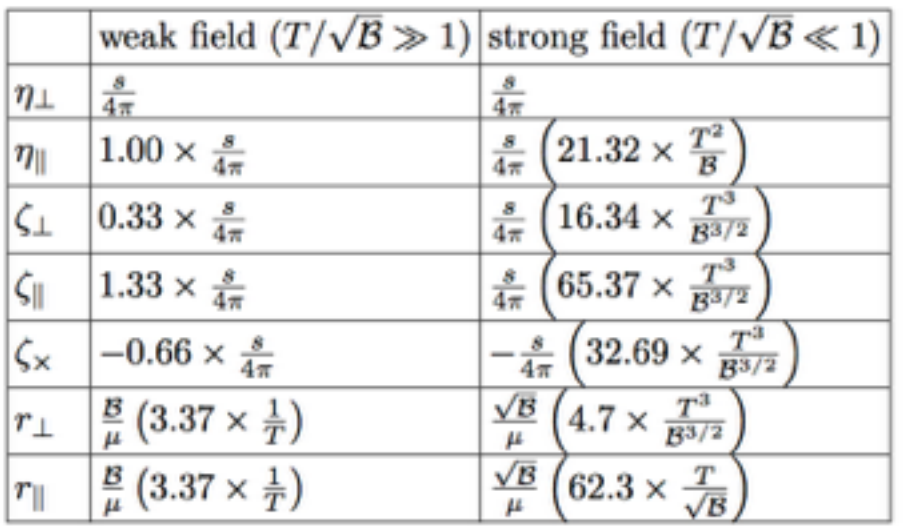


| | weak field $(T/\sqrt{\mathcal{B}} \gg 1)$ | strong field $(T/\sqrt{\mathcal{B}} \ll 1)$ |
|---|---|---|
| $\varepsilon$ | $\frac{N_c^2}{2\pi^2}(74.1 \times T^4)$ | $\frac{N_c^2}{2\pi^2}(5.62 \times \mathcal{B}^2)$ |
| $p$ | $\frac{N_c^2}{2\pi^2}(25.3 \times T^4)$ | $\frac{N_c^2}{2\pi^2}(5.32 \times \mathcal{B}^2)$ |
| $s$ | $\frac{N_c^2}{2\pi^2}(99.4 \times T^3)$ | $\frac{N_c^2}{2\pi^2}(7.41 \times \mathcal{B}T)$ |
| $\mu$ | $\frac{N_c^2}{2\pi^2}(10.9 \times \mathcal{B})$ | $\frac{N_c^2}{2\pi^2}(2.88 \times \mathcal{B})$ |



# TRANSPORT COEFFICIENTS





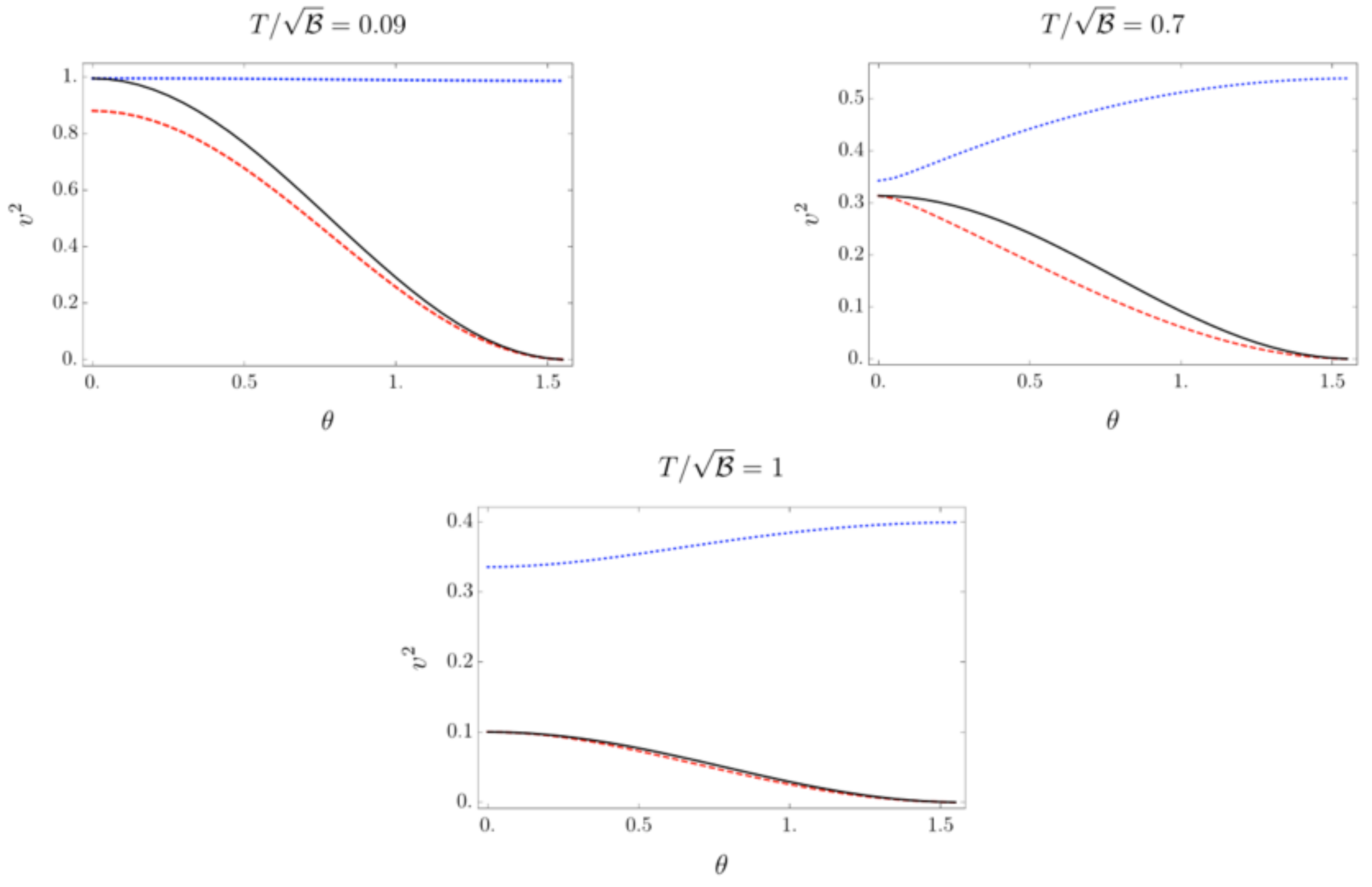
| | weak field ($T/\sqrt{\mathcal{B}} \gg 1$) | strong field ($T/\sqrt{\mathcal{B}} \ll 1$) |
|---|---|---|
| $\eta_\perp$ | $\frac{s}{4\pi}$ | $\frac{s}{4\pi}$ |
| $\eta_\|$ | $1.00 \times \frac{s}{4\pi}$ | $\frac{s}{4\pi}\left(21.32 \times \frac{T^2}{\mathcal{B}}\right)$ |
| $\zeta_\perp$ | $0.33 \times \frac{s}{4\pi}$ | $\frac{s}{4\pi}\left(16.34 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$ |
| $\zeta_\|$ | $1.33 \times \frac{s}{4\pi}$ | $\frac{s}{4\pi}\left(65.37 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$ |
| $\zeta_\times$ | $-0.66 \times \frac{s}{4\pi}$ | $-\frac{s}{4\pi}\left(32.69 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$ |
| $r_\perp$ | $\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$ | $\frac{\sqrt{\mathcal{B}}}{\mu}\left(4.7 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$ |
| $r_\|$ | $\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$ | $\frac{\sqrt{\mathcal{B}}}{\mu}\left(62.3 \times \frac{T}{\sqrt{\mathcal{B}}}\right)$ |

- maximal resistivity
- bulk viscosities

$$\zeta_\perp \zeta_\| = \zeta_\times^2$$

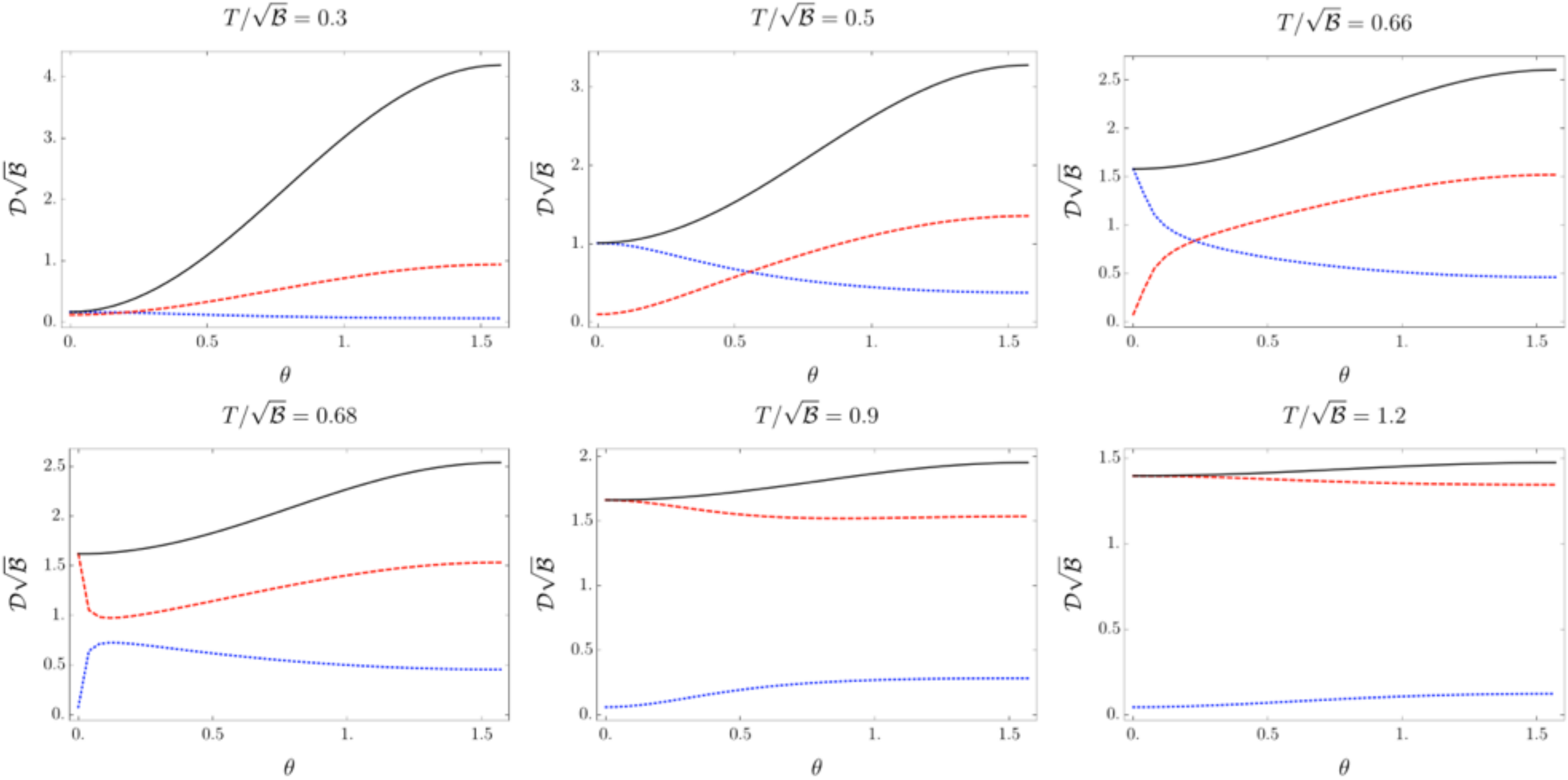
# SPEEDS OF SOUND

$$\omega = \pm v k - i\mathcal{D} k^2$$

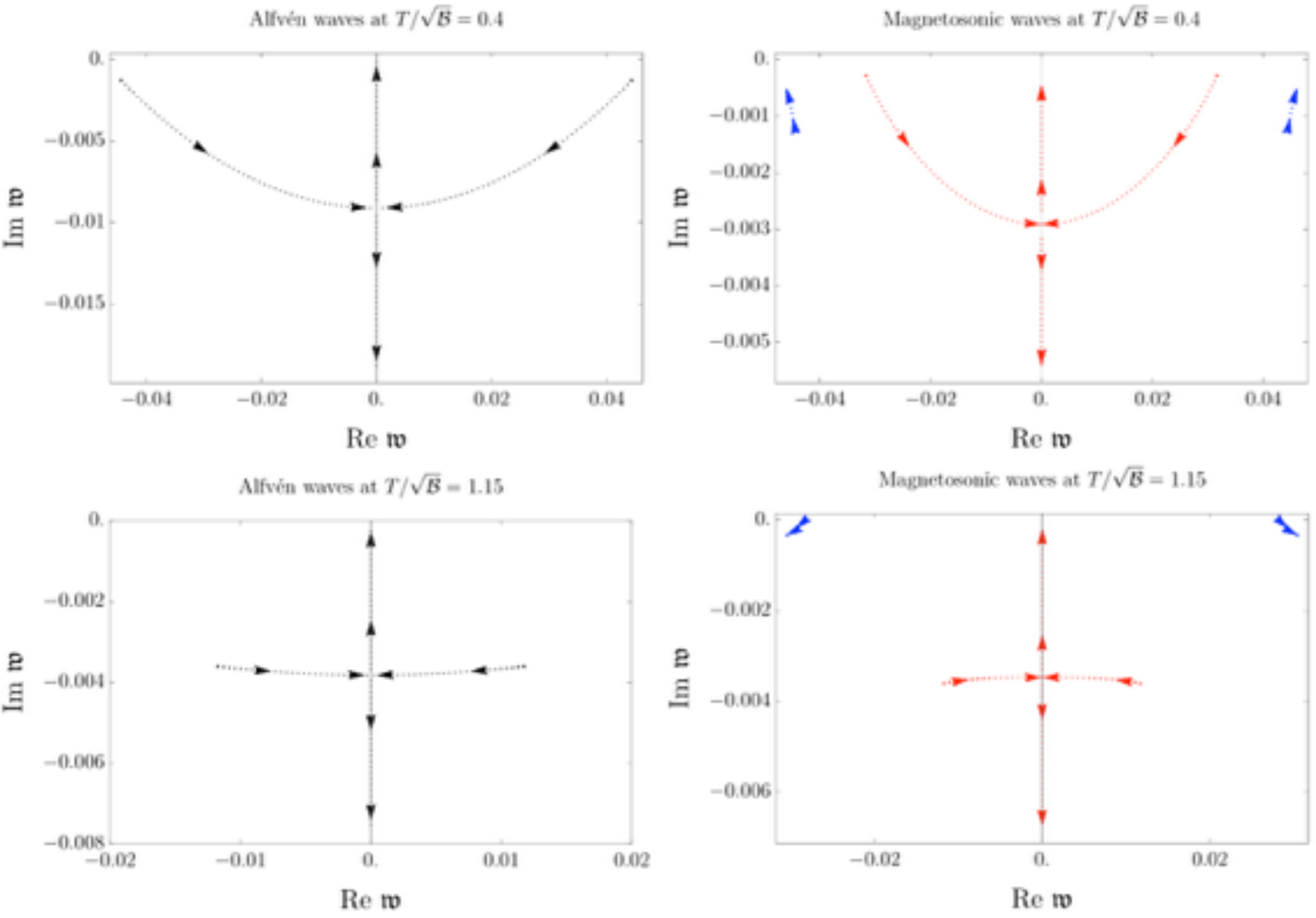


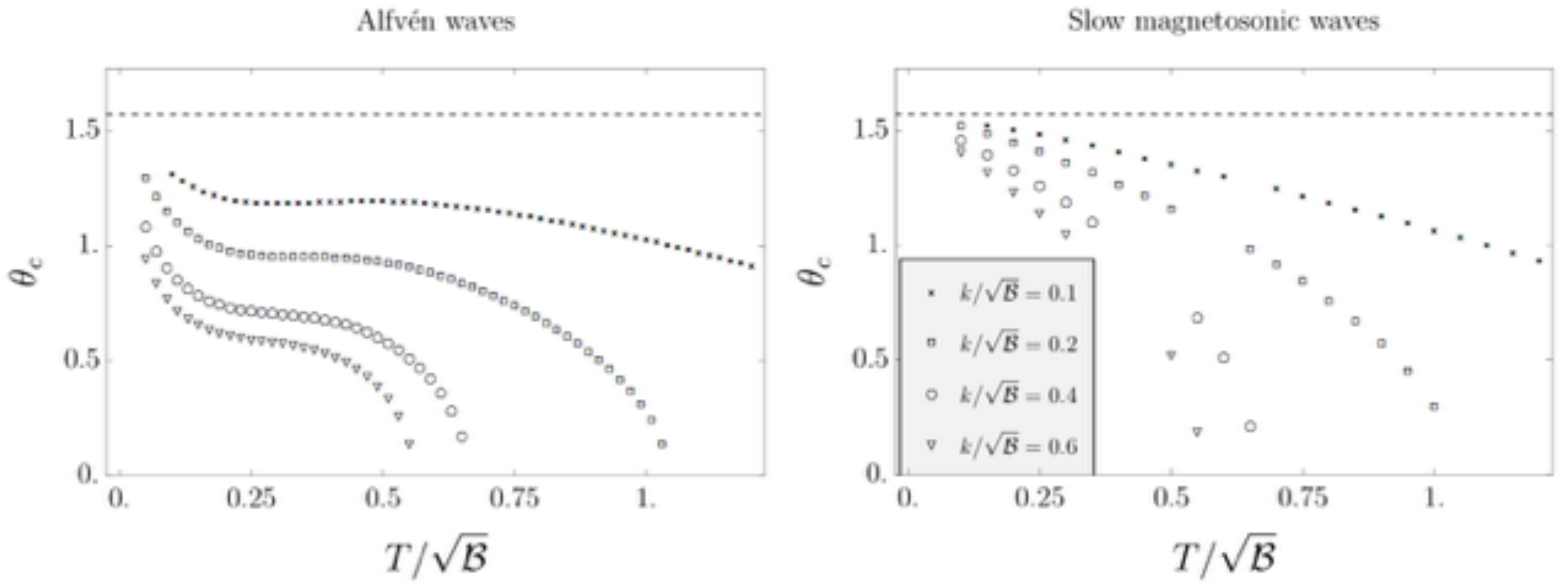
# SOUND ATTENUATION

$$\omega = \pm v k - i \mathcal{D} k^2$$



# TRANSMUTATION OF MODES

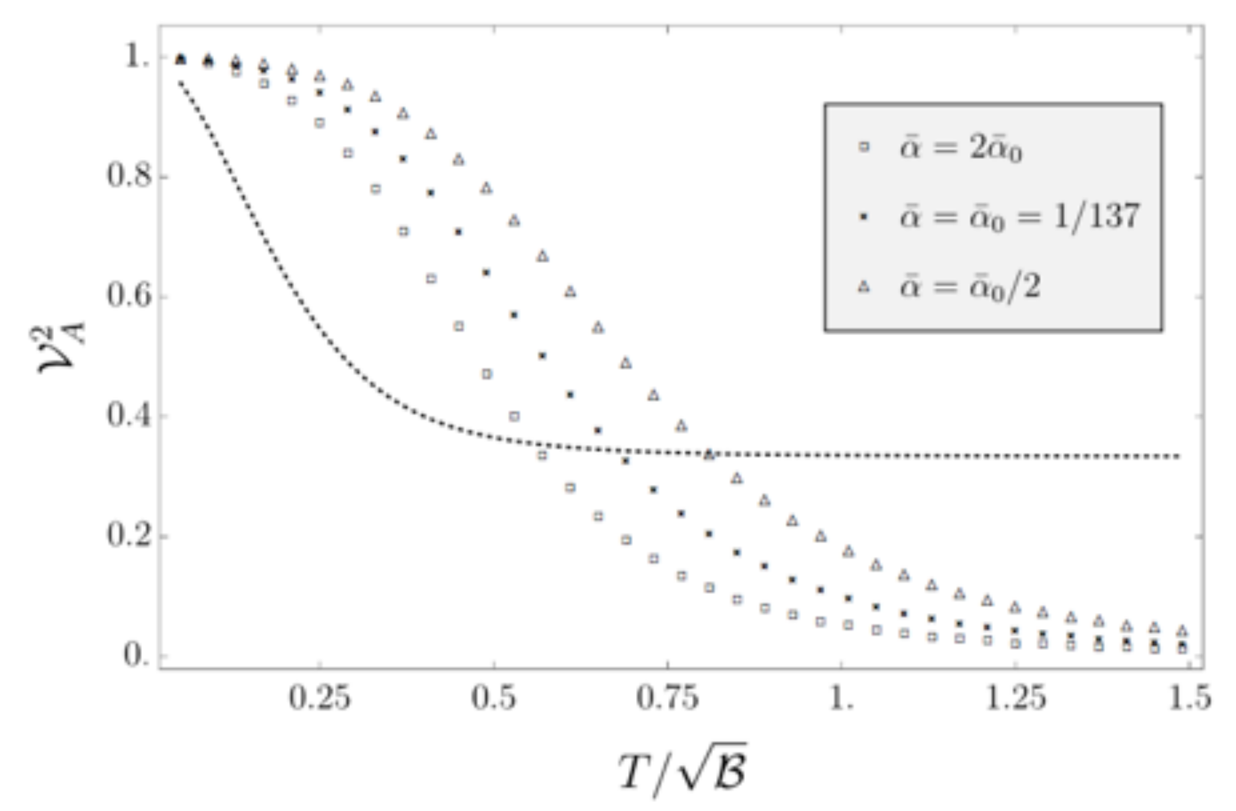


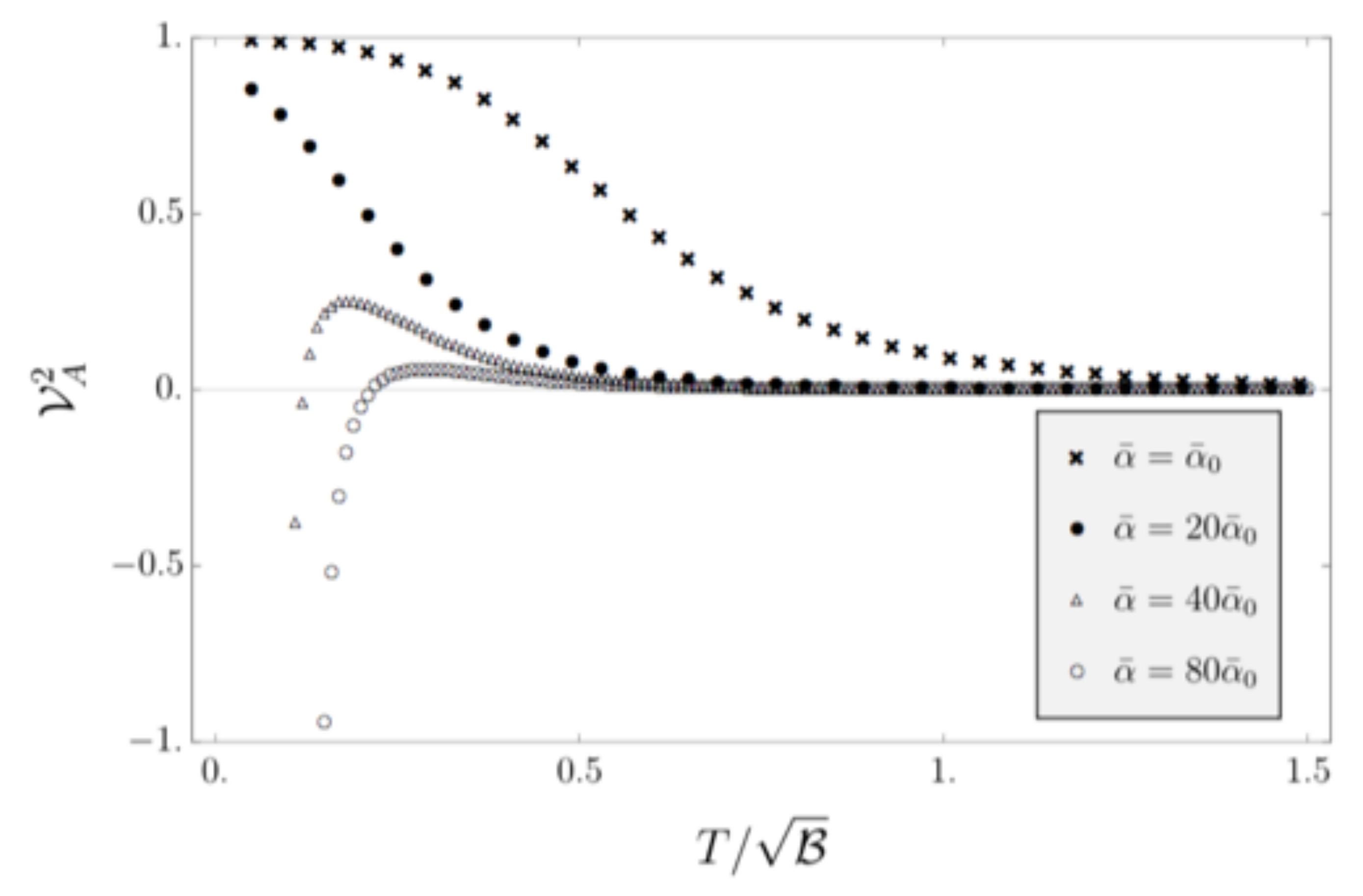


transition from propagating to non-propagating modes (from sound to diffusion) at a $k$-dependent critical angle $\theta_c(k)$

# ELECTRIC CHARGE

- behaviour of waves (e.g. Alfvén) strongly dependent on the renormalisation condition for the boundary electric charge





# SUMMARY

- new, symmetry-based formulation of MHD, with new transport coefficients which agrees with (complete) standard MHD at low magnetic field strength [Hernandez, Kovtun]
- we claim this theory can be used for any plasma
- MHD may survive the zero-*T* limit and become non-dissipative

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- the theory can be studied via a holographic toy model, which includes dynamical electromagnetism (other applications?)
- MHD modes could be examined at any strength of *B*
- transmutation of propagating to non-propagating modes, least-conductive plasma, saturation of bulk viscosity inequality at strong coupling

# OUTLOOK

is any of this useful for realistic plasmas?

# THANK YOU!