Lattice QCD for axion cosmology

Z. Fodor

University of Wuppertal

Borsanyi, Gunther, Kampert, Katz, Kawanai, Kovacs, Mages, Pasztor, Pittler, Redondo, Ringwald, Szabo

July 13, 2017, Oxford (United Kingdom)

Phys. Lett. B752 (2016) 175; quenched results
Nature 539 (2016) 69; dynamical case

SFB TR55

Outline

What are Axions?

Simple candidate for extension of SM:

- add a complex scalar field ϕ
- with symmetry breaking Mexican hat at high scale f_a
- couple the Goldstone mode $\arg(\phi)$ as dynamic θ angle to QCD
- (pseudo)Goldstone mode of the tilted potential is called axion

$$\mathcal{L}_a = \partial_\mu^* \phi \partial^\mu \phi - \frac{\lambda}{8}\left(\phi^*\phi - f_a^2\right)^2 + \chi_t \frac{|\phi|}{f_a} \cos(\arg(\phi))$$

Strong CP Problem

Full QCD can include an effective CP breaking θ term:

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (D_\mu \gamma^\mu + m_f)\psi_f + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - i\theta \frac{g^2}{32\pi^2} \tilde{F}^a_{\mu\nu} F^a_{\mu\nu} \qquad (1)$$

with $-\pi < \theta \leq \pi$, so naturally $\theta \sim \mathcal{O}(1)$

From experiments: $|\theta| < 10^{-10}$, unnatural $\rightarrow$ fine-tuning?

Antrophic principle does not help: $|\theta| < 10^{-2}$ would be still fine

Peccei-Quinn solution

interpret/introduce θ as a dynamical field with minimum at 0

- as phase of a global $U(1)$ symmetric scalar field ϕ
- with spontaneous symmetry breaking potential

redefinition of the angular mode as $\arg(\phi) := \theta_{eff}$

$$\mathcal{Z} = \int \mathcal{D}A_\mu exp(-S_{QCD} - i\theta_{eff} \cdot g^2/32\pi^2 \cdot \tilde{F}^a_{\mu\nu} F^a_{\mu\nu})$$

Z reduced, F raised by phase cancellation unless θ_{eff}=0

one can get the mass of the axion: $m_A^2 \propto \langle Q^2 \rangle \propto \chi_t$

effective potential for ϕ has a tilt & a minimum for $0 = \theta_{eff} = \arg(\phi) = 0$

$$\mathcal{L}_a = \partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda}{8}\left(\phi^*\phi - f_a^2\right)^2 + \chi_t \frac{|\phi|}{f_a} \cos(\theta_{eff})$$

Massive Modes

Two massive oscillations of ϕ

- heavy "string" mode in magnitude; with mass $m_s \approx \sqrt{\lambda} f_a$
- light "axion" mode in phase; with mass $m_a \approx \sqrt{\chi_t}/f_a$

Given χ_t, cosmology gives an abundance of axions

Axions can provide substantial/total amount of dark matter

Two axion production mechanisms:

- dynamics and decay of string/wall networks
- misalignment (sole ingredient in the pre-inflation case)

Topological Structures

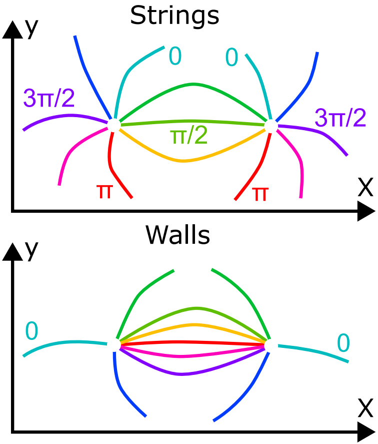
Spontaneous symmetry breaking + causality:

different θ_{eff} in causally disconnected patches

$\Rightarrow$ Strings

with QCD potential $\theta_{eff} \rightarrow 0$ everywhere

$\Rightarrow$ Walls between Strings



String/Wall Networks

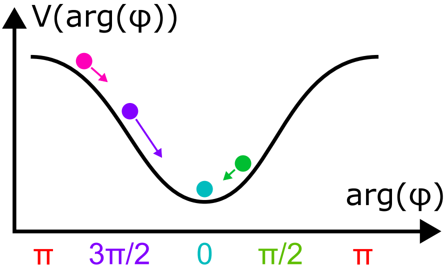
- string-like defects arise and form networks
  $\rightarrow$ axion radiation
- when χ_t becomes relevant, formation of walls between strings
  $\rightarrow$ axion radiation
- walls accelerate annihilation of topological defects
  $\rightarrow$ axion radiation

χ_t influences string dynamics, needed as input for total axion production
only in case of a post-inflationary Peccei-Quinn symmetry breaking

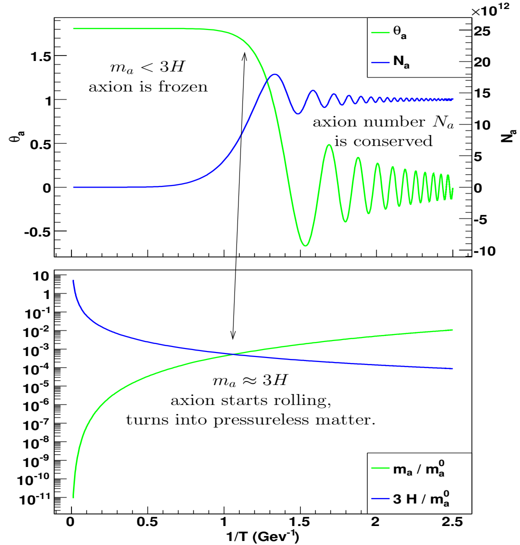
Misalignment

- alignment of misaligned neighbouring patches
  $\rightarrow$ axion radiation
- when χ_t becomes relevant, θ_{eff} "rolls" down to $\theta = 0$
  $\rightarrow$ axion radiation

χ_t influences field dynamics, needed as input for total axion production



Evolution in the expanding universe



Topological Charge

Integral

$$Q = \int_{\mathcal{M}} \mathrm{d}^4 x q(x)$$

over the topological charge density

$$q(x) = \frac{1}{4\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left(F_{\mu\nu}(x) F_{\rho\sigma}(x)\right)$$

- discretized in finite volume on $\mathcal{M} = \mathbb{T}^4$
- sectors with different Q separated by infinite action barrier in continuum
- problem for ergodicity of MC algorithms with small "step" size in field space

Topological Susceptibility

Integral of *qq* correlator

$$\chi = \int_{\mathcal{M}} \mathrm{d}^4 x \langle q(0) q(x) \rangle$$

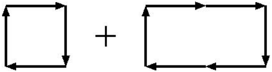
With global translation symmetry on $\mathcal{M} = \mathbb{T}^4$

$$\chi = \frac{1}{V_4} \langle Q^2 \rangle$$

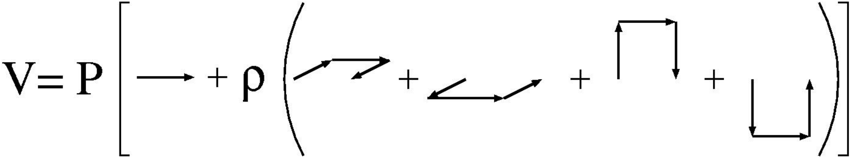
- measurement must sample sectors with $Q \neq 0$
- difficult close to continuum
- difficult when $\chi V_4 = \langle Q^2 \rangle \ll 1$

Choice of the action: improvement

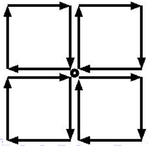
no consensus: which action offers the most cost effective approach
our choice: tree-level $O(a^2)$-improved Symanzik gauge action



2/4-level (stout) staggered or 2/3 (HEX) Wilson or overlap fermions



with tree-level $O(a)$ clover improved fermions (Wilson):



Scale settings and the static potential

raw output of lattice QCD: physical quantities in lattice unit
$\Rightarrow$ measure a dimensionful quantity Q (M_Ω or f_K)
the lattice spacing is given by a=$(aQ^{lat})/Q^{exp}$

today erros below 2% for several lattice predictions
it depends crucially on the error of the lattice spacing
need for a controlled/small error lattice spacing determination

not necessarily directly accesable for experiments e.g. potential
popular choices are:
string tension (strictly speaking doesn't exist: string breaking)
the Sommer-scale $r_x^2 \cdot dV/dr = C_x$
originally r_0 with $C_0 = 1.65$ or MILC choice r_1 with C_1=1

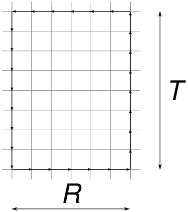
String tension and Sommer scale from the potential

let us take $Q = \sigma$, the string tension

$$\sigma = \lim_{R\to\infty} \frac{\mathrm{d}V(R)}{\mathrm{d}R}$$

"Experimental value:" $\sqrt{\sigma} = 465$ MeV

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln[W(R,T)]\,, \qquad W(R,T) \quad =$$



Static q–$\bar{q}$ potential V(r)

$r_X^2 \cdot dV(r)/dr = C_x$ we need interpolation for the derivative

Sommer-scale, Omega mass, f_π and f_K

unfortunately, the calculations of r_0 & r_1 are quite involved
far more complicated than fitting the masses of particles

complications are reflected in the literature
MILC: r_1=0.3117(22) fm (better than 1% accuracy)
RBC/UKQCD: r_1=0.3333(93)(1)(2) fm
7% difference and 2.3σ tension between them

another popular way is to use the Omega baryon mass
the experimental value of M_Ω is well known
more CPU demanding & sensititve to the strange quark mass
mismatched strange quark mass leads to a mismatched scale

difficulties with f_π (chiral extrapolation) & f_K (mismatched m_s)

suggestion of M. Luscher: use the Wilson flow to set the scale

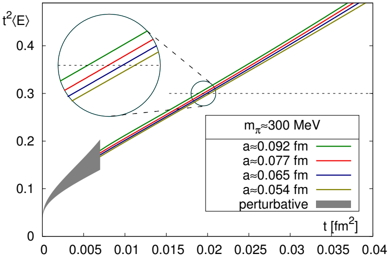
Gauge field flow

flow equation: $\dot{V}_t = Z(V_t)V_t$, where Z is the staple
equivalent to a series of infinitesimal stout smearing steps

M. Luscher, JHEP 1008 (2010) 071

as a representative example $E = G^a_{\mu\nu} G^a_{\mu\nu}/4$ is considered

above the cut-off (small t): lattice and continuum quite different



observed "linearity" for $t^2\langle E\rangle$
one can extract it by $t \cdot dt^2\langle E\rangle/dt$
instead $t^2\langle E\rangle$=0.3 (M. Luscher)
$t \cdot dt^2\langle E\rangle/dt$=0.3 ($w_0$ scale)
it should have less scaling violation

the non-universal part (cut-off) shrinks

Quenched Study

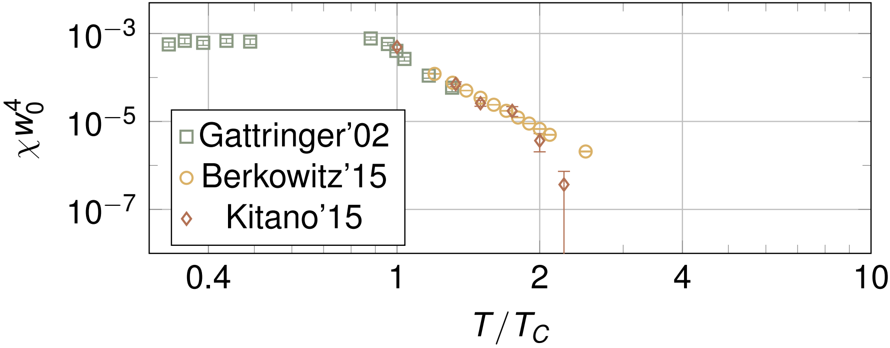
How far can we go with conventional brute force?

$\rightarrow$ test it in the "cheap" quenched case

- learn how to control all errors and apply it for full QCD
- test bed to improve on the brute force strategy
- roughly the same temperature scaling as for full QCD
- estimate the costs for the full result

Previous lattice studies

Alles:1996nm,Gattringer:2002mr etc. 1st gen results
Berkowitz:2015aua large volume/statistics up to $2.5T_c$
Kitano:2015fla HMC up to $2T_c$



Lattice Setup

Pure SU(3)

- Symanzik improved gauge action
- gluonic $q(x)$ from clover field strength tensor $F_{\mu\nu}$
- update sweep: 1 heatbath + 4 overrelaxation

Parameters

- $0.1\ T_c \leq T \leq 4.0\ T_c$
- $n_t = 5, 6, 8$
- spatial volume fixed in physical units $L_{x,y} = 2/T_c$
- $L_z = 2L_{x,y}$ to enable subvolume analysis

Simulations on the Wuppertal-QPACE machine

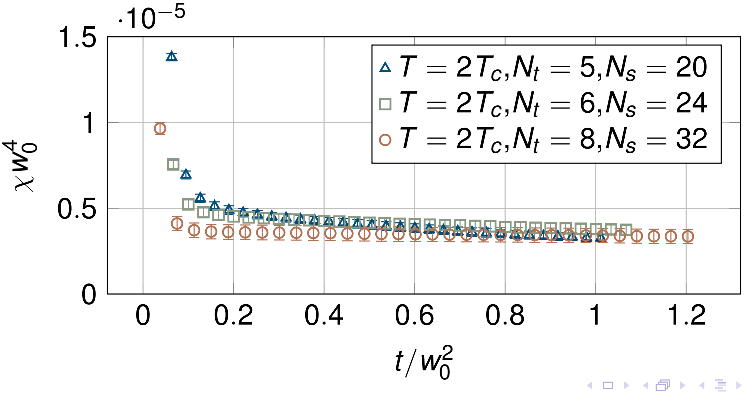
Renormalization of χ

$\chi(t)$ at finite Wilson-flow t is already renormalized [Luscher:2010iy]

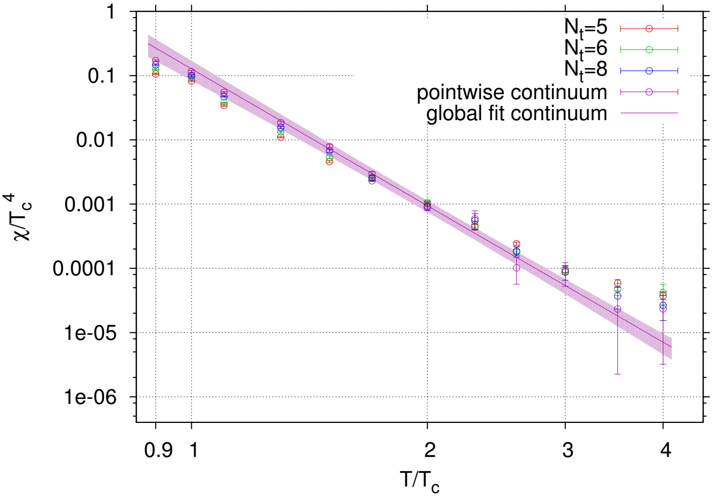
- sufficient to perform a continuum limit at flow time fixed in physical units, e.g. $t = w_0^2$
  (w_0^2: flow time at which $td/dt \cdot [t^2 E(t)] = 0.3$ [Borsanyi:2012zs])
- the choice of t influences the size of the lattice artefacts

Flow dependence of $\chi(t)$

- $\chi(t)$ has weak dependence on the choice of t
- we choose $t = w_0^2 \approx (0.176fm)^2$
- the finer the lattice the weaker the t-dependence



Continuum result: b=7.1(4)(2) & $\chi(4T_c)^{1/4}$=17 MeV



Quenched Lattice $\leftrightarrow$ DIGA

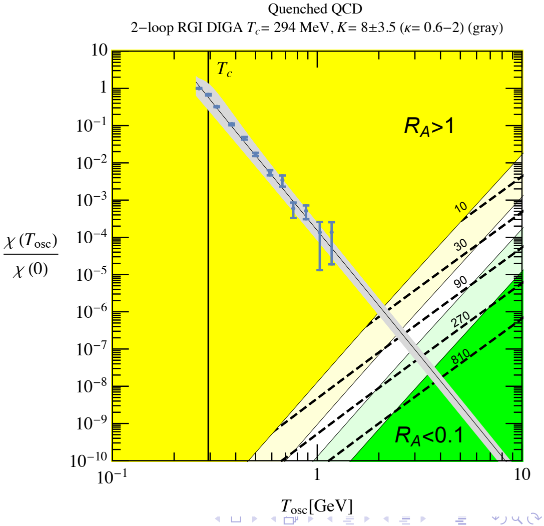
correct T dependence
normalization off by $\mathcal{O}(10)$
fixed by comparison to lattice

how $\chi_t(T)$ determines m_A?
start with an m_A e.g. 30μeV
m_A(T=0) gives the value of f_A

known: Hubble constant H(T)
fix T_{osc} by
$3H(T_{\rm osc}) = m_A(T_{\rm osc})$

using T_{osc} calculate
the amount of dark matter

if it is too much/little iterate



Calibrated guess for dynamical with DIGA

dynamic case with DIGA

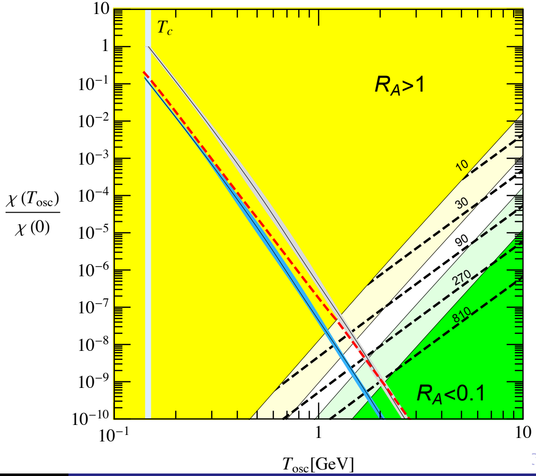
quenched calibrated K-factor is $\mathcal{O}(10)$

cosmology can be used axionic dark matter & m_A can be determined

K-factor uncertainty means a factor two in m_A

dream: predict m_A
ADMX experiment: tune it
(eventually even find it)

Unquenched QCD
2−loop RGI DIGA K = 1 (blue), K = 9.22±0.6 (gray) (κ= 0.6−2)
IILM from Ref. [10] (dashed red)



About costs: quenched case

Cost of the conventional algorithm at relative error $\delta\chi_t$

$$\textit{costs} \propto \frac{1}{(\delta\chi_t)^2\chi_t(T)}$$

relative cost $(4T_c)/(1T_c)$ (our highest T was $4T_c$: not enough)

from measured $\chi_t(T)$	$4^{7.1} \approx 2 \times 10^4$
from measured $\delta\chi_t$	$10^5 - 10^6$

- quenched $\chi_t(T=0)$ calculated $\sim$ 20 years ago
- Moores law leads to a factor of $\sim 10^5$ in 24 years

$\Rightarrow$ Just possible to do (dynamical case is probably hard)

About costs: dynamical QCD

Dynamic relative cost $\$(7T_c)/\$(1T_c)$ ($7T_c \sim 1200 MeV$)

from estimated $\chi_t(T)$	$7^{7-8} \approx 10^6 - 10^7$
increasing τ_{int} with T	$10^7 - 10^9$

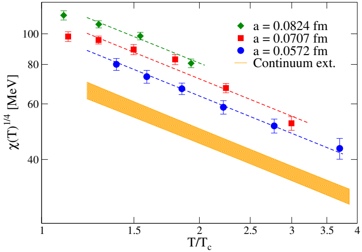
- dynamic $\chi_t(T=0)$ in 2010, Moore factor of ~ 10

$\Rightarrow$ conventional dynamical study not possible (needs 35 years)

Literature: full QCD

C.Bonati, M.d'Elia, G.Martinelli et al. JHEP 229 03, 155 (2016)

- brute force fully dynamic in the continuum up to $\approx 4T_c$



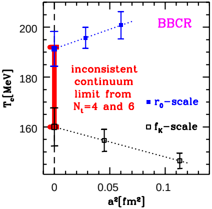
Result: $b \sim 3$ unexpected (DIGA etc. $b \sim 8$)
for T>2 GeV is larger than DIGA by 7-8 orders of magnitude
one order of magnitude shift for the axion dark matter window

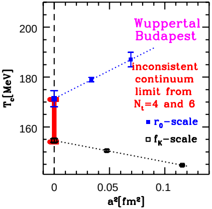
crosses quenched result at $4T_c$ (for quenched $\chi_t^{1/4}(4T_c)$=17 MeV)

$\Rightarrow$ further study is obviously necessary

Consequences of the non-scaling behaviour

for large 'a' no proper a^2 scaling (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the $a \to 0$ limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition



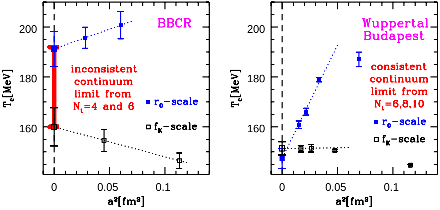


N_t=4,6: inconsistent continuum limit
N_t=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same T_c
signal: extrapolation is safe, we are in the a^2 scaling regime

Consequences of the non-scaling behaviour

no proper a^2 scaling for large 'a' (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the a$\rightarrow$0 limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition



N_t=4,6: inconsistent continuum limit
N_t=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same T_c
signal: extrapolation is safe, we are in the a^2 scaling regime

FLAG review of lattice results

Colangelo et al. Eur.Phys.J. C71 (2011) 1695

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud,\overline{\mathrm{MS}}(2\mathrm{GeV})}$	$m_{s,\overline{\mathrm{MS}}(2\mathrm{GeV})}$
PACS-CS 10	P	★	■	■	★	*a*	2.78(27)	86.7(2.3)
MILC 10A	C	●	★	★	●	–	3.19(4)(5)(16)	–
HPQCD 10	A	●	★	★	★	–	$3.39(6)^{*}$	92.2(1.3)
BMW 10AB	P	★	★	★	★	*b*	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD	P	●	●	★	★	*c*	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum et al. 10	P	●	■	●	★	–	3.44(12)(22)	97.6(2.9)(5.5)

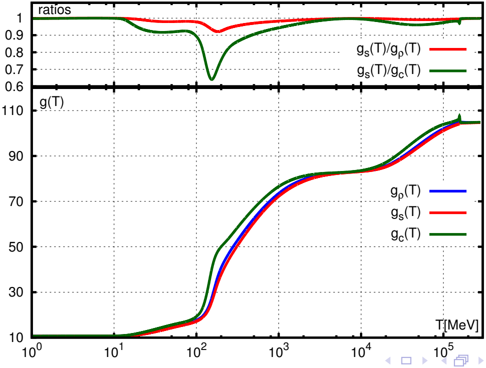
Introduction to the dynamical case

- Strong CP problem $\rightarrow$ axion $\rightarrow$ dark matter candidate
- Two important inputs for axion production:
  equation of state & topological susceptibility at high T
- Determine topological susceptibility at high temperatures
  at the physical point using fixed Q integral
- Exact zero modes of the Dirac operator for $Q \neq 0$ are crucial
  $\rightarrow$ large discretization effects with staggered fermions
- Possible solutions
  1) eigenvalue reweighting
  2) using chiral fermions
- In the following we use
  1) for the 3 flavor theory and
  2) for going down to the physical point

The equation of state

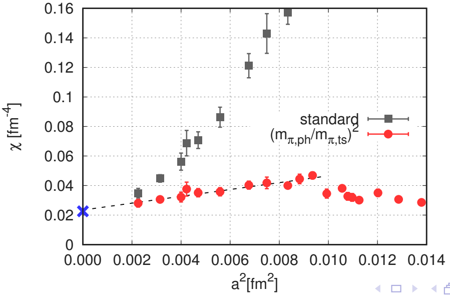
Effective number of degrees of freedom including all SM particles

$$\rho = \frac{\pi^2}{30} g_\rho T^4 \qquad s = \frac{2\pi^2}{45} g_s T^3 \qquad c = \frac{2\pi^2}{15} g_c T^3$$

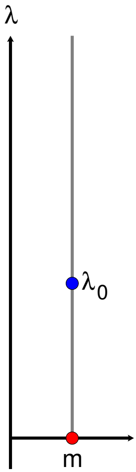


The challenge of computing the susceptibility

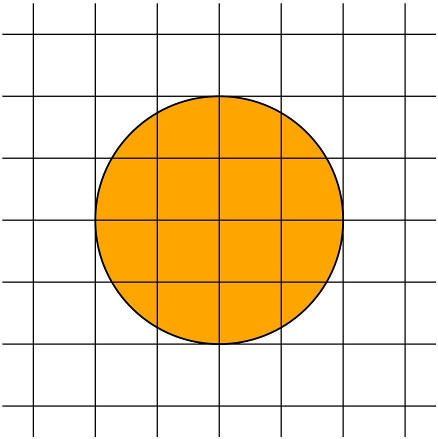
- large autocorrelation of Q on fine lattices (algorithmic problem)
- $\chi(T)$ decreases strongly with temperature
  $\rightarrow$ very few $Q \neq 0$ configurations (physical problem)
  E.g. $\langle Q^2 \rangle = 10^{-6}$ means one $Q = \pm 1$ configuration per million.
  Even $\mathcal{O}(\text{million})$ configurations can lead to large statistical errors
- $\chi(T)$ has large lattice artefacts



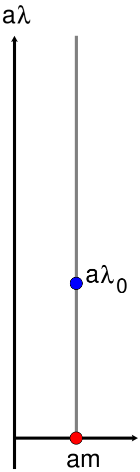
T=0 instanton on the lattice: physical units



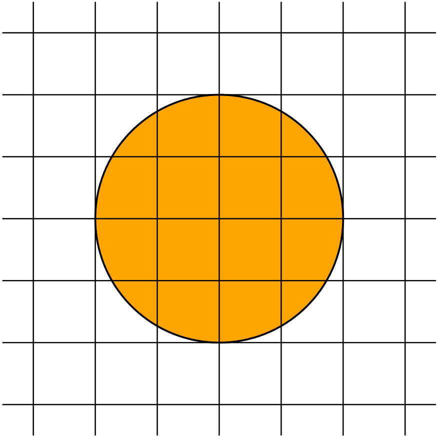
T=0 towards the continuum limit



T=0 instanton on the lattice: lattice units



T=0 towards the continuum limit

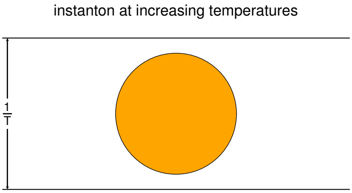


Goal: compute QCD topological susceptibility $\chi(T)$

- Temperature range: $0 < T < 2\text{GeV}$
- Physical quark masses (m_u, m_d, m_s, m_c)
- Continuum limit
- Using
  - $N_f = 2 + 1 + 1$, with isospin splitting correction
  - staggered and overlap quarks
  - Lattices with $N_t = 8, 10, 12, 16, 20$

Physics to be captured

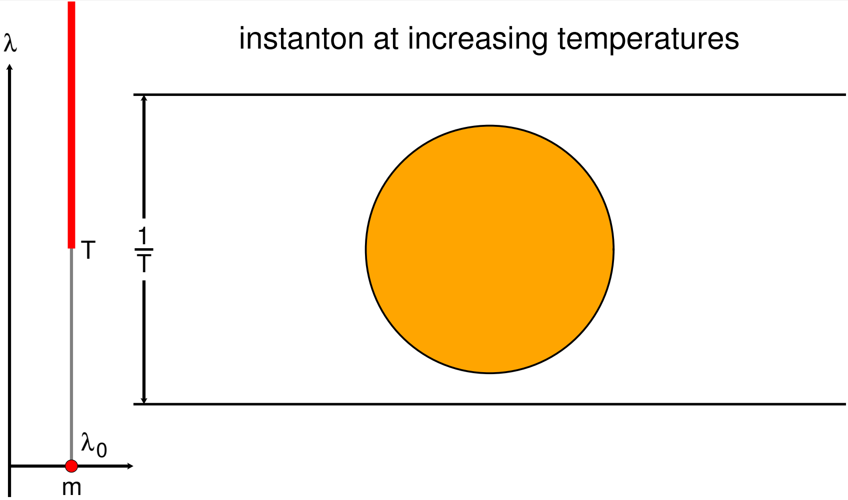
- Typical instanton size $1/T$



- Dilute gas of small ($r \approx 1/T$) instantons remain
- Zero modes in the light quark det suppress topology
- $\Rightarrow \quad \chi(T) \quad$ falls sharply above T_c

T>0 instanton in the continuum

instanton at increasing temperatures

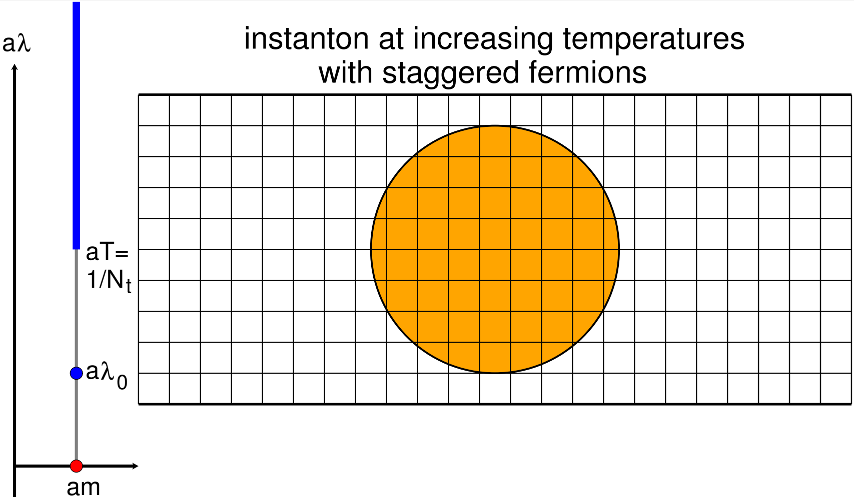


Challenge #1: large cut-off effects

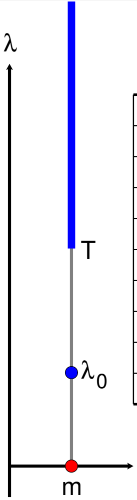
- Small-instanton zero modes badly captured by lattice Dirac operator
- Higher Q sectors not properly suppressed
- Cut-off effects much larger at higher T
- Solution: identify would-be zero eigenvalues and shift them to zero $\rightarrow$ reweighting

T>0 instanton on the lattice: lattice units

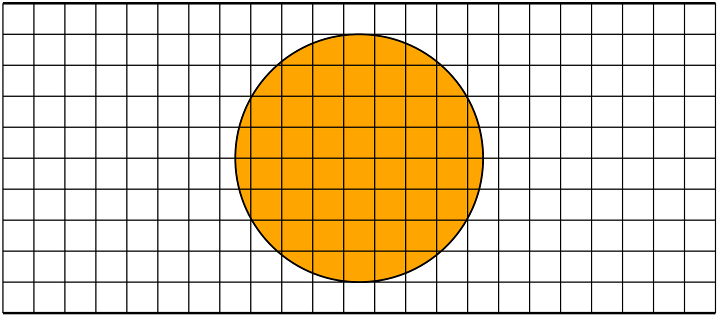
instanton at increasing temperatures
with staggered fermions



T>0 instanton on the lattice: physical units



instanton at increasing temperatures
with staggered fermions



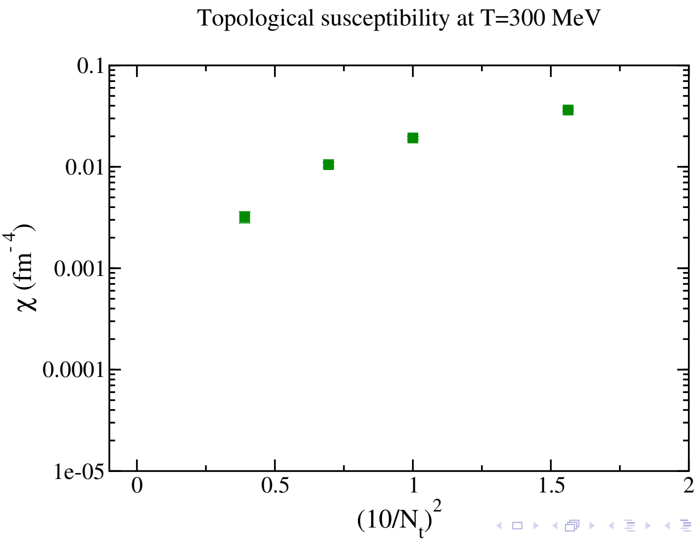
Challenge #1: large cut-off effects

- Small-instanton zero modes badly captured by lattice Dirac operator
- Higher Q sectors not properly suppressed
- Cut-off effects much larger at higher T
- Solution: identify would-be zero eigenvalues and shift them to zero $\rightarrow$ reweighting

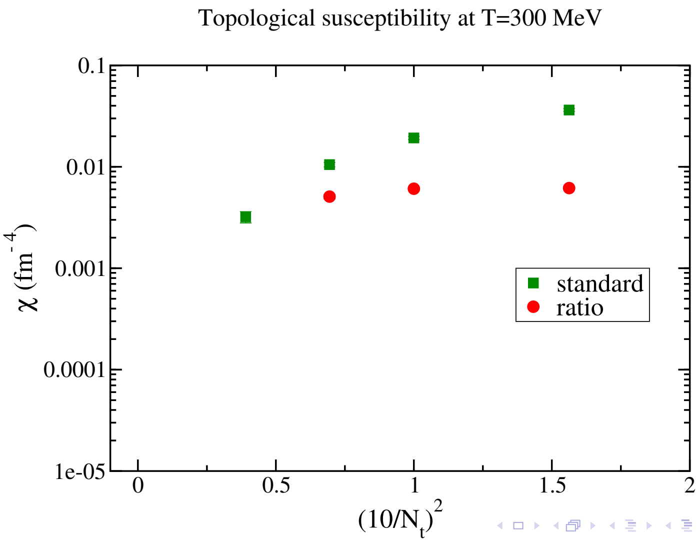
Challenge #2: tiny χ hard to measure

- No statistics for $Q \neq 0$ sectors (dictated by physics)
- Topology change slow on fine lattices (algorithmic)
- Solution:
  Derivative of $\chi(T)$ much easier to measure than χ
- Measure $\chi(T_0)$ at low enough T_0
- Using $d\chi/dT$ integrate up to T $\rightarrow$ integral method
  Also suggested for the quenched case by [Frison et al '16]

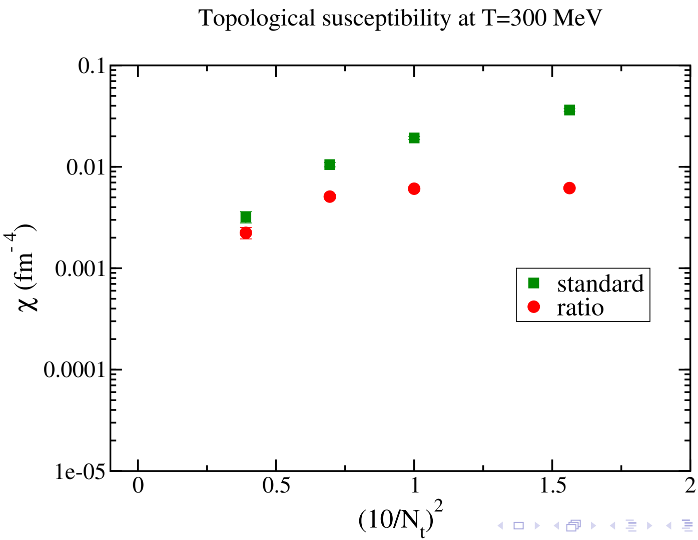
Unusually large cut-off effects: N_f=2+1+1 with 4-stout



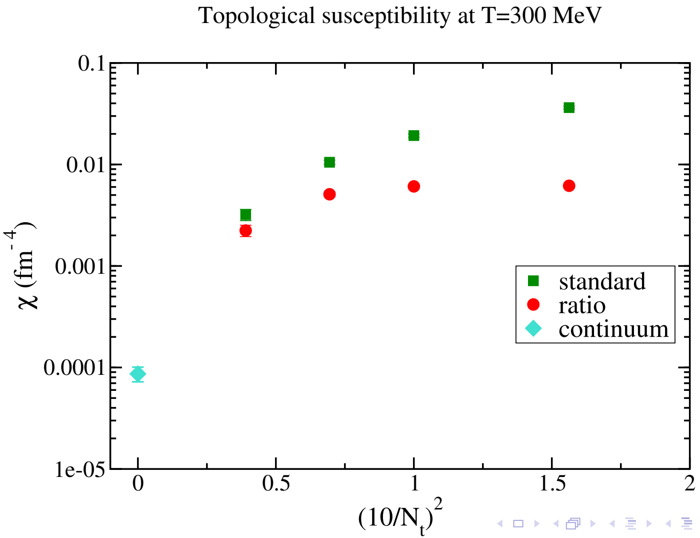
Unusually large cut-off effects: N_f=2+1+1 with 4-stout



Unusually large cut-off effects: N_f=2+1+1 with 4-stout



Unusually large cut-off effects: N_f=2+1+1 with 4-stout



Reason for bad scaling

- Would-be zero eigenvalues too big
- Weight in det is

  Lattice: $\lambda_0 + m_f$

  Instead of continuum: m_f

- Even if $a \propto 1/T$ (fix N_t, increase β)

  λ_0/m increases with T

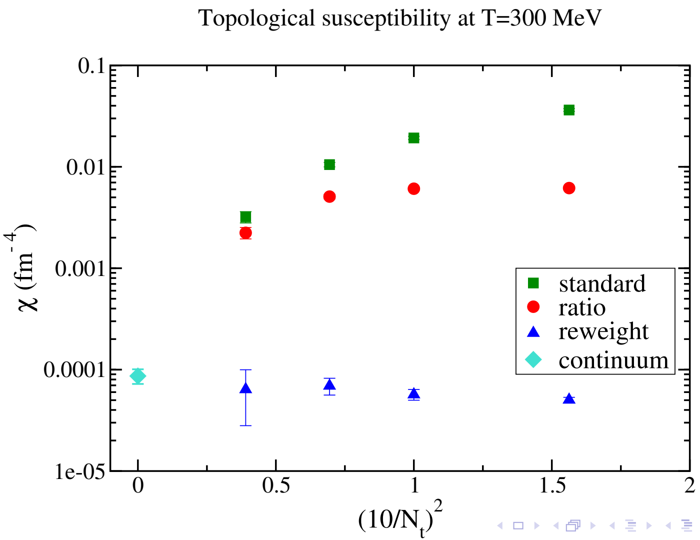
Reweighting

Strong cut-off effects are related to the lack of exact zero-modes.

- In the continuum non-trivial sectors are suppressed by the contribution of zero-modes to the fermion determinant, ie. by the quark mass.
- On the lattice the suppression is altered: $m \to m + \lambda_0$, where λ_0 is a would be zero-mode. Weaker suppression $\to \chi(T)$ overestimated.
- To improve 1. identify would be zero-modes
  2. restore the continuum weight $\to$ reweight

$$w[U] \sim \frac{m}{m + \lambda_0}$$

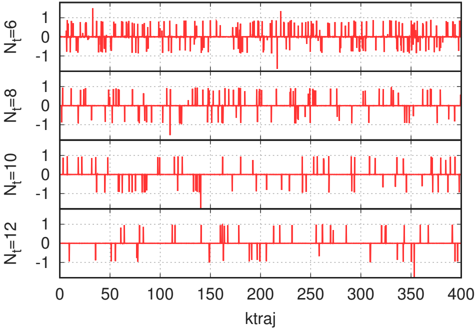
T=300 MeV: susceptibility after reweighting



Topology changing streams

simulation time history of the topological charge
3+1 flavor staggered simulation at T=400 MeV

cutoff effects: N_t=6 fluctuates more than N_t=12
for N_t=6 the rewighting factor is more substantial



Reweighting

- Weight in det is $\lambda_0 + m_f$ instead of m_f

Solution: identify would-be zero modes and shift them to 0

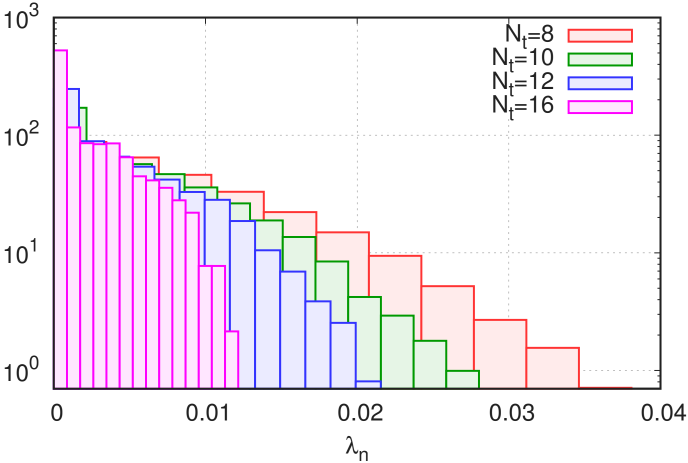
- Compute topological charge Q with Wilson flow [Lüscher '10]
- Identify $4|Q|$ would-be zero eigenvalues $\lambda_1, \lambda_2 ... \lambda_{4|Q|}$
- Modify quark determinant by reweighting with factor

$$w[U] = \prod_f \prod_{n=1}^{4|Q|} \left(\frac{m_f}{\lambda_n[U] + m_f} \right)^{1/4}$$

- Approaching the continuum it is getting better $w[U] \to 1$

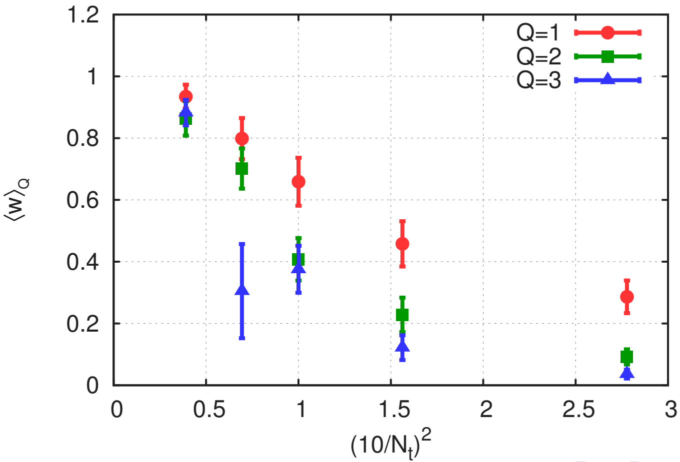
Distribution of the would-be zero modes

$N_f = 2 + 1 + 1$ staggered quarks, $T = 240$ MeV



Average reweighting factors

$N_f = 3 + 1$ staggered quarks, $T = 300$ MeV



Challenge #2: tiny χ hard to measure

- No statistics for $Q \neq 0$ sectors (dictated by physics)
- Topology change slow on fine lattices (algorithmic)
- Solution:
  Derivative of $\chi(T)$ much easier to measure than χ
- Measure $\chi(T_0)$ at low enough T_0
- Using $d\chi/dT$ integrate up to T $\rightarrow$ integral method
  Also suggested for the quenched case by [Frison et al '16]

Fixed sector integral

Instead of waiting for tunneling events,
we make simulations in fixed Q sectors. How to get

$$Z_1/Z_0 = ?$$

First calculate derivative of $\log Z_1/Z_0$:

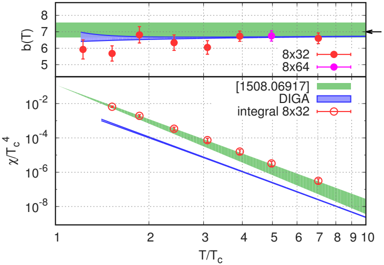
$$b_1(T) \equiv \frac{d \log Z_1/Z_0}{d \log T}$$

Use fixed N_t-approach, ie. $T = (aN_t)^{-1}$ is changed by β:

$$b_1(T) = \frac{d\beta}{d \log a} \left(\langle S_g \rangle_1 - \langle S_g \rangle_0 \right)$$

Fixed Q integral - quenched

Fixed Q simulation: extra acc/rej step at the end of each update, as lattice spacing decreased the acceptance gets better.
Test in quenched case: pure Wilson action upto $7 \cdot T_c$ and 8×64^3



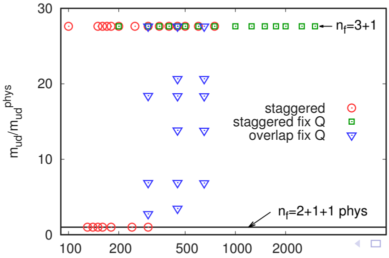
standard method: extrapolation using a fit;
integral method;
Dilute Instanton Gas Approximation:
exponent agrees nicely, but order of magnitude difference in χ

Fixed Q integral with fermions

At high temperatures $\chi(T) \sim T^{-b}$ only $Q = 0, 1$ contribute

$$b_1 = -\frac{d\beta}{d\log a}\langle S_g\rangle_{1-0} - \sum_f \frac{d\log m_f}{d\log a} m_f\langle\overline{\psi}\psi_f\rangle_{1-0}$$

- S_g: small cutoff effects, huge statistics $\rightarrow$ staggered $N_f = 3$
- $m_f\langle\overline{\psi}\psi_f\rangle_{1-0}$: large cutoff effects $\rightarrow$ staggered reweighting for $N_f = 3$, overlap for $N_f = 2+1$



Mass integration with chiral fermions

- There is an exact index theorem on the lattice for fermions which satisfy the Ginsparg-Wilson relation

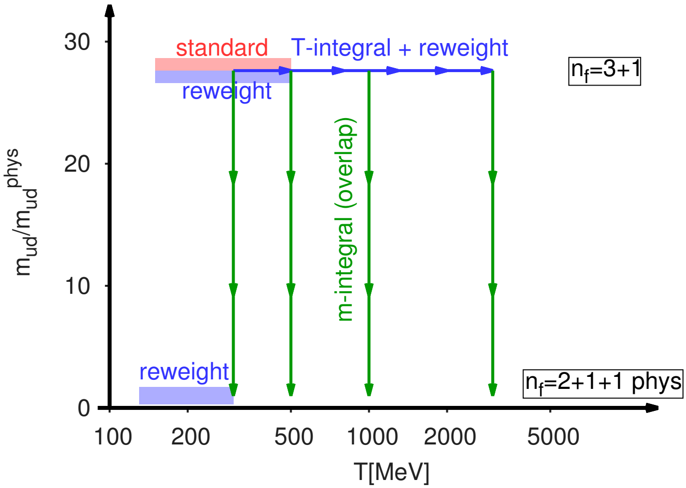
$$\{D, \gamma_5\} = aD\gamma_5 D$$

- lattice artifacts can be largely reduced
- overlap construction with $H_W = \gamma_5(1 - D_W)$:

$$D = (m_0 - m/2)(1 + \gamma_5 \mathrm{sgn}(H_W)) + m,$$

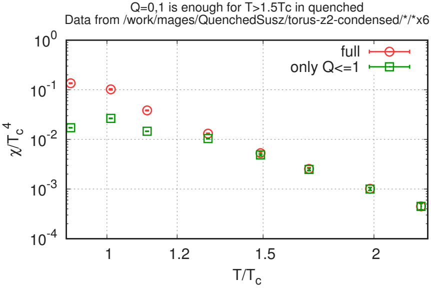
- much more expensive than staggered fermions, but condensate difference can be computed
- we find that $m_f\langle\overline{\psi}\psi_f\rangle_{1-0} = N_f$ for $T \geq 300$ MeV within errors
- this gives the mass exponent required to integrate down to the physical point

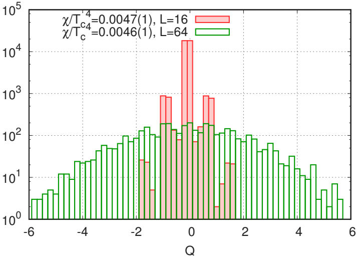
Map of simulations



How many sectors are needed?

for large temperatures (above 300 MeV & N_f=3) χ_t is small
for not too large volumes only q=0,1 and 2 (for N_f=2+1 even less)

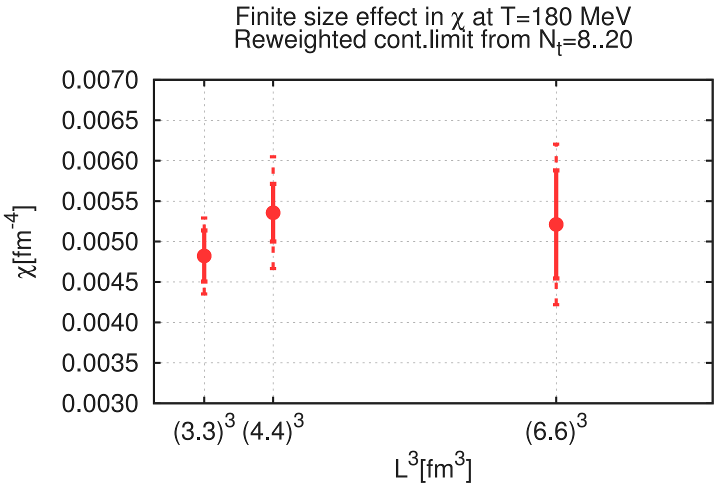




reach saturation ($LT_c \gtrsim 2$) $\Rightarrow$ it is V independent

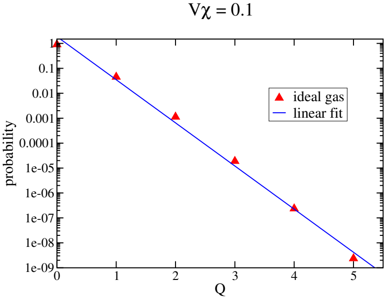
for small T one needs more sectors χ_t is not that small
one should control the contribution of the various sectors

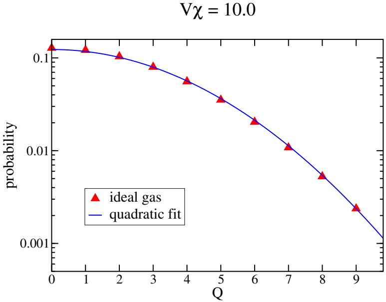
Volume dependence of χ at T=180 MeV for a$\to$0



Weight of different Q sectors

Ideal instanton gas (non-interacting instantons)





$V\chi$ small $\rightarrow$ $\log(Z_Q/Z_0) \propto Q$ $V\chi$ large $\rightarrow$ $\log(Z_Q/Z_0) \propto Q^2$

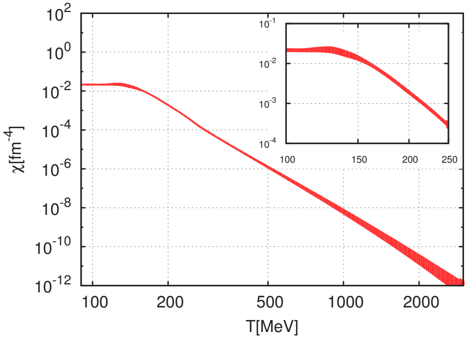
[Azcoiti '16]

Distribution determined by one parameter: $V\chi$

Measure Z_1/Z_0 for any volume V $\rightarrow$ full distribution $\rightarrow \chi$

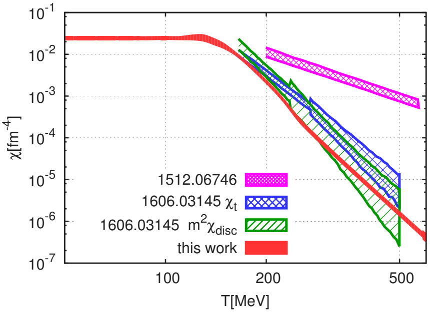
Topological susceptibility at the physical point

very few topology changes (hard): S. Borsanyi et al. Nature 539 (2016) 69



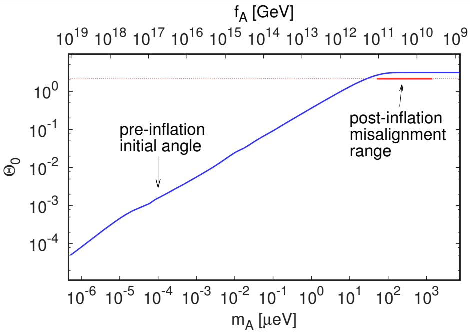
absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2)\ \mu\text{eV}$
assuming 50-99% other (e.g. strings): $m_A = 50 - 1500\ \mu\text{eV}$

Comparison with other work

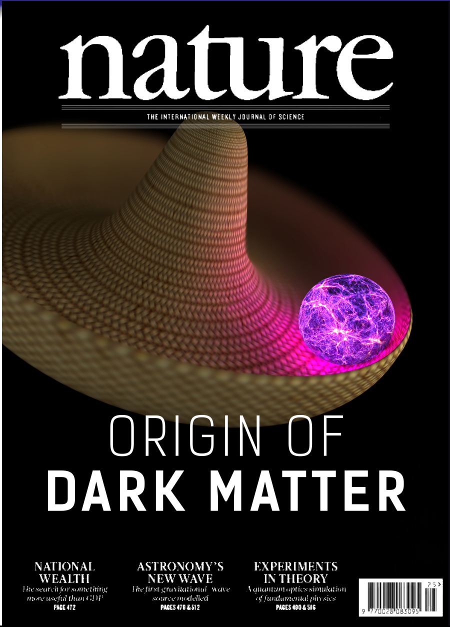


- Bonati et.al (1512.06746): smaller exponent
- Petreczky, Schadler, Sharma (1606.03145): bosonic and fermionic definitions, consistent results with large errors
- Y. Taniguchi et al., (1611.02411) gives 7.2(0.9) and 7.3(1.7)

Constraints on the axion mass



- Pre-inflation scenario: m_A unambiguously determines the Θ_0 initial condition of our Universe
- Post-inflation: Θ_0 average equivalent to $\Theta \approx 2.15$
  absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2)\ \mu\text{eV}$
  assuming 50-99% other (e.g. strings): $m_A = 50 - 1500\ \mu\text{eV}$


nature
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
ORIGIN OF
DARK MATTER
NATIONAL WEALTH
more useful than GDP
PAGE 472
ASTRONOMY'S NEW WAVE
The first gravitational-wave source modelled
PAGES 478 & 512
EXPERIMENTS IN THEORY
A quantum optics simulation of fundamental physics
PAGES 480 & 516


nature
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
Hox gene regulation is central to the evolution of five-digit tetrapods from polydactyl ancestors PAGE 89
DIGITAL ARRAY
ENERGY
AFRICA'S POWER STRUGGLE
Can a continent bypass fossil fuels and adopt renewables?
PAGE 20
SCIENTIFIC METHOD
ONE THEORY IS NOT ENOUGH
There may be another explanation — go look for it!
PAGE 23
COLLEGE EDUCATION
WHAT'S ON THE SYLLABUS?
An open-access resource celebrates ways of learning
PAGE 125
NATURE.COM/NATURE
3 November 2016 £10
Vol 539, No. 7627

Summary: results

- Axion: a solution to a) strong CP b) dark matter problems
- Calculating axion production in the early universe requires the EoS and $\chi(T)$
- Brute force approach expensive: estimates using pure SU(3)
- Both were determined using lattice calculations up to high T
- Axion mass in the post-inflation scenario:
  lower bound: 28(2) μeV
  estimated mass range: $m_A = 50 - 1500\ \mu\mathrm{eV}$

Summary: methods

Calculated T-dependence of the QCD topological susceptibility

- Temperature range: $0 \leq T \leq 2$ GeV
  (follow change of χ over 10 orders of magnitude)
- Physical quark masses
- Continuum limit

Main lesson: keep in mind the physics of the problem

- Large cut-off effects due to instanton zero-modes
- At high T : tiny χ $\rightarrow$ ideal instanton gas