Hydrodynamic theory of quantum fluctuating superconductivity

Sean Hartnoll (Stanford)

St John’s College
Oxford — 2016

Based on: arXiv/1602.08171 [cond-mat.supr-con]
Collaborators

Richard Davison (Harvard)

Luca Delacrétaz (Stanford)

Blaise Goutéraux (Stanford)
Hydrodynamic description of conventional metals

- **Hydrodynamics:**
  - Universal low energy, long wavelength physics.
  - Conserved charges, their currents, Goldstone bosons.

- **Conservation law:**
  \[
  \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0
  \]

- **Constitutive relation** (derivative expansion):
  \[
  j = -D \nabla \rho + \cdots
  \]

- **Conductivity** *(Einstein relation)*:
  \[
  j = -D \nabla \rho = -D \frac{\partial \rho}{\partial \mu} \nabla \mu = D \chi E = \sigma E
  \]
Comment on screening by Maxwell fields

• Charge in a metal does not diffuse, it decays exponentially.

• This comes from solving Maxwell’s equations + Ohm’s law.

• The Einstein relation for the conductivity still holds.

• \( \sigma \) measured with respect to total, not external, electric field:

\[
j = \sigma E_{\text{tot}} = \sigma \frac{E_{\text{ext}}}{\epsilon(\omega, k)} = \frac{\sigma E_{\text{ext}}}{1 - \frac{1}{k^2} \chi(\omega, k)} = \frac{-i\omega D\chi}{i\omega - D(k^2 + \chi)} E_{\text{ext}}
\]
Superfluid hydrodynamics

- Phase $\phi$ of the order parameter appears in hydrodynamics.
- $u_\phi = \frac{1}{m} \nabla \phi$ is the superfluid velocity.
- ‘Josephson relation’:
  \[ m \frac{\partial u_\phi}{\partial t} = \nabla \frac{\partial \phi}{\partial t} = -\nabla \mu + \cdots \]
- Constitutive relation:
  \[ j = \frac{\rho_s}{m} u_\phi - D \nabla \rho + \cdots \]
- (super-)Conductivity:
  \[ j = - \left( \frac{\rho_s}{m^2} \frac{i}{\omega} + D \chi \right) \nabla \mu = \left( \frac{\rho_s}{m^2} \frac{i}{\omega} + D \chi \right) E = \sigma(\omega) E \]
Superconductivity

- $\infty$ conductivity because: diffusion $\rightarrow$ second sound mode.

- In a superconductor, the U(1) symmetry is gauged, i.e. coupled to electromagnetism.

- This gaps out the Goldstone/sound mode in the same way the diffusive mode was previously gapped.

- However, the conductivity is, as before, measured with respect to the total electric field. So the unscreened (superfluid) hydrodynamics determines the conductivities.
Vortices and supercurrent relaxation

- In two space dimensions, above picture incomplete.

- Motion of vortices can wind and unwind the supercurrent.

- Expect supercurrent relaxation rate $\Omega$:

$$\sigma(\omega) = \frac{\rho_s}{m^2} \frac{1}{-i\omega + \Omega}$$

$$\Delta \phi = 2\pi$$
Vortices and supercurrent relaxation

- This problem is well understood in some regimes:
  - Thermal BKT proliferation of vortices above \( T_{BKT} \).

- Classical picture: vortices pushed across sample by ‘superfluid Magnus force’
  - The core of the vortices is in the normal state.
  - Therefore, motion of vortices creates dissipation.
  - Get
    \[
    \Omega \sim \frac{n_f A_v}{\sigma_n} \quad \text{[Bardeen-Stephen '65]}
    \]

- Much controversy, however, about whether (quantum) phase-disordered superconductors exist at \( T = 0 \).
  [review: Phillips-Dalidovich '03]
In the remainder

• Lightening overview of some experiments.

• Develop a fully quantum effective field theoretic formalism for the conductivity of phase-disordered superconductors.

• Illustrate formalism with two examples:

  (i) ‘Check’: Elegant (re)derivation of Bardeen-Stephen result.

  (ii) Phase disordering by a Chern-Simons interaction ['topologically ordered superfluid vortex liquid'].
Superfluid-insulator transitions

• In two (spatial) dimensions, conventional theory suggests that as $T \to 0$ electrons will either localize or pair up.

• That is, the phase of matter one expects to find is either an insulator or a superconductor.

• Indeed, early experiments suggested that disordered thin films undergo superconductor-insulator transitions as a function of magnetic field or thickness ($\approx 1/$disorder).

Destroys superconductivity

Favors localization
Superfluid-insulator transitions

[Hebard and Paalanen '90, α-InO_x]

[Jaeger et al. '89, Pb]
Metallic phases in two dimensions

- Problematically for ‘conventional’ understanding, in weakly disordered films a metallic phase intervenes (at $T = 0!$) between the superconductor and insulator.

[Mason, Kapitulnik ’99, α-MoGe]
Metallic phases in two dimensions

- Often, the residual resistivity of the metallic phase is much smaller than the “normal state” resistivity of the material at temperatures above the “mean field” superconducting temperature.

- Suggests the low energy degrees of freedom of the metallic phases are not the normal state quasiparticles.

- Natural to think of as “failed superconductors” where (quantum!) phase fluctuations have destroyed phase coherence.
Metallic phases in two dimensions

- Direct motivation for our work: observation of a Drude-like peak in the metallic phase of $\text{InO}_x$.

[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage ’13]
Metallic phases in two dimensions

- The width of the Drude-like peak goes to zero at the same magnetic field where superconductivity appears.

[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage ’13]
What we will and won’t do

• We will not answer the question of whether some given microscopic model remains metallic at $T = 0$.

• Our point is that universal aspects of this problem can be isolated from the microscopic models.

• Theory of Drude peaks due to fluctuating superconductivity.

• ‘Classification’ of mechanisms of dissipation at $T=0$. 
Memory matrix formalism

• Most discussions of this physics have involved semi-microscopic models with uncontrolled approximations.

• Instead: work in a limit where a hierarchy of timescales allows an effective field theoretic approach.

• Small parameter will be the supercurrent relaxation rate. I.e. want $\Omega \ll T$, etc.

• (Approach inspired by studies in holographic systems over past few years, where slow mode was momentum.)
Suppose that $H = H_0 + \varepsilon \Delta H$, with $[\Delta H, J_\phi] \neq 0$.

Then the decay of $J_\phi$ is slow and dominates $\sigma$:

$$\sigma(\omega) = \frac{\chi J_\phi J_\phi}{\chi J_\phi J_\phi} \frac{1}{-i\omega + \Omega} + \cdots$$

But now we have a formula for $\Omega!$:

$$\Omega = \varepsilon^2 \lim_{\omega \to 0} \frac{1}{\chi J_\phi J_\phi} \Im G^{R}_{i[\Delta H, J_\phi] i[\Delta H, J_\phi]}(\omega) \bigg|_{\varepsilon=0}.$$
Supercurrent relaxation

• Recap: if an ‘almost conserved’ operator carries current, rate of the decay determines the conductivity.

• In our case of interest today:
  
  \[ J_\phi = \frac{1}{m} \int d^2x \nabla \phi \]

• Need an interaction that doesn’t commute with \( J_\phi \).

• Natural building block:
  
  \[ \pi_\phi = \frac{\partial f}{\partial \phi} = - \frac{\partial f}{\partial \mu} = \rho. \]

  i.e. charge density is canonically conjugate to the phase:

  \[ [\phi(x), \rho(y)] = i\delta(x - y). \]
Supercurrent relaxation

- Thus a simple, generic perturbation of the superfluid state is the short range Coulombic interaction:

\[ \Delta H = \frac{\lambda}{2} \int d^2 x \rho(x)^2. \]

- At first glance looks like commutator is trivial total derivative:

\[ i[\Delta H, J_\phi] = -\frac{\lambda}{m} \int d^2 x \nabla \rho(x) \]

- However, the phase appearing in \( J_\phi \) is only defined outside of vortex cores! Above integral is then also only over the outside of vortex cores. Integral over all space vanishes:

\[ \rightarrow \text{integral over vortex cores.} \]
Supercurrent relaxation

- The memory matrix formula for $\Omega$ becomes an integral of the two point function of $\rho$ over the vortex core.

- Using the diffusive behavior of $\rho$ in normal state, the Bardeen-Stephen formula drops out exactly.

$$\Omega \sim \frac{n_f A_v}{\sigma_n}$$

- So we discover the quantum origin of this formula. Charge interactions enhance phase fluctuations:

$$\Delta \rho \Delta \phi \gtrsim \hbar$$
Beyond Bardeen-Stephen (in progress)

- Real life vortices are not infinitely large. The diffusive form of the charge density correlator is therefore not exact. For small vortices, it will not even be approximately correct.

- Work in progress: generalize Bardeen-Stephen formula allowing for non-diffusive dynamics of the charge density.

- Part of the controversy around T=0 metallic phases is ‘where does the dissipation occur’? From our approach it is manifest that if the phase-relaxing interacting is local, dissipation must be due to vortex cores.
Supercurrent relaxation without parity

• With parity and time-reversal broken, a second very natural $\Delta H$ exists.

• Suppose the low energy effective theory is coupled to an emergent Chern-Simons gauge field:

$$\mathcal{L} = \mathcal{L}_{\text{matter}} + j_\mu (A^\mu + a^\mu) - \frac{1}{2\lambda'} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

• Integrating out the gauge field generates

$$\mathcal{L}' = \frac{\lambda'}{2} j_\mu \frac{\epsilon^{\mu\nu\rho}}{\partial_\sigma \partial_\sigma} j_\nu \quad \Rightarrow \quad \Delta H = \frac{\lambda'}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{\rho_{-k} (\nabla \times j)^z_k}{k^2} + \text{h.c.}$$
Supercurrent relaxation without parity

- **Non-locality** of induced interaction leads to a nonzero time dependence of $J_\Phi$ everywhere. In fact:

  $$i[\Delta H, J_\Phi^i] = -\frac{\chi'}{m} \epsilon^{ij} J^j.$$ 

- Rough physical picture:
  Current = Flow of charge
  → Flow of emergent magnetic flux (CS term)
  → Flow of vortices
  → Relaxation of supercurrent in transverse direction!

- $\Omega$ depends on charge flow in normal component.
Supercurrent relaxation without parity

- Result for conductivities:

\[
\sigma_{xx} = - \frac{m^2}{\lambda' \rho_s} \frac{\omega (\omega \Omega + i (\Omega^2 + \Omega_H^2))}{(-i\omega + \Omega)^2 + \Omega_H^2},
\]

\[
\sigma_{xy} = - \frac{1}{\lambda'} - \frac{m^2}{\lambda' \rho_s} \frac{\omega^2 \Omega_H}{(-i\omega + \Omega)^2 + \Omega_H^2},
\]

- Feature: ‘supercyclotron resonance’ at

\[
\omega_* = \pm \Omega_H - i\Omega = \frac{\lambda' \rho_s}{m^2} \frac{1}{\pm 1 - \lambda' (\pm \sigma_0^H - i\sigma_0)}.
\]

Conductivities of the normal component of superfluid.
Chern-Simons superfluid hydrodynamics

- The expressions for the conductivities can be alternatively derived directly from superfluid hydrodynamics coupled to a Chern-Simons gauge field.

- Dissipation in this case is not due to vortex cores, but to the normal component of the superfluid.

- If the normal component only has a Hall conductivity (e.g. a superfluid coupled to a quantum Hall state), obtain nontrivial dissipationless frequency dependent dynamics.
Recap [see arXiv/1602.08171]

- **Superfluid relaxation** occurs if perturbations of effective Hamiltonian do not commute with the supercurrent.

- Starting with perturbations of superfluid hydrodynamics gives **controlled** entry point. This works even if the underlying microscopic dynamics is strongly correlated.

- Gave two examples, with and without parity: