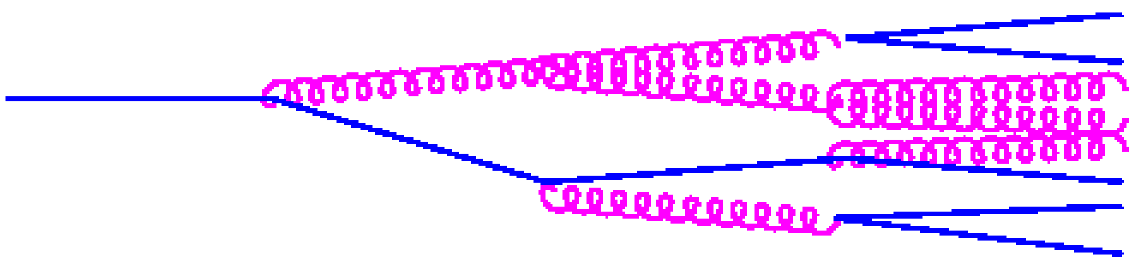
Quantum Interference in Showering: The LPM Effect and What's New

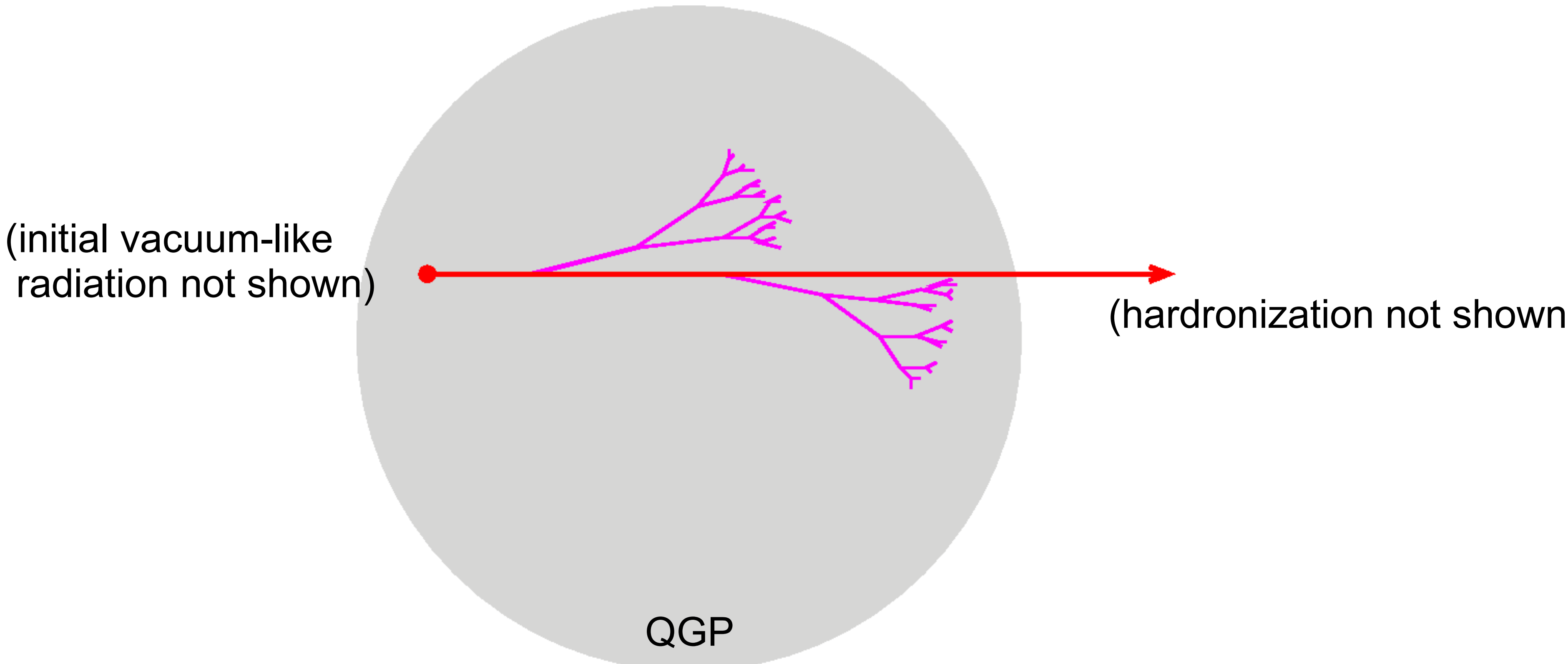
Peter Arnold

University of Virginia



Consider cartoon of

In-medium evolution of a jet



For this talk, simplify discussion by focusing on ...

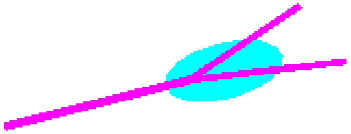
Cascades that stop in-medium

- Qualitative points we'll discuss generalize.
- Formalism generalizeable as well.

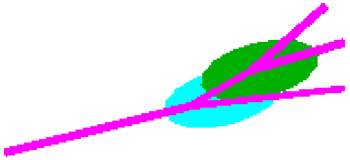
OUTLINE

Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect



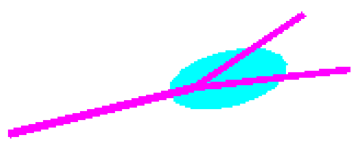
B. interference between splittings (and when/if that's important)



Part II. Formalism for calculations (presented qualitatively)

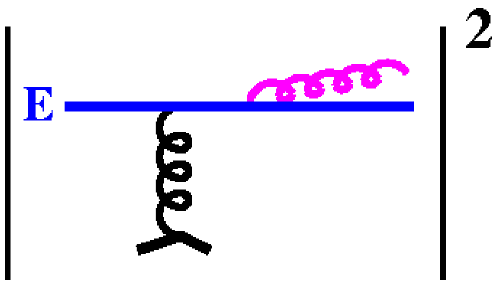
Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect



Landau-Pomeranchuk-Migdal (LPM) Effect

Naively

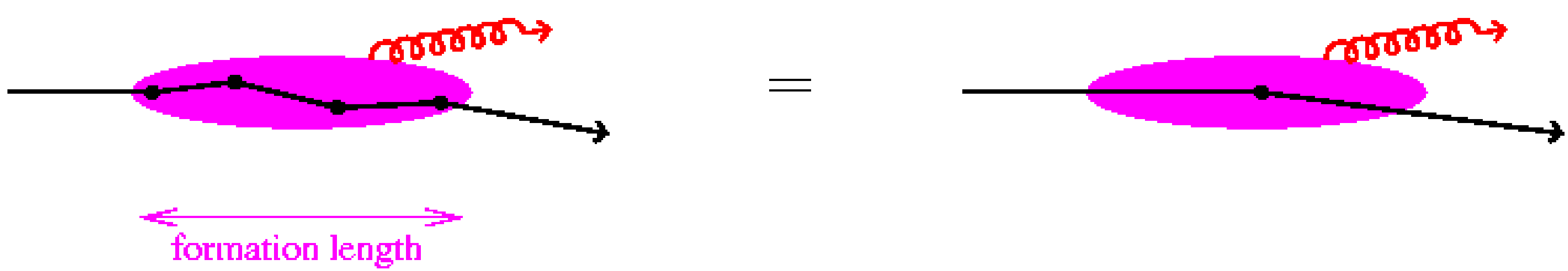
brem rate ~ $n\sigma v$ ~ (density of scatterers) $\times$  $\times\, 1$

Problem

At very high energy,

probabilities of brem from successive scatterings no longer independent;

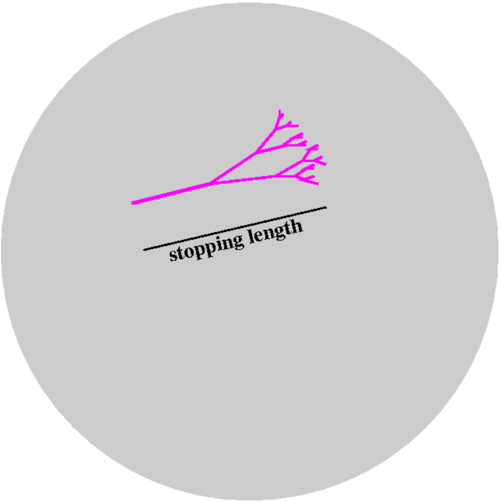
brem from several successive (small angle) collisions not very different from brem from one collision.



$$\text{formation length} \propto \sqrt{E}$$

Result: a reduction of the naive brem rate.

Example: stopping distance



If LPM effect ignored:	stopping distance	$\sim$	E^0	(up to logs)
Actual result (weak coupling):	stopping distance	$\sim$	$E^{1/2}$	(up to logs)

Example: stopping distance

If LPM effect ignored:	stopping distance	$\sim$	E^0	(up to logs)
Actual result (weak coupling):	stopping distance	$\sim$	$E^{1/2}$	(up to logs)

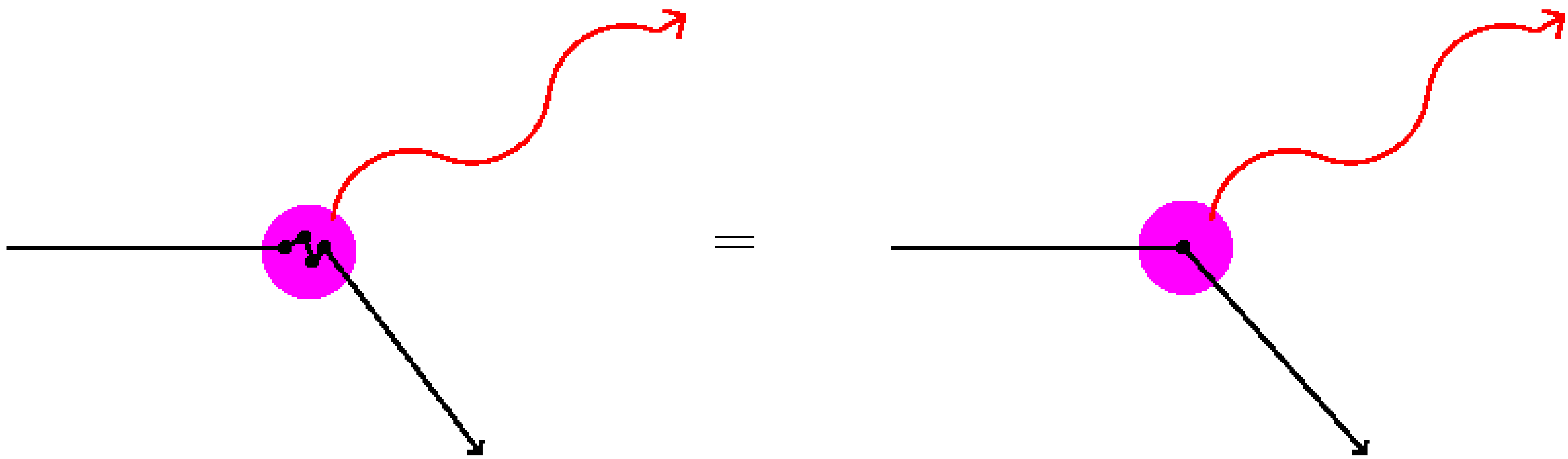
And for later comparison...

QCD-like theories w/ gravity duals (infinitely strong coupling):	max stopping distance	$\sim$	$E^{1/3}$

[Chesler, Jensen, Karch, Yaffe; Gubser, Gulotta, Pufu, Rocha; Hatta, Iancu, Mueller (2008)

The LPM Effect (QED)

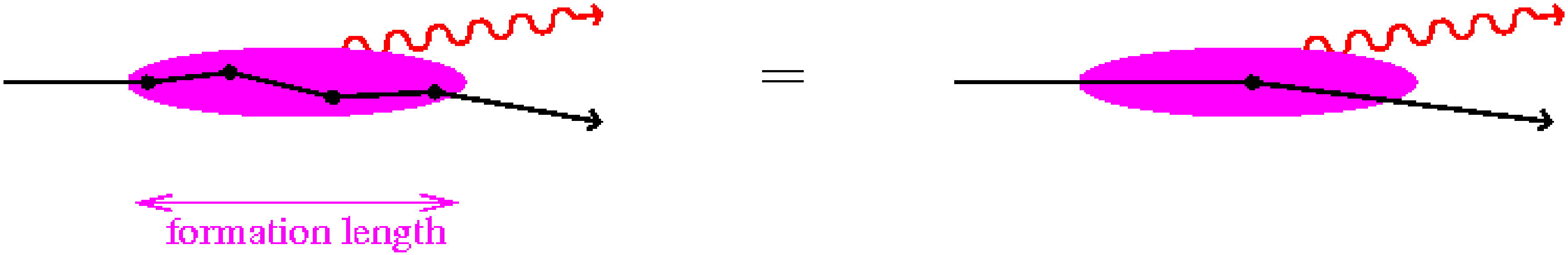
Warm-up: Recall that light cannot resolve details smaller than its wavelength.



[Photon emission from different scatterings have same phase → coherent.]

Now: Just Lorentz boost above picture by a lot!

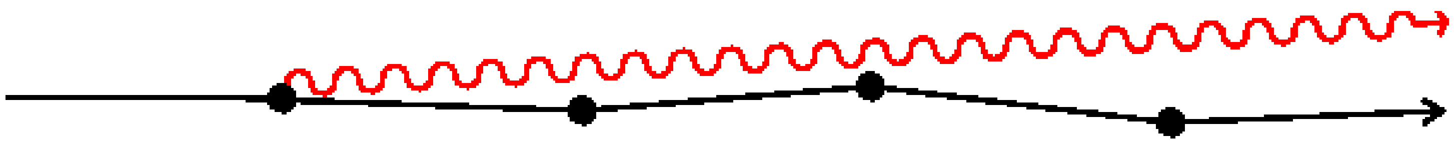
The LPM Effect (QED)



Note: (1) **bigger *E*** requires bigger boost → more time dilation → **longer formation length**

(2) big boost → this process is **very collinear.**

An alternative picture



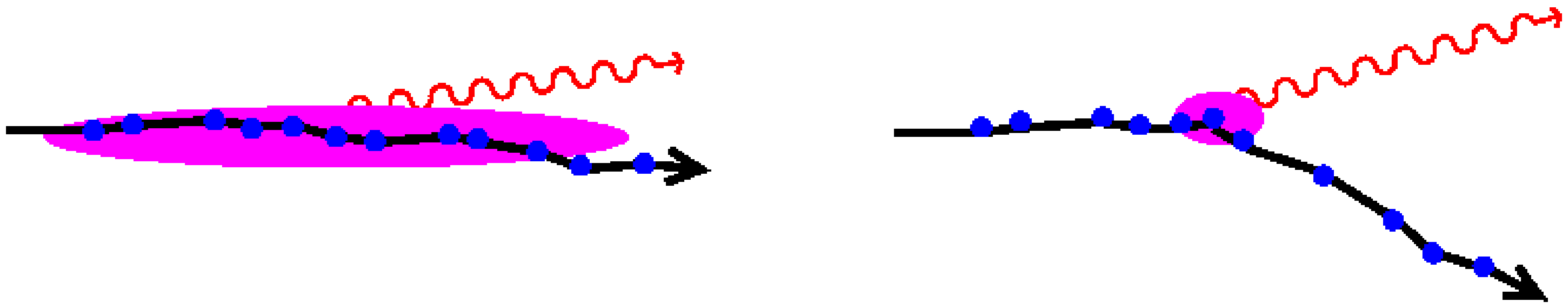
versus

Are these two possibilities in phase? Or does the interference average to zero?

IN PHASE   if   (i) everything is nearly collinear ✓
(ii) particle and photon have nearly same velocity ✓ (*speed of light*)

The important point:

The more collinear the underlying scattering, the longer the formation time.



Note: the formation length

depends on the net angular deflection during the formation length, which ***depends on*** the formation length

[Self-consistency $\rightarrow$ standard parametric formulas for formation length.]

The LPM Effect (QCD)

There is a qualitative difference for ***soft*** bremsstrahlung.:

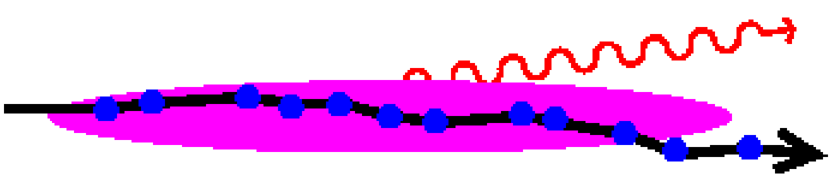
<u>QED</u>

Softer brem photon → longer wavelength
→ less resolution
→ more LPM suppression

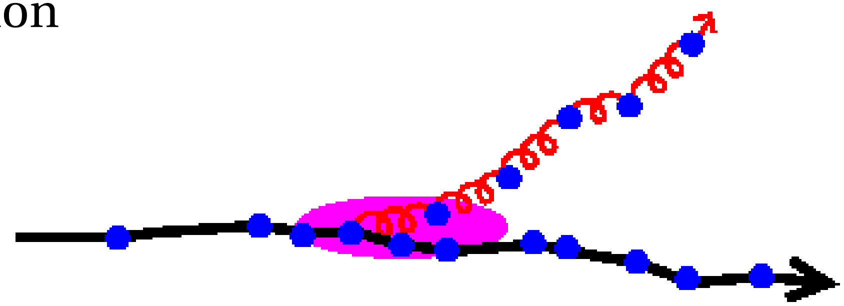
<u>QCD</u>

Unlike a brem photon, a brem gluon can easily scatter from the medium.

Softer brem gluon → easier for brem gluon to scatter
→ less collinearity
→ less LPM suppression

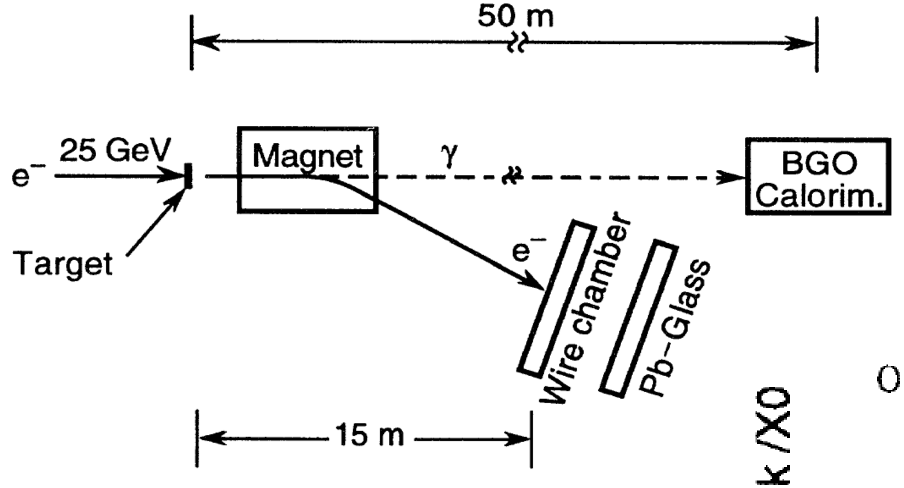


vs.



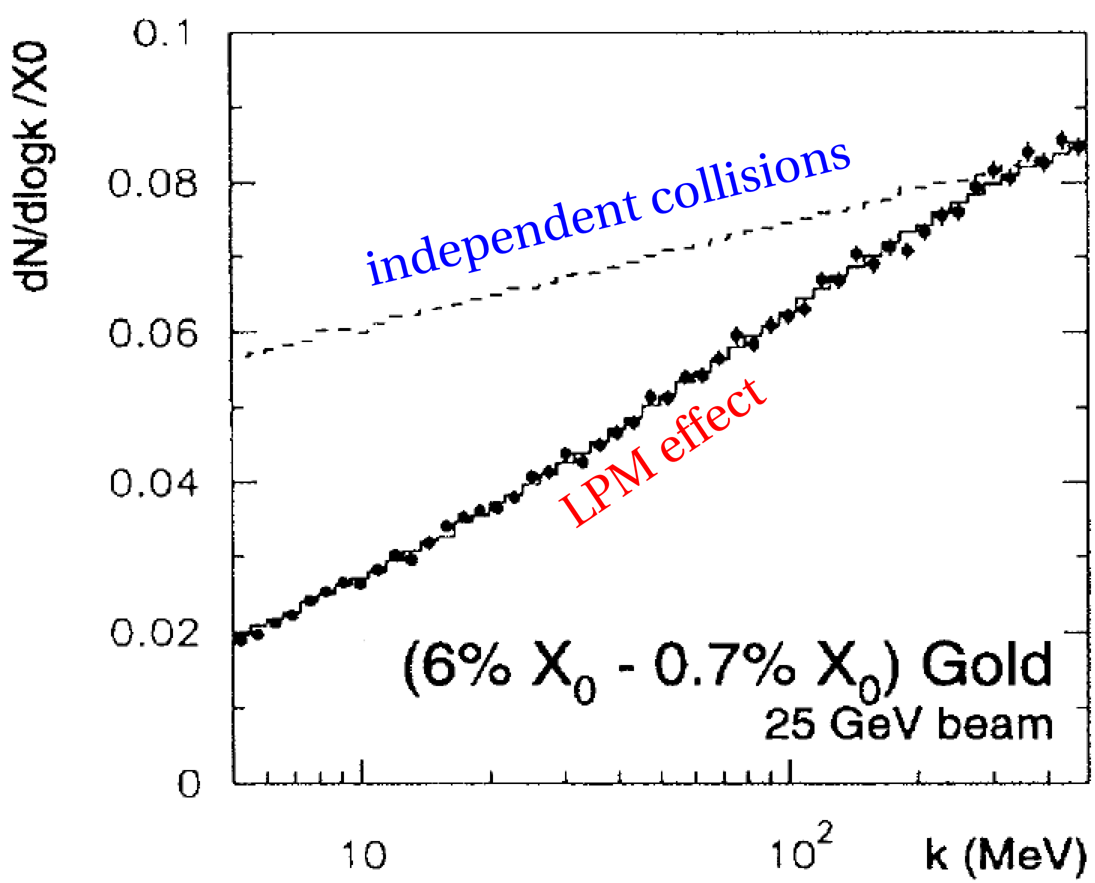
Upshot: Soft brem more important in QCD than in QED (for high-E particles in a medium)

Experimental Measurement of LPM (QED)



SLAC E-146

Phys. Rev. Lett. **75** (1995) 2949.

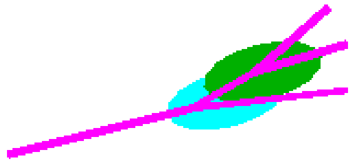


OUTLINE

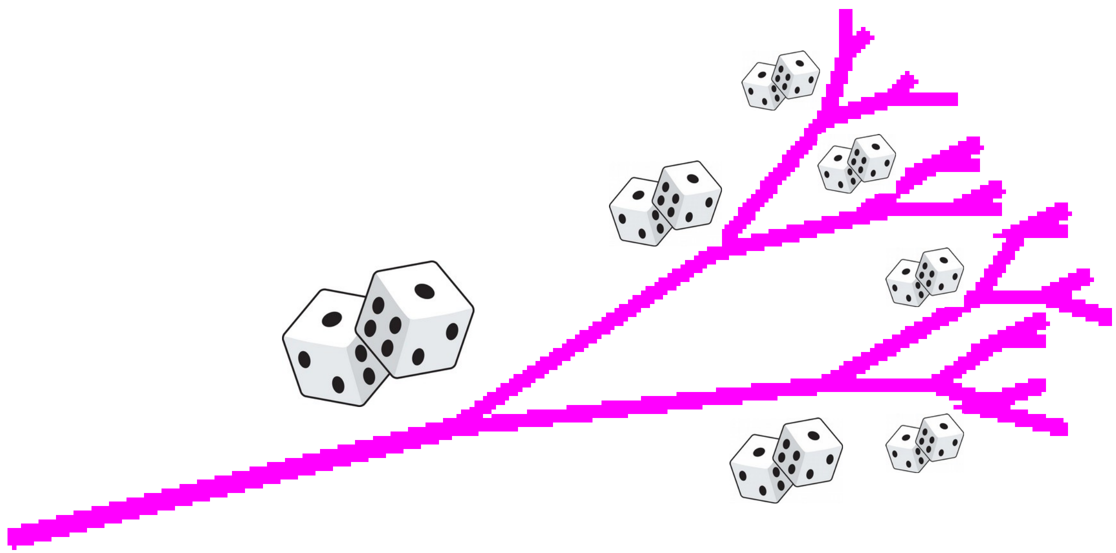
Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect

B. interference between splittings (and when/if that's important)



An idealized Monte Carlo picture of in-medium evolution



As time passes,

roll classical dice for probability of each splitting

weighted by the quantum calculation of the single splitting rate

$\frac{d\Gamma_{\text{brem}}}{dx}$ (<u>including</u> LPM effect) for each vertex  above.

Built-in assumption:

Consecutive splittings are quantum-mechanically independent.

(*But are they*?)

Heuristic attempts to improve on this in real Monte Carlos:

JEWEL
recent versions of MARTINI

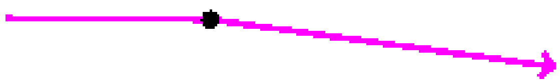
Here, I want to talk about

What's known from first-principles(-ish) QCD calculations?

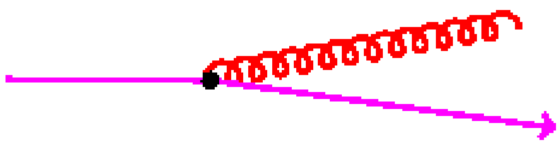
Single splitting rate

Naive picture (ignoring LPM)

Collisions with the medium



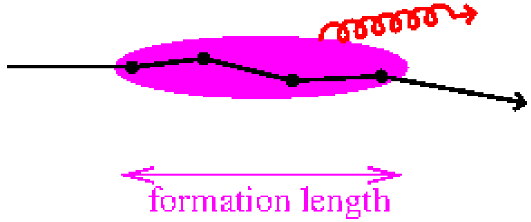
generate chances for bremsstrahlung



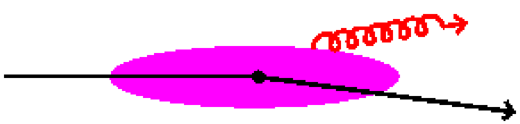
with

prob of emission ~ α per collision

LPM effect



indistinguishable from



So, actually,

prob of emission ~ α per formation length $l_{\text{form}} \propto \sqrt{E}$

Rate calculated quantitatively by

LPM for QED (1950s)
BMDPS-Z for QCD (1990s)

and investigated in numerous ways by many people since.

Dependence on medium:

Units of $l_{\mathrm{form}} \propto \sqrt{E}$ made up as

$$l_{\mathrm{form}} \propto \sqrt{\frac{E}{\hat{q}}}$$

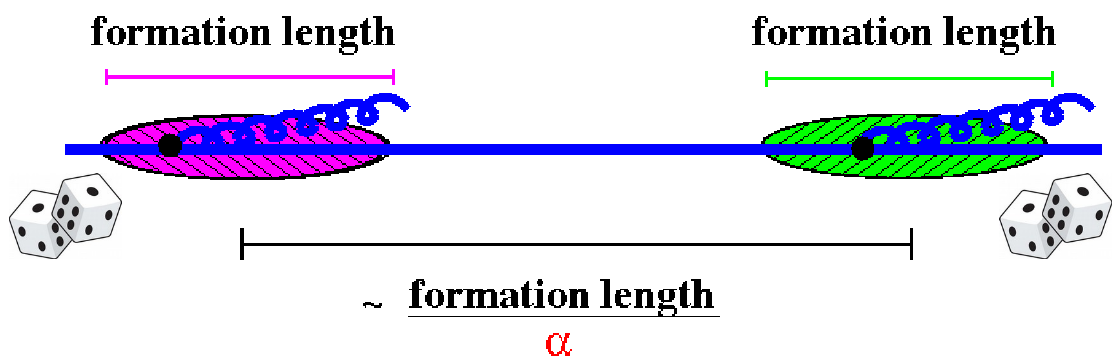
Where medium parameter $\hat{q}$ defined by

$$\left(\begin{array}{c}\text{average net transverse}\\ \text{momentum kick from medium}\end{array}\right) = \hat{q} \text{ times distance traveled}$$

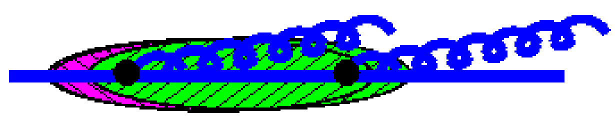
Consecutive emissions

Chance of brem ~ α per formation time

means that two consecutive splittings will typically look like



So chance of overlap (i.e. “rolling dice separately” breaking down) is



$\propto \alpha$

How big is “α” ??

How big is α_s?

Nothing to do with whether medium is

sQGP / perfect liquid

[$\alpha_s(T)$ big]

vs.

weakly-coupled QGP

[$\alpha_s(T)$ small]

α_s on previous slide associated with emission vertex:

costs roughly $\alpha_s(Q_\perp)$ with $Q_\perp \sim (\hat{q}E)^{1/4} \lesssim$ a few GeV

panic and/or fool around
with AdS/CFT energy loss

[$\alpha_s(Q_\perp)$ big]

vs.

LPM-based analysis

[$\alpha_s(Q_\perp)$ small]

Does the wisdom of the ages tell us if α_s(few GeV) is small?

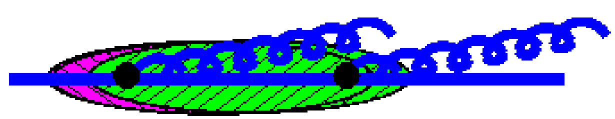
Particle physics in vacuum:

Small for some things, like matching lattice calculations to continuum MS-bar α_s

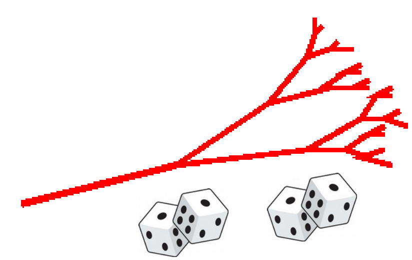
High-temperature physics:

Bad news (except possibly if one does sophisticated resummations of perturbation series)

Overlapping formation times effects on cascade:



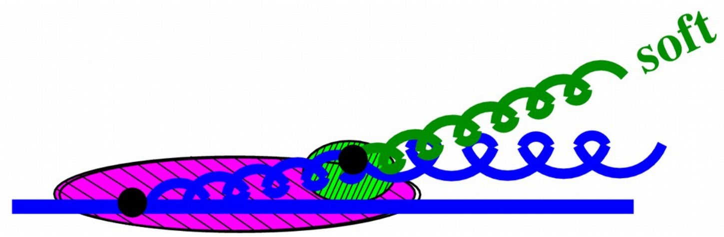
$\propto \alpha$ effect on



We should calculate it and see.

Soft emission

Soft emissions are generally enhanced by logs.
Path-breaking authors found small-*x*-like double logs in this case,



$$\propto \alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\rm mfp}}\right)$$

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

This is a BIG effect for large *E*.

But they found soft emission effects could be absorbed into the medium parameter

$$\hat{q} \to \hat{q}_{\rm eff}(E)$$

following Liou, Mueller, Wu (2013)

Also, leading logs can be resummed to all orders! ...

$$\hat{q}_{\text{eff}}(E) = \hat{q}_0 \times \left\{ 1 + \#\, \alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right) + \# \left[\alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right)\right]^2 + \cdots \right\}$$

$=$ something involving a Bessel fn

$\sim E^{\#\sqrt{\alpha}}$ (for large enough E)

Effect turns out to be

stopping distance $\sim E^{\frac{1}{2}/(1+\#\sqrt{\alpha})}$ or equivalently $E^{\frac{1}{2}(1-\#\sqrt{\alpha})}$

Result for sample values of $\alpha_s(Q_\perp)$:

$\alpha_s \lll 1$	stopping distance	$\sim$	$E^{\frac{1}{2}}$
$\alpha_s = 0.2$	stopping distance	$\sim$	$E^{0.35}$ or $E^{0.28}$

Compare to ...

QCD-like theories w/ gravity duals $N_c\alpha = \infty$	max stopping distance	$\sim$	$E^{1/3}$

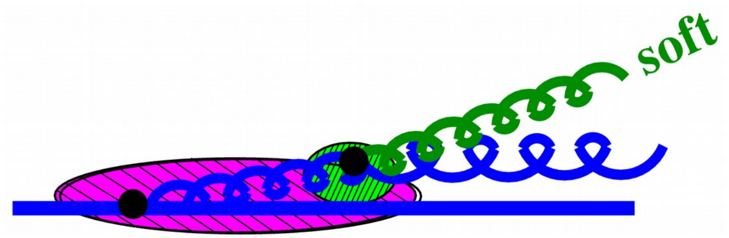
[Aside: A personal gauge-gravity frustration]

What's the $1/(N_c\alpha)$ correction to the infinite-coupling result $E^{1/3}$?

QCD-like theories w/ gravity duals $N_c\alpha \gg 1$	max stopping distance	$\sim$	$E^{\frac{1}{3}+???}$

Back to small(ish) $\alpha_s(Q_\perp)$ ...

Soft emission corrections summary

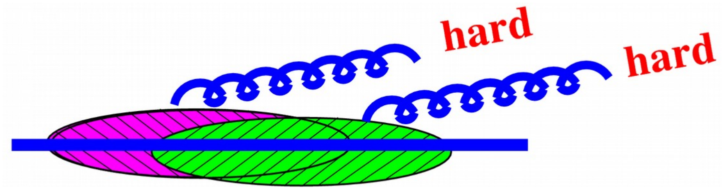


At leading log order

- Absorbable into medium parameter $\hat{q} \to \hat{q}_{\text{eff}}(E)$
- Size of effect controlled by $\sqrt{\alpha_s}$ (after resummation)

Beyond the soft limit

What about overlap effects that *can't* be absorbed into $\hat{q}$?



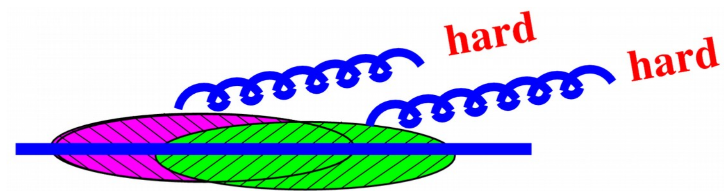
$\propto \alpha_s$

And how big are those effects for relevant sizes of $\alpha_s(Q_\perp)$?

Also, what about sub-leading logs? Are they also resummable into medium parameters?

What we've done

Computed the effect of the overlap for hard emissions



$\propto \alpha_S$

In broad brush: interesting and fun field theory problem.

In calculational detail: a pain in the ass.

OUTLINE

Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect

B. interference between splittings (and when/if that's important)

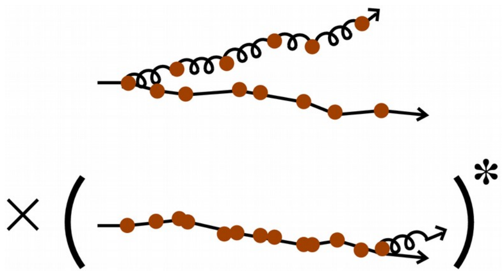
Part II. Formalism for calculations (presented qualitatively)

A. single splitting

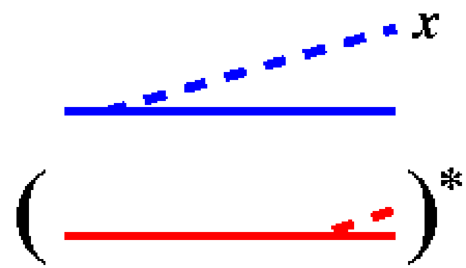
B. consecutive splittings

Formalism for LPM: single brem

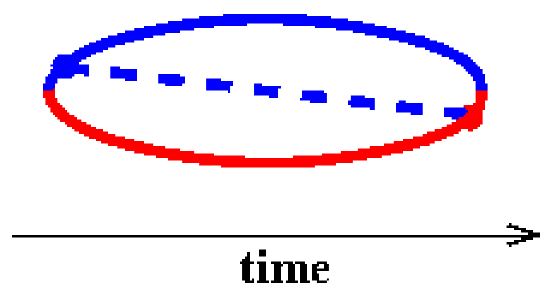
Shorthand henceforth: Draw



as simply

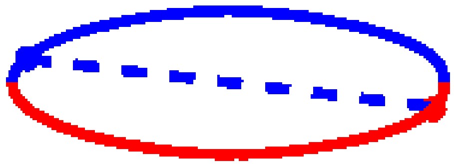


But will be even more convenient to draw as



Can (formally) interpret this as 3 particles moving forward in time [Zakharov 1990's]:

2 particles from the amplitude (evolving with e^{-iHt})
1 particle from the conjugate amplitude (evolving with e^{+iHt})

Will show that evolution in  can be described by

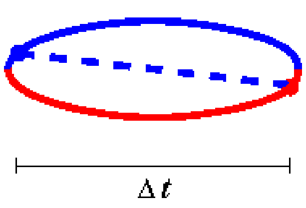
3-particle non-relativistic Quantum Mechanics in 2 dimensions

$$\mathcal{H}_{\text{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

with weird properties:

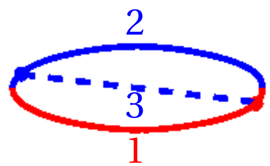
- $m_1 + m_2 + m_3 = 0$
- $V \propto -i$ (i.e. $\mathcal{H}$ is non-Hermitian)

⇨ interference vanishes as $\Delta t \to \infty$, as it must!



Kinetic terms:

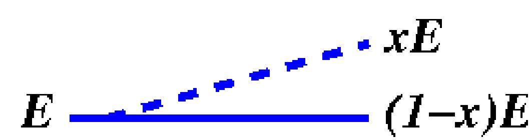
Energy of a high-p_z particle: $$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z}$$

Evolution of  is $e^{-i\mathcal{H}t}$ with

$$\mathcal{H}_{\mathbf{kin}} = -\epsilon_{p_1} + \epsilon_{p_2} + \epsilon_{p_3} \simeq -\frac{p_{\perp 1}^2}{2p_{z1}} + \frac{p_{\perp 2}^2}{2p_{z2}} + \frac{p_{\perp 3}^2}{2p_{z3}}$$

$$\simeq -\frac{p_{\perp 1}^2}{2E} + \frac{p_{\perp 2}^2}{2(1-x)E} + \frac{p_{\perp 3}^2}{2xE}$$

conjugate evolves with $e^{+i\mathcal{H}t}$



This is 2-dimensional non-relativistic QM with

$$(m_1, m_2, m_3) = (-E, (1-x)E, xE)$$

As promised,

$$m_1 + m_2 + m_3 = 0$$

Potential terms:

Accounts for local interactions with medium.

To motivate form, think of something else...

A classical Boltzman analysis of scattering:

$$\frac{d}{dt} f(p_\perp) = \underbrace{\int_{q_\perp} f(p_\perp - q_\perp) \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{gain term}} - \underbrace{f(p_\perp) \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{loss term}}$$

Fourier transform:

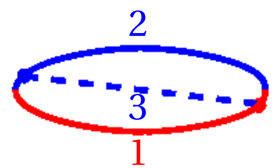
$$\frac{d}{dt} f(b) = f(b) \Big[\Gamma_{\rm el}(b) - \Gamma(0)\Big]$$

$$\text{with} \qquad \Gamma_{\rm el}(b) \equiv \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp} e^{-ib\cdot q_\perp}$$

This looks like a Schrodinger-ish equation:

$$i\frac{d}{dt} f = \mathcal{H}_{\rm boltz}\, f \qquad \text{with} \qquad \mathcal{H}_{\rm boltz} = -i\Big[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b)\Big]$$

In our problem, this physics gives V:



QED: $$V = -ie^2\left[\bar{\Gamma}(0) - \bar{\Gamma}_{\text{el}}(b_2 - b_1)\right]$$

(bar over Γ means charge e^2 factored out)

QCD:

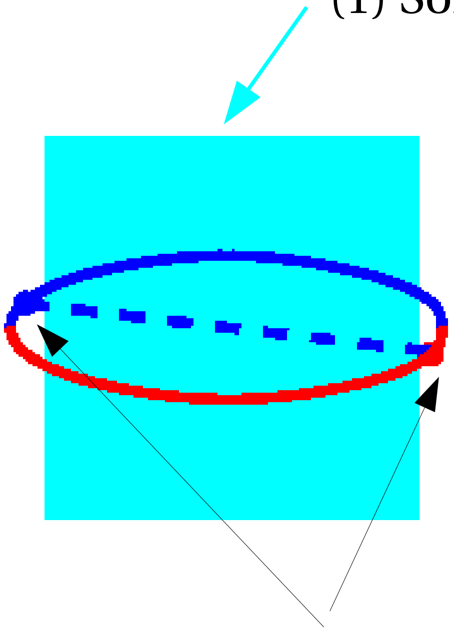
$$V = -ig^2\Big[\tfrac{1}{2}T_1^2\,\bar{\Gamma}_{\text{el}}(0) + \tfrac{1}{2}T_2^2\,\bar{\Gamma}_{\text{el}}(0) + \tfrac{1}{2}T_3^2\,\bar{\Gamma}_{\text{el}}(0)$$
$$+T_2\cdot T_1\,\bar{\Gamma}_{\text{el}}(b_2 - b_1) + T_3\cdot T_2\,\bar{\Gamma}_{\text{el}}(b_3 - b_2) + T_1\cdot T_3\,\bar{\Gamma}_{\text{el}}(b_1 - b_3)\Big]$$

Color factors $T_i \cdot T_j$ are fixed (not dynamical) because $T_1 + T_2 + T_3 = 0$

⇨ e.g. $T_2 \cdot T_1 = -\frac{1}{2}(T_3^2 - T_1^2 - T_2^2) = -\frac{1}{2}(C_1 - C_2 - C_3)$

How to put the calculation together:

(1) Solve for propagation in 3-particle QM in shaded region.



(2) Tie together with QFT matrix elements for vertices

$$\propto \sqrt{\textbf{DGLAP splitting functions}}$$

$$\propto \sqrt{P_{i\to j}(x)}$$

Simplifcation: 3-particle QM → 1-particle QM

$$\mathcal{H}_{\text{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

- Translation invariance:

  Factor out COM motion → 2-particle QM

- Results should not depend on exact choice of z axis:

→ can factor out d.o.f. associated with tiny changes of z axis
→ 1-particle QM

[In 2-dim QM language, the last simplification depends on a special property of the case $m_1+m_2+m_3 = 0$.]

Solving 1-particle QM:

$$\mathcal{H} = \frac{P_B^2}{2M} + V(B)$$

[BPMPS-Z (1990's)]

Method 1. Can solve numerically.

[Zakharov (2004+); Caron-Huot & Gale (2010)]

Method 2. High energies → very collinear → b's small.

So make small b approximation to

$$-i\Big[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b)\Big] \simeq -i\hat{q}b^2$$

$$\hat{q} \equiv \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp} q_\perp^2$$

→ a harmonic oscillator problem

$$\mathcal{H} = \frac{P_B^2}{2M} + \frac{1}{2} M\Omega_0^2 B^2$$

[Baier *et al*. (1998)]

(a non-Hermitian one: $\Omega_0^2 \propto -i$)

OUTLINE

Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect

B. interference between splittings (and when/if that's important)

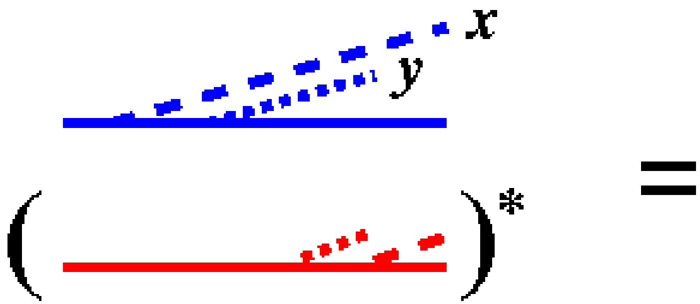
Part II. Formalism for calculations (presented qualitatively)

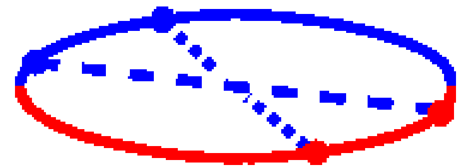
A. single splitting

B. consecutive splittings

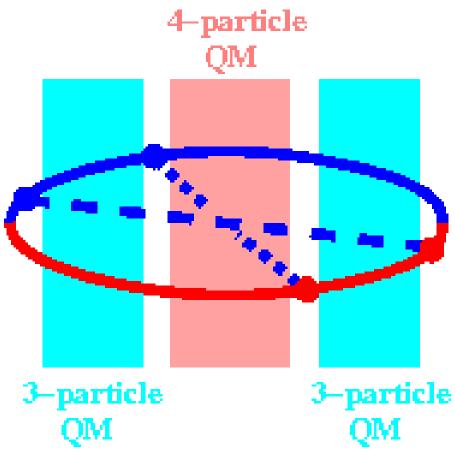
Formalism for LPM: double brem

Example of an interference contribution:

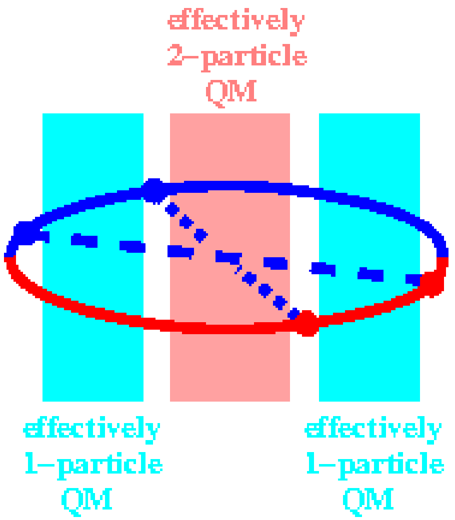


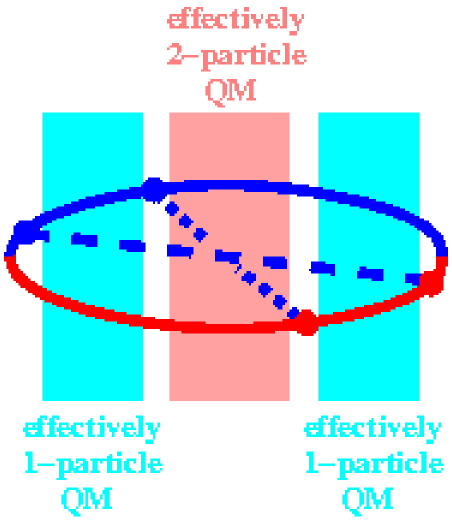


To compute : Sew together QFT matrix element for vertices with QM evolution in between.



Simplify : Using symmetries, as before.





ugliest bit = 2-particle QM evolution

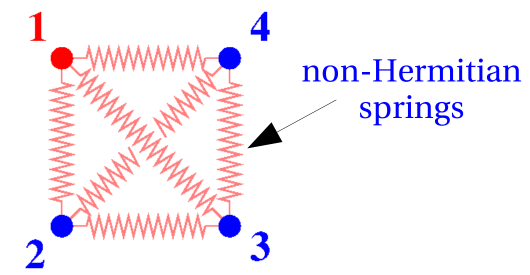
Can imagine

- numerics [have not done]
- harmonic osc. approximation [have done!]

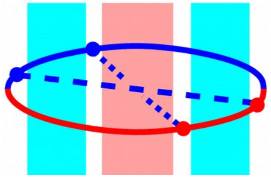
Harmonic osc. ***sounds*** very straight-forward, but in fact quite complicated.

What do we do?

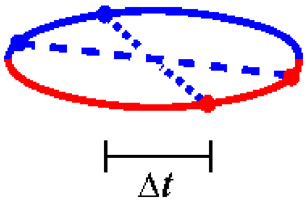
- For 4-particle (effectively 2-particle) evolution, find eigenmodes and frequencies of



- Construct corresponding propagator for 4-particle (2-particle) evolution.
  [Also do the same for 3-particle (1-particle) evolution.]
- Combine with QFT matrix elements for splitting vertices.



- Analytically integrate over all vertex times except Δt :
- Analytically integrate over all vertex transverse positions.



Result:

$$\text{answer} = \int_0^\infty d(\Delta t)\ \text{complicated formula}$$

- Final Δt integral easy to do numerically.

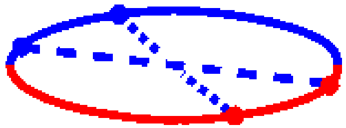
Complications

Formalism: Getting straight the formalism for 4-particles → effectively 2 particles.

Color: During 4-particle evolution, $T_1+T_2+T_3+T_4 = 0$ is not enough to fix color factors $T_i \cdot T_j$.

Color dynamics is non-trivial!

For now: Work in large N_c limit. [Not necessary if the brems are soft.]

Helicities: Helicities of high-energy particles contract non-trivially in 

Must use helicity-dependent DGLAP splitting functions at vertices.

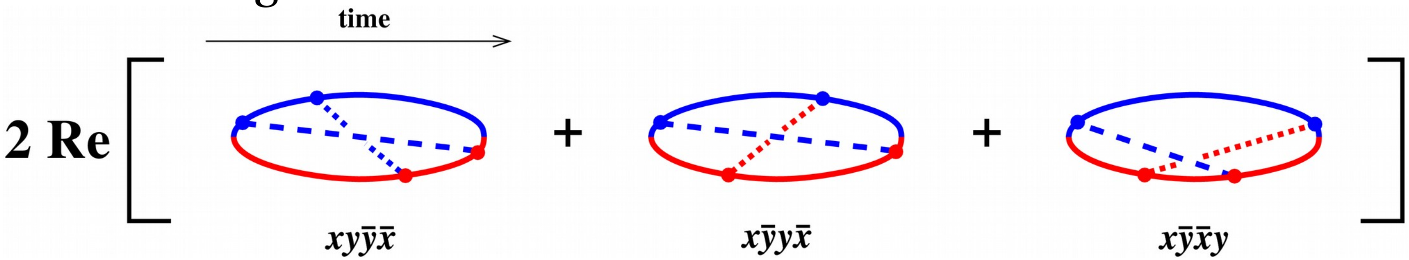
Divergences: Each time-ordered diagram diverges as $\Delta t \to 0$.

Must handle carefully (and non-trivially), even though the amplitude (blue) is just a tree diagram!

Published Work with Shahin Iqbal and Han-Chih Chang

All diagrams for overlap of two *real* gluon emissions [all for $g \to gg \to ggg$]

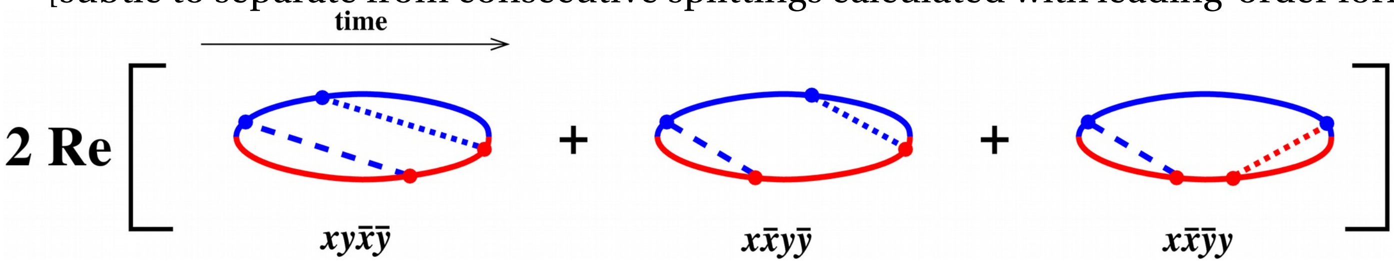
Crossed diagrams:



\+ permutations of $(x, y, 1\text{-}x\text{-}y)$

Sequential diagrams:

[subtle to separate from consecutive splittings calculated with leading-order formalism!]



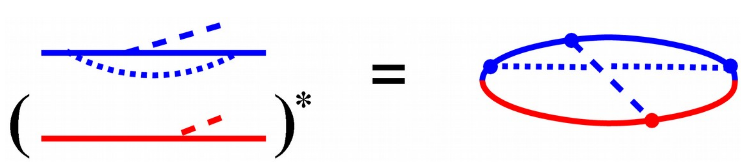
\+ permutations of $(x, y, 1\text{-}x\text{-}y)$

Diagrams with 4-gluon vertices:

e.g.

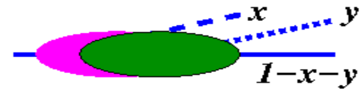
Still in progress

virtual corrections, e.g.



correct single brem rate

Results for real double brem

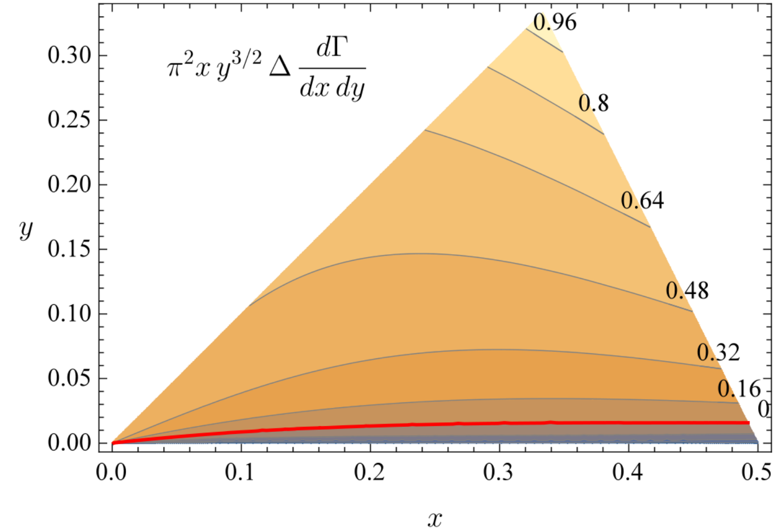


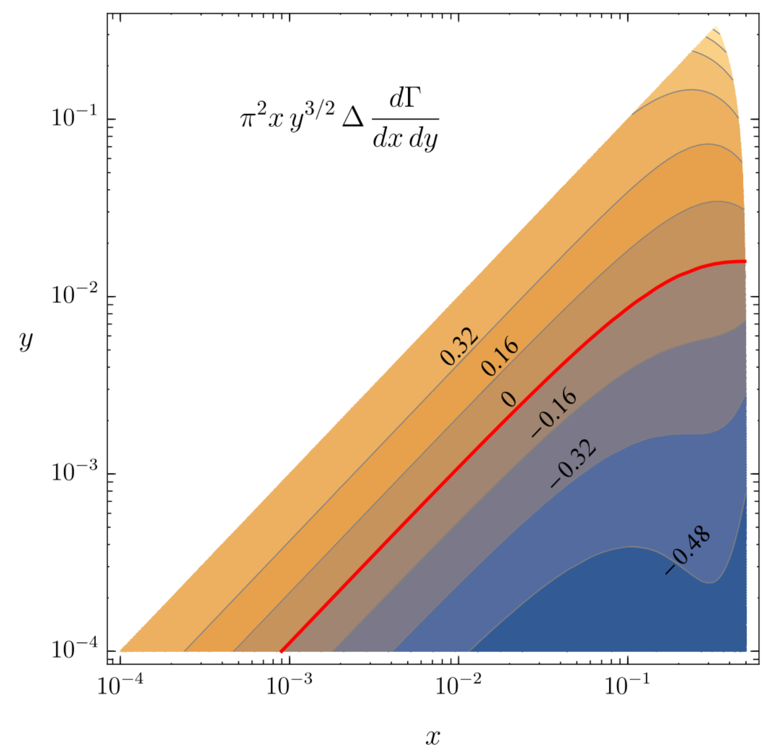
$$\Delta \frac{d\Gamma}{dx\,dy} \equiv$$ correction to double brem due to overlapping formation times

$$= f(x,y)\,\frac{C_A^2\alpha_s^2}{\pi^2 x y^{3/2}}\sqrt{\frac{\hat{q}_A}{E}}$$

$$(y < x < 1{-}x{-}y)$$

where $f(x,y)$ varies from 1.05 to -0.90 and is shown on the right.





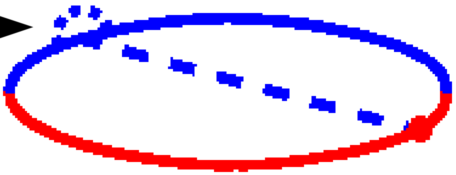
Qualitative Point

Effect of overlapping formation times enhances the rate except when one gluon is very soft.

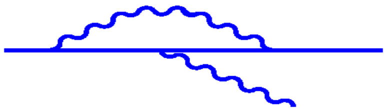
Virtual diagrams: what's the holdup?

Example:

UV divergent vertex correction:
renormalization of charge, etc.



But can't I look up in my favorite textbook how to compute UV part of the amplitude



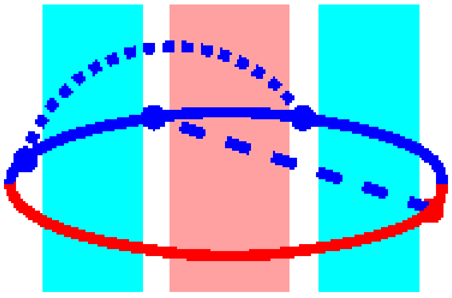
in QFT?

Yes, but I earlier treated these particles in the approximation

$$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z}$$

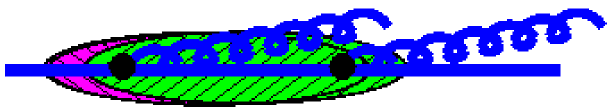
2-dim QM with "mass" p_z

So I need to <u>match</u> UV renormalization of underlying gauge theory to calculations
In the effective QM theory used for



Summary

Subtle problems in the field theory description of very-high energy showering

e.g. 

can be reduced to problems in

2-dimensional non-relativistic non-Hermitian quantum mechanics

and even

2-dimensional non-relativistic non-Hermitian harmonic oscillators!

(Just when you thought you couldn't learn anything more from the harmonic oscillator...)

Coming in the future

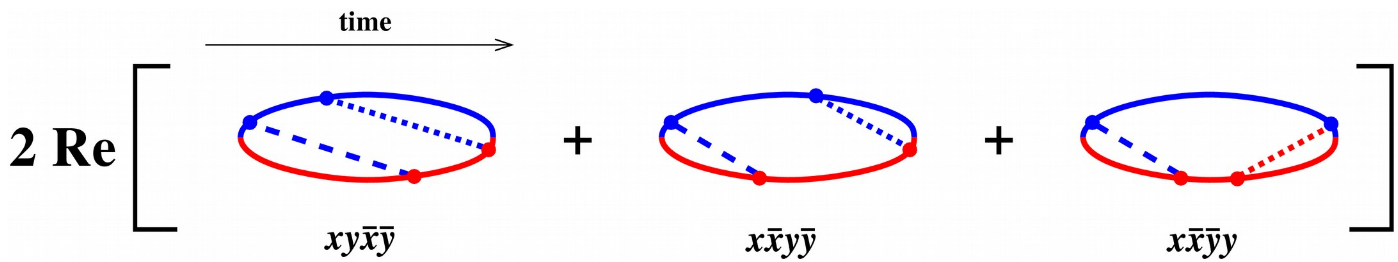
Are the $O(\alpha_s)$ corrections to physical, infrared-safe quantities characterizing shower development small (after accounting for the known running of $\hat{q}(E)$ due to soft brem)?

To wit, is the basic physical assumption behind in-medium Monte Carlo simulations on firm ground?

EXTRA

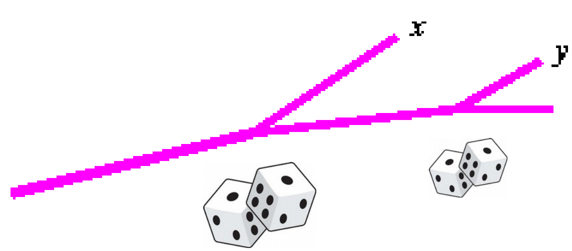
What was subtle about

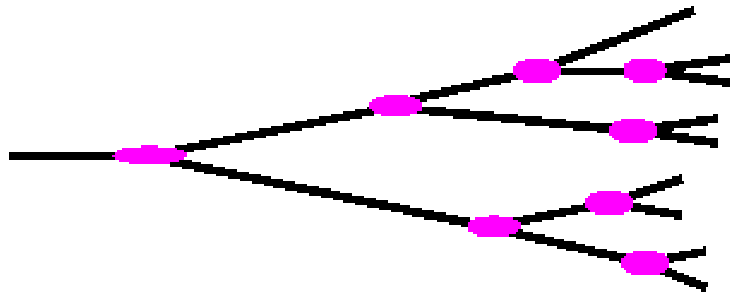
Sequential diagrams:



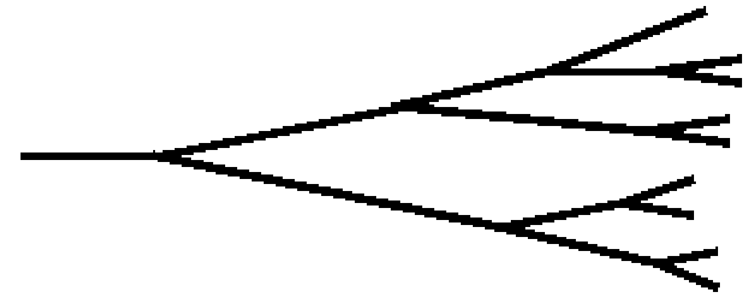
+ permutations of (x, y, 1-x-y)

A: Have to avoid double counting with Monte Carlo based on single-splitting rates:



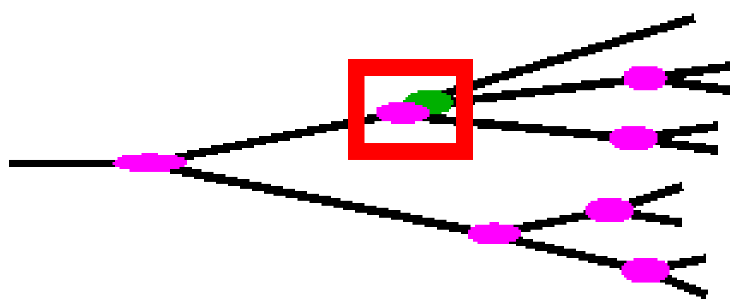


vs



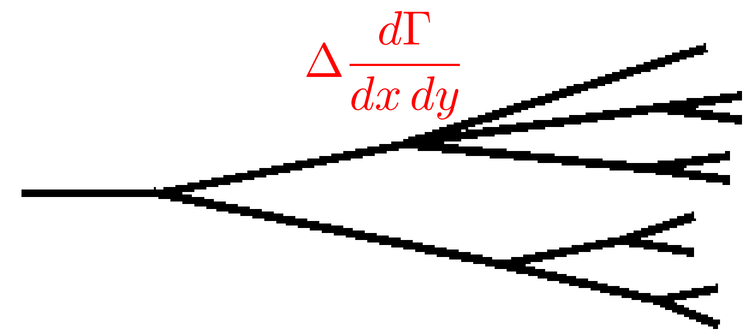
Monte Carlo (MC)

How to account for correction from



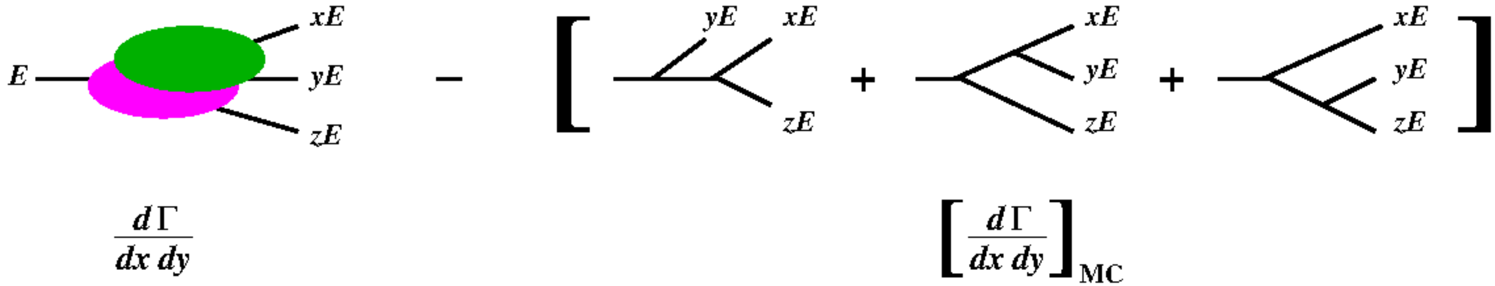
?

Add a $g \to ggg$ Monte Carlo possibility to account for correction:



where

$$\Delta \frac{d\Gamma}{dx\,dy} =$$



RESERVE

Landau-Pomeranchuk-Migdal (LPM) effect

What is the LPM Effect?

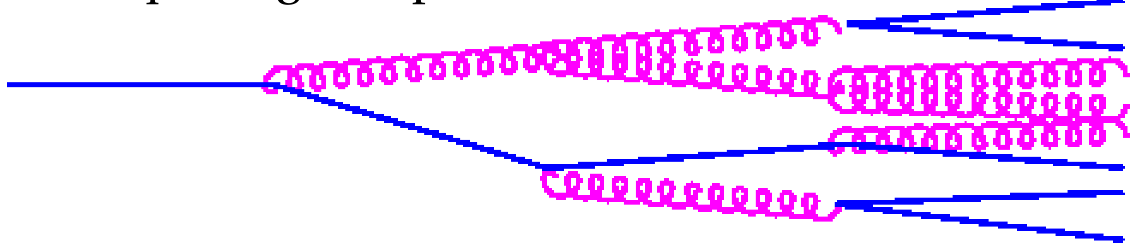
A coherence effect that complicates calculations of bremsstrahlung or pair production when a very high energy particle scatters from a medium.

Places it comes up in QED

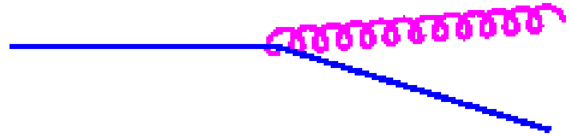
- Very high energy cosmic rays showering in the atmosphere.
- Certain beam dump experiments designed to measure the LPM effect.

Places it comes up in QCD

- Energy loss of high energy jets in a quark-gluon plasma.



- Complete leading-order calculations of the viscosity and other transport coefficients of a weakly-coupled quark-gluon plasma.



$$\hat{q}_{\text{eff}}(E) = \hat{q}_0 \times \left\{ 1 + \#\, \alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right) + \# \left[\alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right)\right]^2 + \cdots \right\}$$

$$= \hat{q}_0 \times \frac{I_1\left(\# \left[\alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right)\right]^{1/2}\right)}{\# \left[\alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\text{mfp}}}\right)\right]^{1/2}}$$

$\sim E^{\#\sqrt{\alpha}}$ (for large enough *E*)

[adapted from Liou, Mueller, Wu (2013)]

OUTLINE

Part I. Qualitative Discussion

A. interference within a single splitting: the LPM effect

B. interference between splittings (and when/if that's important)