A BOUND ON THE SPEED OF SOUND FROM HOLOGRAPHY

0905.0900 Hohler + Stephanov

0905.0903 Cherman, Cohen, Nellore

0905.2969 Cherman + Nellore

1408.5116 Bedaque + Steiner

I. Review: Sound Modes in Fluids

Consider (3+1)-dimensional Minkowski space (although nothing that follows ~~depends~~ is ~~crucially~~ unique to that dimension)

~~Consider a relativistic, homogeneous, isotropic~~

$[\eta_{\mu\nu}] = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ $\quad (t, x^i) \quad i = 1,2,3$

Consider a relativistic fluid with conserved stress-energy tensor

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0 \qquad \langle T^{\mu\nu} \rangle = \langle T^{\nu\mu} \rangle$$

with an isotropic, homogeneous thermal equilibrium state

$$\langle T^{tt} \rangle \equiv \varepsilon \qquad \text{energy density}$$

$$\langle T^{ij} \rangle \equiv P \delta^{ij} \qquad \text{pressure}$$

and equation of state $\varepsilon(P)$ or $P(\varepsilon)$

Consider a small fluctuation of stress-energy

$$\langle T^{tt} \rangle = \varepsilon + \langle \delta T^{tt} \rangle$$

$$\langle T^{ij} \rangle = P \delta^{ij} + \langle \delta T^{ij} \rangle$$

$$\langle T^{ti} \rangle = \langle \delta T^{ti} \rangle$$

with $\langle \delta T^{tt} \rangle, \langle \delta T^{ij} \rangle, \langle \delta T^{ti} \rangle \ll \varepsilon, P$

The fluctuations depend on time and space

Fourier transform

$$\langle \delta T^{\mu\nu} \rangle \equiv \int \frac{d^4 q}{(2\pi)^4} \, e^{-iq\cdot x} \langle \delta \tilde{T}^{\mu\nu} \rangle$$

$$q \cdot x = -\omega t + \vec{q} \cdot \vec{x}$$

From the conservation equation

$$-i\omega \langle \delta \tilde{T}^{tt} \rangle + i q_i \langle \delta \tilde{T}^{ti} \rangle = 0$$

$$-i\omega \langle \delta \tilde{T}^{ti} \rangle + i q_j \langle \delta \tilde{T}^{ji} \rangle = 0$$

We want the <u>eigenmodes</u> of $\langle \delta \tilde{T}^{\mu\nu} \rangle$

Among other things, these will include <u>sound</u>

Simplest example: <u>ideal fluid</u>

Landau + Lifshitz vol. 6 <u>Fluid Mechanics</u>

Chapter <u>VIII</u> "Sound"

"... a sound wave in an ideal fluid is, like any motion in an ideal fluid, adiabatic. Hence the small change [...] in the pressure is related to the small change in [energy] density by"

$$\langle \delta \tilde{T}^{ij} \rangle = \frac{\partial P}{\partial \varepsilon} \langle \delta \tilde{T}^{tt} \rangle \, \delta^{ij}$$

Plugging into the conservation equations

$$-i\omega \langle \delta \tilde{T}^{tt} \rangle + i q_i \langle \delta \tilde{T}^{ti} \rangle = 0$$

$$-i\omega \langle \delta \tilde{T}^{ti} \rangle + i q_j \, \delta^{ij} \frac{\partial P}{\partial \varepsilon} \langle \delta \tilde{T}^{tt} \rangle = 0$$

Solving the second equation for $\langle \delta \tilde{T}^{ti} \rangle$ and plugging the result into the first equation

$$\left(\omega^2 - \frac{\partial P}{\partial \varepsilon} \vec{q}^{\,2} \right) \langle \delta \tilde{T}^{tt} \rangle = 0$$

$\Rightarrow$ eigenmodes with $\boxed{\omega = \pm \sqrt{\frac{\partial P}{\partial \varepsilon}} \, |\vec{q}|}$

These are fluctuations of the energy density that propagate with speed V_s given by

$$\boxed{V_s^2 = \frac{\partial P}{\partial \varepsilon}}$$

These are the <u>sound</u> <u>waves</u>

(the <u>only</u> non-trivial excitation of an <u>ideal</u> fluid)

V_s determined <u>entirely</u> by <u>equilibrium</u> <u>thermodynamics</u>!

<u>i.e.</u> the equation of state $P(\varepsilon)$

For a <u>non-ideal</u> <u>fluid</u> in a <u>hydrodynamic</u> regime

to leading order in spatial gradients

$$\langle \delta T^{ij} \rangle = -\frac{1}{\varepsilon + p}\Big[\eta\left(\partial_i \langle \delta T^{ti}\rangle + \partial_j \langle \delta T^{ti}\rangle - \frac{2}{3}\delta^{ij}\partial_k \langle \delta T^{tk}\rangle\right) + \zeta\, \delta^{ij}\,\partial_k \langle \delta T^{tk}\rangle\Big]$$

$\eta \equiv$ shear viscosity

$\zeta \equiv$ bulk viscosity

the term $\propto \eta$ is <u>traceless</u>

the term $\propto \zeta$ has trace $\propto \zeta$

Solving for the eigenmodes again:

1. Transverse fluctuations of $\langle \delta T^{ti} \rangle$

shear mode

$$\omega = -i \frac{\eta}{\varepsilon + p} |\vec{q}|^2 + \dots$$

Purely imaginary $\Rightarrow$ non-propagating dissipative

2. Sound Modes

$$\omega = \pm v_s |\vec{q}| - \frac{i}{2} \frac{1}{\varepsilon + p} \left(\zeta + \frac{4}{3}\eta\right) |\vec{q}|^2 + \dots$$

Dissipation $\Rightarrow$ non-zero dispersion

<u>i.e.</u> the sound wave now <u>decays</u>

Other ways to write $v_s^2 = \frac{\partial P}{\partial \varepsilon}$

just using thermodynamic identities

WHEN ALL CHEMICAL POTENTIALS VANISH

$$\boxed{\mu = 0}$$

$F = -P$ = free energy density

$S \equiv -\frac{\partial F}{\partial T} = \frac{\partial P}{\partial T}$ = entropy density

$c_v \equiv \frac{\partial \varepsilon}{\partial T}$ heat capacity density

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$$v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\partial P/\partial T}{\partial \varepsilon/\partial T} = \frac{S}{c_v} = \frac{S}{T}\left(\frac{\partial S}{\partial T}\right)^{-1} = \frac{\partial \ln T}{\partial \ln S}$$

$w \equiv \varepsilon + P$ enthalpy

$\langle T^{\mu}{}_{\mu} \rangle = \Theta = \varepsilon - 3P$

$\varepsilon = (3w + \Theta)/4$ $P = (w - \Theta)/4$

$$v_s^2 = \frac{1 - d\Theta/dw}{3 + d\Theta/dw}$$

Special values of $v_s^2 = \frac{\partial p}{\partial \varepsilon}$

1. $v_s^2 \geq 0$ because if $v_s^2 < 0$

then v_s must be complex, and

for the sound mode $\mathrm{Im}\,\omega \approx \mathrm{Re}\,\omega$

$\Rightarrow$ No longer a well-defined quasi-particle

(when $\mu = 0$, thermodynamic stability $\Rightarrow$ $v_s^2 = \frac{S}{c_V} \geq 0$)

2. For a theory with dilatation invariance

$T^\mu{}_\mu = 0$ as an <u>operator</u>

Thermal equilibrium $\langle T^\mu{}_\nu \rangle = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$

$\langle T^\mu{}_\mu \rangle = 0 \quad \Rightarrow \quad -\varepsilon + 3p = 0 \quad \Rightarrow \quad \varepsilon = 3p$

$$\boxed{v_s^2 = \frac{1}{3}}$$

First-order hydrodynamics:

$\langle T^\mu{}_\mu \rangle = 0 \quad \Rightarrow \quad \zeta = 0$

Both v_s^2 and ζ measure <u>deviation</u>

from scale invariance (conformality)

3. Causality clearly requires $v_s^2 \leq 1$

A survey of systems suggests $v_s^2 = \frac{1}{3}$ may be an upper bound, however!

(1408.5116 Bedaque + Stecher)

- Non-relativistic systems have $v_s^2 \ll 1$
- Relativistic free massless particles have $v_s^2 = \frac{1}{3}$
- Relativistic free massive particles have $v_s^2 < \frac{1}{3}$
- Weak coupling among the particles $\Rightarrow v_s^2 < \frac{1}{3}$

Ex) $\lambda \phi^4$ with $\lambda \ll 1$ and mass m

when $T \gtrsim m/\sqrt{\lambda}$

hep-ph/9409250 S. Jeon

hep-ph/9512263 Jeon + Yaffe

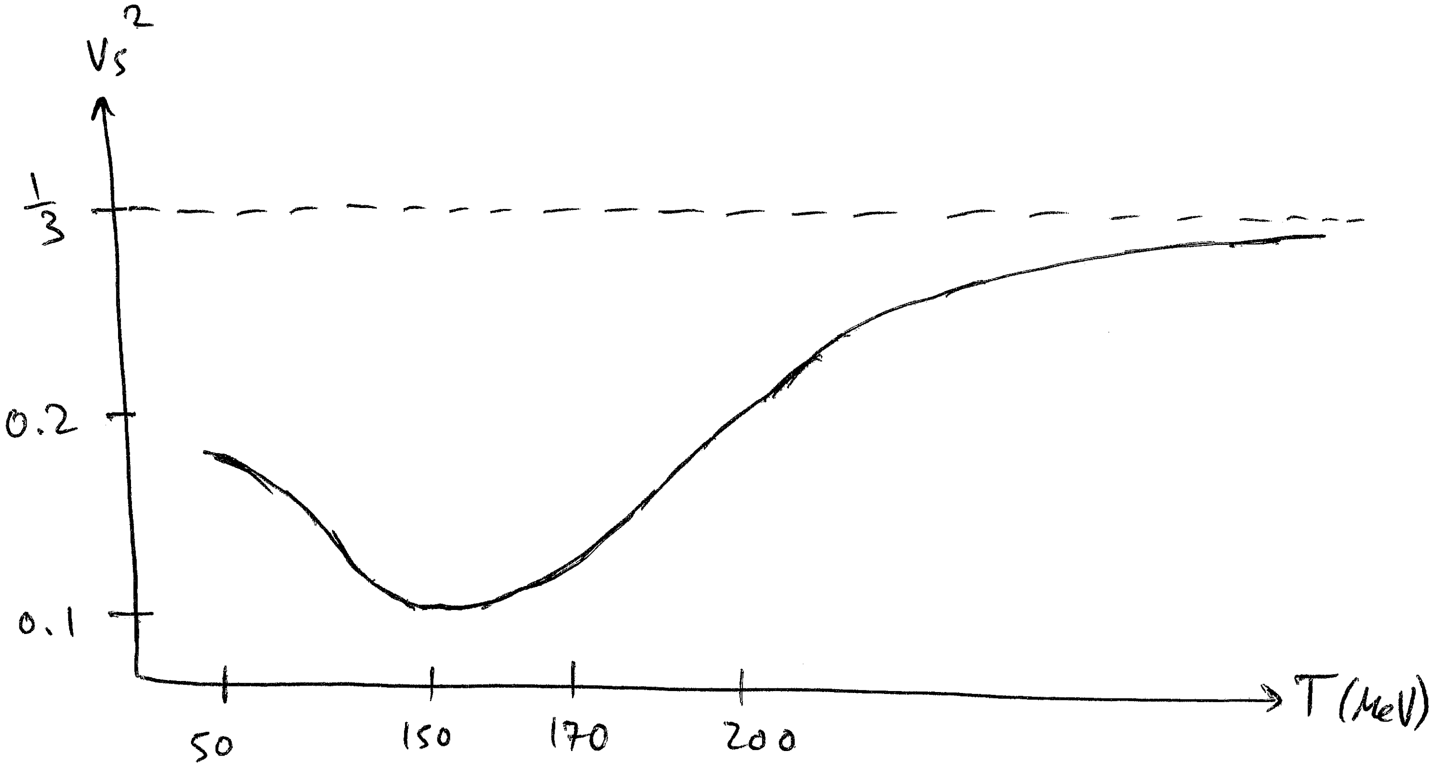
$$v_s^2 = \frac{1}{3} - \frac{5}{12\pi^2} \frac{m^2}{T^2} + O(\lambda^{3/2})$$

- Quantum Chromodynamics has $v_s^2 < \frac{1}{3}$

Lattice QCD results for v_s^2

hep-lat/0601013 Karsch

1007.2580 Borsányi et al.



- ALL KNOWN HOLOGRAPHIC SYSTEMS (for which v_s^2 has been computed) with <u>FIVE</u> exceptions discussed in section III

COUNTER-EXAMPLE

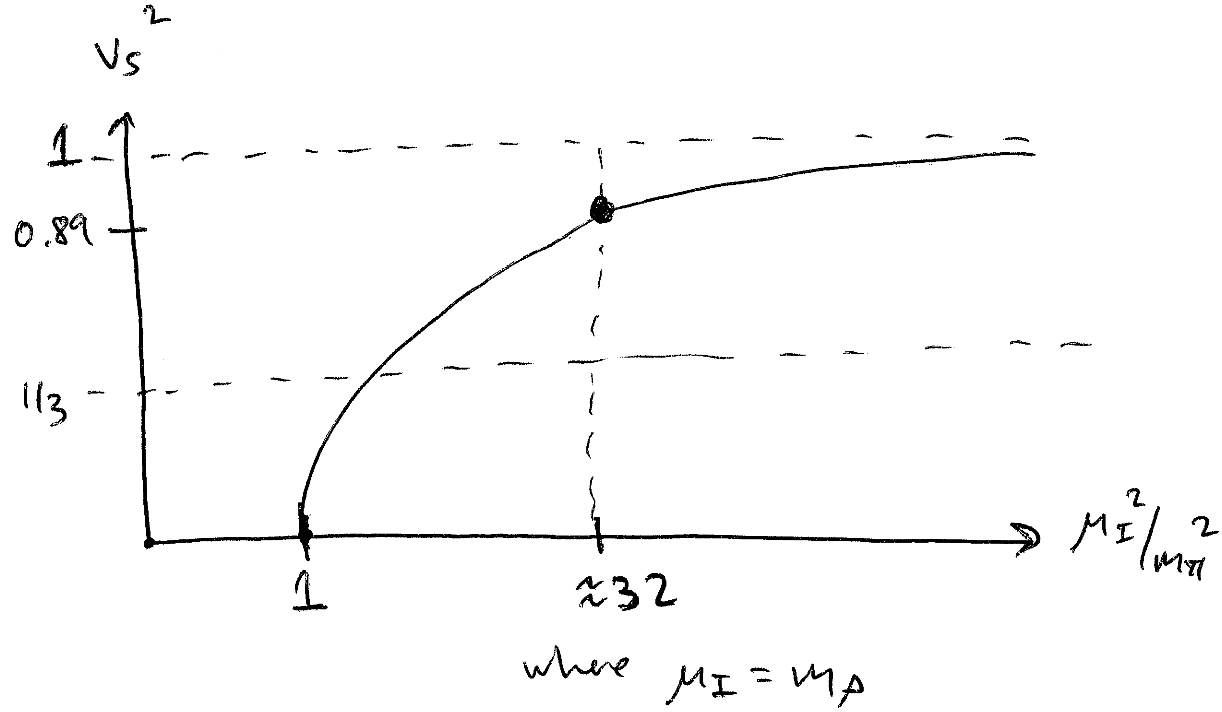
hep-ph/0011365 Son + Stephanov

QCD with $T = 0$ and non-zero <u>isospin chemical potential</u> μ_I

in regime $m_\pi \simeq 140\ \mathrm{MeV} \ll \mu_I \ll m_\rho \simeq 775\ \mathrm{MeV}$
$\simeq 5.66\, m_\pi$

Chiral Perturbation Theory reveals the ground state is a <u>Bose-Einstein Condensate of Pions</u>

$$V_s^2 = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3 m_\pi^2}$$



COUNTER - EXAMPLE

1408.5116 Bedaque + Steiner

the bound $v_s^2 \leq \frac{1}{3}$ puts an upper bound on the masses of neutron stars

$$M \lesssim 2 M_\odot$$

Recently, two neutron stars discovered with $M \gtrsim 2 M_\odot$

suggests bound does not exist

Question: Under what circumstances is $v_s^2 = \frac{1}{3}$ an upper bound?

For what classes of systems?

In what classes of states?

II. A Bound on the Speed of Sound from Holography

0905.0900 Hohler + Stephanov

0905.0903 Cherman, Cohen, Nellore

Consider classical Einstein gravity on asymptotically AdS_5 manifold M minimally coupled to scalar ϕ with <u>any</u> <u>potential</u> $V(\phi)$

$$g_{\mu\nu} \longleftrightarrow T_{\mu\nu}$$

$$\phi \longleftrightarrow \mathcal{O}$$

$\mathcal{O}$ = relevant scalar operator responsible for breaking conformality (<u>i.e.</u> introduce dimensionful <u>source</u> for $\mathcal{O}$)

<u>Theorem</u>: For such systems,

as $T \to \infty$, $v_s^2 \to \frac{1}{3}$ <u>from</u> <u>below</u>

In other words, if we start at $T = \infty$ (14)

and then reduce T,

the correction to $v_s^2 = \frac{1}{3}$ is

ALWAYS NEGATIVE

PROOF OF THE THEOREM

$$S_{grav} = \frac{1}{2\kappa^2} \int_M d^5x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] + S_{\partial M}$$

κ^2 = gravitational constant $\propto 1/N_c^2$ typically

g = determinant of $g_{\mu\nu}$

R = Ricci scalar

$S_{\partial M}$ = boundary terms
(Gibbons - Hawking + counterterms)

Want solutions describing non-zero T

Phase structure depends on $V(\phi)$

For sufficiently high T, we expect a black brane solution

The most general metric that is static, isotropic and homogeneous in field theory directions, asymptotically approaches AdS_5 and has a regular, non-extremal horizon takes the form

$$ds^2 = A(z)^2 e^{2B(z)} \frac{dz^2}{f(z)} + A(z)^2 \left(-f(z)\, dt^2 + d\vec{x}^2\right)$$

with $z \in (0, z_H]$

<u>Boundary conditions</u>: asymptotically AdS_5

$\lim_{z\to 0} f(z) = 1$ $\lim_{z\to 0} e^{2B(z)} = 1$

as $z \to 0$ $A(z) \to \frac{L}{z}$

$\lim_{z\to 0} \phi(z) = 0$

<u>Boundary condition</u>: non-extremal horizon

$f(z_H) = 0$ (simple zero)

Demand that ~~all metric functions~~ $A(z), B(z)$ and $\phi(z)$ are regular at z_H (except g_{zz})

Hawking Temperature

$$T = \frac{|A(z_H)^2 f'(z_H) e^{-B(z_H)}|}{4\pi}$$

Bekenstein - Hawking entropy

$$S = \frac{2\pi}{\kappa^2} |A(z_H)|^3$$

where $S \propto \frac{1}{\kappa^2} \propto N_c^2$

suggests a <u>deconfined</u> phase

The scalar ϕ depends only on z

and vanishes as $z \to 0$

<u>Assumption</u>: $\lim_{z \to 0} V(\phi) = -\frac{12}{L^2} + \frac{1}{2} m^2 \phi^2 + O(\phi^4)$

with $m^2 = \Delta(\Delta - 4)/L^2$

$$\Rightarrow \quad \Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$$

Δ_+ is the dimension of the dual $\mathcal{O}$

Breitenlohner - Freedman stability bound

$$m^2 \ell^2 \geq -4$$

Stay <u>above</u> that : $m^2 \ell^2 > -4$

which $\Rightarrow$ $\Delta_+ > 2$

Restrict to <u>relevant</u> operators

which $\Rightarrow$ $\Delta_+ < 4$

Asymptotically : $\phi(z) = c_- z^{\Delta_-} + c_+ z^{\Delta_+} + \ldots$

c_- = source for dual $\mathcal{O}$

c_+ = VEV for dual $\mathcal{O}$

with choice $2 < \Delta_+ < 4$

these are <u>both</u> dimensionful

$[c_-] = \Delta_-$ $\qquad$ $[c_+] = \Delta_+$

$$\langle \mathcal{O} \rangle = - \frac{\delta S_{grav}}{\delta c_-} \bigg|_{\text{on-shell}} = -(\Delta_+ - \Delta_-)\, c_+$$

Here I will follow 0905.0900 Kohler + Stephanov

Strategy: Compute $\langle T^{tt}\rangle$ and $\langle T^{xx}\rangle$

compute $\frac{\partial P}{\partial \mathcal{E}}$

Asymptotically $g_{\mu\nu}(z) = \frac{L^2}{z^2} \overset{\circ}{g}_{\mu\nu} + \ldots$

$$\langle T^{\mu\nu}\rangle = 2 \frac{\delta S_{grav}}{\delta g^{0}_{\mu\nu}}\Bigg|_{on\text{-}shell}$$

~~Theq~~ "Bare" correlator diverges

could "Renormalize" using counterterms in $S_{\partial M}$

Kohler + Stephanov instead

renormalize by subtracting $T=0$ values

$$\mathcal{E} \equiv \langle T^{tt}\rangle_{T>0} - \langle T^{tt}\rangle_{T=0}$$

$$P \equiv \langle T^{xx}\rangle_{T>0} - \langle T^{xx}\rangle_{T=0}$$

$$w \equiv \mathcal{E} + P \qquad \text{enthalpy}$$

A straightforward calculation gives

$$\mathcal{E} = (3w - c_- c_+ \Delta_- (\Delta_+ - \Delta_-))/4$$

$$P = (w + c_- c_+ \Delta_- (\Delta_+ - \Delta_-))/4$$

which also immediately gives

$$\langle T^\mu{}_\mu \rangle = -\mathcal{E} + 3P = c_- \Delta_- \langle \mathcal{O} \rangle$$

as required by a Ward identity, and

$$v_s^2 = \frac{\partial P}{\partial e} = \frac{1 + c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w}}{3 - c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w}}$$

So far we haven't really used the high-T limit

Do so now, in the form

$$c_- / T^{\Delta_-} \ll 1$$

since c_- is the only dimensionful scale besides T

in the limit $c_-/T^{\Delta} \ll 1$

$$V_s^2 = \frac{1}{3} + \frac{4}{9} c_- \Delta_- (\Delta_+ - \Delta_-) \frac{\partial c_+}{\partial w} + \dots$$

We can <u>linearize in ϕ</u>

The metric is then approximately AdS_5-Schwarzschild

$$B = 0 \quad \forall z$$

$$f(z) = 1 - \frac{w}{4} z^4 \qquad A(z) = \frac{L}{z}$$

The most general solution to

ϕ's linearized equation is then

$$\phi(z) = c_- z^{\Delta_-} {}_2F_1(\Delta_-/4, \Delta_-/4, \Delta_-/2, wz^4/4)$$
$$+ c_+ z^{\Delta_+} {}_2F_1(\Delta_+/4, \Delta_+/4, \Delta_+/2, wz^4/4)$$

BOTH terms diverge logarithmically at $z = z_H$

regularity of $\phi(z_H)$ fixes c_+ in terms of c_-

$$c_+ = - c_- \, w^{(\Delta_+ - \Delta_-)/4} \, D(\Delta_-)$$

$$D(\Delta_-) \equiv \frac{\pi 2^{\Delta_-}}{2-\Delta_-} \cot\left(\frac{\pi \Delta_-}{4}\right) \frac{\Gamma(\Delta_-/2)^2}{\Gamma(\Delta_-/4)^4}$$

$$\phi(z_H) = c_- \, w^{\Delta_+/4 - 1} \, 2^{-\Delta_+/2} (2\Delta_+ - 4) \frac{\Gamma(\Delta_+/4)^2}{\Gamma(\Delta_+/2)}$$

with these results, we find

$$V_s^2 = \frac{1}{3} - \frac{1}{9} c_-^2 \Delta_- (\Delta_+ - \Delta_-)^2 w^{-\Delta_-/2} D(\Delta_-) + \ldots$$

$\uparrow$

NEGATIVE FOR ANY $V(\phi)$

In terms of $\phi(z_H)$

$$V_s^2 = \frac{1}{3} - \frac{1}{18\pi} (4-\Delta_+)(4-2\Delta_+) \tan\left(\frac{\pi \Delta_+}{4}\right) \phi(z_H)^2 + \ldots$$

0905.0903 Cherman, Cohen, Nellore

compute $\phi(z)$'s leading order back-reaction (keeping fixed $c_- L$ and z_H)

and compute $V_s^2 = \frac{\partial \ln T}{\partial \ln s}$ OBTAIN THE SAME RESULT

GENERALIZATIONS

$\boxed{\text{0912.2100 Yarom}}$

generalize AdS_5 to AdS_{d+1}

$$V_s^2 = \frac{1}{d-1} - (d-2\Delta_+)\tan\left(\frac{\pi\Delta_+}{d}\right)\frac{C_-^2}{T^{2(d-\Delta_+)}} D(d,\Delta_+)$$

$$D(d,\Delta_+) \equiv \frac{16^{1-\Delta_+/d}\,(d-\Delta_+)\,d^{2(d-\Delta_+-1)}}{2(4\pi)^{2(d-\Delta_+)}\,(d-1)^2}\left[\frac{\Gamma\left(\frac{\Delta_+}{d}\right)}{\Gamma\left(\frac{\Delta_+}{d}+\frac{1}{2}\right)}\right]^2$$

$\boxed{\text{0905.2969 Cherman + Nellore}}$

generalize from one to multiple scalars

Find the "least relevant" ~~...~~ $\mathcal{O}$

Plug ~~that~~ into the above

that $\mathcal{O}$'s Δ_+

(always assuming ~~...~~ ~~...~~ $\mathcal{O}$ is relevant)

III. Counterexamples from Holography

I checked on the order of 150 papers that explicitly compute v_s^2 in holographic systems

Including finite μ and $T=0$ cases

various phases, including superfluids (s- and p-wave)

~~These included~~ Improved Holographic QCD

Hard-wall + Soft-wall AdS/QCD

Klebanov-Strassler

$\mathcal{N}=2^*$ SYM

Various probe brane systems

also back-reaction

ALL OBEYED the bound, even away from the high-T limit

WITH FIVE EXCEPTIONS

(and ONE SPECIAL CASE)

① 0811.2262 Kulaxizi + Parnachev

Sakai-Sugimoto at $T=0$ (from EOS and QNM)

ZERO SOUND $v_s^2 = \frac{2}{5} > \frac{1}{3}$

② 0908.3493 Karch, Kulaxizi + Parnachev

Dp/Dq intersections

if the defect has d_s spatial dimensions

~~a~~ define $\nu \equiv \dfrac{(p-7)(q-2d-4+p)}{4} + q - d - 1$

ZERO SOUND

$\boxed{\text{from EOS}}$ $V_s^2 = \dfrac{\partial P}{\partial \varepsilon} = \dfrac{1}{\nu}$

ALL values with $p \geq 4$ violate the bound

~~ALL OTHERS obey the bound~~

ALL with $p = 3$ saturate the bound

ALL OTHERS OBEY the bound

③ 0904.1716 Buchel + Pagnutti

asymptotically AdS_4 + two scalars

~~are~~ dual to relevant + irrelevant operators

with particular potential

~~at high T~~ find ~~these~~ solutions where $\overset{\text{breaks } Z_2}{\text{irrelevant}}$ has VEV

for $T \gtrsim T_c$ (high T)

Locally thermo. stable, but not globally

$V_s^2 > \frac{1}{2}$

(4) 1007.3431 Albrecht + Erlich

Hard-wall AdS/QCD
with non-zero isospin μ_I

pion condensation

$$V_s^2 = \frac{\mu_I^4 - m_\pi^4}{\mu_I^4 + 3m_\pi^4}$$

similar to Son + Stephanov, but with
$\mu_I^2, m_\pi^2 \longrightarrow \mu_I^4, m_\pi^4$

(5) 1206.4725 Gorsky + Zayakin

AdS_5 - Maxwell - Chern - Simons
with magnetic field
Chern - Simons $\Rightarrow$ anomaly in CFT current

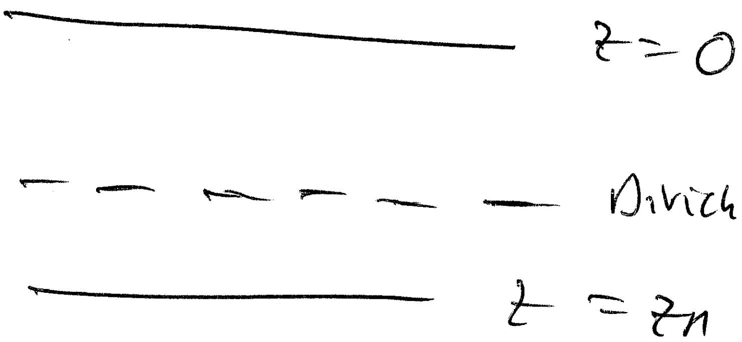
Find sound splits into two sound modes,

one with $V_s^2 \leq \frac{1}{3}$ another with $V_s^2 > \frac{1}{3}$

THIS <u>PAPER IS WRONG</u>

SPECIAL CASE

Wilsonian RG / "Dirichlet brane"



Compute Brown-York stress-energy tensor on Dirichlet brane

Find fluid form, with parameters that depend on z

Find:

1. $T \to \infty$ as $z \to z_H$
2. $v_s^2 \to \infty$ as $z \to z_H$

1011.2451 Strominger et al.
1105.4530 Kapustin + Mulc.
1106.2577 Rangamani
1201.1233 Marolf + Rangamani
1307.1367
1812.0753 Matsuo + Sasai

claim this is consistent with causality!

DUAL CFT always has $v_s^2 = \frac{1}{d-1}$ (!)

0210270 PSS $\mathcal{N}=4$ SYM

0302086 Nezbag M2/M5

0506002 Sharing, Buchel, Yaron — Klebanov-Strassler (using EOS)

0506144 Parnachev & Starinets — NS5 — $v_s^2 = 0$ at Hagedorn temp.

0507026 Benincasa, Buchel, Starinets — $\mathcal{N}=2^*$

0509083 Buchel — KS (using QNM)

0510041 Benincasa & Buchel — $\mathcal{N}=4$ SYM + α' corr. find leading correction vanishes (CFT for any λ!)

0602059 Kovtun & Starinets — spectral functions of $T_{\mu\nu}$

0701132 Mateos, Myers, Thomson — D3/D7 — obeys bound → −∞ at transition Dp result

0605076 Benincasa + Buchel — Sakai-Sugimoto $v_s^2 = \frac{1}{5}$

0607233 Caceres, Natsuume, Okamura — Dp value

0703093 Mas + Tarrio — Dp value, EOS ~~and~~ QNM

0712.2289 Fujita — Dp value, membrane paradigm

0712.2456 Fluid-Gravity — conformal value

0712.2451 2^{nd} order hydro. — conformal value

0804.0434 Gubser + Nellore — AdS + single scalar, obeys bound

0806.0407 Gubser Pufu Rocha — " "

0806.3796 Karch Son Starinets — ZERO SOUND conformal value

0807.2663 Erdmenger, Kaminski, Kerner — D3/D7 w/ $U(1)_B$ + isospin μ

0808.3953 Kulaxizi + Parnachev $v^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2} \leq \frac{1}{3}$ $\mu > m$ OBEYS BOUND (eq. 3.21 + fig. 12)

②

0810.4094 Buchbinder, Buchel, Vázquez — AdS_4 + B field hydro. + gravity bound. OBEYED

0810.4354 Springer — AdS + ~~...~~ multiple scalars, bound OBEYED

• 0811.2262 Kulaxizi + Parnachev — Sakai-Sugimoto $v^2 = 2/5$

0811.4325 Buchbinder + Buchel — AdS_4 + B field OBEYED (but imaginary part)

0812.0792 Kiritsis et al. — IHQCD thermo. OBEYED

0901.0610 Matsuo, Sin et al. — AdS-RN. $T>0$ $v^2 = 1/3$

0901.1487 Kanitscheider + Skenderis — Dp-brane values

0901.2013 David, Mahato, Wadia — D1 value $v^2 = 1/2$ (M2)

0901.2191 Burstein + Medved — EH + corrections in AdS $v^2 = 1/3$

0902.0409 Herzog + Pufu — p-wave 2^{nd} sound

0902.2566 Springer — EH + scalars

0902.3170 Buchbinder + Buchel — AdS_4 + B field OBEYED

0903.1353 Yarom — 4^{th} sound AdS_4 OBEYED $v^2 = 1/2$

0903.1458 de Wolfe + Rosen — AdS + scalar (Kapustin model) OBEYED

0903.3605 Buchel — Klebanov-Strassler ~~...~~ OBEYED

0906.4810 Herzog + Yarom — mostly 2^{nd} + 4^{th} sound

• 0908.3493 Karch, Kulaxizi, Parnachev — Dp/Dq $\nu = \frac{(p-7)(q-2ds-4p)}{4} \cdot \ldots q-ds-1$ $v_s^2 = \frac{1}{\nu}$ VIOLATED FOR $p \geq 4$

0909.2865 Bigazzi et al. — D3/D7 with smeared back-reaction OBEYED

0909.3526 Hung + Sinha — D3/D3 with $U(1)_B$ zero sound $v^2 = 1$

0912.2100 Yarom — AdS + scalar in any d OBEYED

0912.3212 Buchel + Pagnutti — same as 0904.1716

• 0904.1716 Buchel + Pagnutti — AdS_4 + two scalars: at high T, irrelevant scalar has VEV, thermodynamically unstable, but $v^2 > 1/2$

0912.3256 Bigazzi, Cotrone, Tarrio — D3 on cone + smearing, OBEYED

1002.1088 Nunez, Paredes, Ramallo — smeared back-reaction OBEYED

1005.0819	Buchel	AdS + two scalars + μ	OBEYED (3)
1005.4075	Edalati, Jottar, Leigh	Extremal AdS_4-RN	$v^2 = 1/2$
1006.1902	Strominger et al.	Wilsonian cutoff	$v^2 \to \infty$ at horizon
1006.4915	Lee + Pang	probe branes in Lifshitz	$v^2 = \frac{z}{p}$
1006.5461	Kiritsis et al.	IHQCD review	OBEYED
• 1007.3431	Albrecht + Erlich	Hard-wall AdS/QCD with isospin μ_I pion condensation	$v^2 = \frac{\mu^4 - m_\pi^4}{\mu^4 + 3m_\pi^4}$
1008.4350	David, Wadia et al.	D1s, $v^2 = 1/2$ (H2)	
1009.3094	Nickel + Son	zero sound again	
1009.3966	Lee + Pang + Park	zero sound in Lifshitz	
• 1011.2451	Strominger et al.	see 1006.1902	$v^2 \to \infty$ at horizon
• 1105.4530	Kuperstein + Mukhopadhyay	"	
• 1106.2577	Rangamani et al.	Dirichlet surface	$v^2 \to \infty$ at horizon
1107.0931	Kelley	AdS + two scalars,	OBEYED
1109.6343	Davison + Starinets	D3/D7 zero sound + T	OBEYED
1110.1744	Bigazzi et al.	smeared flavor back-reaction	OBEYED
1111.0660	Davison + Kaplis	AdS_4-RN finite T	$v^2 = \frac{1}{2}$ $\forall T$
• 1201.1233	Marolf + Rangamani	Dirichlet surface causality OK	
• 1204.6232	Goykhman, Parnachev, Zaanen $b = \bar{B}/d$ $\bar{B} = 2\pi\alpha' B$	D3/D5 + μ and B zero sound develops $\underline{\text{gap}}$ (AdS_4 codim. one) also normal sound	$v^2 = \frac{1}{2}\frac{1+2b^2}{1+b^2} > \frac{1}{2}$ $\boxed{m=0}$
• 1206.4725	Gorsky + Zayakin	AdS_5-Maxwell-Chern Simons	$v^2 > \frac{1}{3}$
1207.4208	Faulkner + Iqbal	AdS_3 + Maxwell	OBEYED
1209.0009	Brattan, Davison, Gentle, O'Bannon	D3/D5 + D3/D7 μ, B, T	OBEYED
1211.0630	Jokela, Mas, Ramallo, Zoakos	ABJM + D6s	OBEYED
1303.6334	Davison + Parnachev	AdS_4-RN	OBEYED
1306.3816	Pang	AdS-Maxwell-Dilaton	Lifshitz hyperscaling violation
1307.0808	Edalati + Pedraza	ibid.	$v^2 = \frac{z}{d} + \frac{1}{z}$ $z = -\frac{\theta}{z}$
1307.0195	Dey + Roy	ibid.	
• 1307.1367	Kuperstein + Mukhopadhyay	Dirichlet surface	$T \to \infty$, $v^2 \to \infty$ at horizon
1311.6675	Finazzo + Noronha	AdS + Scalar	OBEYED
1312.0463	Davison + Parnachev	$\mathcal{N}=4$ SYM + 2 equal μ	OBEYED $v^2 = \frac{1}{3}$
+ 1312.0753	Matsuo + Sasai	Dirichlet, not asymptotically AdS	$v^2 \to \infty$

1312.2902 Tarrio back-reacting DBI $v^2 = \frac{1}{d}$ OBEYED

1403.5263 Chang, Fujita, Kaminski AdS_3 Maxwell-CS $v^2 = 1$ OBEYED

1409.6357 Arias + Landea p-wave probe 2^{nd} sound AdS_4 OBEYED

Dp-branes non-zero T

$\mu = 0$

$$V_s^2 = \frac{5-p}{9-p}$$

p	V_s^2		
1	$1/2$	< 1	
2	$3/7$	$< 1/2$	
3	$1/3$	$\le 1/3$	
4	$1/5$	$< \frac{1}{4}$	(MS value)
5	0		