Effective field theory, holography, and non-equilibrium physics

Hong Liu

Equilibrium systems

Microscopic description

Λ

low energy effective field theory:

Express free energy in terms of appropriate low energy d.o.f., constrained only by symmetries

Renormalization group, universality

Macroscopic phenomena

Direct computation: almost always impossible

For equilibrium systems: Ginsburg-Landau-Wilson paradigm

Could in principle be generalized to non-equilibrium systems, but ....

Non-equilibrium systems

Microscopic description

Macroscopic phenomena

Many theoretical approaches,

"Too" microscopic:
like Liouville equation, BGKKY hierarchy, Boltzmann equation ......

(too many d.o.f or too formal)

"Too" phenomenological:
hydrodynamics, stochastic systems ...

In this talk: two new approaches

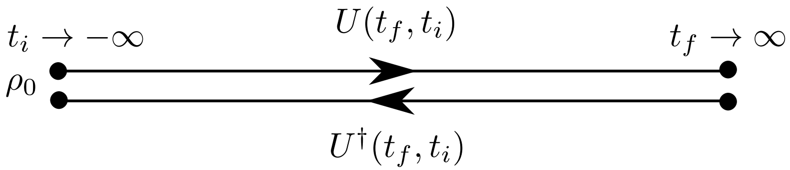
1. non-equilibrium effective field theory
2. Holographic duality

First principle, with small number of d.o.f.

Non-equilibrium effective field theory

Path integral in a general state

Non-equilibrium problems: interested in describing nonlinear dynamics around a state.



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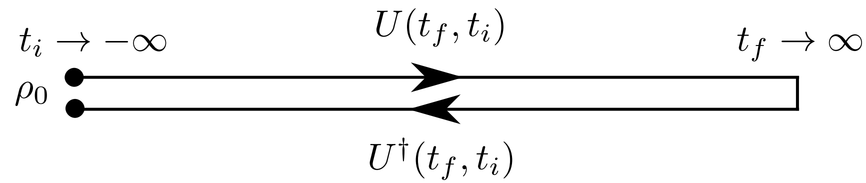


$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i)$$

$$\mathrm{Tr}\left(\rho_0 \cdots\right)$$

Closed time path (CTP) or Schwinger-Keldysh contour

Should be contrasted with path integral for transition amplitudes,

$$\langle f| \cdots |i\rangle$$

$t = -\infty$ → $t = +\infty$

Non-equilibrium effective field theory

Microscopic Schwinger-Keldysh path integral: (double # of d.o.f.)

$$\mathrm{Tr}\,(\rho_0 \cdots) = \int_{\rho_0} D\psi_1 D\psi_2\, e^{iS[\psi_1]-iS[\psi_2]} \ldots$$

not manageable beyond perturbation theory

Integrate out all "massive" modes: Low energy gapless modes (two sets)

$$\mathrm{Tr}\,(\rho_0 \cdots) = \int D\chi_1 D\chi_2\, e^{iS_{\mathrm{EFT}}[\chi_1,\chi_2;\rho_0]} \ldots$$

1. What are χ ?
2. What are the symmetries of S_{EFT} ?

Depend on class of physical systems and ρ_0

Non-equilibrium EFT

Microscopic description

Λ

low energy effective field theory:

$$\int D\chi_1 D\chi_2 \, e^{iS_{\mathrm{EFT}}[\chi_1,\chi_2;\rho_0]} \dots$$

Renormalization group, universality

Macroscopic phenomena

Not much explored

Example: effective field theory for general dissipative fluids.

Effective field theory for dissipative fluids

Paolo Glorioso

Michael Crossley

arXiv: 1511.03646

See also Haehl, Loganayagam, Rangamani
arXiv: 1510.02494, 1511.07809

Fluid dynamics

Consider a long wavelength disturbance of a system in thermal equilibrium

non-conserved quantities: relax locally, $\tau_{\rm relax} \sim \tau_{\rm mfp}$

conserved quantities: cannot relax locally, only via transports

$$\lambda \to \infty, \quad \Rightarrow \quad \tau_{\rm relax} \to \infty \quad \Rightarrow$$

Gapless and long-lived modes (universal)

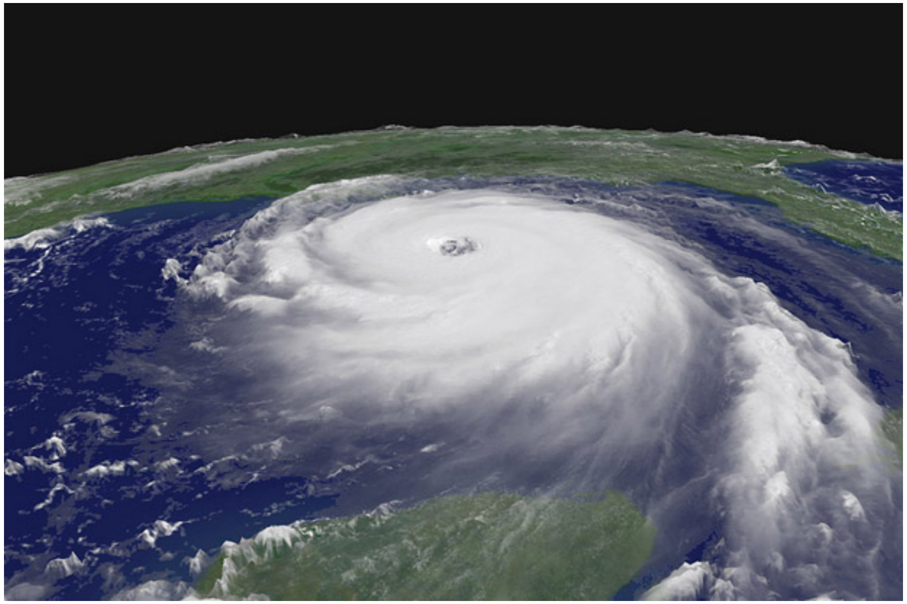
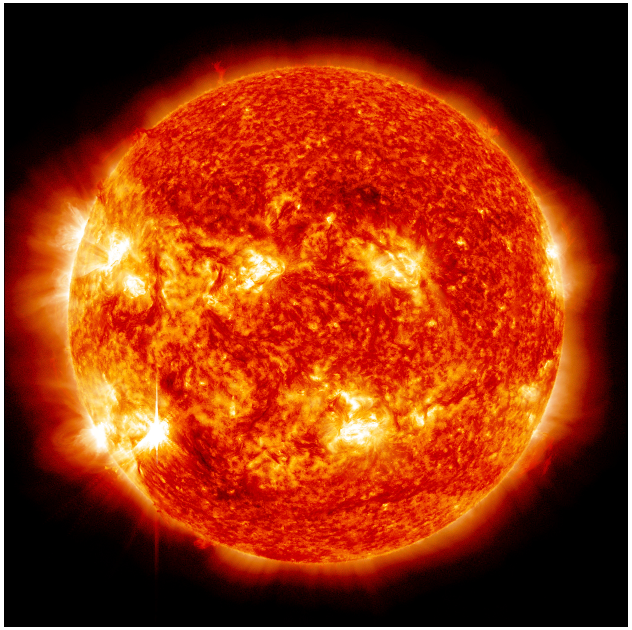
Phenomenological description:

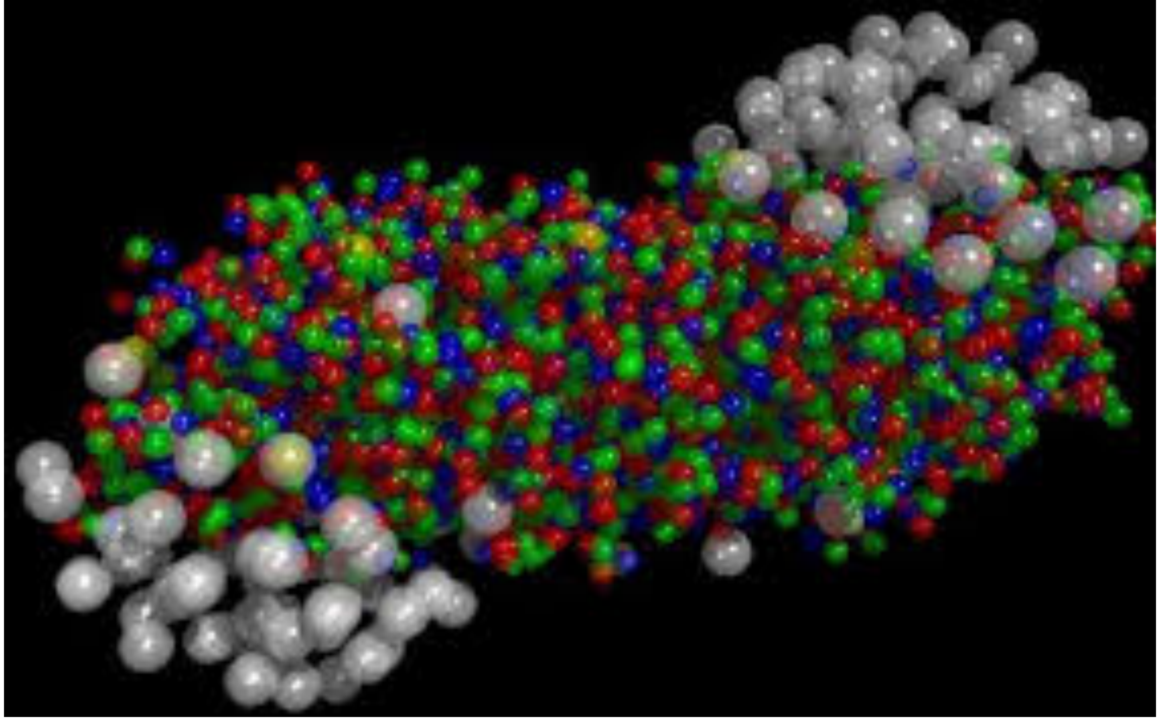
Hydrodynamics $\partial_\mu \langle T^{\mu\nu} \rangle = 0, \qquad \partial_\mu \langle J^\mu \rangle = 0$

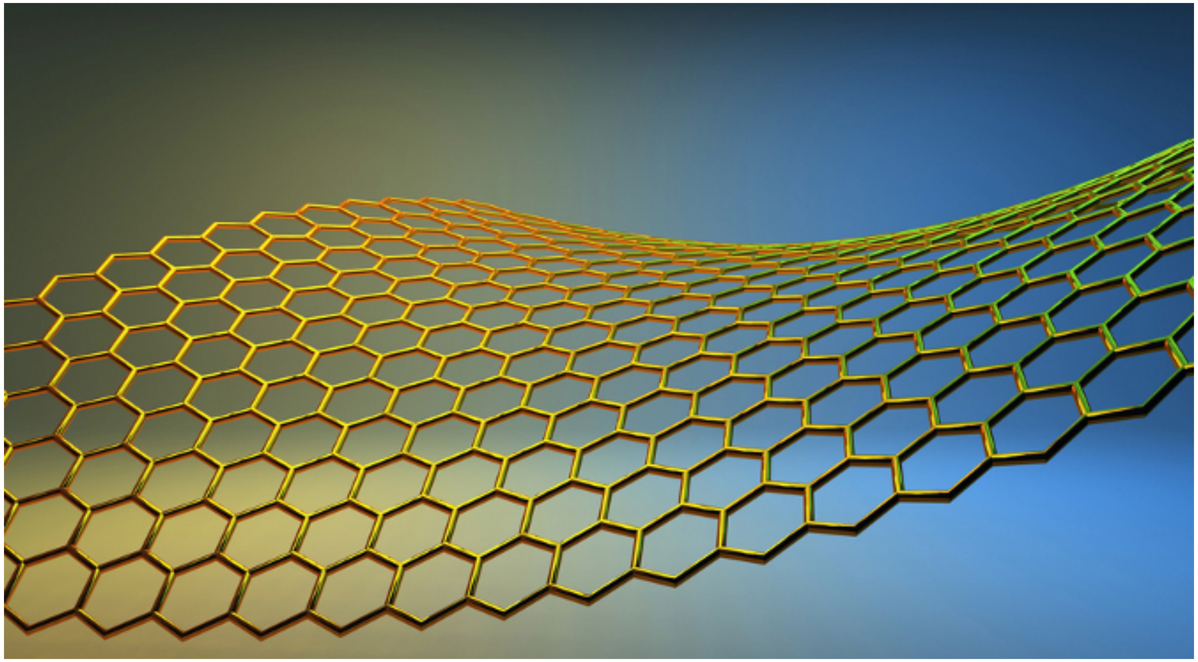
dynamical variables: (local equilibrium)

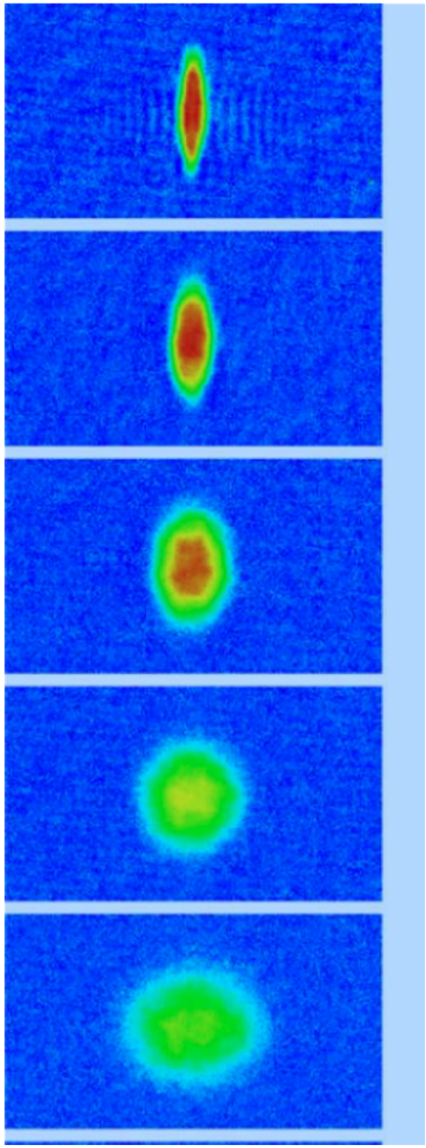
$$\beta(t, \vec{x}),\ u^\mu(t, \vec{x}),\ \mu(t, \vec{x})$$

slowly varying functions of spacetime





O'Hara et al (2002)

Despite the long and glorious history of hydrodynamics

It is like a mean field theory, does not capture fluctuations.

There are always statistical fluctuations .....

Important in many contexts:

Long time tail, transports,

dynamical aspects of phase transitions,

non-equilibrium states,

turbulence,

finite size systems ....

At low temperatures, quantum fluctuations can also be important.

Phenomenological level: stochastic hydro (Landau, Lifshitz)

$$\partial_\mu \langle T^{\mu\nu} \rangle = \xi^\nu, \quad \partial_\mu \langle J^\mu \rangle = \zeta$$

ξ^μ, ζ : noises with local Gaussian distribution, fluctuation-dissipation relations

Good for near-equilibrium disturbances

Far-from-equilibrium:

1. interactions among noises
2. interactions between dynamical variables and noises
3. fluctuations of dynamical variables themselves

non-equilibrium fluctuation-dissipation relations?

Until now no systematic methods to treat such nonlinear effects.

We will be able to address these issues by developing hydrodynamics as a non-equilibrium effective field theory of a general many-body system at a finite temperature.

Searching for an action principle for dissipative hydrodynamics has been a long standing problem, dating back at least to the ideal fluid action of G. Herglotz in 1911.

Subsequent work include Taub, Salmon; Jackiw et al.; Andersson et al.

The last decade has seen a renewed interest: including, Dubovsky, Gregoire, Nicolis and Rattazzi in hep-th/0512260 and further developed by Dubovsky, Hui, Nicolis and Son, Grozdanov et al, Haehl et al, Kovtun et al ….

Holographic derivation: Nickel, Son; de Boer, Heller, Pinzani-Fokeeva; Crossley, Glorioso., HL, Wang.

Many activities since 70's to understand hydrodynamic fluctuations, long time tails …

Searching for an EFT description should be distinguished from searching an action which just reproduces constitutive relations (which may not capture fluctuations correctly).

Hydro effective field theory

At long distances and large times:

All correlation functions of the stress tensor and conserved currents in thermal density matrix

$$= \int D\chi_1 D\chi_2 \, e^{iS_{\rm hydro}[\chi_1,\chi_2]} \ldots$$

hydrodynamic modes (2 sets)

1. What are χ ? $\beta(t,\vec{x}), u^{\mu}(t,\vec{x}), \mu(t,\vec{x})$ do not work

2. What are the symmetries of $S_{\rm hydro}$?

3. Integration measure?

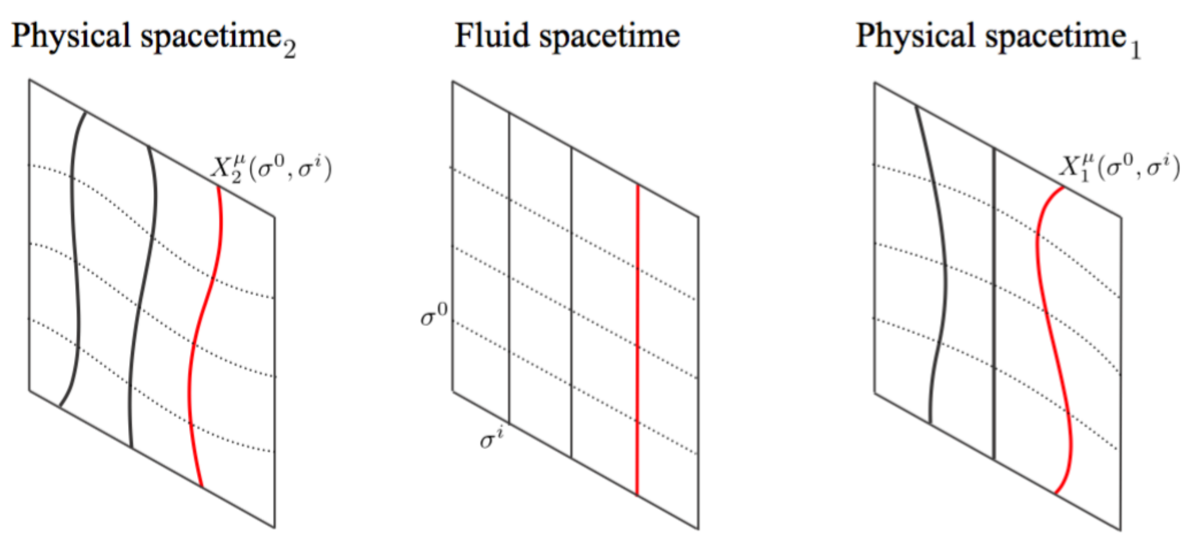
Dynamical variables

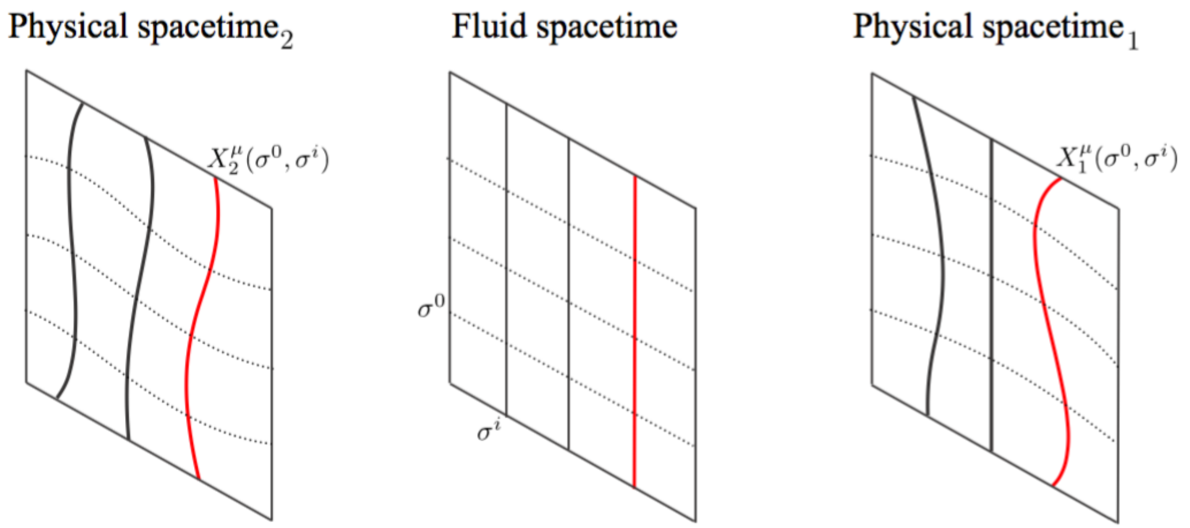
Recall Lagrange description of hydrodynamics:

σ^i : label fluid elements $\quad x^i(\sigma^i, t)$: describe fluid motion

For a general many-body system in a generic density matrix state, we developed an "integrating-in" procedure to show that gapless d.o.f. associated with a conserved stress tensor can be described by:

$$X_1^\mu(\sigma^i, \sigma^0), \quad X_2^\mu(\sigma^i, \sigma^0) \qquad \sigma^0 \text{ : internal time}$$





Standard hydro variables (which are now derived quantities)

$$u^\mu = \frac{1}{b}\frac{\partial X^\mu}{\partial \sigma^0}, \quad X^\mu = \frac{1}{2}(X_1^\mu + X_2^\mu) \quad e^{-\tau} = \frac{T}{T_0},$$

Noise: $X_a^\mu = X_1^\mu - X_2^\mu$

A significant challenge: ensure the eoms from the action of X can be solely expressed in terms of these velocity. (e.g. solids v.s. fluids)

Symmetries

σ^i label individual fluid elements, σ^0 internal time

Require the action to be invariant under:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

define what is a fluid!

It turns out these symmetries indeed do magic for you:

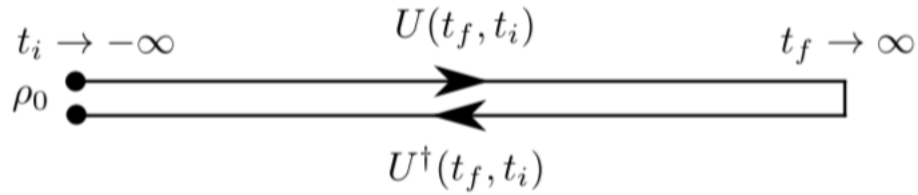
at the level of equations of motion, they ensure all dependence on dynamical variables can be expressed in u^μ and temperature.

→ Recover standard formulation of hydrodynamics

(modulo phenomenological constraints)

Consistency conditions and symmetries

We are considering EFT for a system defined with CTP:



When coupled to external sources:

- Reflectivity condition: $W^*[g_1, A_1; g_2, A_2] = W[g_2, A_2; g_1, A_1]$

  Requires the action to satisfy a Z_2 reflection symmetry

  ⇒ complex action, fluctuations of noises are damped.

- Unitarity condition: $W[g, A; g, A] = 0$

  Introduce fermionic partners each for dynamical variables and require the action to have a BRST type symmetry.

- KMS condition plus PT imply a Z_2 symmetry on W:

$$W[g_1(x), A_1(x); g_2(x), A_2(x)] = W[g_1(-x), A_1(-x); g_2(-t - i\beta_0, -\vec{x}), A_2(-t - i\beta_0, -\vec{x})]$$

Local KMS condition: Z_2 symmetry

⇒ All the constraints from entropy current condition and linear Onsager relations

New constraints on equations of motion from nonlinear Onsager relations.

Non-equilibrium fluctuation-dissipation relations

supersymmetry

Summary

1. Hydrodynamics with classical statistical fluctuations

   is described by a supersymmetric quantum field theory

$$\hbar_{\rm eff} \propto \frac{1}{s} \qquad s : \text{entropy density}$$

2. Hydrodynamics with quantum fluctuations also incorporated

   is described by a “quantum-deformed” (supersymmetric) quantum field theory.

Example: nonlinear stochastic diffusion

Consider the theory for a single conserved current, where the relevant physics is diffusion.

Dynamical variables: $\varphi_{1,2}$ (or φ_a, φ_r)

Roughly, φ_r : standard diffusion mode, φ_a: the noise.

$$\mathcal{L} = iT(\partial_i\varphi_a)^2(\sigma + \sigma_1\partial_0\varphi_r) + \partial_0\varphi_a\partial_0\varphi_r(\chi + \chi_1\partial_0\varphi_r) - \partial_i\varphi_a\partial_0\partial_i\varphi_r(\sigma + \sigma_1\partial_0\varphi_r)$$
$$+c_a(\chi\partial_0 - \sigma\partial_i^2)\partial_0 c_r - \chi_1\partial_0 c_a\partial_0\varphi_r\partial_0 c_r - \sigma_1\partial_i^2 c_a\partial_0\varphi_r\partial_0 c_r$$
$$-iT\sigma_1(\partial_i c_a\partial_i\varphi_a\partial_0 c_r + (\partial_0 c_a\partial_i\varphi_a - \partial_i c_a\partial_0\varphi_a)\partial_i c_r),$$

If ignoring interactions of noise

$$(\partial_0 - D\partial_i^2)n - \left(\lambda_1\partial_0 - \frac{\lambda}{2}\partial_i^2\right)n^2 = \xi$$

A variation of Kardar-Parisi-Zhang equation

Applications

Long time tail,

transports,

dynamical aspects of phase transitions,

non-equilibrium states,

turbulence

……………

Holographic duality
and
non-equilibrium systems

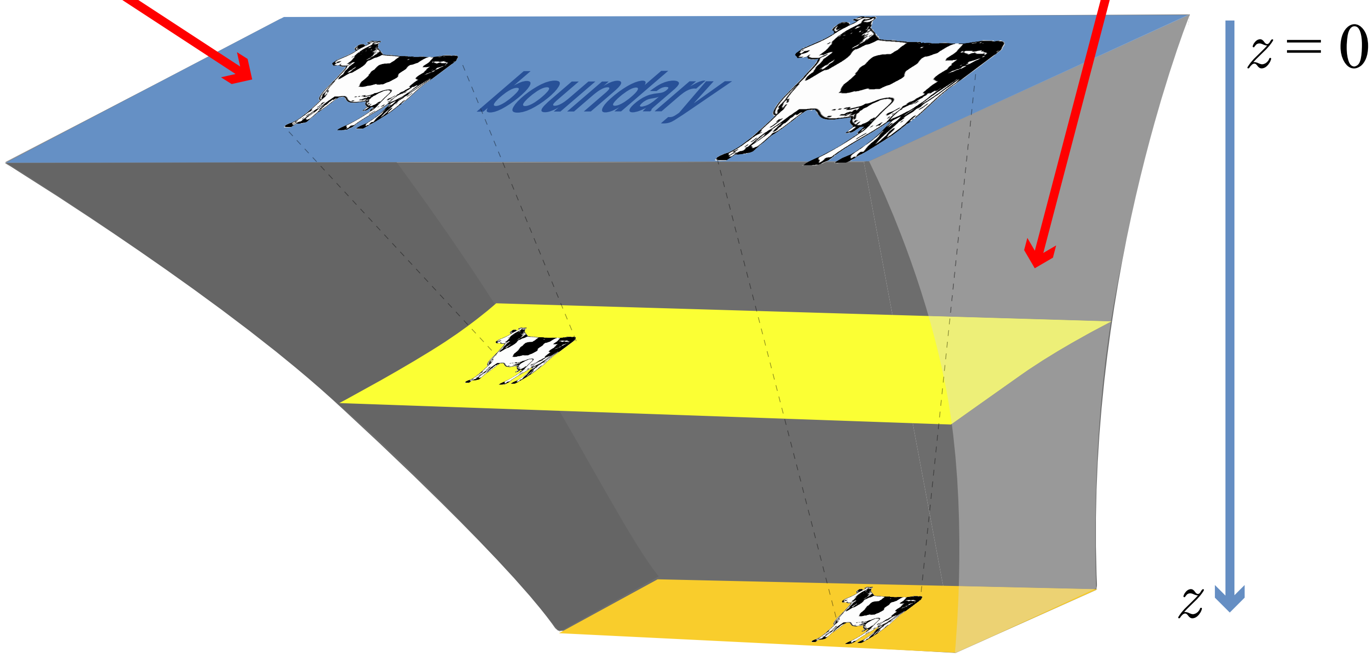
Holographic duality

Maldacena; Gubser, Klebanov, Polyakov; Witten

Many-body system without gravity in d spacetime dimensions

=

String theory, quantum gravity in d+1 dimensions



Many-body system without gravity in d spacetime dimensions

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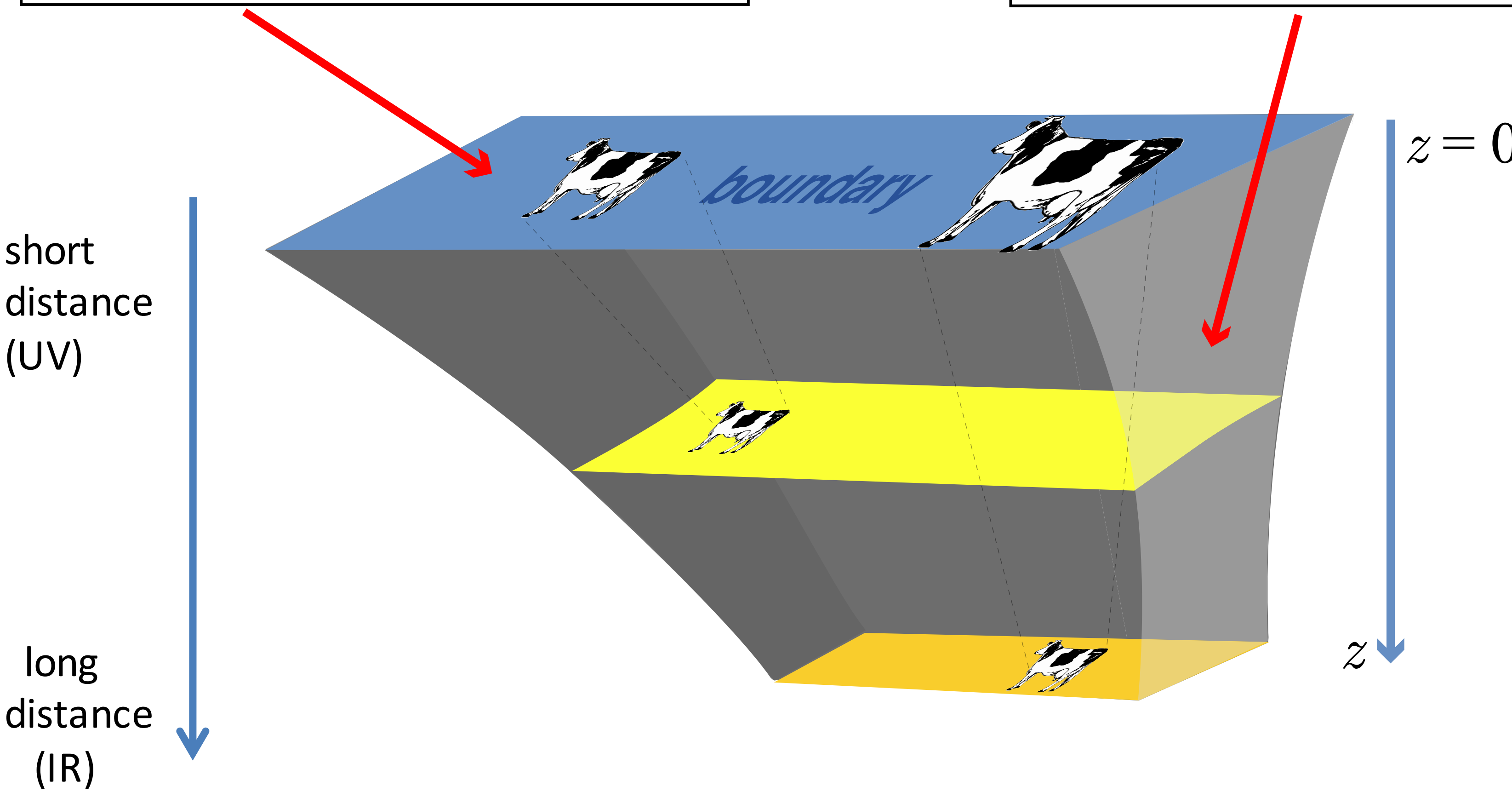
String theory, quantum gravity in d+1 dimensions



Extra dimension: geometrization of renormalization group flow!

A prototype example

Maldacena (1997)

$\mathcal{N} = 4$ Super-Yang-Mills theory in (3+1) dimensions with gauge group SU(N)

=

A string theory in 5-dimensional anti-de Sitter spacetime

A relative of QCD

scale invariant

anti-de Sitter (AdS): homogeneous spacetime with negative cosmological constant.

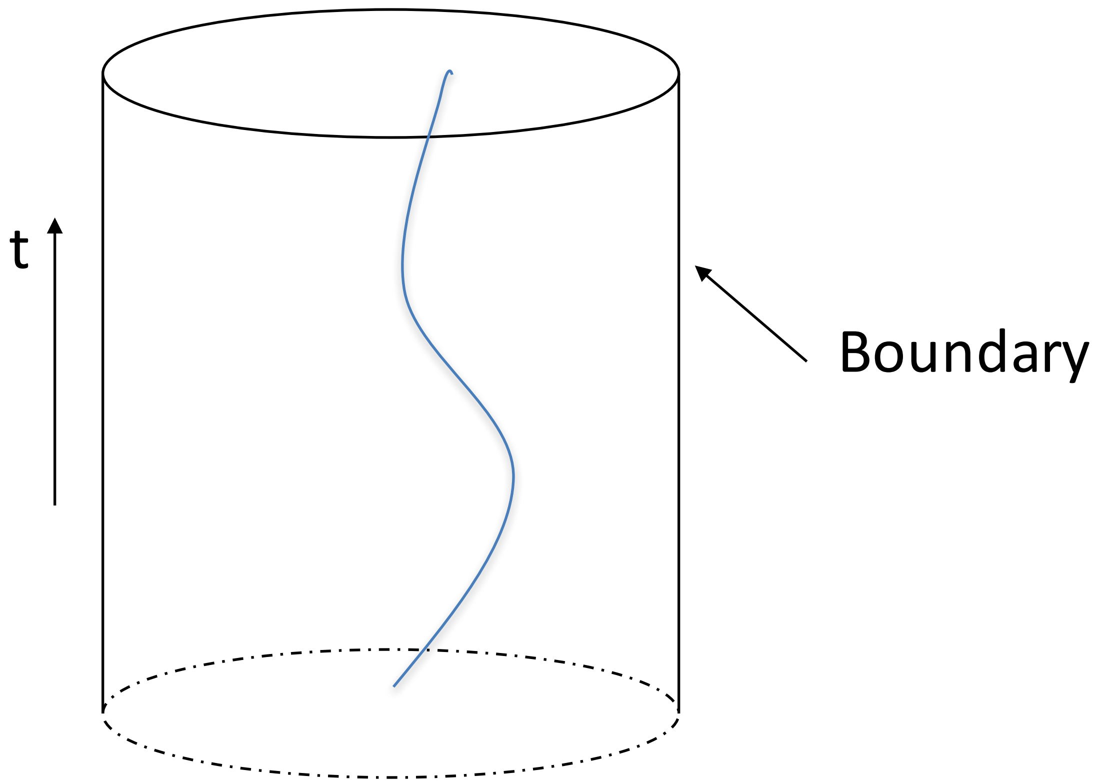
↓ ↓

Strongly coupled and large N limit	=	Classical gravity limit (Einstein gravity plus matter fields)

Many examples in different spacetime dimensions known.

Previously impossible problems in strongly coupled systems can now be mapped to solvable problems classical gravity !

Anti-de Sitter (AdS) spacetime



AdS spacetime is a like a finite size box, confined by gravitational potential

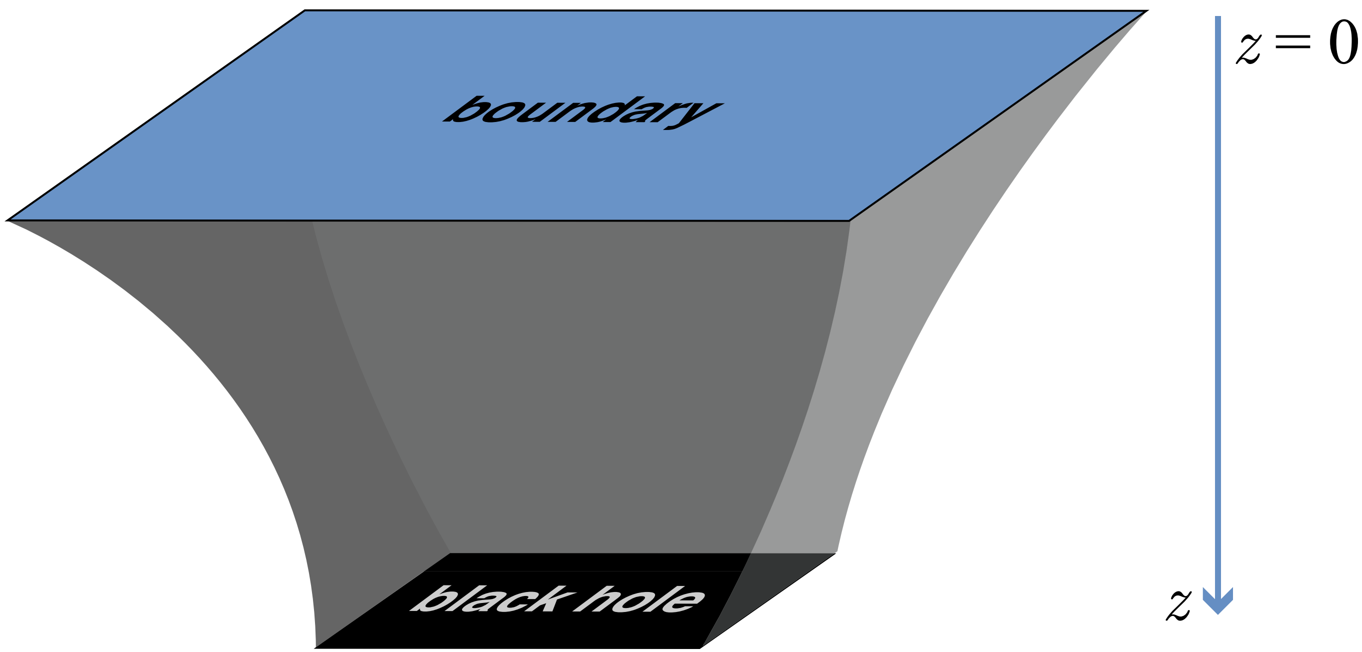
Dictionary

Boundary ⟷ Bulk

Boundary	Bulk
states	states/geometries
operators $\mathcal{O}(x)$	fields $\phi(z,x)$
• Spin, charge	• Spin, charge
• dimension	• mass
• $T^{\mu\nu}$	• metric
• J^{μ}	• Gauge field
Partition function	Partition function
Correlation functions	Scattering amplitudes
……..	……..

Thermal state

Thermal state ⟷ Schwarzschild black hole



Finite charge density state ⟷ Charged black hole

Macroscopic behavior dictated by black hole geometry

Power of holographic duality

strong coupling limit ⟺ Classical gravity

1. Greatly reduces the number of degrees of freedom

Quantum many-body ⟺ classical few-body

⟹ Can track real time evolution

2. highly quantum mechanical, strong coupling phenomena often follows from simple geometric picture or gravitational dynamics

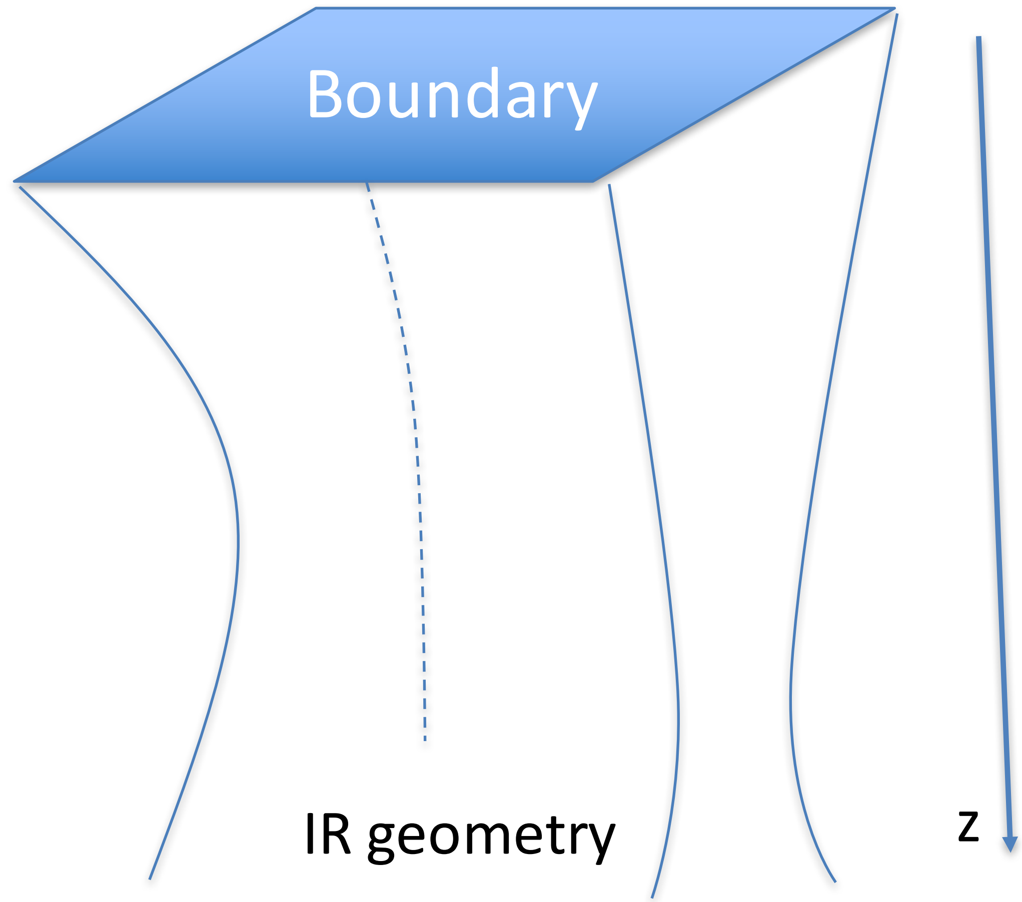
3. Geometrization of RG: like a magnifying glass helping understand physics at every scale

IR physics

Microscopic description

↓

Infrared?



IR fixed point ← IR scaling geometry

Gapped ← IR geometry ends at finite proper distance

Finite temperature ← Black hole horizon

Holography and non-equilibrium physics

Holographic duality already led to many new insights into non-equilibrium physics

Near-equilibrium: transports

Far from-equilibrium: Quantum turbulence

Many other topics:

Relaxation: quasi-normal modes

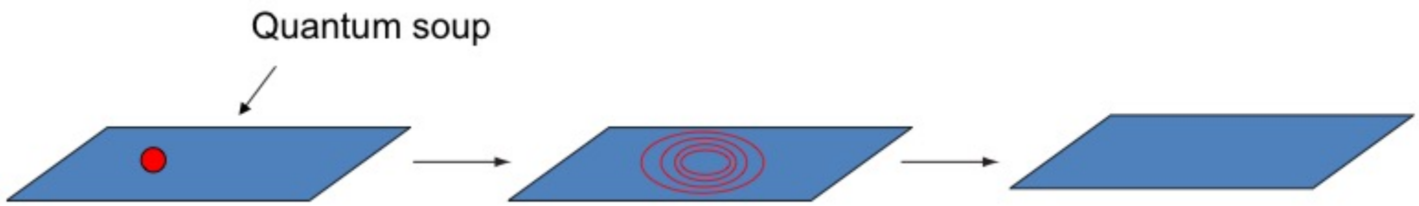
Non-equilibrium steady states

Quantum quenches

Thermalization: unreasonable effectiveness of hydro

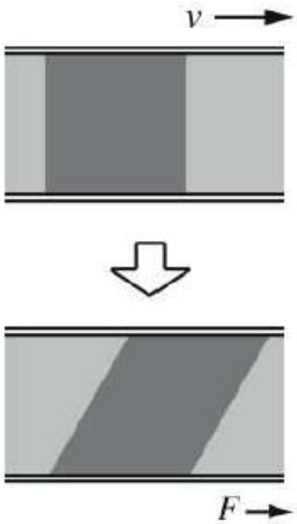
……….

Dissipation



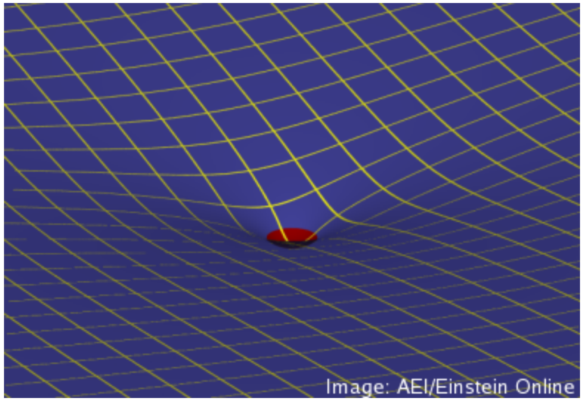
Shear viscosity from gravity

Kovtun, Policastro, Son, Starinets

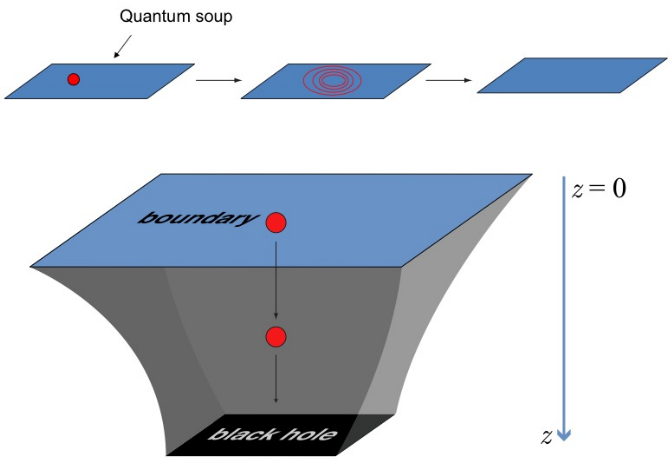


$$\frac{F}{A} = \eta \frac{v}{L} \, .$$

Field theory: slightly deform the metric of spacetime.



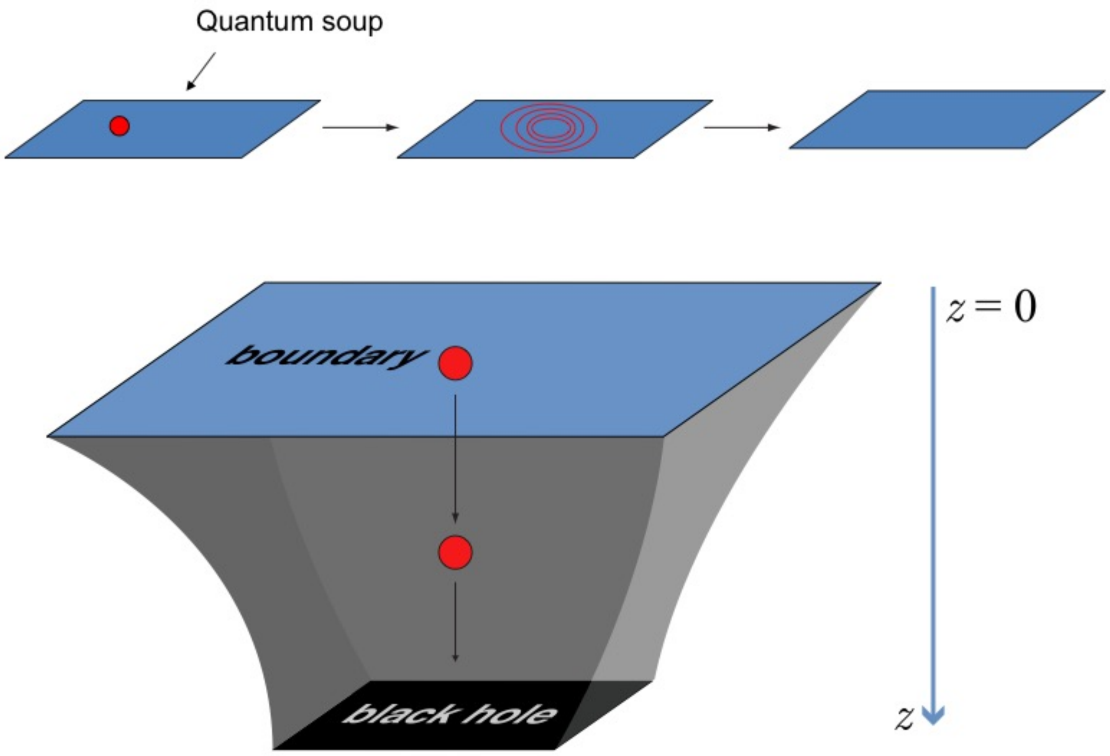
Gravity picture:



$$\eta = \frac{\lim_{\omega \to 0} \sigma_{\rm BH}}{16\pi G} = \frac{1}{4\pi} s$$

$\sigma_{\rm BH}$: absorption cross section of gravitons by a black hole.

Transports



DC electric conductivity, thermal conductivities
and bulk viscosity all captured by geometries at the horizon.

DC conductivity

$$\sigma = \sigma_Q + \frac{Q^2 \tau}{\rho_M}$$

"anti-Matthiessen's rule"

Blake, Tong
Gauntlett, Donos,
....

Quantum Turbulence

Irregular, chaotic motion of superfluid vortices

Feynman 1955
Viven, 1957

"Quasi-classical" quantum turbulence

e.g. Kolomogorov scaling

Mauer and Tabeling, 1997

Smith, Donnelly, Goldenfeld, Viven, 1993

……….

Some outstanding questions:

Does the observed Kolomogorov scaling has a classical origin?

an energy cascade and direction of the cascade? Especially 2+1 d

Dissipation mechanism?

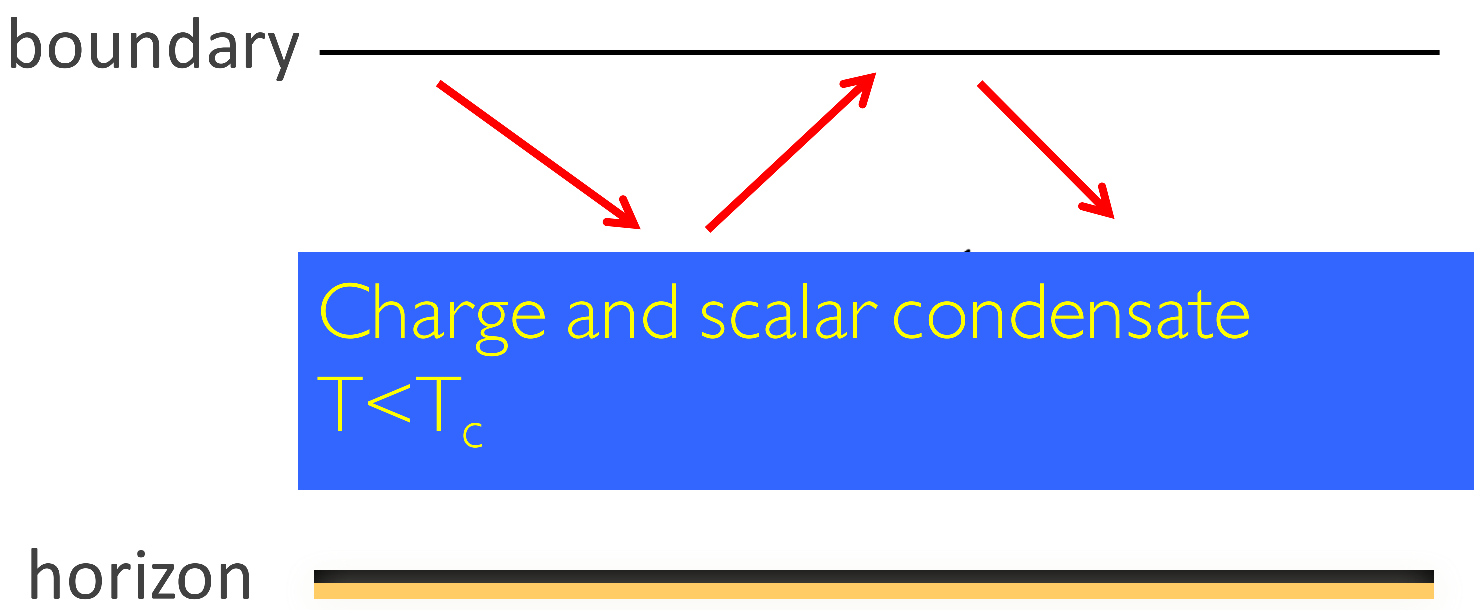
Holographic duality (first principle calculation) can provide definite answers to these questions

Chesler, HL, Adams, Science 341, 368 (2013)

Holographic Superfluid

Gubser, Hartnoll, Herzog, Horowitz

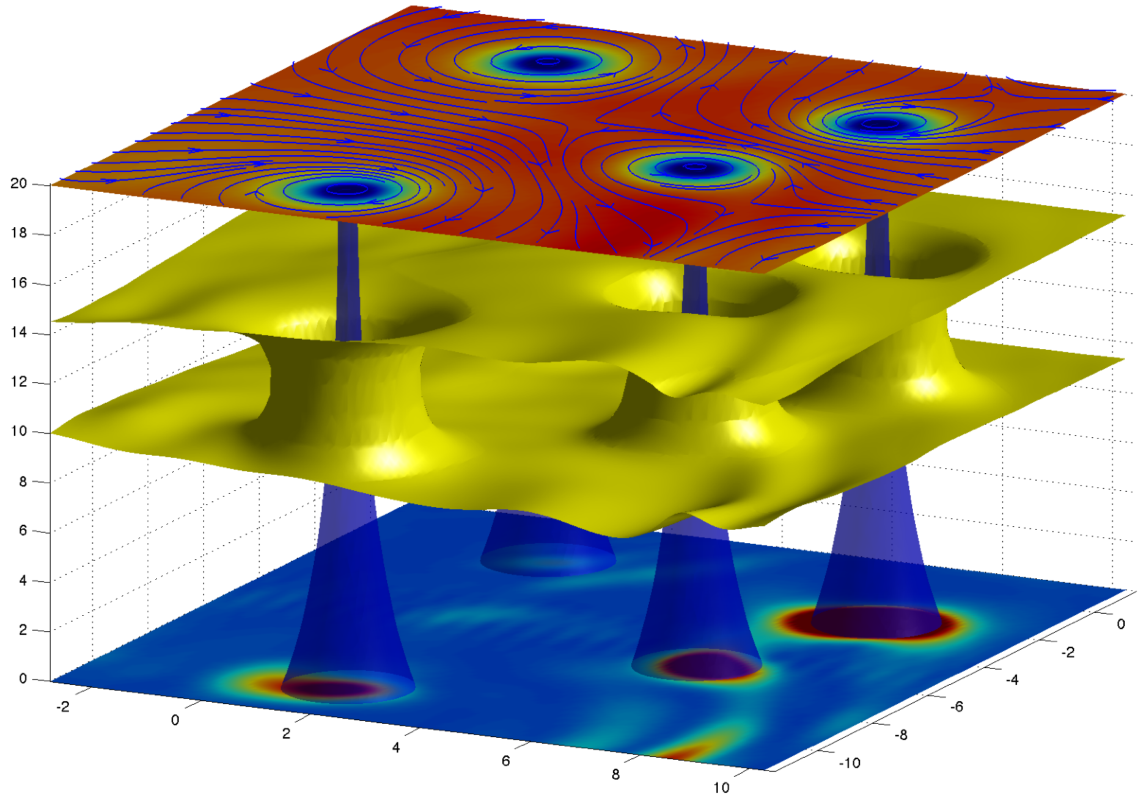
Holography relates a superfluid in 2+1 dimensions to classical electrodynamics + charge scalar + gravity in 3+1 dimensions.



Bulk charge acts like a screen, preventing charged excitations from falling into the horizon and dissipating: it is a superfluid.

Superfluid component: bulk charge; normal: black hole geometry

Holographic superfluid with vortices



2+1 dimensional boundary

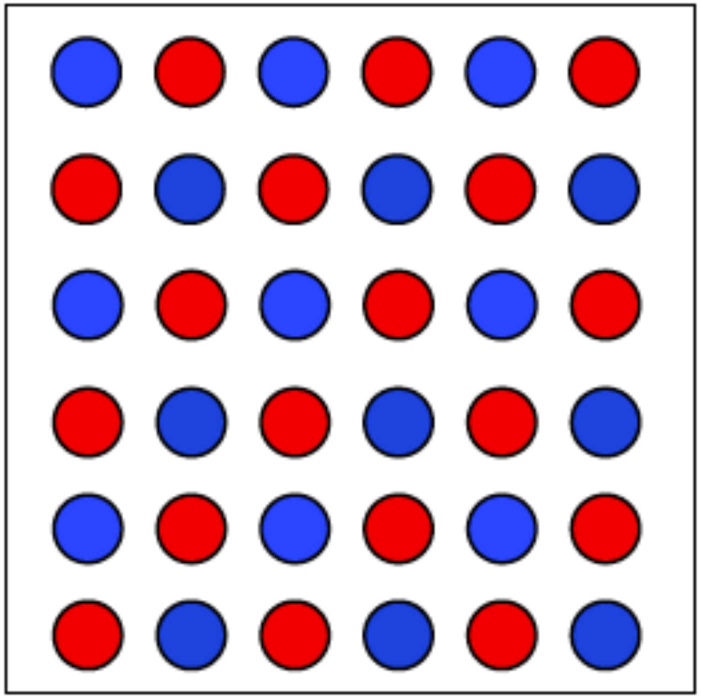
Vortices in the superfluid (blue disks on top) extend to flux tubes --- holes in the screen.

Energy can fall through these holes into the black hole. Vortices thus allow dissipation.

Initial conditions

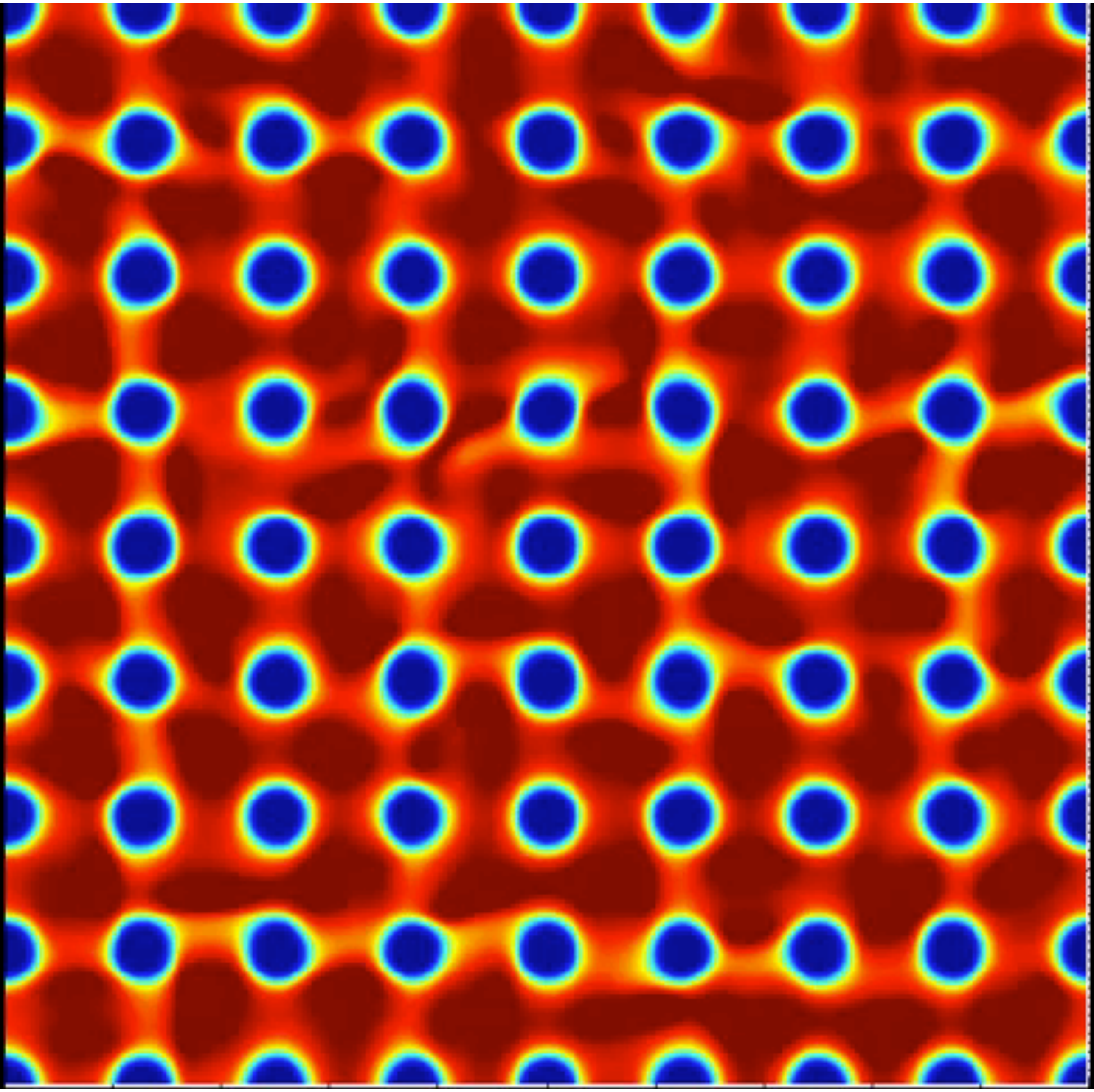
Initial data: periodic lattice of winding number n = ±6 vortices.

Superfluid can dissipate into the normal component, but the effects on the normal component are neglected. (works for T not too low)



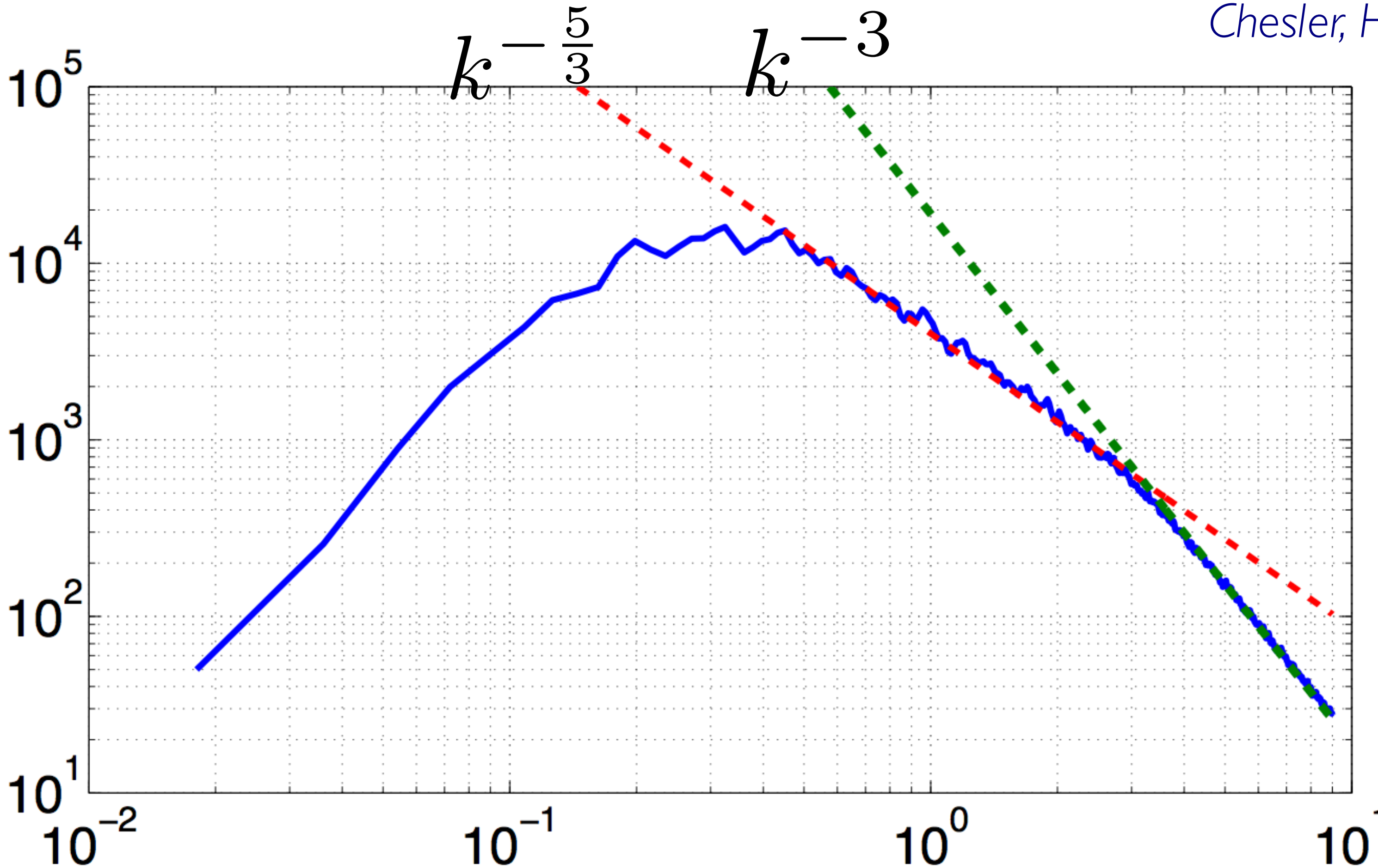
We consider $T \approx 0.6 T_c$ $T_c \approx 0.06 \mu$

Superfluid: 77% Normal: 23%



Kinetic Energy Spectrum Scaling with k

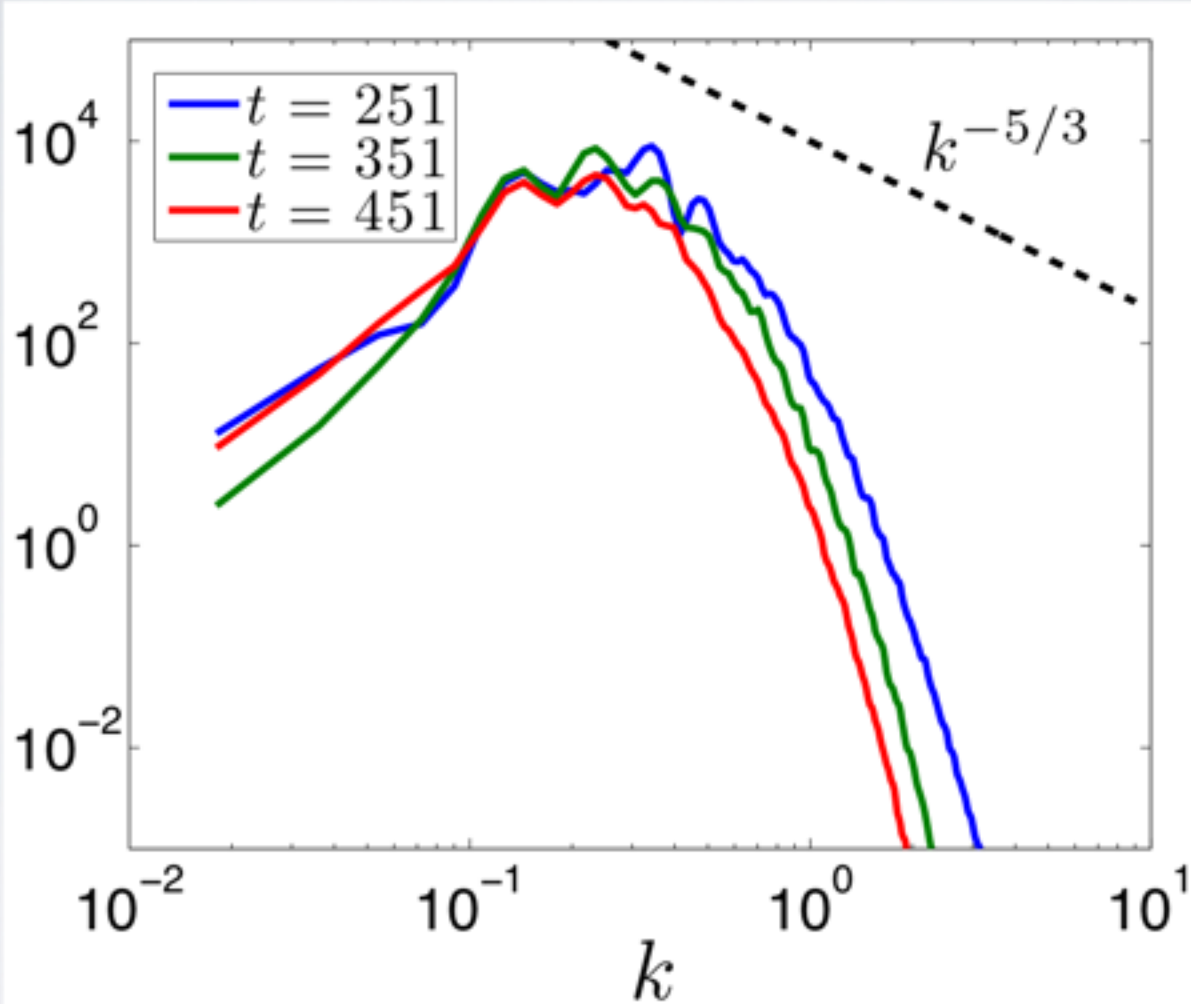
Chesler, HL, Adams



Scaling range: $k \in (0.4, 3)$

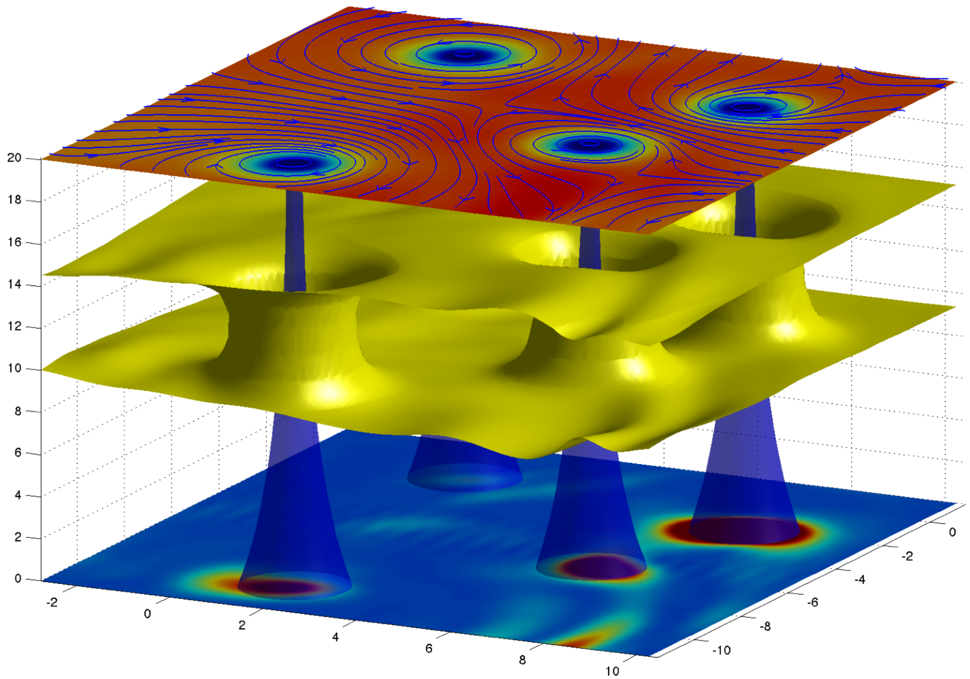
Average vortex spacing $k \sim 0.6$

Indicates quantum effects significant!



No Scaling Without Vortices

Adams, Chesler, Liu



Vortices: holes in the screen

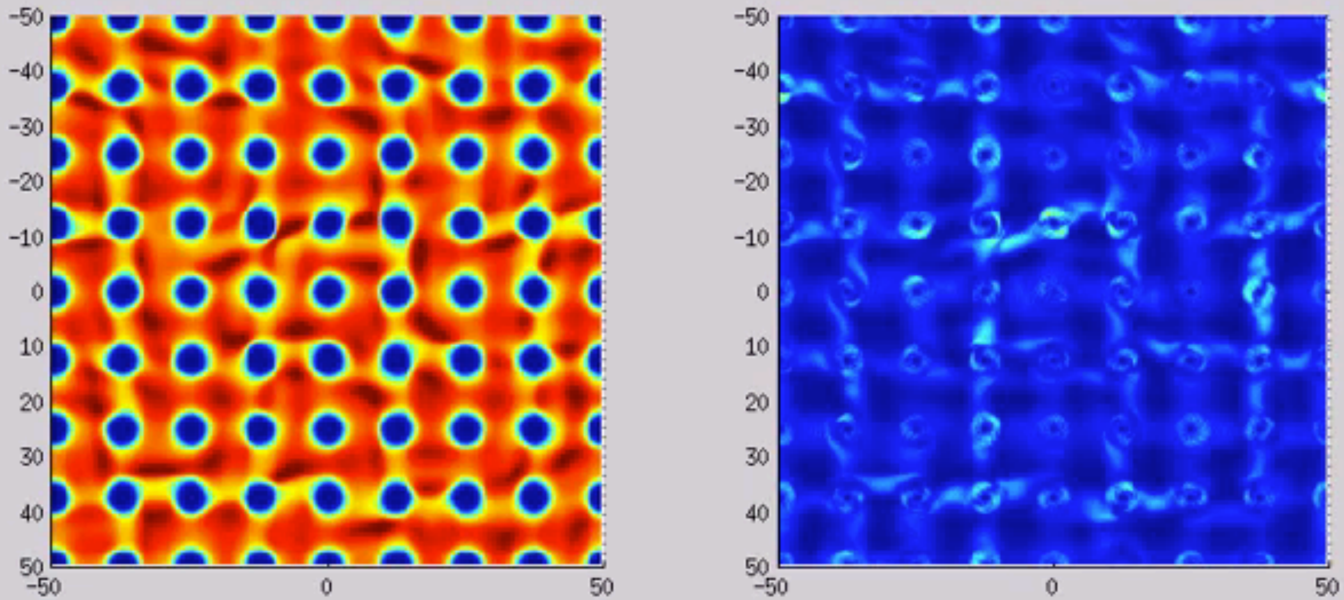
Energy can fall through these holes into the black hole.

Dissipation scale:

$$k_{\mathrm{diss}} = \frac{2\pi}{\text{vortex size}}$$

Vortex size $\approx 1 \;\; k_{\mathrm{diss}} \approx 2\pi > \Lambda_{+} \approx 3$

Direct energy cascade ! **confirmed by direct driving**



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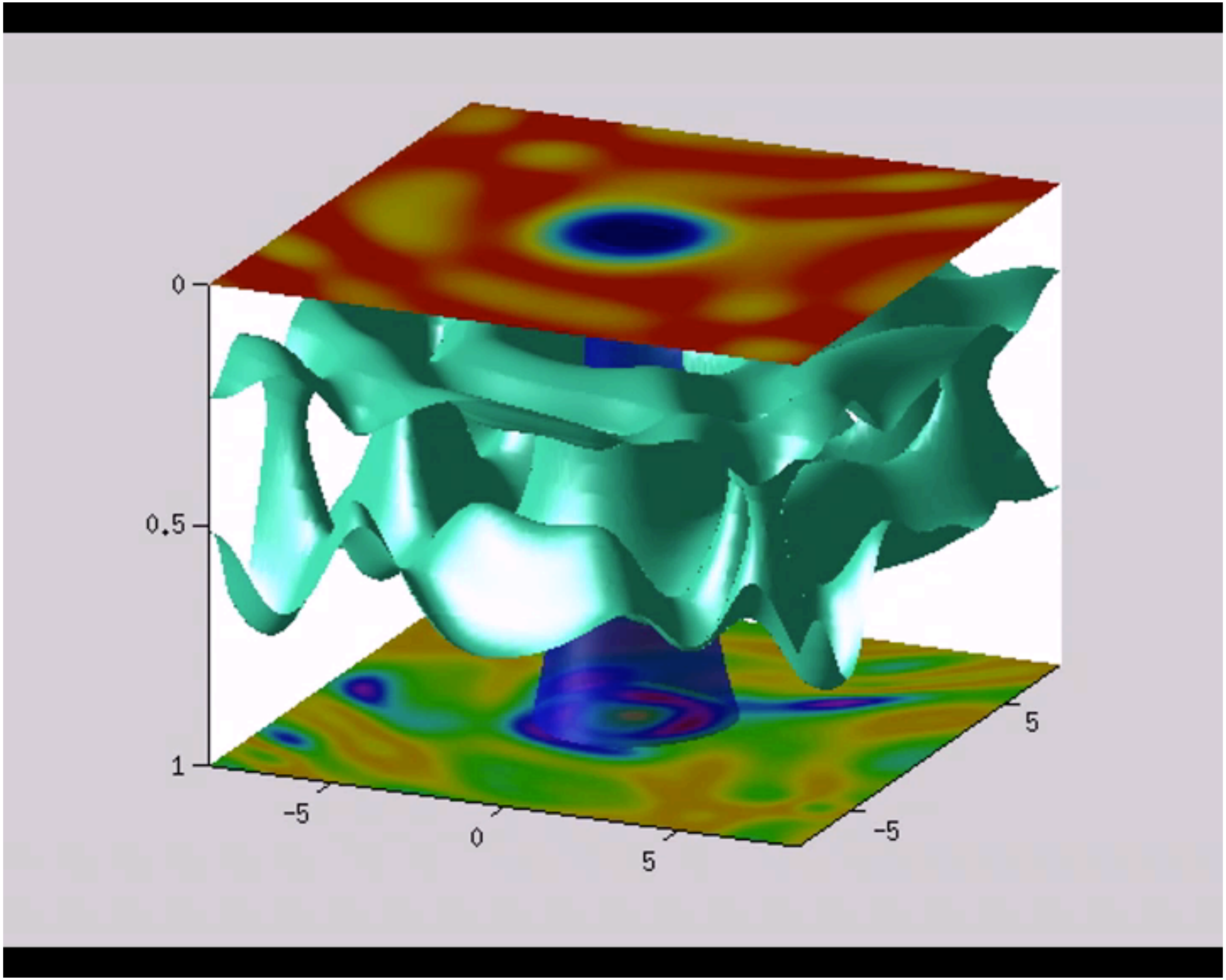

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-5
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5
5
-5

Summary

Kolomogorov scaling

Energy dissipation through vortex annihilations by leaking through vortex core

Direct cascade in (2+1)-dimension in contrast to the inverse cascade of ordinary fluid turbulence

Must be of quantum origin

Thank You