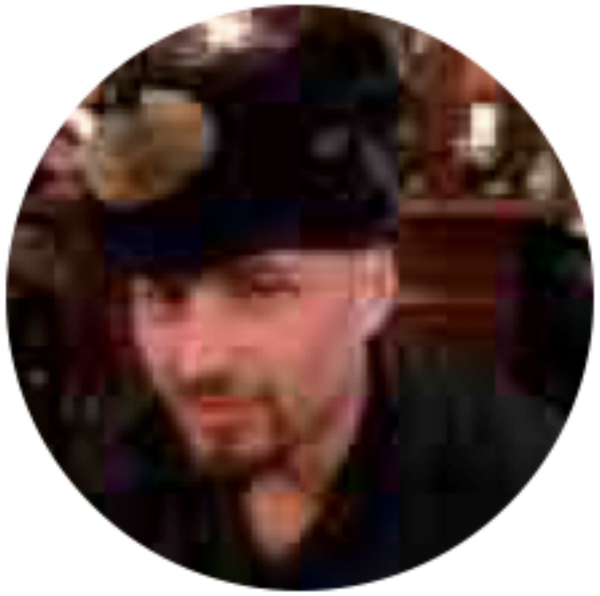
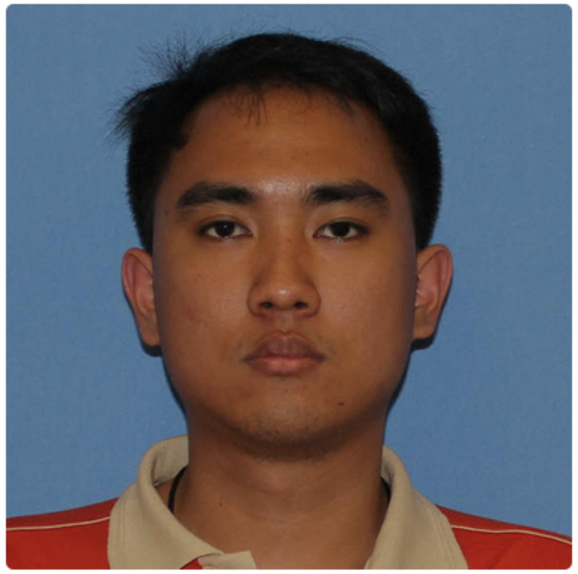
Non-Local Actions and Anomalous Dimensions: Application to the Strange Metal

thanks to

Gabriele La Nave





Kridsangaphong Limtragool

NSF

arxiv:1605.07525/

strange metal

$[J] = \Phi$

fractional not d-1

non-local action

$[A_\mu] = d_A \neq 1$

probe by Aharonov-Bohm effect on underdoped cuprates

dimensions of current

$$\phi(x) \to \phi(x) + \delta\phi(x)$$

$$[J_0(x), \phi(y)] = \delta\phi(x)\delta^d(x-y)$$

$$[J_0] = d$$

$$\nabla \cdot J_\mu = \partial_t J_0$$

$$\xi_\tau \approx \xi$$

$$[J_\mu] = d$$

can the dimension of the current change?

$$\nabla \cdot J_\mu = \partial_t J_0$$

$$\xi_\tau \propto \xi^z$$

$$[J_\mu] = d - \theta + z - 1$$

$$d \to d - \theta$$

hyperscaling violation
Sachdev, Kiritsis

consistent actions

$$\int d^dx\sqrt{-g}\left\{R-\frac{1}{2}(\partial_\mu\phi)^2+V(\phi)-\frac{1}{4}Z(\phi)F^{\mu\nu}F_{\mu\nu}-\frac{W(\phi)}{2}A^2\right\}$$

$$A_t = Qr^{\zeta - z}$$

$$\theta \neq 0, z > 1, \zeta \neq 0$$

$$W \neq 0$$

$$J_\mu = \partial_\nu Z(\phi) F_{\mu\nu}$$

dimension changes

boundary terms?

are the boundary terms finite?

$$\nabla_a(Z(\phi)F^{ab}) - W(\phi)A^b = 0$$

$$S_{\rm bound} = -\frac{1}{2}\int d^3x\sqrt{-\gamma}Z(\phi)n_\mu A_\nu F^{\mu\nu}$$

$$\sqrt{-\gamma}n_r = \sqrt{-g} = l^{-\theta}Lr^{2\theta-z-3},$$

$$Z(\phi) = Z_0\left(\frac{r}{l}\right)^{4-\theta},$$

$$A_tF_{rt}g^{rr}g^{tt} = \frac{2(z-1)}{Z_0}r^{-3}l^{2z+5}L^{-2}$$

$$S_{\rm bound} \approx 2(z-1)l^{2z}L^{-1}\int d^3x r^{2\theta-z-2}$$

typically divergent

J. Tarrio, W=0

are there Lifshitz-type solutions that modify
the scaling of A that
have finite boundary actions? $\zeta \neq 0$

units of gauge field

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu \mathcal{G}$$

has no units

$$[A_\mu] = \frac{1}{L} = 1$$

dimensions of current can change without any change in the dimension of A

$$J_\mu \propto \frac{\delta \ln Z}{\delta A_\mu}$$

↓

dimensions of J and A are related

physics beyond HSV and Lifshitz geometries how can the scaling of J and A be linked directly?

gauge symmetry

$$\nabla^\alpha \cdot J_\mu = \partial_t J_0$$

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial^\alpha_\mu \mathcal{G}$$

both J and A have anomalous dimensions!

why would A have an anomalous dimension?

Hartnoll/Karch

$$[j] = d - \theta + \Phi + z - 1$$

$$[j^Q] = d - \theta + 2z - 1$$

$$[A] = 1 - \Phi$$

$$[B] = 2 - \Phi$$

$$\Phi = -2/3$$
$$\theta = 0$$

strange metal

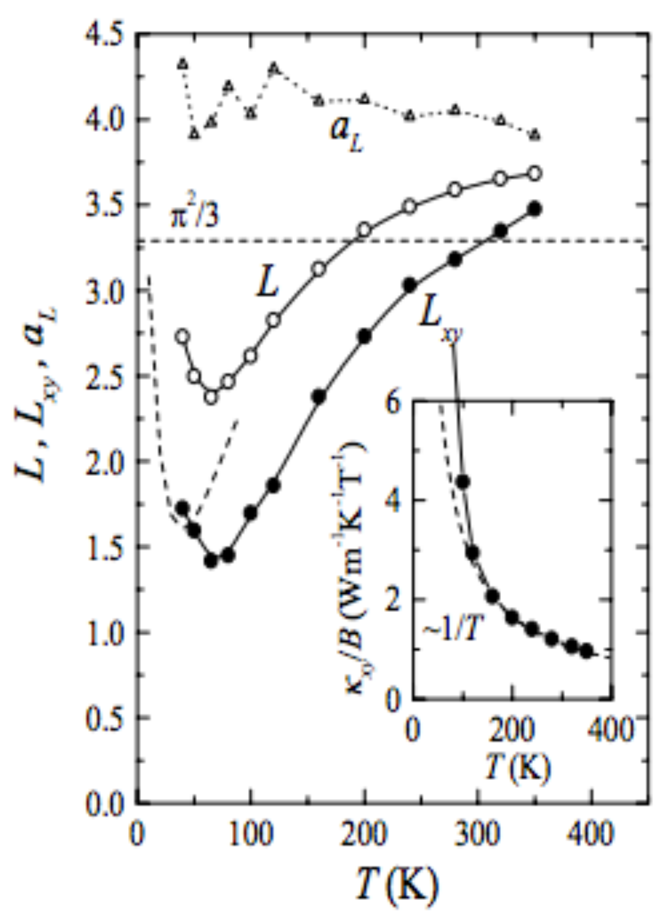
strange metal: experimental facts

Hall Angle

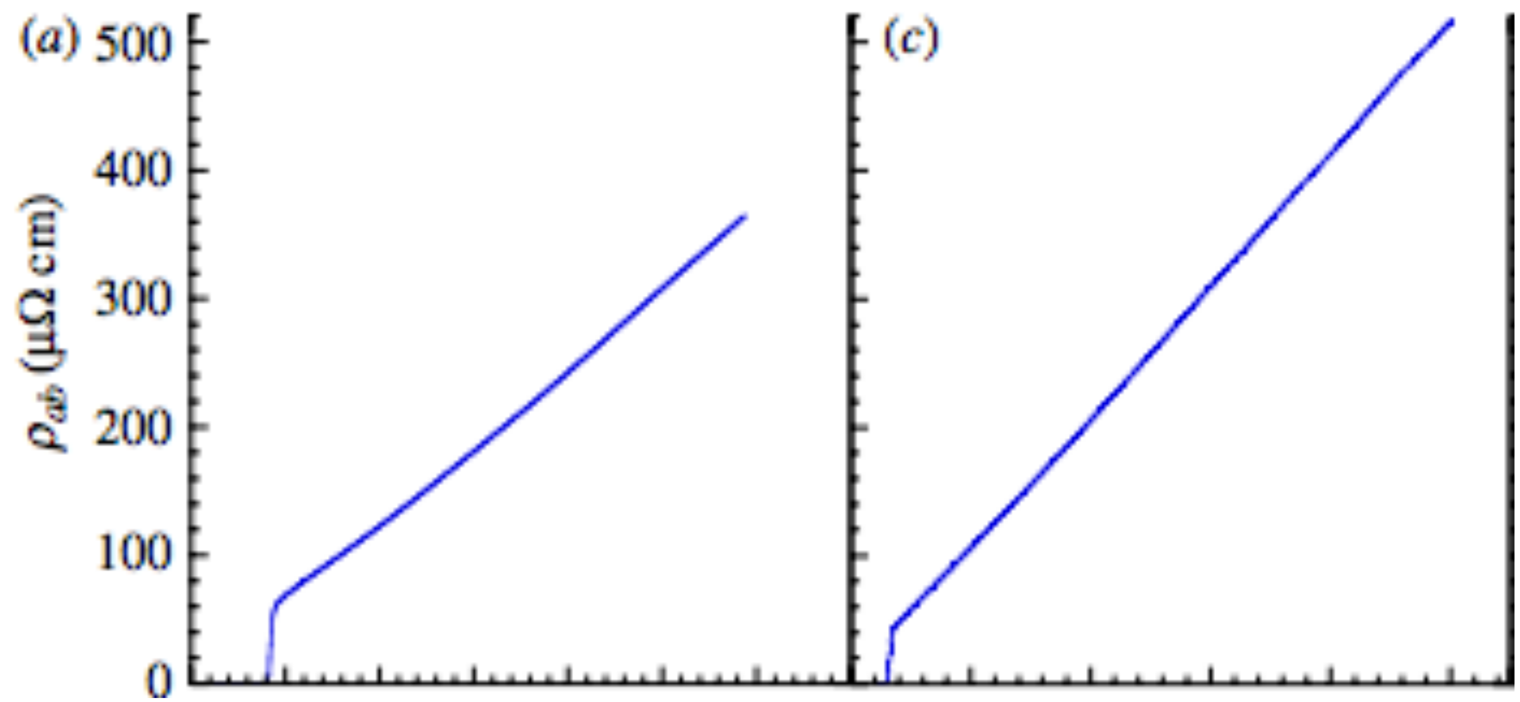
$$\cot \theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



T-linear resistivity



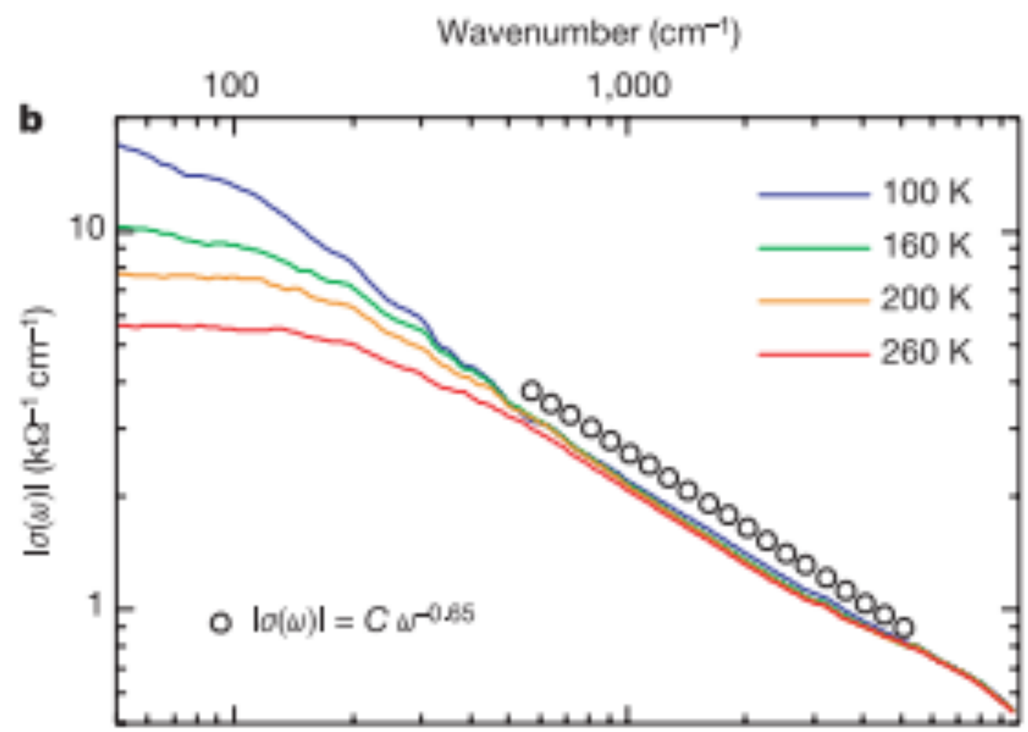
all explained if

$$\Phi = -2/3$$

what else can be explained with
an anomalous dimension for A and J?

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]



Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

$$\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)}$$
$$\gamma = 1.35$$

take experiments seriously

$$\sigma^i(\omega) = \frac{n_i e_i^2 \tau_i}{m_i} \frac{1}{1 - i\omega\tau_i}$$

continuous mass(scale invariance)

$$\sigma(\omega) = \int_0^M \frac{\rho(m)e^2(m)\tau(m)}{m} \frac{1}{1 - i\omega\tau(m)} dm$$

variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

$$e(m) = e_0 \frac{m^b}{M^b}$$

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

Karch, 2015

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0}{M^{a+2b+c}} \int_0^M dm \frac{m^{a+2b+c-2}}{1 - i\omega\tau_0 \frac{m^c}{M^c}}$$

perform integral

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\sigma(\omega) = \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M} \frac{1}{\omega^{\frac{2}{3}}} \int_0^{\omega\tau_0} dx \, \frac{x^{-\frac{1}{3}}}{1-ix}$$

$$\downarrow \; \omega\tau_0 \to \infty$$

$$\sigma(\omega) = \frac{1}{3}(\sqrt{3}+3i)\pi \frac{\rho_0 e_0^2 \tau_0^{\frac{1}{3}}}{M\omega^{\frac{2}{3}}}$$

$$\tan\sigma = \sqrt{3}$$

$$60^\circ$$

are anomalous dimensions necessary

$$\frac{a+2b-1}{c} = -\frac{1}{3}$$

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

hyperscaling violation

$$e(m) = e_0 \frac{m^b}{M^b}$$

anomalous dimension

$$\tau(m) = \tau_0 \frac{m^c}{M^c}$$

momentum loss

$c = 1$

$a + 2b = 2/3$ → $b = 0$, $a = 2/3$

No

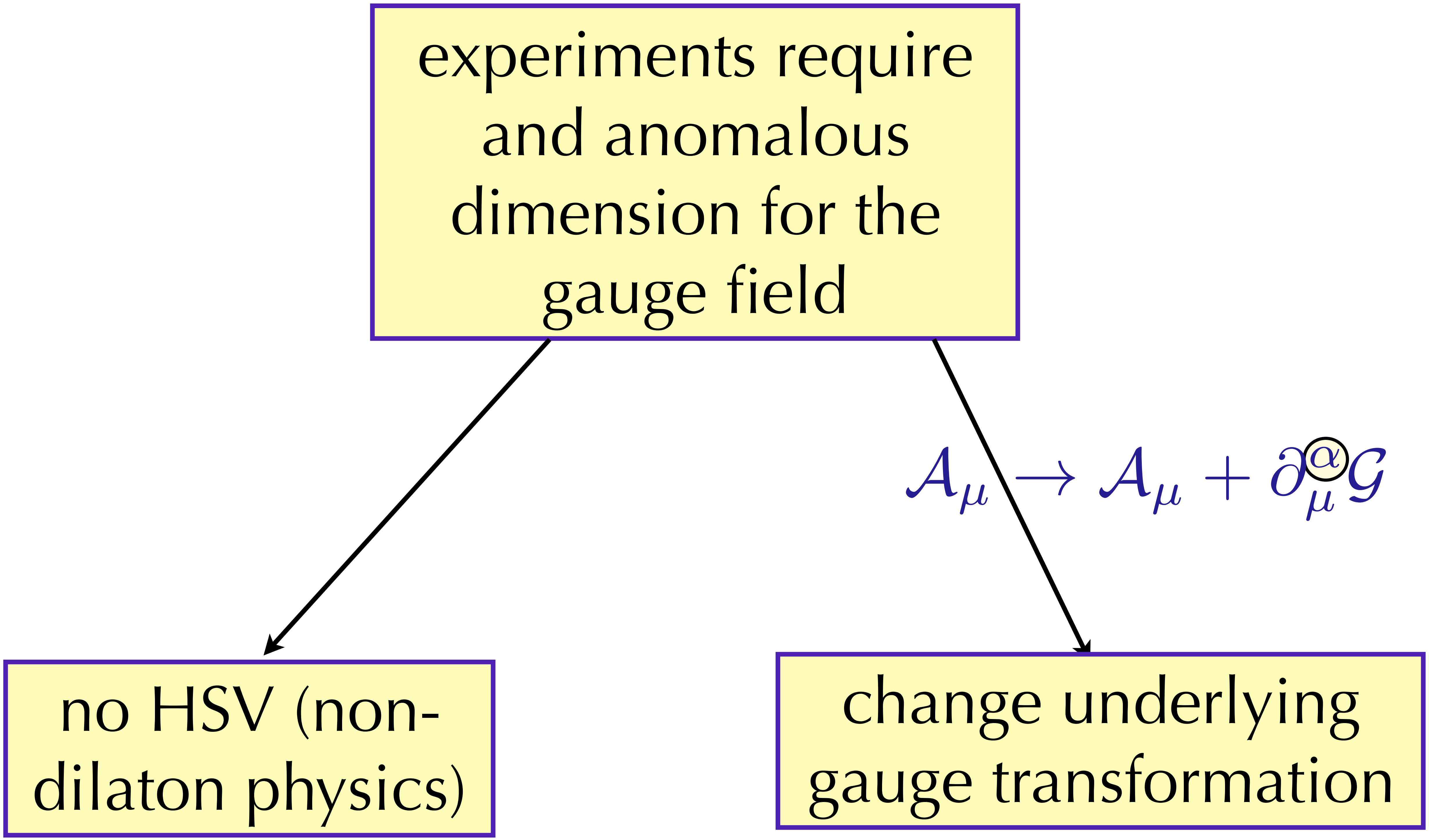
but the Lorenz ratio is not a constant

$$L_H = \frac{\kappa_{xy}}{T\sigma_{xy}} \sim T \equiv T^{-2\Phi/z}$$

Hartnoll/Karch

$$\Phi = bz = -2/3$$

$$\rho \propto T^{-2\Phi/z}$$


experiments require and anomalous dimension for the gauge field
$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial^{\alpha}_\mu \mathcal{G}$
no HSV (non-dilaton physics)
change underlying gauge transformation

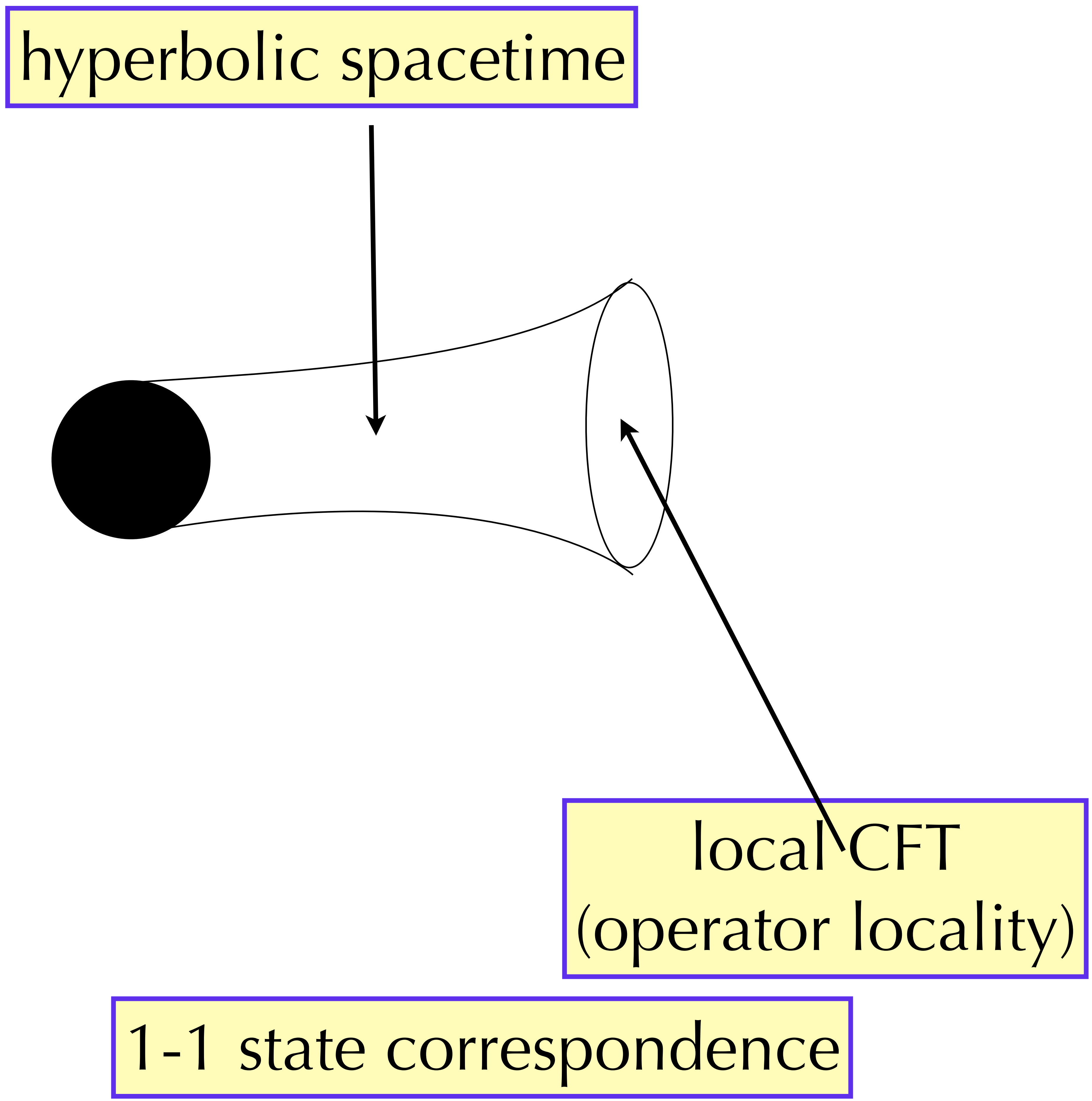
$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial^\alpha_\mu \mathcal{G}$$

Reisz fractional Laplacian

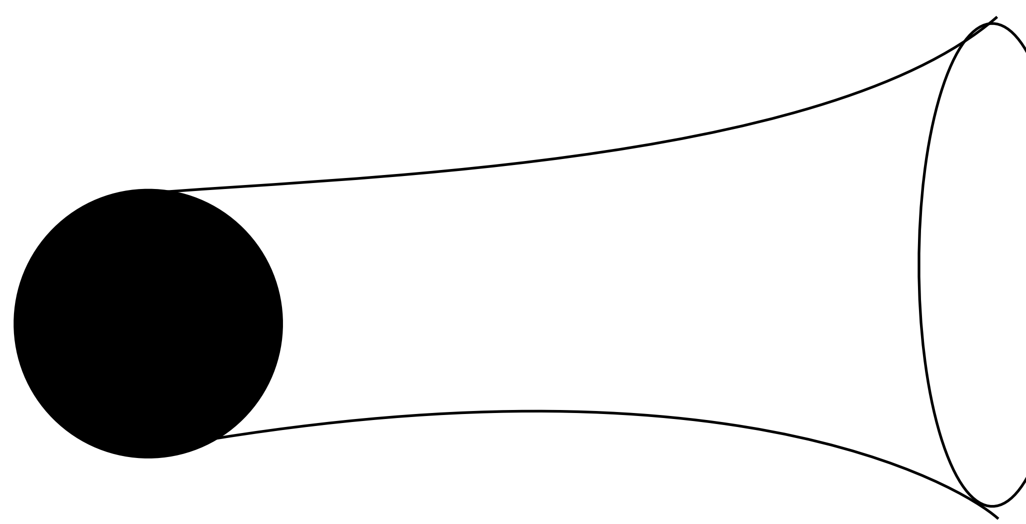
$$(-\Delta)^\gamma f(x) = C_{d,s} \int_{\mathbf{R^d}} \frac{f(x) - f(\xi)}{|x-\xi|^{d+2\gamma}} d\xi$$

non-local: f must be known everywhere

what theories have such non-local interactions?


hyperbolic spacetime
local CFT
(operator locality)
1-1 state correspondence

any theory with gravity
has less observables
than a theory without it!



how can a
local CFT
emerge at the
boundary?

quantum gravity $\stackrel{?}{=}$ boundary local QFT

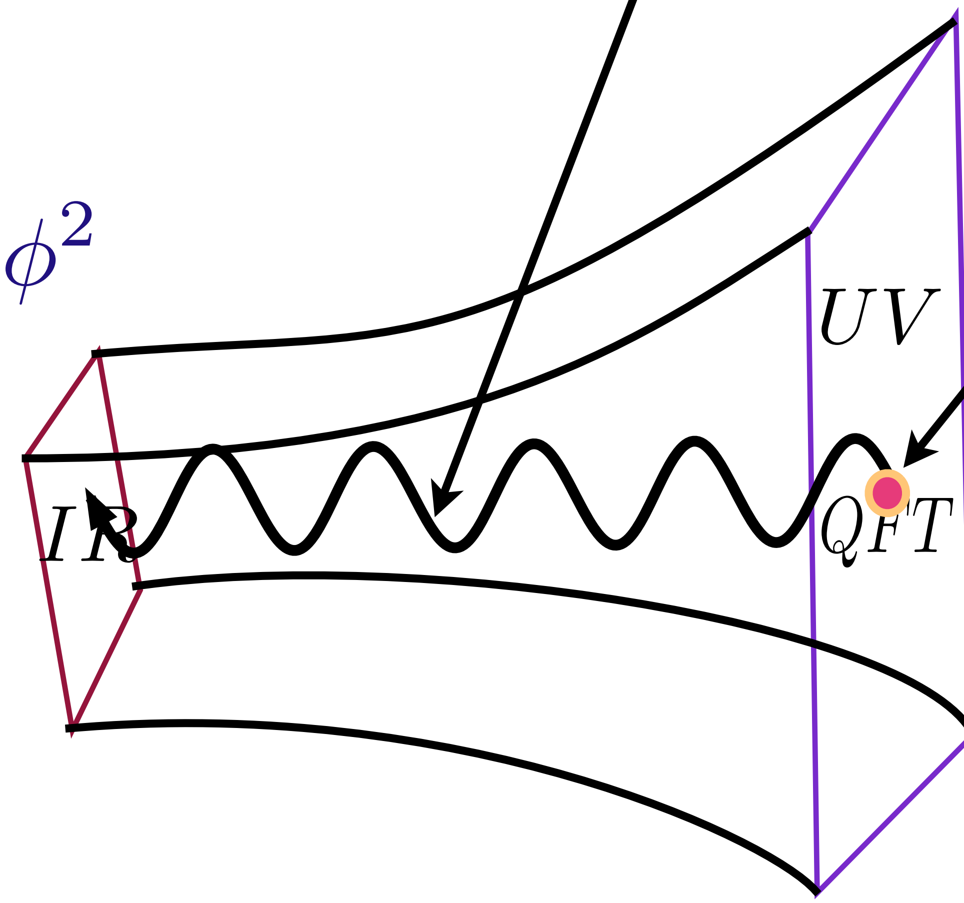
is

standard holography

$$S = S(g_{\mu\nu}, A_\mu, \phi, \cdots)$$

operators $\mathcal{O}$

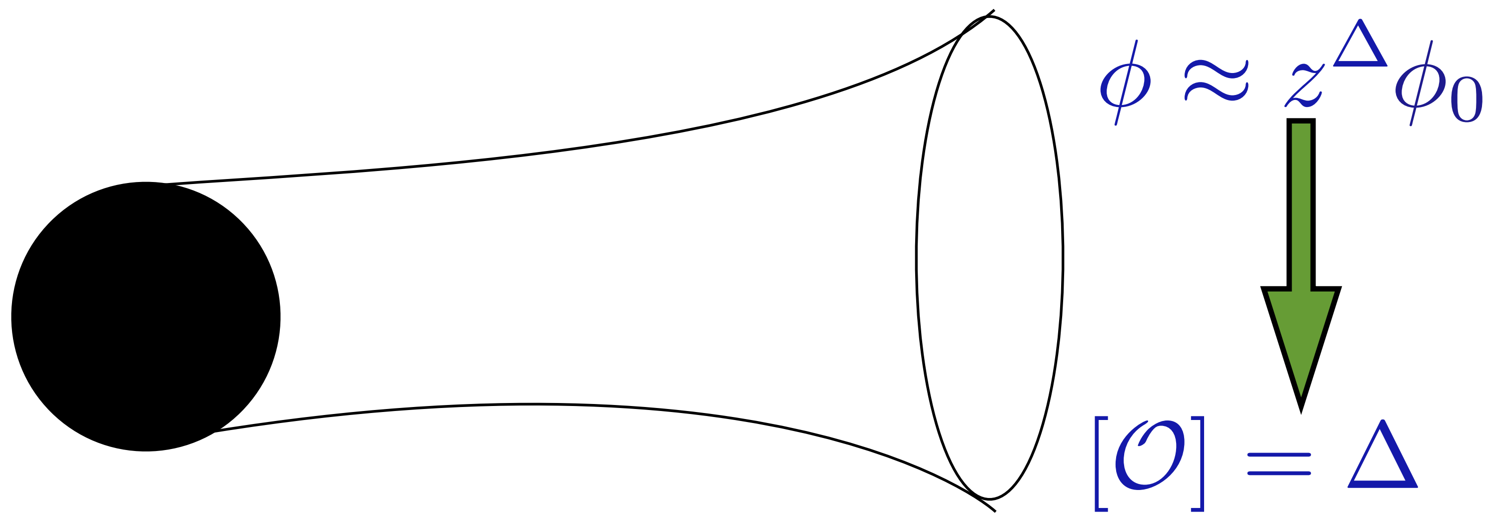
$$(\partial_\mu \phi)^2 + m^2\phi^2$$



$$\int d^4x \phi_0 \mathcal{O}$$

AdS=CFT claim: $\langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0)$

can $\mathcal{O}$ be determined exactly in some cases?



$$\mathcal{O} = C_{\mathcal{O}} \lim_{z\to 0} z^{-\Delta} \boxed{\phi(x,z)}$$

Polchinski: 1010.6134

Local bulk operators in AdS/CFT:
a boundary view of horizons and locality

Alex Hamilton[1], Daniel Kabat[1], Gilad Lifschytz[2] and David A. Lowe[3]

$$\phi_0(x) \leftrightarrow \mathcal{O}(x)\ .$$

This implies a correspondence between local fields in the bulk and *non-local* operators in the CFT.

$$\phi(z,x) \leftrightarrow \int dx'\, K(x'|z,x)\mathcal{O}(x')\ .$$

smearing function

construct $\mathcal{O}$

exactly

consistent with
Polchinski prescription

redo Witten's massive scalar field calculation explicitly

$$S_\phi = \frac{1}{2}\int \underbrace{d^{d+1}u\sqrt{g}}_{dV_g}\left(|\nabla\phi|^2 + m^2\phi^2\right)$$

to establish correspondence

$$\langle e^{\int_{S^d}\phi_0\mathcal{O}}\rangle_{\text{CFT}} = Z_S(\phi_0)$$

$$\downarrow$$

$$(-\nabla)^\gamma\phi_0$$ Reisz fractional Laplacian

$$S_\phi = \frac{1}{2}\int d^{d+1}u\sqrt{g}\left(|\nabla\phi|^2 + m^2\phi^2\right)$$

integrate by parts

$$S_\phi = \frac{1}{2}\int dV_g\left(-\phi\partial_\mu^2\phi + m^2\phi^2 + \phi\partial_\mu\phi\right)$$

equations of motion

$$-\Delta\phi - s(d-s)\phi = 0 \qquad -\Delta\phi = \nabla_i\nabla^i\phi$$

$$m^2 = -s(d-s) \qquad s = \frac{d}{2} + \frac{1}{2}\sqrt{d^2 + 4m^2}$$

bound

$$m^2 \geq -d^2/4$$

BF bound

solutions

$$\phi = F z^{d-s} + G z^{s}, \quad F, G \in \mathcal{C}^{\infty}(\mathbb{H}),$$

$$F = \phi_0 + O(z^2), \quad G = g_0 + O(z^2)$$

restriction

$$\phi_0 = \lim_{z \to 0} \phi$$

boundary of AdS_{d+1}

$$S_\phi = \frac{1}{2} \int dV_g \left(-\phi \partial_\mu^2 \phi + m^2 \phi^2 + \phi \partial_\mu \phi \right)$$

↓

$$\int_{z>\epsilon} dV_g \phi \partial_\mu \phi$$

restriction

$$\text{pf} \int_{z>\epsilon} \left(|\partial\phi|^2 - s(d-s)\phi^2\right) dV_g = -d \int_{z=0} \phi_0\, g_0$$

finite part from integration by parts

use Caffarelli-Silvestre extension theorem (2006)

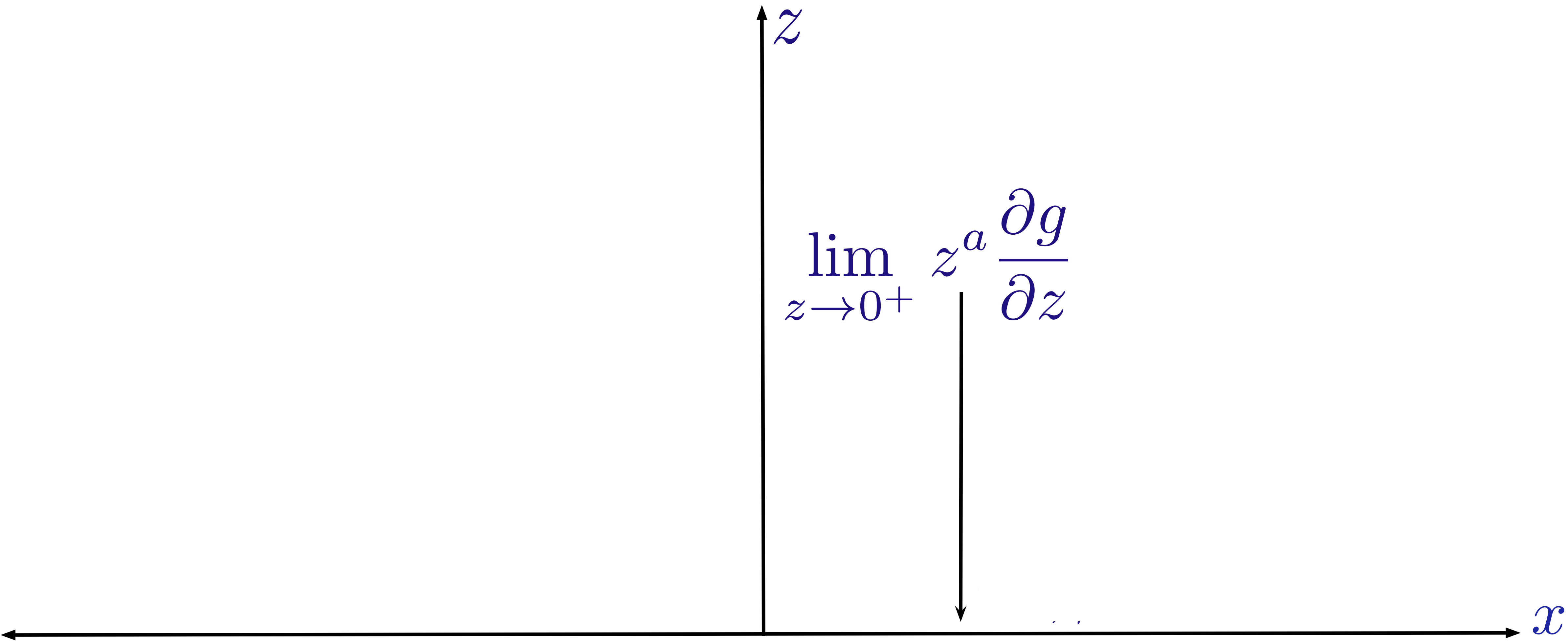
$$g(x,0) = f(x)$$

$$\Delta_x g + \frac{a}{z}\partial_z g + \partial_z^2 g = 0$$

$$\lim_{z\to 0^+} z^a \frac{\partial g}{\partial z} = C_{d,\gamma}\, (-\nabla)^\gamma f$$

$$\gamma = \frac{1-a}{2}$$

non-local



$$g(z = 0, x) = f(x)$$

$$\gamma = \frac{1 - a}{2}$$

$$\phi$$

solves massive scalar problem

$$g = z^{\gamma - d/2}\phi$$

solves CS extension problem

$$\gamma := \frac{\sqrt{d^2 + 4m^2}}{2}$$

$$\mathcal{O} = (-\Delta)^{\gamma}\phi_0$$

the $\mathcal{O}$ for massive scalar field consistency with Polchinski

$$\mathcal{O} = C_{\mathcal{O}} \lim_{z\to 0} z^{-\Delta}\phi(x,z)$$
$$= C_{\mathcal{O}} \lim_{z\to 0} z^{-\Delta+1}\partial_z\phi(x,z)$$

use Caffarelli/ Silvestre

$$\lim_{z\to 0^+} z^a \frac{\partial g}{\partial z} = C_{d,\gamma}\,(-\nabla)^\gamma f$$
$$\gamma = \frac{1-a}{2}$$

$$\mathcal{O} = (-\Delta)^{\gamma}\phi_0 \longrightarrow |x - x'|^{-d-2\gamma}$$

2-point correlator

$$\langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0)$$

AdS-CFT correspondence but operators are non-local !!

$$(-\nabla)^{\gamma}$$

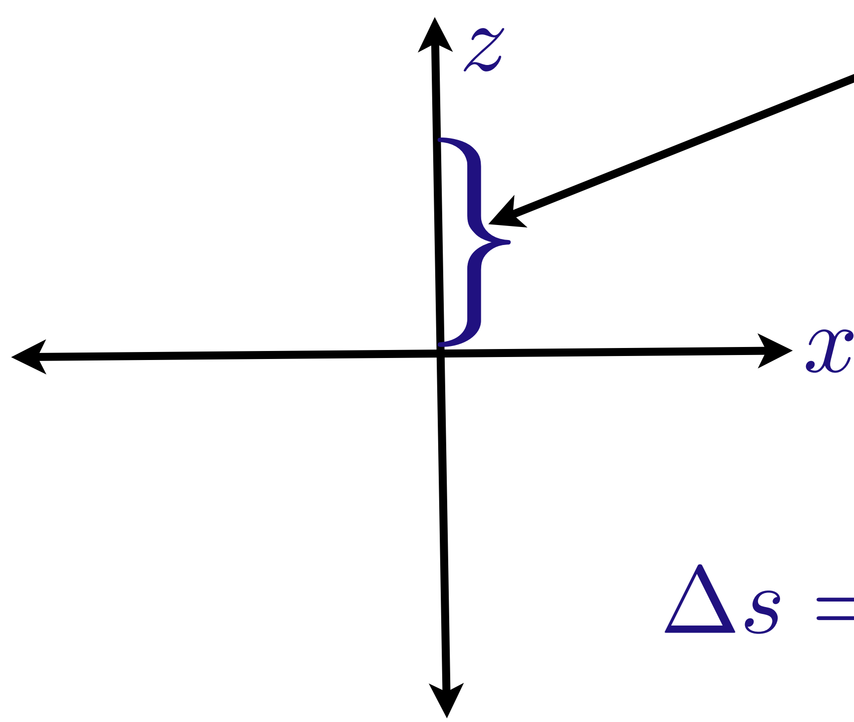
is not conformal if spacetime is curved

Why should the boundary be conformal?

AdS metric: Euclidean signature

$$ds^2 = \frac{dz^2 + \sum_i dx_i^2}{z^2}$$

what is the length of this segment?



$$\Delta s = \int_0^{z_0} \frac{dz}{z} = \ln(z_0/0) = \infty$$

metric at boundary is not well defined

$$z^2 ds^2 = dz^2 + \sum_i dx_i^2$$

solves problem

$$ds^2 \to e^{2w} ds^2$$

works for any real w

↓

boundary can only be specified conformally bulk conformality

$$S = S_{\rm gr}[g] + S_{\rm matter}(\phi)$$

$$S_{\rm matter} = \int_M d^{d+1}x\sqrt{g}\mathcal{L}_m$$

conformal sector

$$\mathcal{L}_m := |\partial\phi|^2 + \left(\underbrace{m^2 + \frac{d-1}{4d}}R(g)\right)\phi^2$$

`conformal mass'

scalar curvature

on Riemannian (M,g) manifold of dimension N=d+1

conformal Laplacian

$$L_g = -\Delta_g + \frac{N-2}{4(N-1)} R_g = -\Delta_g + \frac{d-1}{4d} R_g$$

conformal change

$$A_w(\varphi) = e^{-bw} A(e^{aw} \varphi)$$

$$\hat{g} = y^2 g$$

$$L_g(\varphi) = y^{\frac{d+3}{2}} L_{\hat{g}} \left(y^{-\frac{d-1}{2}} \varphi \right)$$

hyperbolic metric

$$L_g = -\Delta_g + \frac{N-2}{4(N-1)} R_g = -\Delta_g + \frac{d-1}{4d} R_g$$

$$R_{g_{\mathbb{H}}} = -d(d+1)$$

$$L_{g_{\mathbb{H}}} = -\Delta_{g_{\mathbb{H}}} - \frac{d^2-1}{4}$$

$$m^2 - \frac{d^2-1}{4} = -s(d-s)$$

$$s = \frac{d}{2} + \frac{\sqrt{4m^2+1}}{2} \quad \Longrightarrow \quad m^2 > -1/4$$

stability independent of dimensionality

Chang/Gonzalez
1003.0398

$$P_\gamma \in (-\Delta_{\hat{g}})^\gamma + \Psi_{\gamma-1}$$

Panietz operator

pseudo-differential operator

in general

$$P_k = (-\Delta)^k + \text{lower order terms}$$

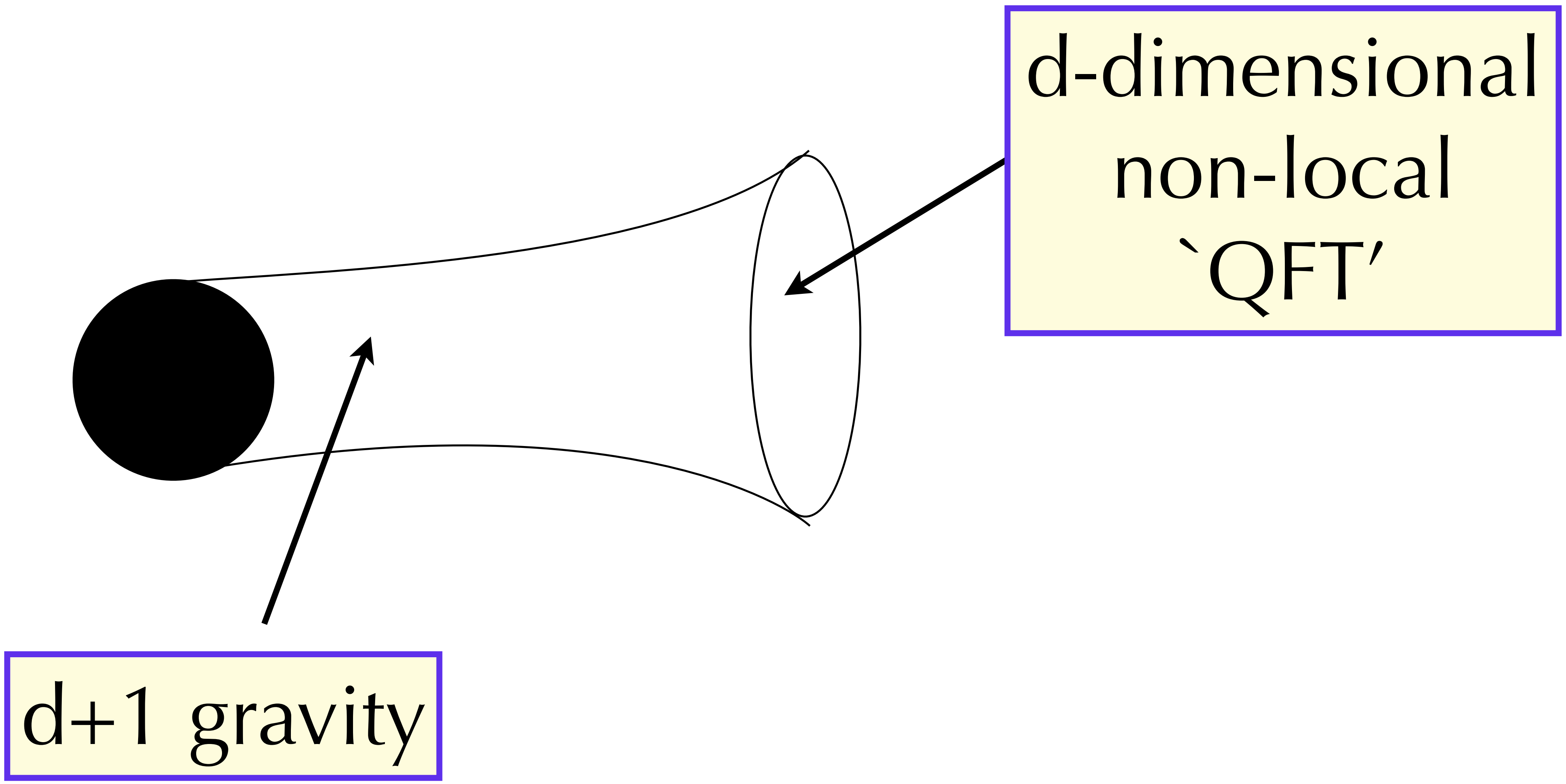
$$P_1 = -\Delta + \frac{d-1}{4(d-1)} R_g$$

scattering problem

$$P_\gamma f = d_\gamma S\left(\tfrac{d}{2}+\gamma\right) = d_\gamma \; h$$

$$\mathrm{pf} \int_{y>\epsilon} [|\partial\phi|^2 - \left(s(d-s) + \frac{d-1}{4d} R(g)\right)\phi^2] dV_g = -d \int_{\partial X} dV_h f \, P_\gamma[g^+, \hat{g}] f$$

fractional conformal Laplacian


d-dimensional
non-local
`QFT'
d+1 gravity

What about Maldacena conjecture?

Type IIB String `action'

$$S = \int d^{10}x\sqrt{-g}\left(e^{-2\phi}(R+4|\nabla\phi|^2) - \frac{2e^{2\alpha\phi}}{(D-2)}F^2\right)$$

$D = 7$ — extremal solution

$$ds_L^2 = H^{-1/2}(r)\,\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)\,\delta_{mn}dx^m dx^n$$

horizon at r=0

D3-branes

AdS

$$H = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g N\alpha'^2, \; r^2 = \delta_{mn}x^m x^n$$

more generally

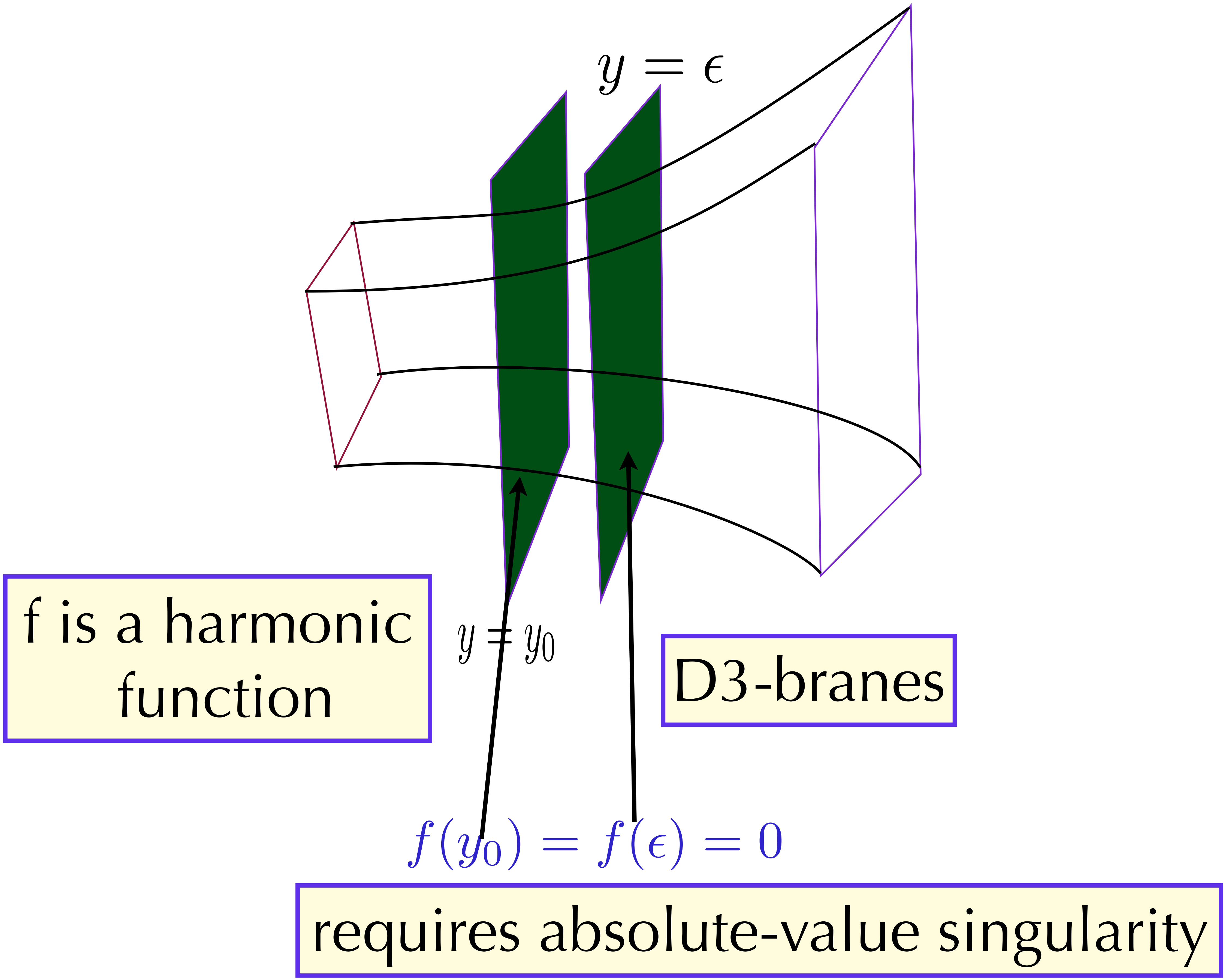
$$\mathbb{R}^{3,1} \times K_6$$

$$ds^2 = f^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + f^{1/2}\delta_{mn}dx^m dx^n$$

$$\Delta f = (2\pi)^4 \alpha'^2 g \, \rho$$

$N\delta(r)$

density of D3-branes



$y = \epsilon$
f is a harmonic function
$y = y_0$
D3-branes
$f(y_0) = f(\epsilon) = 0$
requires absolute-value singularity

|y| singular metrics (GI)

Randall-Sundrum

$$ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

$$y \in [-\pi R, \pi R]$$

massive-particle action at Brane at πR

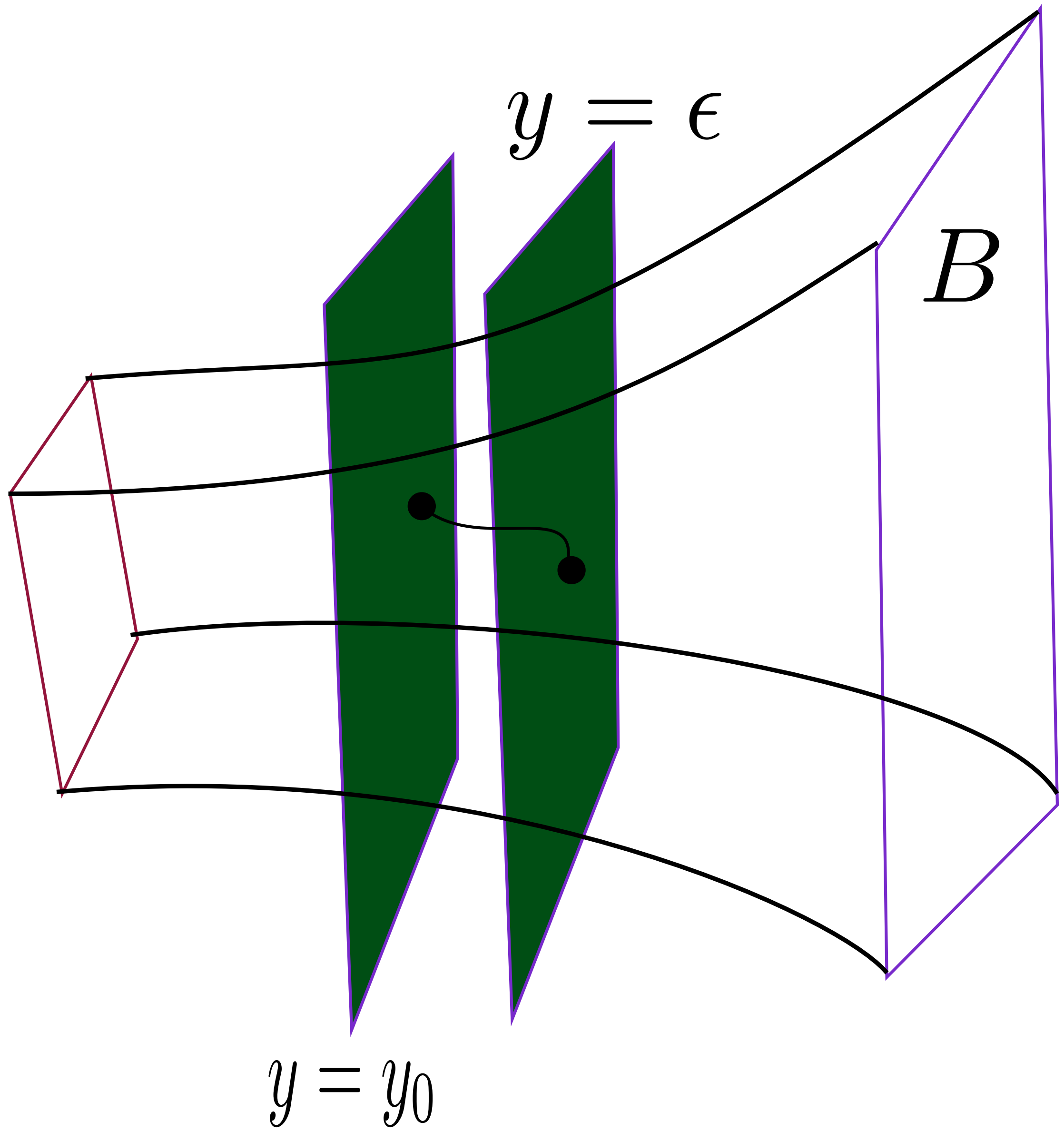
$$\int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} + m^2 e^{-2\pi R/L} \hat{\phi}^2 \right),$$

$$\hat{\phi} = e^{-\pi R/L} \phi$$

$$\lim_{R/L\to\infty} m^2 e^{-2\pi R/L} \to 0$$

$$\gamma = \frac{1}{2}$$

non-locality vanishes



$$m^2 = -\frac{1}{\alpha'} + (\ln \epsilon)^2/(2\pi\alpha')^2$$

positive mass

$$|\ln \epsilon| > 2\pi\sqrt{\alpha'}$$

non-locality vanishes

Branes in Type IIB
string theory
eliminate non-local boundary
interactions

geodesic incompleteness

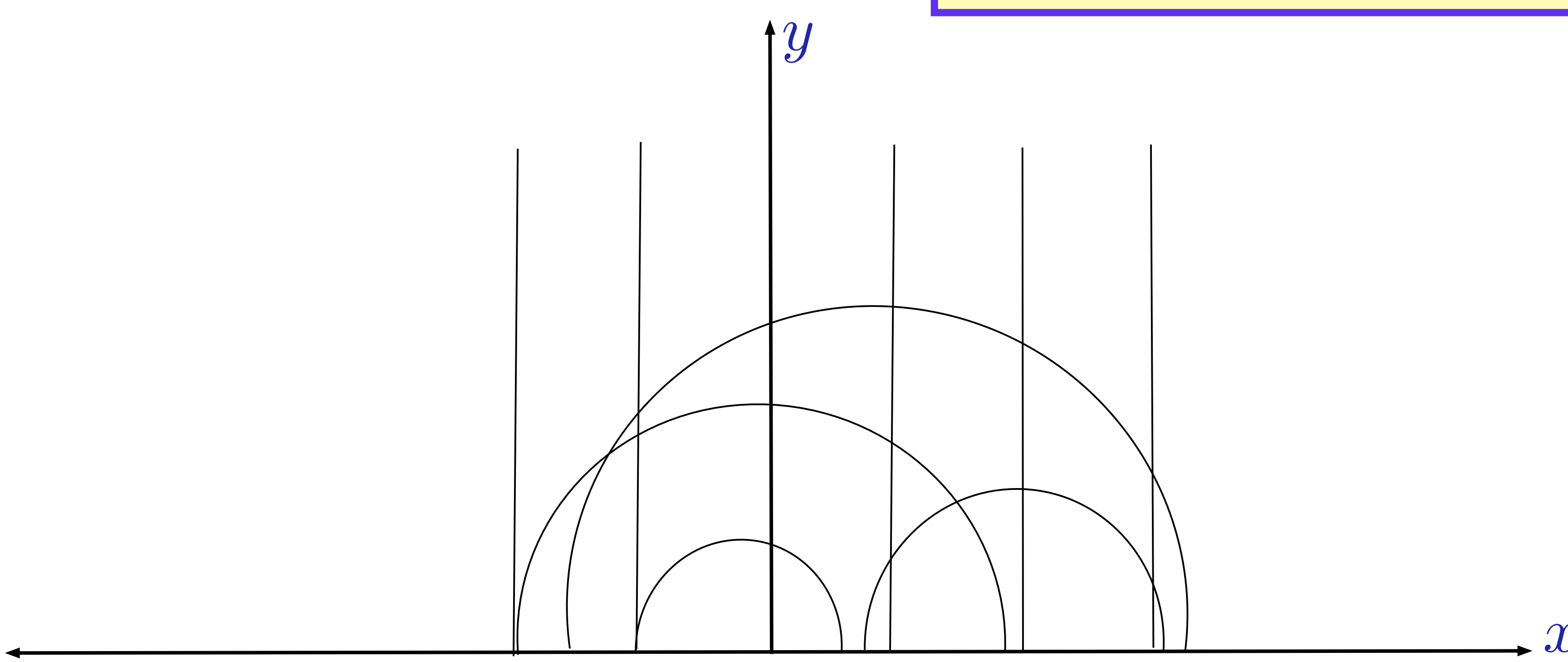
$$ds^2_{\mathbb{H}} = \frac{1}{y^2}(dx^2 + dy^2) \qquad \mathbb{H}^2$$

non-zero Christoffel symbols

$$\begin{cases} \Gamma^x_{xy} = \Gamma^x_{yx} = -\dfrac{1}{y} \\ \Gamma^y_{xx} = -\Gamma^y_{yy} = \dfrac{1}{y} \end{cases}$$

geodesics

cover all spacetime



what if?

$$ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

singularity

$\Gamma^\rho_{\mu\nu}$ ill-defined (GI)

$+$ non-compactness

boundary locality

physical consequences of anomalous dimension for $\mathcal{A}_\mu$

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu^{\alpha}\mathcal{G}$$

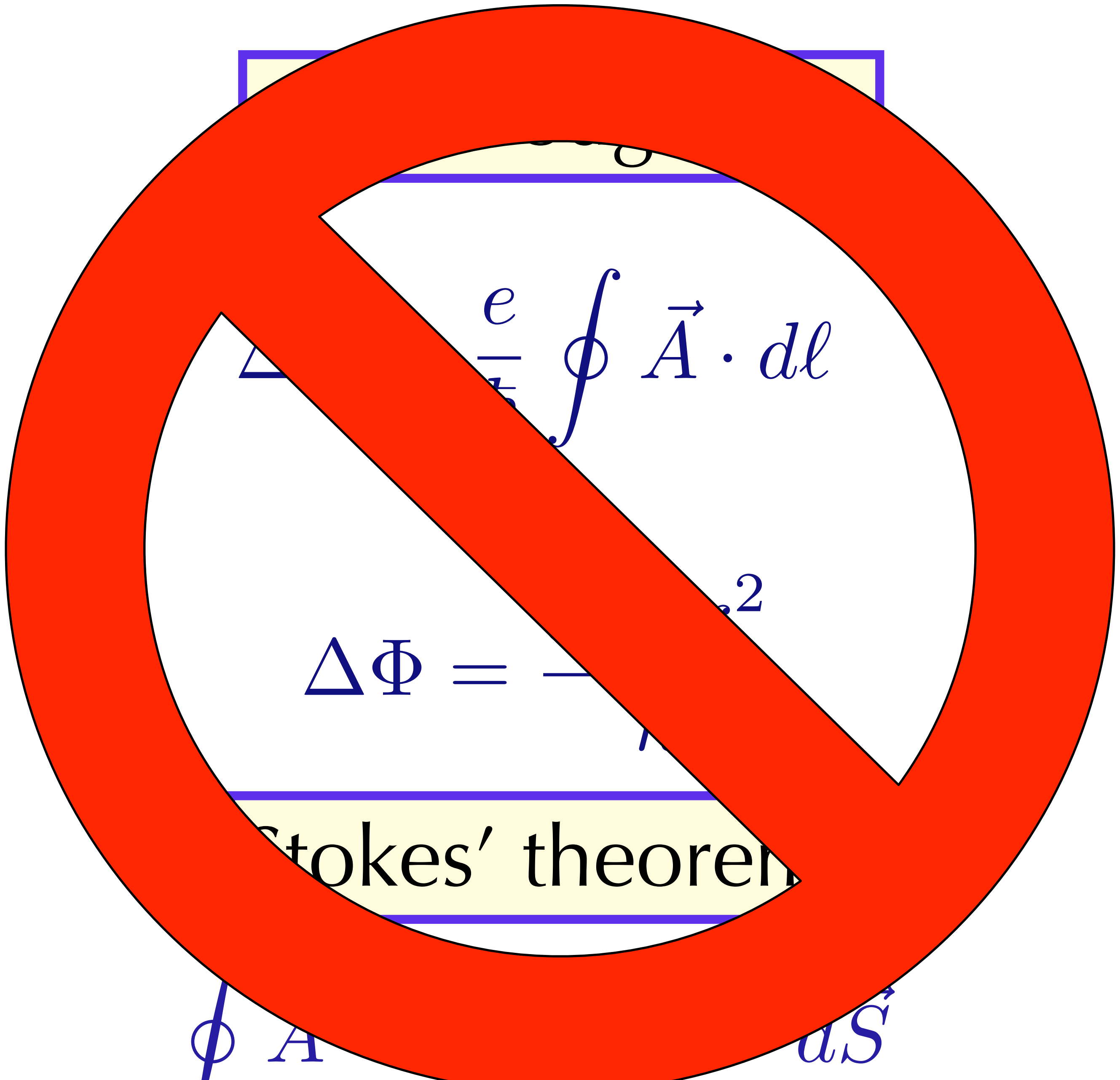
$${}_{\alpha}F_{\mu\nu} = \partial_\mu^{\alpha_\mu}{}_{\alpha_\nu}A_\nu - \partial_\nu^{\alpha_\nu}{}_{\alpha_\mu}A_\mu$$

$$\vec{\nabla}^{\alpha} \times \vec{A} = \vec{B}$$

no Stokes' theorem

$$\oint \vec{A}\cdot d\ell \neq \int_S B \cdot d\vec{S}$$

Aharonov-Bohm Effect must change



$$\Delta \ldots \frac{e}{\hbar} \oint \vec{A} \cdot d\ell$$

$$\Delta\Phi = - \ldots$$

Stokes′ theore…

$$\oint A \ldots \int_S \ldots d\vec{S}$$

physical gauge connection

$$a_i \equiv [\partial_i, I_i^\alpha A_i] = \partial_i I_i^\alpha A_i$$

$$A_\mu \to A_\mu + \partial_\mu^\alpha \Lambda$$

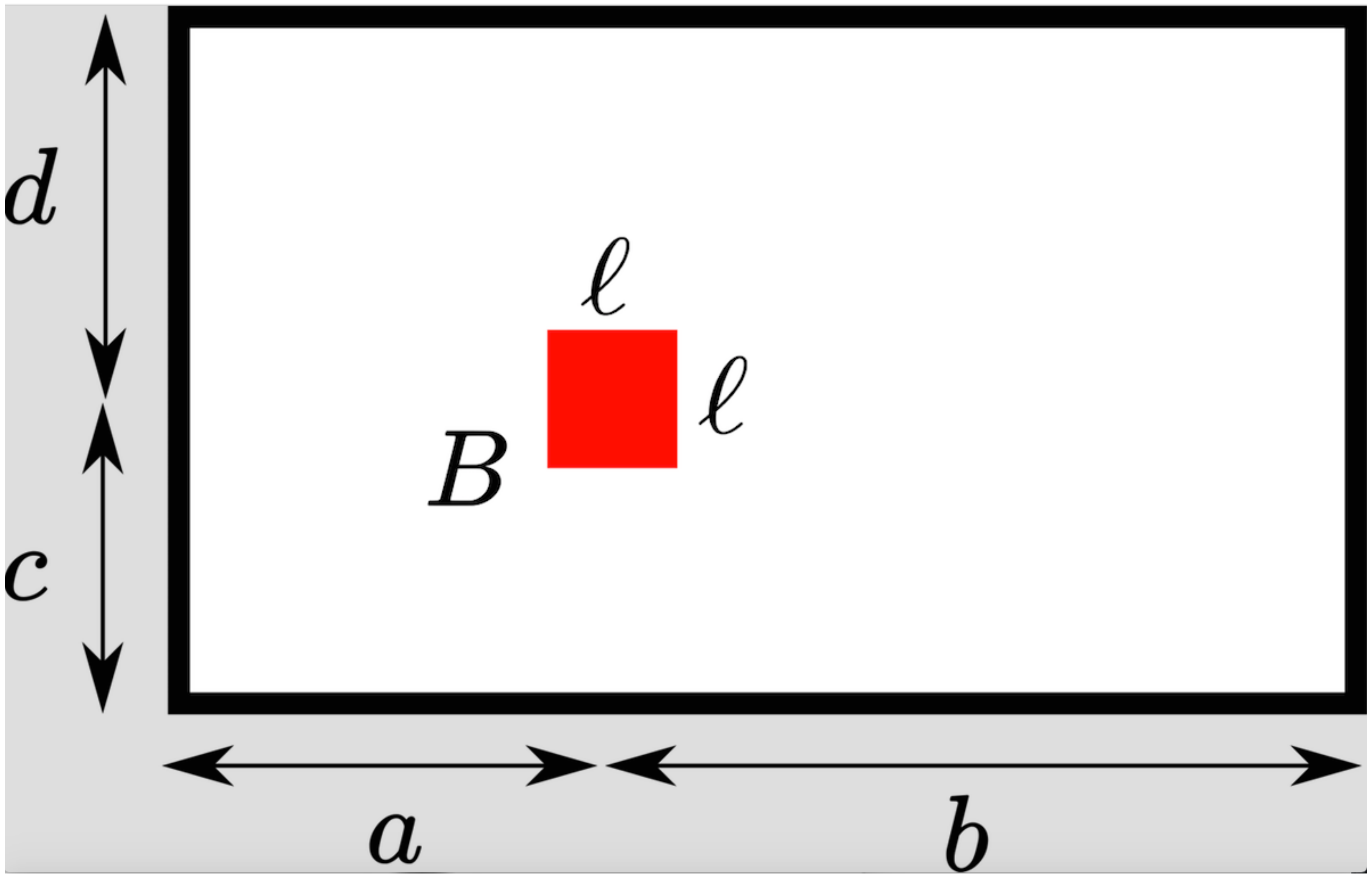
$$a_\mu \to a_\mu + \partial_\mu \Lambda$$

$$-\frac{\hbar^2}{2m}(\partial_i - i\frac{e}{\hbar}a_i)^2\psi = i\hbar\partial_t\psi.$$

compute AB phase

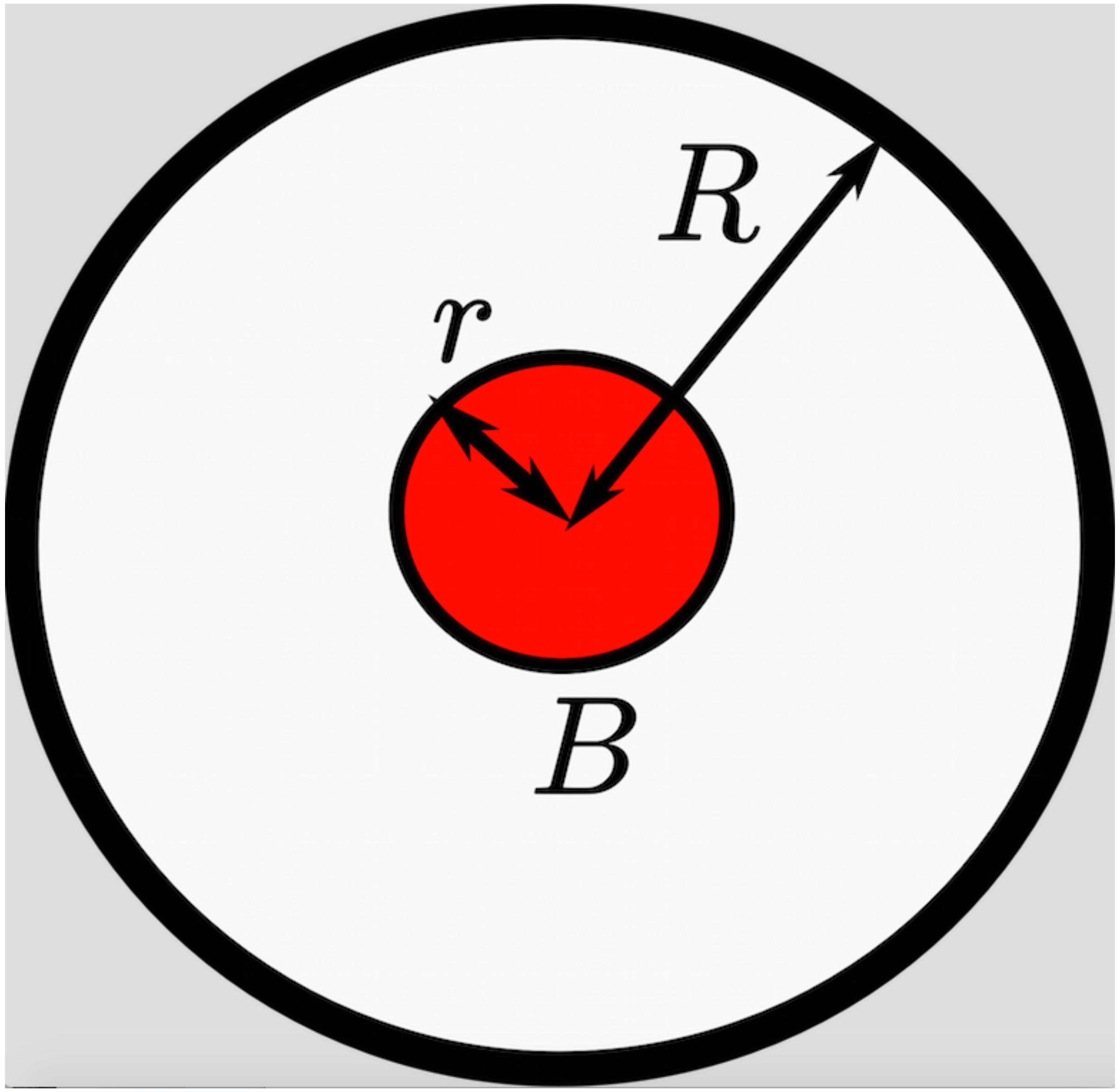
compute AB phase

$$\Delta\Phi = \frac{e}{\hbar}\oint \vec{a}(r')\cdot\vec{d}\ell'$$



use fractional calculus

$$\Delta\phi_{\rm R} = \frac{eB\ell^2}{\hbar}\left(\frac{b^{\alpha-1}d^{\alpha-1}}{\Gamma^2(\alpha)}\right) c \gg l, d \gg l$$



$$\Delta\phi_{\rm D} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)$$

is the correction large?

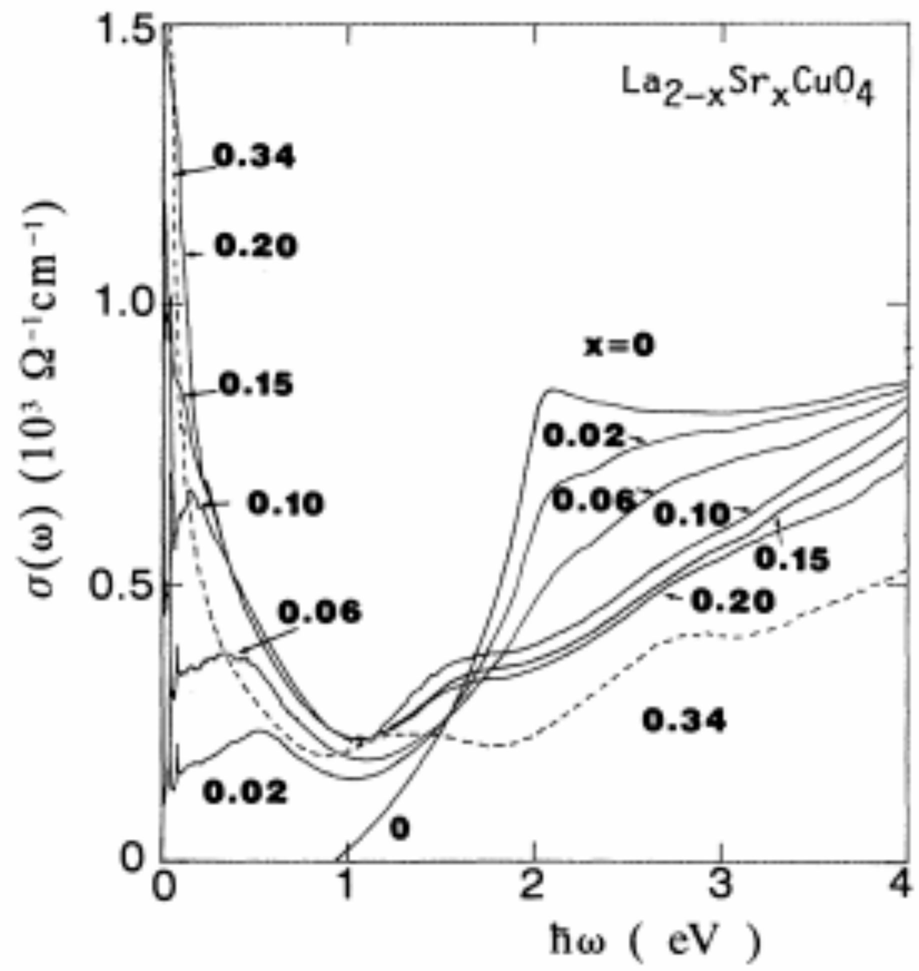
$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

sum rules

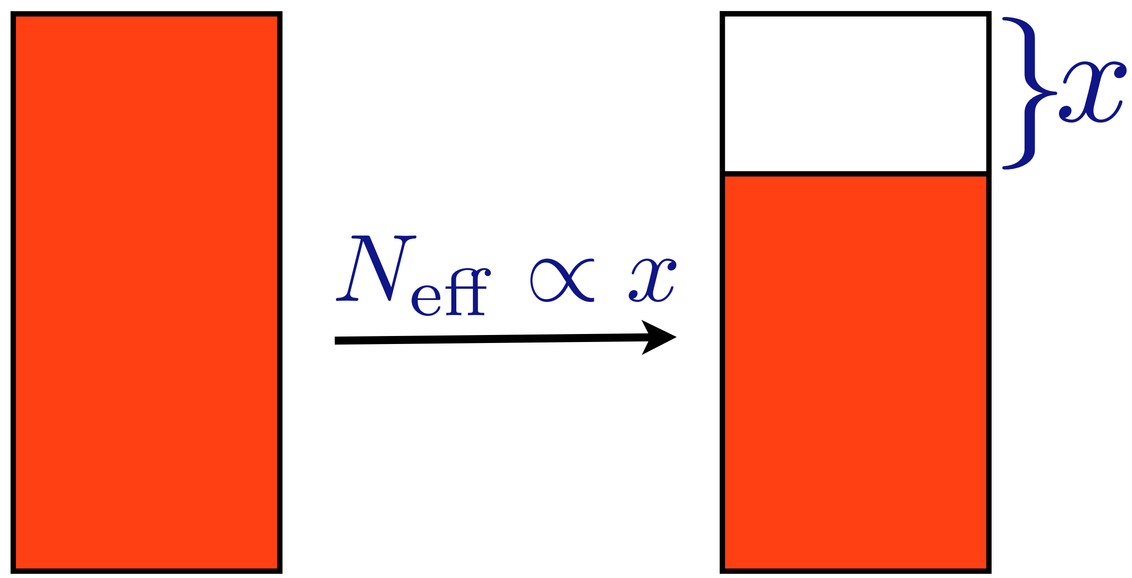
sum rules



$La_{2-x}Sr_xCuO_4$
0.34
0.20
0.15
0.10
0.06
0.02
x=0
0.02
0.06
0.10
0.15
0.20
0.34
0
$\sigma(\omega)$ (10^3 $\Omega^{-1}cm^{-1}$)
$\hbar\omega$ (eV)
1.5
1.0
0.5
0
0
1
2
3
4

optical gap

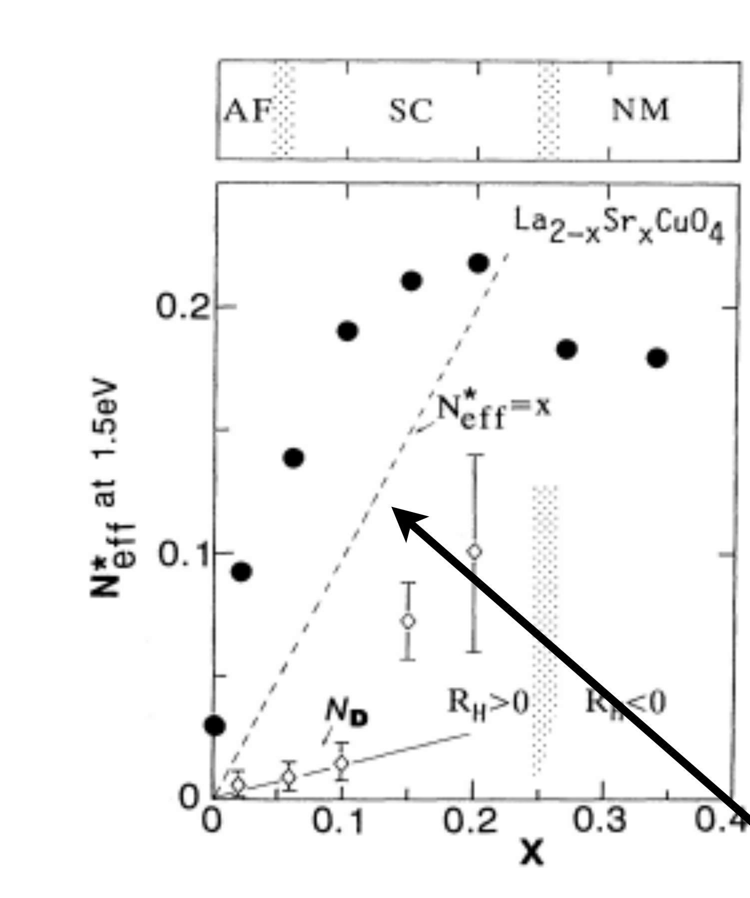
$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2}\int_0^{\Omega}\sigma(\omega)d\omega$$

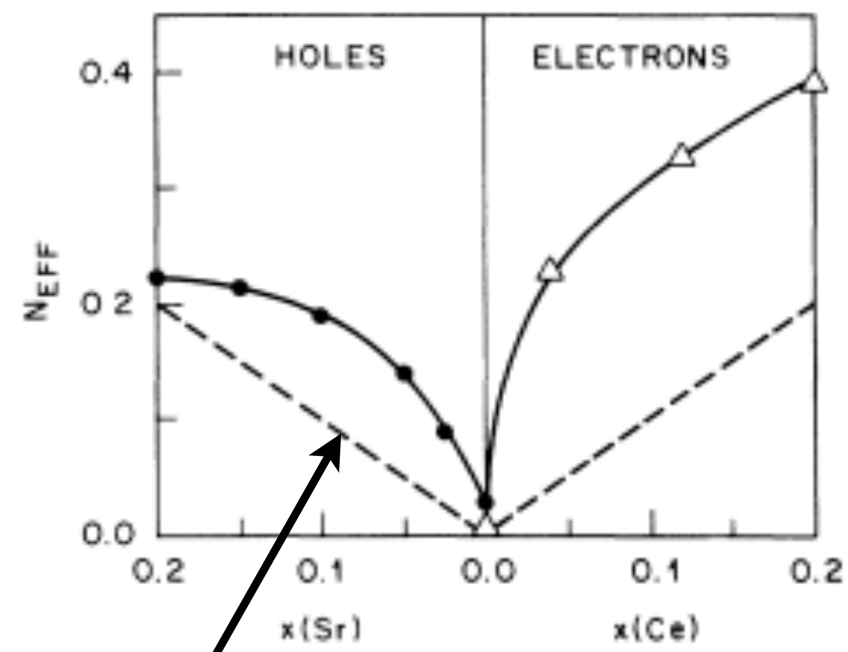


$$N_{\text{eff}}(\Omega) = \frac{2mV_{\text{cell}}}{\pi e^2} \int_0^{\Omega} \sigma(\omega) d\omega$$

Uchida, et al.

Cooper, et al.





x

low-energy model for $N_{\text{eff}} > x$??

f-sum rule

$\text{K.E.} = p^2/2m$

$N_{\text{eff}} = x$

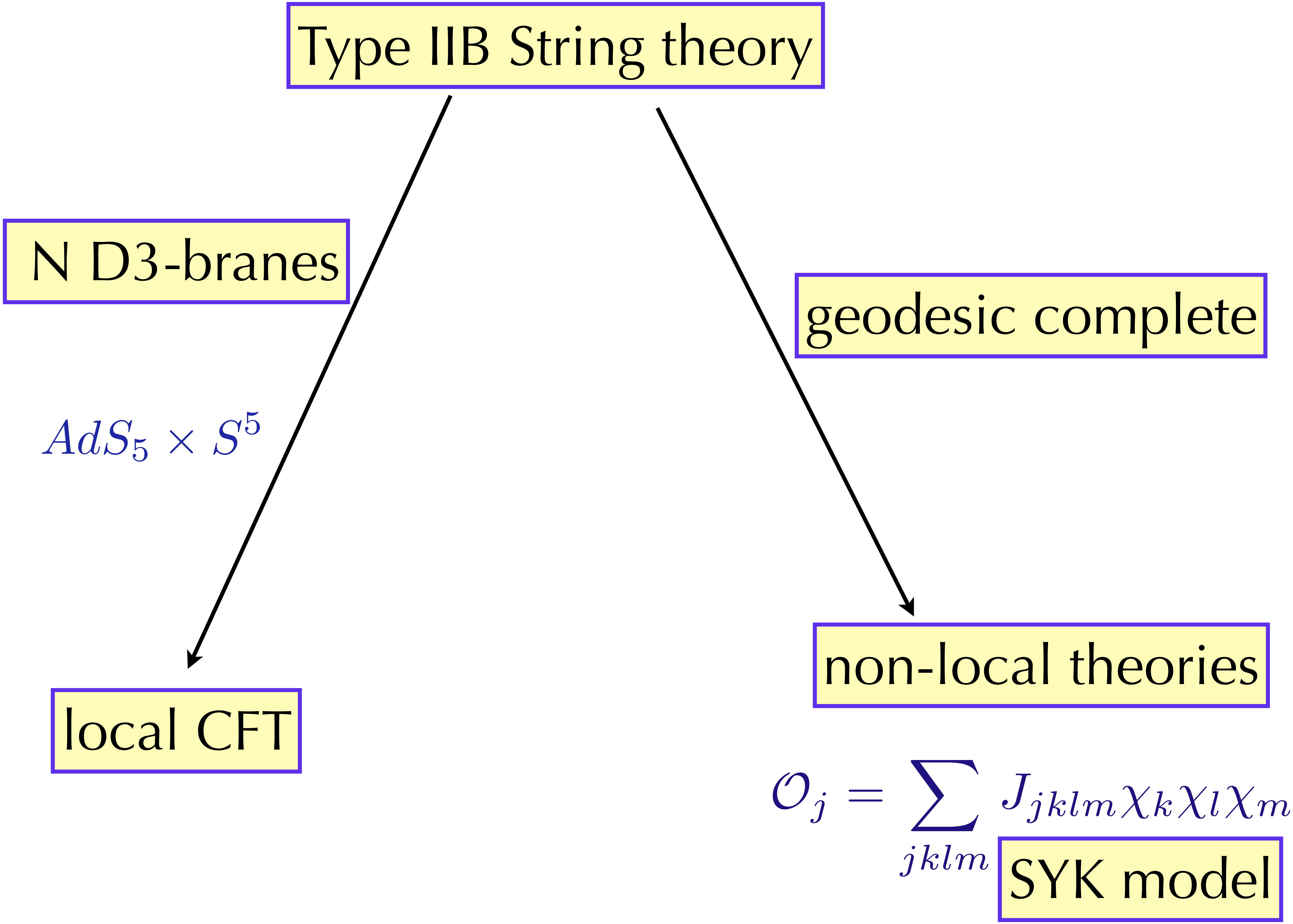
what if?

$$\text{K.E.} \propto (\partial_\mu^2)^\alpha$$

f-sum rule

$$\frac{W(n,T)}{\pi c e^2} = A n^{1+\frac{2(\alpha-1)}{d}} + \cdots$$

W>n if $\alpha < 1$


Type IIB String theory
N D3-branes
$AdS_5 \times S^5$
local CFT
geodesic complete
non-local theories
$\mathcal{O}_j = \sum_{jklm} J_{jklm} \chi_k \chi_l \chi_m$
SYK model

combine AC
+DC transport

fixes all exponents
a,b,c

boundary
non-local action

probe with fractional
Aharonov-Bohm effect

$$[J] = d_U$$