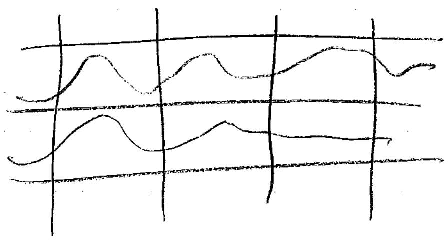
Non-Fermi liquids from black holes

The main reference we follow in the latter part is 1003.1728v1 "From Black Holes to Strange Metals," Faulkner, Iqbal, Liu, Mcgreevy, Vegh.

We shall start from Landau-Fermi liquid theory following Polchinski 9210046 "Effective Field Theory and the Fermi surface".

Landau-Fermi liquids

Goal



Metal: Lattice of ions & Cloud of shared electrons

We would like to determine macroscopic properties of the electron cloud. The main example being conductivity.

Results:

$R \sim T^2$ at small T in any reasonable 3+1 dim. metal

$R \sim T$ in "strange metals"

Want to understand why T^2 is relatively robust and why $R \sim T$ is surprising. Then finally we will get $R \sim T$ from holography of AdS-RN black holes. The holographic calculation is almost identical to LF liquid one $\Rightarrow$ do LF in detail.

Build an effective field theory for electrons.
Start from free theory and gradually add interactions

$$S_0 = \int dt\, d\bar{x}\, \psi^\dagger_\sigma \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_\sigma$$

$$Z = \int [d\psi]\, e^{-S_0} = \mathrm{Tr}\left(e^{-\beta(\hat{H} - \mu \hat{N})} \right)$$

$$\hat{N} = \int d\bar{x}\, \psi^\dagger_\sigma \psi^\dagger = \text{fermion } \#$$

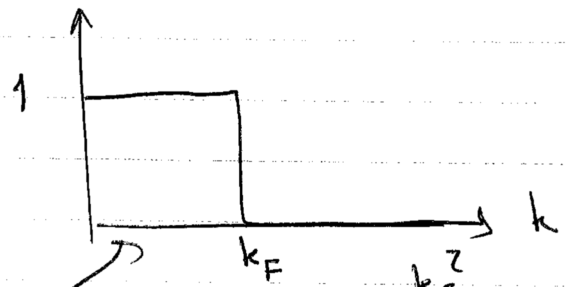
$$\hat{H} = \int d\bar{x}\, \psi^\dagger_\sigma \left(-\frac{\nabla^2}{2m} \right) \psi_\sigma = \int \frac{d\bar{k}}{(2\pi)^3} \frac{\bar{k}^2}{2m} \hat{a}^\dagger_{k,\sigma} \hat{a}_{k,\sigma}$$

$$\psi_\sigma(x) = \int \frac{d\bar{k}}{(2\pi)^3} e^{-i\bar{k}\cdot\bar{x}} \hat{a}_{k,\sigma}$$

$$\frac{1}{\beta} \frac{\partial}{\partial \mu} \log Z = \langle \hat{N} \rangle = 2V_3 \int \frac{d\bar{k}}{(2\pi)^3} n(k)$$

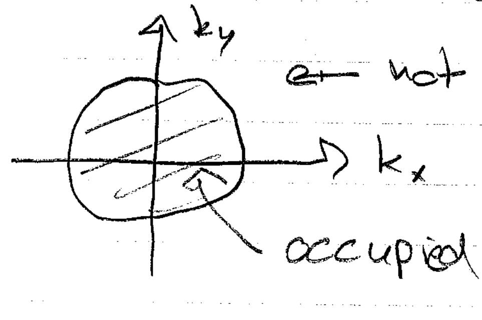
$$n(k) = \frac{1}{\left(e^{\beta(\frac{k^2}{2m} - \mu)} + 1 \right)} = \text{occupation \# of states}$$

As $\beta \to \infty$



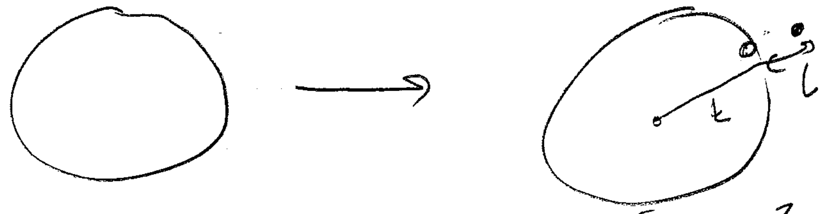
$$\frac{k_F^2}{2m} = \mu.$$

This is the Fermi surface.



Low energy excitations near the Fermi surface

E.g.



Excitation energy $\omega = \frac{(k_F + l)^2}{2m} - \frac{k_F^2}{2m}$

$= \frac{k_F}{m} l + O(l^2) \equiv v_F l + O(l^2)$

Fermi velocity.

Gapless d.o.f at finite momentum.

A simple way to see them (generalizes to holography)

$$G_E(i\omega_n, k) = \frac{1}{i\omega_n - \frac{\vec{k}^2}{2m} + \mu} = \langle T_E \, \phi(k, i\omega_n) \phi^\dagger(-k, -i\omega_n) \rangle$$

Analytic continuation to real time

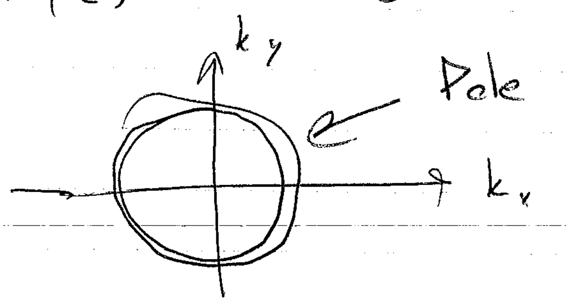
$$\boxed{G_R(\omega, k) = G_E(i\omega_n = \omega + i\varepsilon)}$$

$$G_R(\omega, k) = \frac{1}{\omega - \frac{\vec{k}^2}{2m} + \mu + i\varepsilon} \approx \frac{1}{\omega - v_F l + i\varepsilon}$$

Has pole at $\omega = \frac{k^2}{2m} - \mu \approx v_F l$.

Diagnosis for a Fermi surface

$$G_R(0, k)^{-1} = 0$$

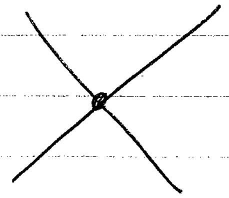


Pole of $G_R(0, k)$

Add interactions (Polchinski 9210046)

Upshot: At low energies all interactions can be taken into account by adding

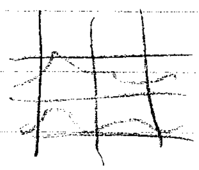
$$S_{int} = \int dt\, d\bar{x}\, \lambda\, \psi^\dagger_\sigma \psi_\sigma \psi^\dagger_{\sigma'} \psi_{\sigma'}$$



Local ϕ^4 interaction.

$\lambda > 0$ irrelevant
$\lambda < 0$ relevant $\Longrightarrow$ Fermion binding & BCS

What are the interactions? Electrodynamic



e – e Coulomb → Screened
e — N Coulomb

~~Impurities in the lattice~~

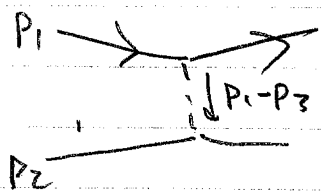
Scattering with lattice vibrations i.e. phonons

Otherwise seem local except phonons.

Quick argument against phonons:

$$\text{---} = \frac{1}{\omega^2 - k^2}$$

Important: Low E limit is at $\omega \to 0$, $|\bar{k}_j| \to k_F$



$$\sim \frac{1}{\omega^2 - (\bar{k}_1 - \bar{k}_3)^2}$$

total energy

$$\longrightarrow \frac{1}{2k_F^2(1 - \hat{n}_1 \cdot \hat{n}_3)} = \text{const.}$$

Except unimportant forward scattering

→ RG implies that we are doomed to end up with almost free fermions.

Interactions 2: Origin of dissipation

2-loop correction to G_R

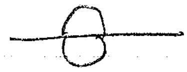
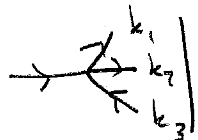
$\Sigma' =$ 

$$G_R = \frac{-1}{\omega - \frac{k^2}{2m} + \mu + \Sigma}, \quad \mathrm{Im}\,\Sigma = \frac{1}{\tau}$$

Lifetime of quasiparticles

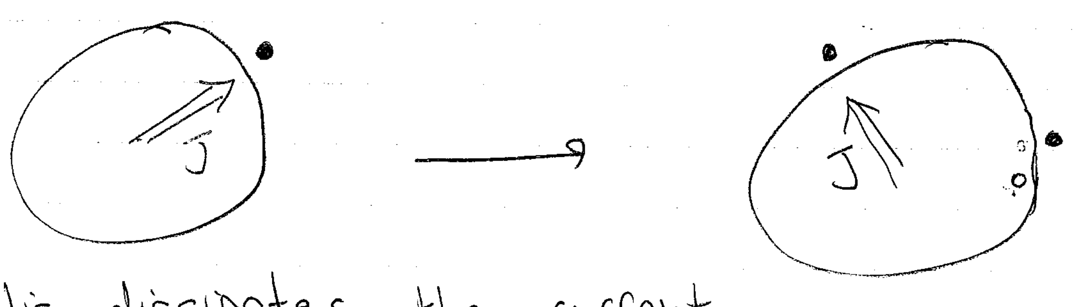
Le Bellac "Thermal field theory" p. 72–

Use optical theorem to simplify calculation

$\frac{1}{\tau} = \mathrm{Im}\Big($  $\Big) = \int d\Pi \,\Big|$  $\Big|^2 \delta^4(k - k_1 - k_2 - k_3)$

Phase space of k_1, k_2, k_3

Physically: Particle decays to two particles & 1 hole



This dissipates the current

$$\frac{1}{\tau} \sim \lambda^2 \int d\bar{k}_1 d\bar{k}_2 d\bar{k}_3\, \delta(\omega + \omega_1 - \omega_2 - \omega_3)\, \delta^3(\bar{k} - \bar{k}_1 - \bar{k}_2 - \bar{k}_3)$$

$$\times\, n(k_1)\big(1 - n(k_2)\big)\big(1 - n(k_3)\big)$$

No particle — Particle — Particle

Separate the integral into energies and angles

$$\frac{1}{\tau} \sim \frac{\pi^2 T^2 + \omega^2}{1 + e^{-\beta\omega}} \overset{\omega = 0}{\simeq} T^2$$

The integral is evaluated in Nozieces & Pines around p. 62 - 63 "The theory of quantum liquids"

$$\boxed{\operatorname{Im} \Sigma = \frac{1}{\tau} \sim T^2}$$

Appendix on decay rate

$$\int d\bar k_1 d\bar k_2 d\bar k_3 \ \delta^3(\bar k - \bar k_1 - \bar k_2 - \bar k_3)$$

$$\bar k_j = \hat n_j k \ , \qquad \omega = \frac{k^2}{2m} - \mu \ , \qquad \frac{d\omega}{dk} = \frac{k}{m}$$

$$\int d\bar k_j = \int d\hat n_j \int dk_j \, k_j^2 = \int d\hat n_j \int d\omega_j \underbrace{\left(\frac{dk_j}{d\omega_j} k_j^2\right)}_{\frac{k_j^3}{m}}$$

$$\approx \frac{k_f^3}{m} \int d\hat n_j \, d\omega_j$$

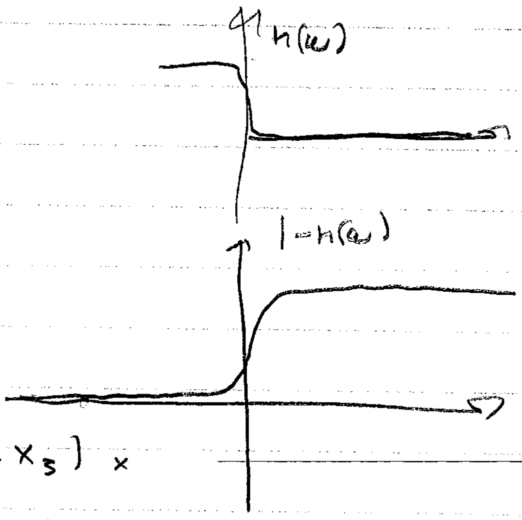
$$\overbrace{\int d\hat n_1 d\hat n_2 d\hat n_3 \frac{1}{k_F^3} \delta^3(\hat n - \hat n_1 - \hat n_2 - \hat n_3)}^{\text{Geometric factor}} \times$$

$$\times \int d\omega_1 d\omega_2 d\omega_3 \ \frac{k_f^3}{m}$$

$$\Rightarrow \frac{1}{\tau} \sim \int d\omega_1 d\omega_2 d\omega_3 \ \delta(\omega + \omega_1 - \omega_2 - \omega_3) \times$$

$$\times \ n(+\omega_1)\big(1 - n(\omega_2)\big)\big(1 - n(\omega_3)\big)$$

$$n(\omega) = \frac{1}{e^{\beta\omega} + 1}$$



$$x_j = \beta\omega_j \ , \quad \omega \to 0$$

$$\frac{1}{\tau} \sim \frac{1}{\beta^3} \int dx_1 dx_2 dx_3 \ \beta \, \delta(x_1 - x_2 - x_3) \times$$

$$\times \ n(x_1)\big(1 - n(x_2)\big)\big(1 - n(x_3)\big)$$

$$\sim T^2 \ (\text{integral})$$

Conductivity

Use linear response

$$\sigma(\omega) = \frac{1}{i\omega} G_R^{JJ}(\omega, \bar{k}=0)$$

Quick & dirty almost derivation:

$$\langle j_x(\omega,k)\rangle = \left\langle j_x(\omega,k)\, e^{i\int d\omega' dk' A_x j_x(\omega',k')} \right\rangle_J \qquad E_x = i\omega A_x$$

$$\approx \langle j_x(\omega,x)\rangle + i\int d\omega' dk' A_x(\omega',k') \langle j_x(\omega,k)\, j_x(\omega',k')\rangle + \ldots$$

$$\langle \delta j_x(\omega,k)\rangle = \frac{1}{i\omega}\, i\omega A_x(-\omega,-k)\, \underbrace{i\langle j_x(\omega,k)\, j_x(-\omega,-k)\rangle}_{G(\omega,k)}$$

$$\langle \delta j_x \rangle = \frac{G(\omega)}{i\omega} E_x \equiv \sigma(\omega) E_x(\omega)$$

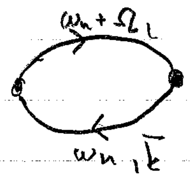
$$\sigma(\omega) = \frac{1}{i\omega} G_R(\omega)$$

Appeared from my hat!

Conductivity

Assume

$$T \sum_{\substack{\omega_m \\ \bar{k}}} \frac{1}{i(\omega_m+\Omega_l)-\omega_1} \frac{1}{i\omega_m-\omega_2} = \frac{n(\omega_1)-n(\omega_2)}{\omega_1 - i\Omega_l - \omega_2}$$



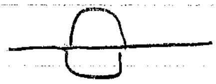
$$\hat{J}_x = \frac{q}{m} \hat{\psi}^{+}(-i\nabla_x)\hat{\psi}$$

$$\langle \hat{J}_j(\bar{o},\Omega_l)\, \hat{J}_{j'}(\bar{o},-\Omega_l) \rangle = -\int \frac{d\bar{k}}{(2\pi)^D}\, T \sum_{\omega_m} \left(\frac{q}{m}\right)^2 k_j k_{j'}$$

$$\times\, G_E(\omega_n+\Omega_l, \bar{k})\, G_E(\omega_n, \bar{k}) = G_E^{JJ}$$

$$G_E(\Omega_l, \bar{k}) = G_R(\omega = i\Omega_l, k)$$

$$= \frac{h}{\omega - v_F l - \Sigma(\omega, k)}$$

$$\Sigma =$$  $$+ \dots = \dots + i(a\omega^2 + bT^2)$$

Spectral function

$$\rho_B = 2\,\mathrm{Im}(G_R)\Big|_{\omega\in\mathbb{R}} = \frac{2h\,\mathrm{Im}\,\Sigma}{(\omega - v_F l - \mathrm{Re}\,\Sigma)^2 + \mathrm{Im}\,\Sigma^2}$$

$$G_R(\omega, k) = \int \frac{d\omega'}{2\pi} \frac{\rho_B(\omega', \bar{k})}{\omega - \omega' + i\varepsilon}$$

$$G_E(\Omega_l, k) = \int \frac{d\omega'}{2\pi} \frac{\rho_B(\omega', k)}{i\Omega_l - \omega'}$$

$$\frac{h(\omega - v_F l - \mathrm{Re}\,\Sigma + i\,\mathrm{Im}\,\Sigma)}{(\omega - v_F l - \mathrm{Re}\,\Sigma)^2 + \mathrm{Im}\,\Sigma^2}$$

$$G_E^{JJ} = -T\sum_{\omega_n}\left(\frac{q}{m}\right)^2\int\frac{d\bar{k}}{(2\pi)^D}k_ik_j\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\times$$

$$\times\frac{1}{i(\omega_n+\Omega_l)-\omega_1}\;\frac{1}{i\omega_n-\omega_2}\;\rho_B(\omega_1)\rho_B(\omega_2)$$

$$= T\left(\frac{q}{m}\right)^2\int\frac{d\bar{k}}{(2\pi)^D}k_ik_j\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\;\frac{n(\omega_1)-n(\omega_2)}{\omega_1-i\Omega_l-\omega_2}\;\rho_B(\omega_1)\rho_B(\omega_2)$$

Now we can take $i\Omega_l \to i\varepsilon + \omega$

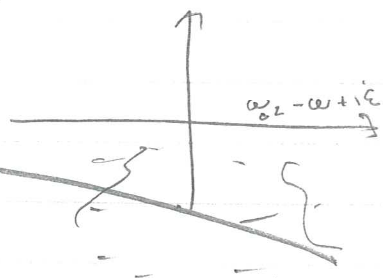
$$\sigma(\omega) = +\frac{1}{i\omega}G_R^{JJ}(\omega) = -\frac{p}{i\omega}\left(\frac{q}{m}\right)^2\int\frac{d\bar{k}}{(2\pi)^D}k_ik_j\times$$

$$\times\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\;\frac{n(\omega_1)-n(\omega_2)}{\omega_1-\omega-\omega_2-i\varepsilon}\;\rho_B(\omega_1)\rho_B(\omega_2)$$

~~$\rho_B(\omega)$ analytic in the upper half plane~~

$$\cancel{\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\;\frac{n(\omega_2)\rho_B(\omega_2)\rho_B(\omega_1)}{\omega_1-\omega-\omega_2-i\varepsilon}}$$

$$\cancel{= i\;n(\omega_2)\rho_B(\omega_2)\rho_B(\omega_2-\omega)}$$



Doesn't seem to work.

$$\frac{1}{\omega_1-\omega_2-\omega-i\varepsilon} = \frac{\omega_1-\omega_2-\omega}{(\omega_1-\omega_2-\omega)^2+\varepsilon^2} + i\frac{\varepsilon}{(\omega_1-\omega_2-\omega)^2+\varepsilon^2}$$

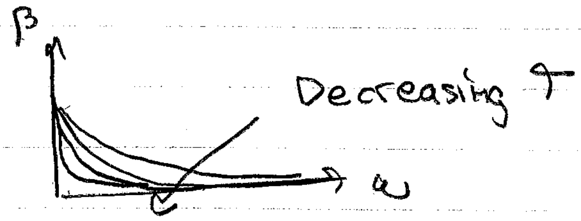
$$= P\left(\frac{1}{\omega_1-\omega_2-\omega}\right) + i\pi\,\delta(\omega_1-\omega_2-\omega)$$

~~Formal argument~~ $\cancel{G_R^*(\omega) = -G_R(-\omega)}$

$\cancel{\Rightarrow G_R^*(0) = -G_R(0) \Rightarrow}$ ~~Has to be imaginary~~

$$\sigma^{ij}_{DC} = -\frac{1}{2}\left(\frac{q}{m}\right)^2 \int \frac{d\bar{k}}{(2\pi)^D} k^i k^j \int \frac{d\omega}{2\pi} \frac{\partial n(\omega)}{\partial \omega} \rho_B(\omega, k)^2$$

$$n(\omega) = \frac{1}{e^{\beta\omega}+1}$$



$$\sigma^{ij}_{DC} = -\int \frac{d\omega}{2\pi} \frac{\partial n(\omega)}{\partial \omega} I(\omega, k, T)$$

$$I(\omega, k, T) = \frac{1}{2}\left(\frac{q}{m}\right)^2 \int \frac{d\bar{k}}{(2\pi)^D} k^i k^j \rho_B(\omega, k, T)^2$$

Biggest contribution from where the states are & where $\partial n/\partial\omega \cancel{\approx} 0$
$\Rightarrow$ Near fermi surface

$$\rho_B(\omega, k) = \frac{2h \,\mathrm{Im}\Sigma}{(\omega - v_F l - \mathrm{Re}\Sigma)^2 + (\mathrm{Im}\Sigma)^2}$$

Small ω, Σ = const. & assume not a fast function of momentum

$$\int d\bar{k}\, k^i k^j \sim \delta^{ij} k_F^{D+1} \int dl$$

We should perform the integral

$$\int_{-\infty}^{\infty} dl \left(\frac{1}{(l+a)^2 + b^2}\right)^2 = \int_{-\infty}^{\infty} \frac{dl}{(l^2+b^2)^2} = \qquad x = \frac{l}{b}$$

$$= \frac{1}{b^3} \underbrace{\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}}_{\pi/2} = \frac{\pi}{2} \frac{1}{(\mathrm{Im}\Sigma)^3}$$

$$\Gamma^{ij} \sim \delta^{ij} \left(\frac{q}{m}\right)^2 k_F^{D+1} \, h^2 \, \frac{1}{(\mathrm{Im}\,\Sigma)}$$

$$\int \frac{d\omega}{2\pi} \frac{\partial n(\omega)}{\partial \omega} = \int \frac{dw}{2\pi} \frac{\partial}{\partial w} \left(e^{w}+1\right)^{-1}$$

$w = \frac{\omega}{T}$

$$= -\int_{-\infty}^{\infty} \frac{dw}{2\pi} \frac{e^{w}}{(e^{w}+1)^2} = -\frac{1}{2\pi}$$

$$\sigma_{DC}^{ij} \propto \delta^{ij} \left(\frac{q}{m}\right)^2 k_F^{D+1} \, \tau \,, \quad \tau^{-1} \equiv \mathrm{Im}\,\Sigma$$

$$\boxed{R = \frac{1}{\sigma_{DC}^{xx}} \sim \frac{1}{\tau} \sim T^2}$$

In a Fermi liquid current dissipation directly related to lifetime of carriers, and $R \sim T^2$.

In a theory with with fermions only the RG arguments suggest that one can only get higher powers of T in R, making the T^2 scaling at low T a robust prediction of fermion systems.

Holographic setup

$U(1)$ chemical potential A_μ, $A_t \neq 0$

Fermions Ψ

$$S = \int d^{d+1}x \sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}(\not{D} - m)\Psi\right)$$

Finite T, μ state, AdS - RN.

Assume no scalars will destabilize it.

$$ds^2 = r^2\left(-h\,dt^2 + d\vec{x}^2\right) + \frac{dr^2}{r^2 h}$$

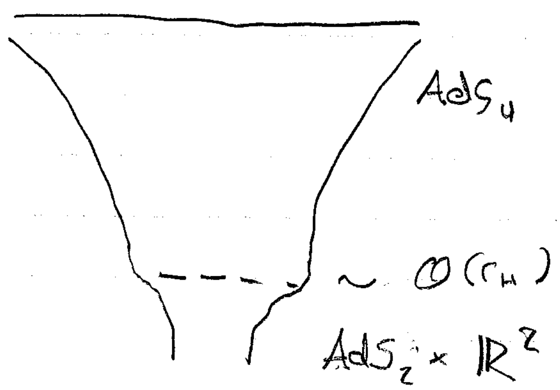
$$h = 1 - (1+\rho^2)\left(\frac{r_H}{r}\right)^3 + \rho^2\left(\frac{r_H}{r}\right)^4$$

$$M = (1+\rho^2)r_H^3, \quad Q = \rho r_H^2, \quad T_H = \frac{|h'(r_H)|}{4\pi}$$

Increase ρ^2 from 0 to 3

$$h \overset{\rho^2=3}{=} \underbrace{\left(1 - \frac{r_H}{r}\right)^2}_{\text{Double pole + vanishes}}\left(1 + 2\frac{r_H}{r} + 3\frac{r_H}{r}\right), \quad A = \mu\left(1 - \frac{r_H}{r}\right)dt$$

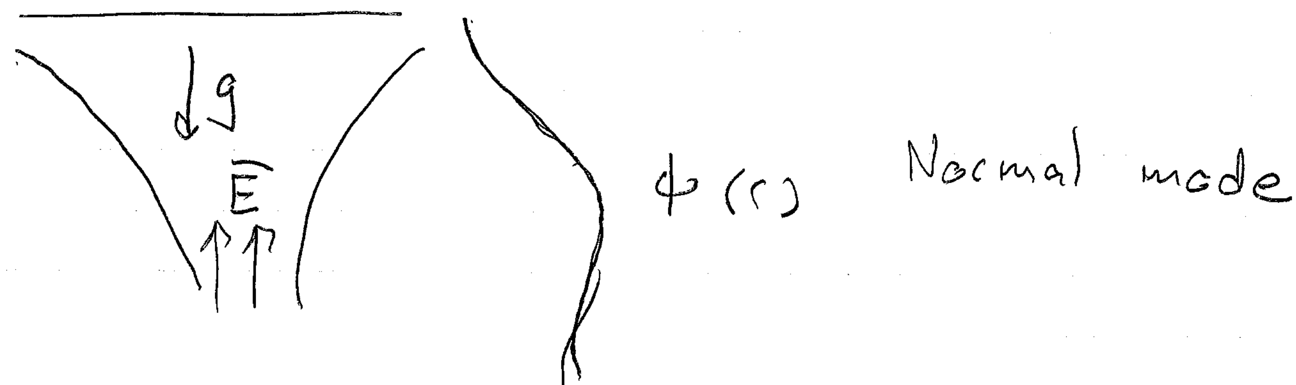
$$ds^2 \sim \underbrace{-(r-r_H)^2 dt^2 + \frac{dr^2}{(r-r_H)^2}}_{AdS_2} + \underbrace{r_H^2 d\vec{x}^2}_{\mathbb{R}^2}$$



Look for a Fermi surface from singularities of Fermion 2-pt. function

$$G_R(\omega=0, k=k_F)^{-1} = 0$$

Fermion solution which satsfies ingoing boundary conditions <u>and</u> is normalizable near AdS boundary. Only possible to satisfy for some specific values of k.



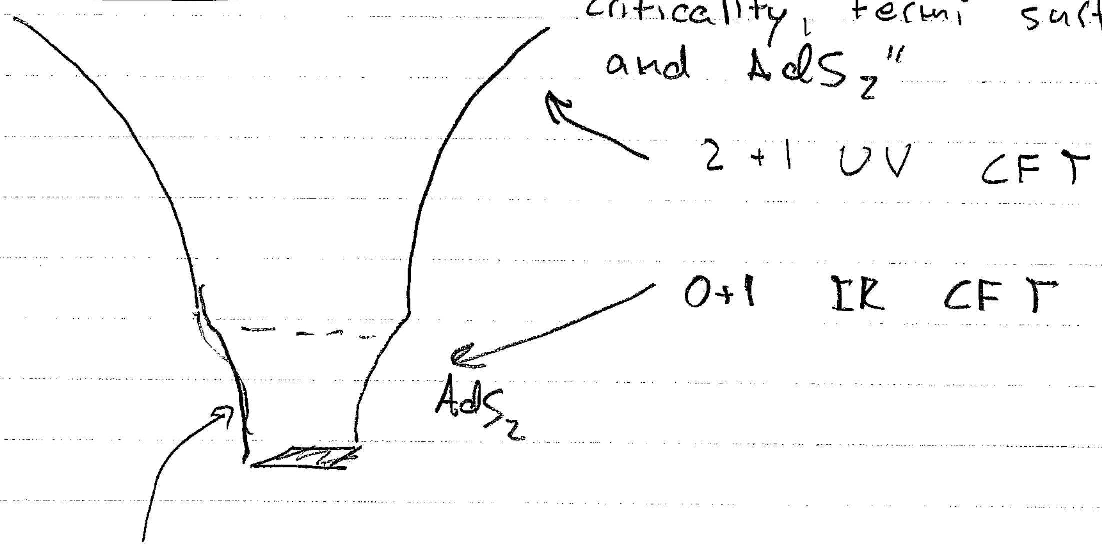
Competition btwn. two effects gravity & Electric field.

If $\vec{E} = \partial_r A_t = 0$ Fermion falls down due to gravity ⇒ No static normal mode

Need to turn on E to balance gravity.
⇒ Fermi surface is a finite density effect.

Solve Fermion eq numerically
⇒ One spherical singular surface.
Finite T & ω smooth the singularity.
Liu, Mcgreevy, Vegh "Non-Fermi liquids from holography" 0903.2477v3

Matching method: 0907.2694 "Emergent quantum criticality, Fermi surfaces and AdS_2"



Solve Fermion eq analytically in AdS_2 & match into a perturbative expansion in ω & T in UV

$$G_R(\omega,k) \sim \frac{h_1}{\omega - v_F l + \Sigma(\omega,T)}$$

$$\Sigma(\omega,T=0) \sim \omega^{2\nu}, \quad \nu = \sqrt{m^2 + k_F^2 - q^2}$$

q: Fermion charge

$\omega - v_F l$ "UV effect"
Σ "IR effect"

$\omega^{2\nu}$ follows from symmetries of AdS_2
$\Delta(\hat{\mathcal{O}}_F) = \frac{1}{2} + \nu$

$$\begin{cases} \nu < 1/2 & \hat{\mathcal{O}} \text{ relevant} \Rightarrow \text{no quasiparticles} \\ \nu = 1/2 & \text{marginal} \\ \nu \geq 1/2 & \hat{\mathcal{O}} \text{ irrelevant} \Rightarrow \text{quasiparticles} \end{cases}$$

$\nu = \frac{1}{2}$

$$\Sigma \sim a\omega + b\omega \log \omega$$ "Marginal Fermi liquid"

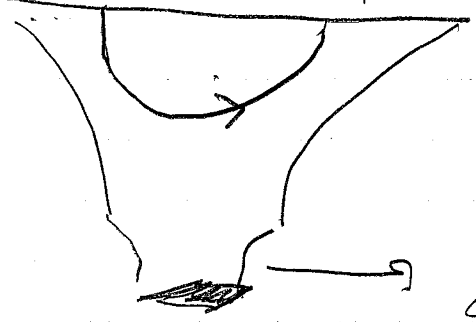
$$\Sigma(\omega=0,T) \sim T^{2\nu}$$

$\boxed{\text{1101.0597v1}}$

$\nu_k = \frac{1}{2}$ agrees with "marginal Fermi liquid" ansatz for cuprates; Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein, PRL 63, 1996 (1989) "Phenomenology of the normal state of Cu-O High T superconductors"

1-loop conductivity

$G_F \longrightarrow$ Fermi surface poles $\frac{1}{k-k_F}$



$\longrightarrow$ Quasiparticle decay controlled by near horizon region. $\omega^{2\nu_k}$

Fermi surface in the bulk
$\longrightarrow$ Fermi surface in the boundary
To see it no need to construct it explicitly, just use grand canonical Hamiltonian, to find poles.

Fermi surface in bulk is a bulk quantum effect. Fermion charge density $\mathcal{O}(1/N^2)$ smaller than BH charge density,

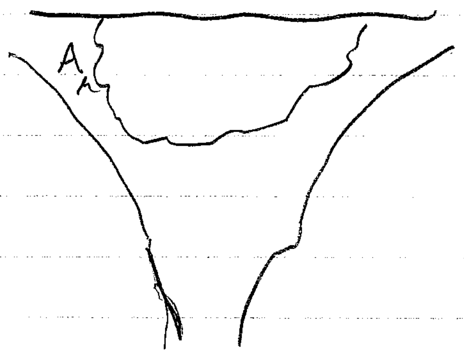
$$\int [dg\, dA\, d\Psi]\, e^{-N^2 S_{grav}}$$

$$= \int [dg\, dA]\, e^{\underbrace{-N^2 S_{grav}}_{\mathcal{O}(N^2)} + \underbrace{\operatorname{Tr}\log(\not{D} - m)}_{\mathcal{O}(1)}}$$

For example the fermion backreaction to the geometry is an $1/N^2$ effect.

Conductivity $\langle J^\mu J^\nu \rangle \rightarrow \langle A^\mu A^\nu \rangle$

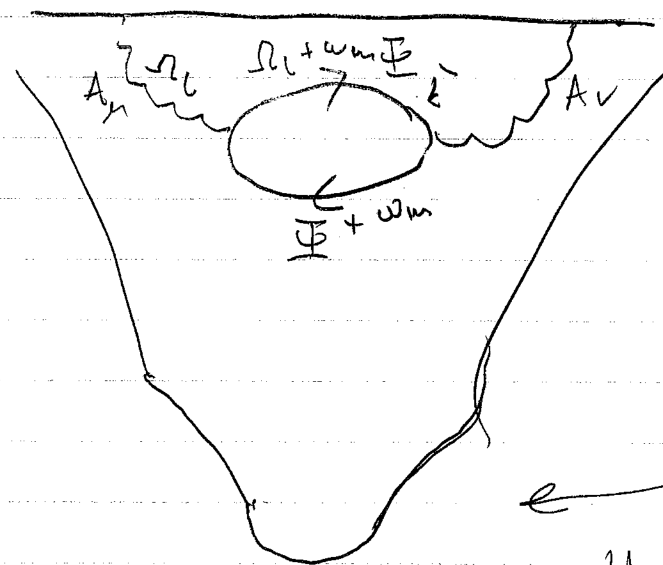
$$S_{bulk} = N^2 \int d^{d+1}x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$



$$\sigma(\omega) = A \frac{i}{\omega} + B\omega^2 + C T^2$$

$$\sigma_{DC} = \delta(0) + C T^2 + \dots$$

This doesn't know anything about bulk fermions.

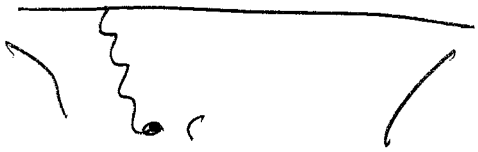


Do Euclidean calculation & in the end analytically continue.

$$G_R(\omega, k) = G_E(\Omega_l, k)\Big|_{\Omega_l \to \varepsilon - i\omega}$$

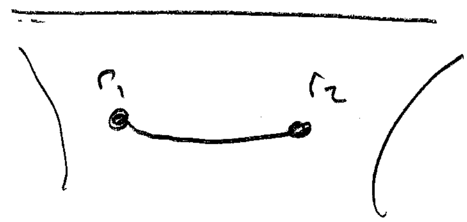
Don't have to use real time formalisms or what's behind the horizon.

$K_A(r, i\Omega_l) =$



Bulk to boundary propagator for A_μ

$D_E(r_1, r_2; i\omega_n) =$



Bulk to bulk propagator for Φ

$$G_E \sim T \sum_{\omega_n} \int d\bar{k}\, dr_1 dr_2\, D_E(r_1, r_2; i\Omega_l + i\omega_n, \bar{k}) \times$$
$$K_A(r_1; i\Omega_l)\, D_E(r_2, r_1; \underset{\substack{\uparrow \\ \text{Fermion Matsubara}}}{i\omega_n}, \bar{k})\, K_A(r_2; \underset{\substack{\uparrow \\ \text{Boson Matsubara}}}{-i\Omega_l})$$

Important technicalities:

1) $D_E(r_1, r_2; i\omega_n, \bar{k}) = \int \frac{d\omega}{2\pi} \frac{\rho(r_1, r_2; \omega, \bar{k})}{i\omega_n - \omega}$

Spectral repr.

2) $\underset{\substack{\uparrow \\ \omega, k}}{\psi(r)}$ normalizable spinor wavefunction in Lorentzian bulk

$$\rho(r, r'; \omega, k) = \psi(r; \omega, k)\, \underset{\substack{\uparrow \\ \text{Boundary spectral function}}}{\rho_B(\omega, k)}\, \bar{\psi}(r'; \omega, k)$$

Can perform r integrals

$$G_E \sim T \sum_{\omega_n} \int d\bar{k} \frac{d\omega_1 d\omega_2}{2\pi \cdot 2\pi} \int dr_1 dr_2 \times$$

$$\times \frac{\Phi(r_1; \omega_1, k)\, \rho(\omega_1, k)\, \bar{\Phi}(r_2, \omega_1, k)}{i\Omega_l + i\omega_n - \omega_1} K_A(r_1, i\Omega_l)$$

$$\times \frac{\Phi(r_2; \omega_2, k)\, \rho(\omega_2, k)\, \bar{\Phi}(r_1, \omega_2, k)}{i\omega_n - \omega_2} K_A(r_2, -i\Omega_l)$$

$$\Lambda(\omega_1, \omega_2, i\Omega_l, \bar{k}) = \int dr_1\, \Phi(r_1; \omega_1, k) K_A(r_1, i\Omega_l) \bar{\Phi}(r_1, \omega_2, k)$$

Perform Matsubara sum

$$T \sum_{\omega_n} \frac{1}{i\Omega_l + i\omega_n - \omega_1} \frac{1}{i\omega_n - \omega_2} = \frac{n(\omega_1) - n(\omega_2)}{\omega_1 - i\Omega_l - \omega_2}$$

Then analytically continue $i\Omega_l \to \Omega + i\varepsilon$
to Minkowski

$$G_R^{(\omega)} \sim T \int d\bar{k} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \Lambda(\omega_1, \omega_2, \Omega, k) \Lambda(\omega_2, \omega_1, -\Omega, k) \times$$

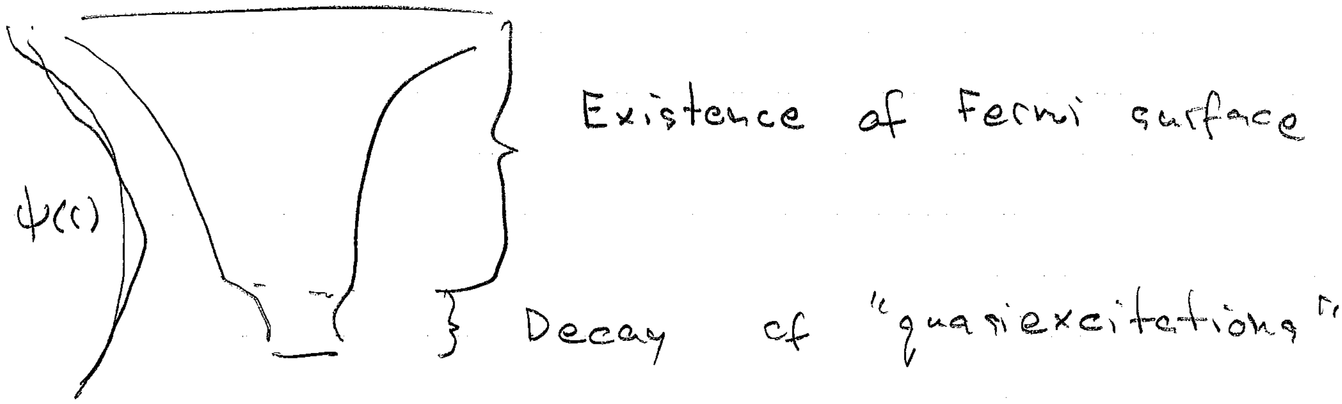
$$\times \rho_B(\omega_1, k)\, \rho_B(\omega_2, k) \frac{n(\omega_1) - n(\omega_2)}{\omega_1 - \Omega - \omega_2 - i\varepsilon}$$

This is formally of the same form as Fermi liquid with "effective vertices" Λ.

All the complication is in Λ.

This simplifies dramatically by arguing that the dominant low T contribution comes from near the Fermi surface.

Still Λ could be terrible. The key point is that there is a "separation of scales",



$\psi(r)$ & K_A have no specifically fast ω, α, k dependence near Fermi surface.

$\Rightarrow$ Near Fermi surface can simply approximate Λ by constant. This is argued in more detail in the paper.

$\Rightarrow$ We get exactly the Fermi gas result

$$\sigma_{DC} \sim \frac{h_1^2}{\mathrm{Im}\,\Sigma}$$

except now h_1 & $\mathrm{Im}\,\Sigma$ have different T dependence.

$$\mathrm{Im}\,\Sigma \sim T^{2\nu_k} \quad , \; h_1 \sim \text{const}$$

$$\sigma_{DC} \sim T^{-2\nu_k}$$

$$R = \frac{1}{\sigma_{DC}} \sim T^{2\nu_k}$$

"Marginal Fermi liquid" case $\nu_k = \frac{1}{2}$

$$\boxed{R \sim T}$$

In holographic non-Fermi liquids, current dissipation controlled by 1-particle decay rate (not always the case e.g. Fermions coupled to gapless bosons).