An Introduction to the Chiral Magnetic Effect

Andy O’Bannon

University of Oxford
February 24, 2014
Outline:

• The Chiral Magnetic Effect (CME)
• The CME and Heavy-Ion Collisions
• The CME and Holography
The Chiral Magnetic Effect

“Discovered” many times in condensed matter, astrophysics, cosmology...

Relevance for heavy-ion collisions:

Kharzeev, McLerran, Warringa 0711.0950
Kharzeev 0911.3715, 1312.3348
Fukushima, Kharzeev, Warringa 0808.3382
Kharzeev, Warringa 0907.5007
STAR Collaboration 0909.1717
The Chiral Magnetic Effect

(3+1)d Dirac fermions

magnetic field $B$

Net chirality

$\Rightarrow$ CURRENT parallel to $B$
\( S_{\text{Dirac}} = \int d^4 x \bar{\psi} \left[ i \partial \gamma - m \right] \psi \)

\[ U(1)_V \times U(1)_A \]

\( \psi \rightarrow e^{i \alpha} \psi \)

\( \psi \rightarrow e^{i \alpha \gamma^5} \psi \)

\( J^\mu_V = \bar{\psi} \gamma^\mu \psi \)

\( J^\mu_A = \bar{\psi} \gamma^\mu \gamma^5 \psi \)

\( \partial_\mu J^\mu_A = 2im \bar{\psi} \gamma^5 \psi - \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \)
\( \psi_R = \frac{1}{2} (1 + \gamma^5) \psi \quad \psi_L = \frac{1}{2} (1 - \gamma^5) \psi \)

\[
J^\mu_A = \bar{\psi}_R \gamma^\mu \psi_R - \bar{\psi}_L \gamma^\mu \psi_L
\]

\[
\int d^3 x \ J^t_A = N_R - N_L
\]

**Net chirality**

\[
\mu_A = \mu_R - \mu_L
\]
Example: Massless Fermions + $U(1)_V$ Gauge Field

Kharzeev, McLerran, Warringa 0711.0950

\[ J^t_V = 0 \quad J^t_A = 0 \quad J^z_V = 0 \quad J^z_A = 0 \]
Example: Massless Fermions $+$ $U(1)_V$ Gauge Field

$J^t_V = 0 \quad J^t_A \neq 0 \quad J^z_V \neq 0 \quad J^z_A = 0$

Kharzeev, McLerran, Warringa 0711.0950
\[ m = 0 \]

\[ \mathcal{L} = i \bar{\psi} \slashed{D} \psi + \mu_A \bar{\psi} \gamma^t \gamma^5 \psi \]

\[ \psi \rightarrow e^{i\alpha(t) \gamma^5} \psi \]

\[ \mathcal{L} = i \bar{\psi} \slashed{D} \psi + \mu_A \bar{\psi} \gamma^t \gamma^5 \psi - (\partial_t \alpha) \bar{\psi} \gamma^t \gamma^5 \psi \]

\[ \alpha(t) = \mu_A t \]
\[ \mathcal{L} = i \bar{\psi} \slashed{D} \psi - \frac{e^2}{16\pi^2} \alpha(t) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \]

\[ J^i_V = \frac{\delta \mathcal{L}}{\delta A_i} = \frac{e^2}{4\pi^2} \partial_t \alpha(t) \epsilon^{ijk} F_{jk} \]

\[ F_{xy} = B \quad \alpha(t) = \mu_A t \]

\[ J^z_V = \frac{e^2}{2\pi^2} \mu_A B \]
\[ \mathcal{L} = i \bar{\psi} \slashed{D} \psi - \frac{e^2}{16\pi^2} \alpha(t) \varepsilon^{\mu \nu \alpha \beta} F_{\mu \nu} F_{\alpha \beta} \]

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\[ F_{xy} = B \quad \alpha(t) = \mu_A t \]

**CME requires P and CP breaking**
Nonzero mass

\[ \psi \rightarrow e^{i\alpha \gamma^5} \psi \]

\[ m \bar{\psi} \psi \rightarrow m \bar{\psi} e^{i2\alpha \gamma^5} \psi \]

\[ = m \cos(2\alpha) \bar{\psi} \psi + im \sin(2\alpha) \bar{\psi} \gamma^5 \psi \]

\[ \alpha(t) = \mu_A t \implies \text{“rotating mass”} \]
Nonzero mass

\[ \partial_{\mu} J_{A}^{\mu} = 2im\bar{\psi}\gamma^{5}\psi - \frac{e^{2}}{16\pi^{2}}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \]

Net chirality will **decay** in time


Mass works against the CME
Chiral Magnetic Conductivity

Kharzeev and Warringa 0907.5007

$$J^z_V(\omega, k) = \sigma_\chi(\omega, k) B(\omega, k)$$

Kubo formula

$$\Pi^{jk}_R(\omega, k) = i \int d^4x \, e^{i p x} \langle [J^j(x), J^k(0)] \rangle \theta(t)$$

$$\sigma_\chi(\omega, k) = \frac{1}{2i} \frac{1}{k^i} \epsilon^{ijk} \Pi^{jk}_R(\omega, k)$$
Chiral Magnetic Conductivity

The anomaly fixes

\[
\left[ \frac{\partial}{\partial B} J^z_V (\omega, k, B) \right]_{\omega, k, B=0} = \frac{\mu A}{2\pi^2}
\]

“Non-dissipative”
(does not contribute to entropy production)

Otherwise sensitive to interactions
The imaginary part of the Chiral Magnetic conductivity by applying the momentum $p$ to particle-antiparticle pair production in the time order. The Chiral Magnetic conductivity becomes resonant at large temperatures.

At zero temperature we can now recover the real part of the leading order Chiral Magnetic conductivity as a function of frequency, at $T > 0$.

For large temperatures ($T > \mu$), the imaginary part of the leading order Chiral Magnetic conductivity becomes resonant at

$$\sigma^{'} = \sigma^{''} + i \sigma^{'''}$$

Free fermions

$$\sigma^{'} = \sigma^{''} + i \sigma^{'''}$$

$T = 0$

Pair production

Kharzeev and Warringa 0907.5007
### Chiral Magnetic Conductivity

Kharzeev and Warringa 0907.5007

\[
\frac{\sigma_\chi(\omega)}{\sigma_0}
\]

\[
\sigma_\chi = \sigma'_\chi + i\sigma''_\chi
\]

\[
T > 0
\]
Chiral Symmetry Breaking
(Dynamical mass generation)

and

Confinement

Do not FORBID the CME...

...but work AGAINST it.
CME in a Confined Phase
Asakawa, Majumder, Mueller 1003.2436

\[ \mathcal{L}_{\text{eff}} \supset \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi} \frac{1}{f_i} \psi_i F \wedge F = \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi} \frac{1}{f_i} d\psi_i \wedge A \wedge F \]

\[ J^\mu = \frac{\delta}{\delta A_\mu} \int d^4 x \mathcal{L}_{\text{eff}}[A] = \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi} \frac{1}{f_i} \epsilon^{\mu\nu\alpha\beta} \partial_\nu \psi_i F_{\alpha\beta} \]

\[ F_{xy} = B \]

\[ J^z = \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi} \frac{1}{f_i} \partial_t \psi_i B \]

B deflects quark and anti-quark in opposite directions INSIDE pseudo-scalar meson: internally polarized Is the current observable?
\[ \mathcal{L}_{\text{eff}} \supset \frac{e g V}{2 m_V} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} (\partial^\alpha \psi) V^\beta \]

\[ V = \rho, \omega, \phi \]

```
\[ \begin{array}{c}
\text{j(x)} \otimes \rho \\
\rightarrow \pi \otimes B
\end{array} \]
```

“\[ F_{xy} + \partial_t \pi = \rho^z \]”
Vector meson dominance

\[ \mathcal{L}_{\text{eff}} \supset \frac{e g V}{2 m_V} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} (\partial^\alpha \psi) V^\beta \]

\[ V = \rho, \omega, \phi \]

Describes interactions of hadrons with photons

Hadronic components of photon polarization tensor are lightest neutral vector mesons

\[ \rho, \omega, \phi \]

J.J. Sakurai 1960
\[ \mathcal{L}_{\text{eff}} \supset \frac{e g_V}{2m_V} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} (\partial^\alpha \psi) V^\beta \]

\[ V = \rho, \omega, \phi \]

**Vector meson dominance**

\[ J^\mu = - \frac{e m_\rho^2}{g_\rho} \rho^\mu \]
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• The CME and Heavy-Ion Collisions
• The CME and Holography
Relativistic Heavy-Ion Collider

$Au + Au$

$\sqrt{s_{NN}} = 200 \text{ GeV}$
Large Hadron Collider

$Pb + Pb \quad \sqrt{s_{NN}} = 2.76 \text{ TeV}$
Nearly-ideal hydrodynamics

$\eta/s = 0.1 \pm 0.1$

Luzum, Romatschke 0804.4015
Nearly-ideal hydrodynamics

Strongly-coupled Quark-gluon plasma (QGP)

Large elliptic flow

Jet suppression

$t \approx 0.5 - 1 \text{ fm/c}$

$t \approx 10 - 15 \text{ fm/c}$
The CME in Heavy-Ion Collisions

Kharzeev, McLerran, Warringa 0711.0950

$(3+1)d$ Dirac fermions?

QUARKS

B field?

Net chirality?
Positive charges + Angular momentum = Current

\[ dN/d\eta = \sum_{n} a_n \cos(n \eta) + b_n \sin(n \eta) \]

\[ a_n \geq 0, b_n \leq 0 \]

\[ a_2 \geq 0, b_2 \leq 0 \]

\[ a_4 \geq 0, b_4 \leq 0 \]

\[ a_n \geq 0, b_n \leq 0 \]

\[ a_2 \geq 0, b_2 \leq 0 \]

\[ a_4 \geq 0, b_4 \leq 0 \]

\[ a_n \geq 0, b_n \leq 0 \]

\[ a_2 \geq 0, b_2 \leq 0 \]

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\[ a_2 \geq 0, b_2 \leq 0 \]

\[ a_4 \geq 0, b_4 \leq 0 \]

\[ a_n \geq 0, b_n \leq 0 \]

\[ a_2 \geq 0, b_2 \leq 0 \]

\[ a_4 \geq 0, b_4 \leq 0 \]

\[ a_n \geq 0, b_n \leq 0 \]
Magnetic Fields in Heavy-Ion Collisions

\[ b = 4 \text{ fm} \]
\[ b = 8 \text{ fm} \]
\[ b = 12 \text{ fm} \]

\[ eB (\text{MeV}^2) \]
\[ \tau (\text{fm}) \]

\[ Au + Au \]
\[ \sqrt{s_{NN}} = 200 \text{ GeV} \]

\[ CENTER \text{ of QGP} \]

Kharzeev, McLerran, Warringa
0711.0950
<table>
<thead>
<tr>
<th>System</th>
<th>$B$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy-ion Collision</td>
<td>$10^{14} - 10^{15}$</td>
</tr>
<tr>
<td>Neutron star</td>
<td>$10^6 - 10^8$</td>
</tr>
<tr>
<td>Produced in lab</td>
<td>$10^2 - 10^3$</td>
</tr>
<tr>
<td>Levitating frog</td>
<td>$15$</td>
</tr>
<tr>
<td>MRI</td>
<td>$10^0 - 10^1$</td>
</tr>
<tr>
<td>refrigerator magnet</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Earth</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>
Magnetic Levitation of Frogs

Experiment: 1997
Ig Nobel Prize: 2000
for “unusual or trivial achievements in scientific research”

Sir Michael Berry
Berry’s Phase

Andre Geim
Nobel Prize 2010
Magnetic Fields in Heavy-Ion Collisions

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

We will consider the spectators, then we will discuss an approximation for the participants. We will perform both approximations at the origin ($x_\perp = 0$ and $\eta = 0$). In that case the magnetic field is pointing in the $y$-direction, $eB = eB_y$. Especially for large impact parameters the magnetic field at the origin will be a good estimate for the magnetic field at the surface of the interacting region, since the magnetic field in the overlap region is to a good degree homogeneous in the transverse plane.

A.1 Spectator Contribution for $\tau \gg R/\sinh(Y_0)$

For $\tau \gg R \sinh(Y_0)$ the denominator of the integrand of the spectator contribution Eq. (A.6) can be approximated by $\tau^3 \sinh(Y_0)$. Hence we find

$$eB_s \approx Z \alpha_{EM} \exp\left(-\frac{2}{3} Y_0 \frac{R}{b} \tau^3 g(b/R)\right), \quad (A.9)$$

where

$$g(b/R) = \sum_{\pm} g_{\pm}(b/R) = \frac{b}{R} \int d^2 x \rho_{\pm}(x_{\perp}')(1 - \theta_{\pm}(x_{\perp}')). \quad (A.11)$$

We find that to very good approximation $g_{\pm}(b/R) = b/R$. As a result

$$eB_s \approx Z \alpha_{EM} \exp\left(-\frac{2}{3} Y_0 \frac{R}{b} \tau^3\right). \quad (A.12)$$

What is INTEGRATED magnetic field?

Over time?

Over space?
Net Chirality in Heavy-Ion Collisions

Strong interactions preserve P and CP

NET CHIRALITY requires FLUCTUATIONS...

...of WHAT?

“Topological charge density”
Pure $SU(N_c)$ YM

Vacuum: zero-energy, pure gauge configurations

\[ A_t = 0 \quad A_i = \frac{i}{g} \Omega \partial_i \Omega^{-1} \]

\[ \Omega \in SU(N_c) \]

\[ N_{CS} = \frac{g^2}{8\pi^2} \int d^3x \, \epsilon^{ijk} \text{tr} \left[ A_i \partial_j A_k - \frac{2ig}{3} A_i A_j A_k \right] \]
FIG. 1. A schematic representation of the (bosonic) potential energy along a particular direction (labeled $z$) in field space, corresponding to topologically non-trivial transitions between vacua.

A. Review of the standard picture

Fig. 1 is the standard visual aid for thinking about anomalous transitions. Consider the theory in Hamiltonian formalism or in $A_0 = 0$ gauge, where the degrees of freedom are $A(x)$ and the conjugate momenta are $E(x) = -\frac{\partial A(x)}{\partial t}$. The horizontal axis represents one particular direction in the infinite-dimensional space of gauge configurations $A(x)$. The minima represent the vacuum $A = 0$ and large gauge transformations of it, labeled by their Chern-Simons number $N_{cs}$. The vertical axis represents the potential energy of the configurations.

Whenever a transition is made from the neighborhood of one minimum to another (which we call a topological transition), the electroweak anomaly causes baryon number to be violated by an amount proportional to $\Delta N_{cs}$:

$\Delta B \sim \frac{1}{\Lambda} \Delta N_{cs}$

More precisely, the non-fermionic contribution to the potential energy. When a transition is made, there will also be the perturbative energy cost of the fermions created by that transition.

There are also other directions in configuration space along which $N_{cs}$ changes that have nothing to do with the vacuum structure of the theory and exist even in $U(1)$ theories. These directions are not ultimately relevant to baryon number violation. To make the issue more precise, imagine starting with a cold system with some baryon number, heating it up for a time to make anomalous transitions possible, and then quickly cooling it. The system will cool into the nearest vacuum state shown in fig. 1. So the net change in baryon number is:

$B \rightarrow B + \frac{1}{\Lambda} \Delta N_{cs}$

Pure $SU(N_c)$ YM

Vacuum: zero-energy, pure gauge configurations

$A_t = 0$ \quad $A_i = \frac{i}{g} \Omega \partial_i \Omega^{-1}$

$\Lambda$
\[
\frac{g^2}{16\pi^2} \text{tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial_{\mu} K^\mu
\]

\[
K^\mu = \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} \left[ A_\nu \partial_\alpha A_\beta - \frac{2ig}{3} A_\nu A_\alpha A_\beta \right]
\]

\[
N_{CS} = \int d^3 x \ K^t
\]

\[
\Delta N_{CS} = \frac{g^2}{16\pi^2} \int d^4 x \ \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu}
\]
Adding Fermions

\[ \partial_\mu J^\mu_A = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2} F^{EM\mu\nu} \tilde{F}^{EM\mu\nu} - \frac{g^2}{16\pi^2} \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

Ignoring first two terms...

\[ \Delta(N_R - N_L) = -\frac{g^2}{16\pi^2} \int d^4x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

What gauge field configurations have nonzero \( \Delta(N_R - N_L) \)?
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$$\Delta N_{CS} = \frac{g^2}{16\pi^2} \int d^4 x \, \text{tr} \, F^{\mu\nu} \tilde{F}_{\mu\nu}$$

More precisely, the non-fermionic contribution to the potential energy. When a transition is made, there will also be the perturbative energy cost of the fermions created by that transition.

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= Solution of classical equations of motion

= Instanton

= QUANTUM TUNNELING EVENT between vacua
\[ \Delta N_{CS} = \frac{g^2}{16\pi^2} \int d^4x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

The instanton density is exponentially suppressed at high $T$.

Gross, Pisarski, Yaffe Rev. Mod. Phys 53 (1981) 43
\[ \Delta N_{CS} = \frac{g^2}{16\pi^2} \int d^4x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

**UNSTABLE solution of classical equations of motion**

Energy = height of barrier

Klinkhammer and Manton PRD 28 (1983) 2019
FIG. 1. A schematic representation of the (bosonic) potential energy along a particular direction (labeled \( z \)) in field space, corresponding to topologically non-trivial transitions between vacua.

A. Review of the standard picture

Fig. 1 is the standard visual aid for thinking about anomalous transitions. Consider the theory in Hamiltonian formalism or in \( A_0 = 0 \) gauge, where the degrees of freedom are \( A(x) \) and the conjugate momenta are \( E(x) = -\frac{\partial A(x)}{\partial t} \). The horizontal axis represents one particular direction in the infinite-dimensional space of gauge configurations \( A(x) \). The minima represent the vacuum \( A = 0 \) and large gauge transformations of it, labeled by their Chern-Simons number \( N_{cs} \). The vertical axis represents the potential energy of the configurations.

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\[
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\[
N_{CS} = \frac{g^2}{16\pi^2} \int d^4x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu}
\]

Thermally excite sphaleron (false vacuum) which then decays.
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\[
\mathcal{O}(x) \equiv \frac{g^2}{32\pi^2} \text{tr} F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x)
\]

\[
\Gamma \equiv \lim_{t \to \infty} \frac{\langle [\Delta (N_R - N_L)]^2 \rangle}{Vt} = \int d^4x \langle \mathcal{O}(x)\mathcal{O}(0) \rangle
\]

Moore and Tassler 1011.1167
Sphaleron (transition) rate
Chern-Simons diffusion rate
Baryon violation rate

Estimate from perturbation theory:

Arnold, Son, Yaffe hep-ph/9609481

\[ \Gamma \approx 10^2 \alpha_s^5 T^4 \]

\[ \alpha_s = \frac{g^2}{4\pi} \]
\[ \Gamma \approx 10^2 \alpha_s^5 T^4 \]

NAIVELY, this SUGGESTS that in a heavy-ion collision at RHIC or LHC, the rate of chirality production in a region of

1 fm\(^3\)

per 1 fm/c

MIGHT be order one...

...but nobody knows for sure.

Remember: perturbative results for transport coefficients way off!
Final state: charge separation

Chiral Symmetry Breaking and Confinement work against CME

CME in QGP?

Final state: charge separation

Observe P and CP breaking in strong interactions?

Kharzeev, McLerran, Warringa 0711.0950
For experimentalists

**Strong interactions preserve P and CP**

Over many events, all P and CP-odd effects average to zero

Look for event-by-event P and CP-odd effects

Which observables?

Look for charge correlations between particles
For experimentalists

If we see charge $+1$ on one side, do we see charge $-1$ on the other side?
For experimentalists

At both RHIC and LHC, detect charge separation in the final state!

STAR 0909.1717, 0909.1739, 1302.3802, 1303.0901
PHENIX Proc. of the RBRC Workshops vol. 96, 2010
ALICE 1106.2826, 1207.0900, 1211.0890

Signal consistent with estimates based on CME:

Size, centrality dependence, center-of-mass energy dependence, etc.

Existing event-generators cannot reproduce the signal suggests the signal is NEW physics
For experimentalists

At both RHIC and LHC, detect charge separation in the final state!

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Questions

Other (P-even) sources of charge separation?
What about other observables? “Smoking gun?”
U-U collisions?
Can we COMPUTE $\sigma_\chi$ and $\Gamma$ for QCD at $T \approx 2 - 4 \times T_c$?
For theorists

Can we COMPUTE $\sigma_{\chi}$ and $\Gamma$

For QCD at $T \approx 2 - 4 \times T_c$?

NO.
For theorists

Can we COMPUTE $\sigma \chi$ and $\Gamma$?

For QCD at $T \approx 2 - 4 \times T_c$?

Perturbation theory unreliable

Lattice simulation unreliable
For theorists especially difficult to compute from lattice

In lattice QCD, no way to satisfy two conditions simultaneously

\[ \Gamma \]

\[ \text{Moore and Tassler 1011.1167} \]

\[ \text{is topological (second Chern number)} \]

\[ \text{is local (compact support, over a finite number of lattice sites)} \]

\[ \int d^4x \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \]
For theorists

\( \Gamma \) especially difficult to compute from lattice

Moore and Tassler 1011.1167

“We believe that this additional complication will make \( \Gamma \) even harder than other transport coefficients to extract from Euclidean lattice calculations.”
Lattice simulations
Introduce magnetic field, and...

Non-zero topological charge
Buividovich, Chernodub, Polikarpov, et al.
0907.0494, 1011.3795, 1309.2850

\[ qB = 1.8 \text{ GeV}^2 \]

Charge separation
Blum et al. 0911.1348
Bali et al. 1401.4141

Buividovich, Chernodub, Polikarpov, et al.
0907.0494, 1011.3795, 1309.2850
We can parametrize its functional form as
\[ \text{rent} \[6\]. \]

the dielectric correction, which reduces the induced current on the lattice \[15\]. This is very different from the different candidate is a correction by the renormalization. The overall constant is numerically determined as a linearly rising function both of \( \mu \) and \( \text{external magnetic field} \) induces a finite current density as a function of the chiral chemical potential \( \mu \). For simplicity, we consider only the longitudinal direction. These results suggest that an external magnetic field cannot induce the global current in observable amount.

\[ J^z_f \sim \mu A B \]

Lattice simulations
Introduce magnetic field, and...

Axial chemical potential

Yamamoto
1105.0385, 1111.4681, 1207.0375

Buividovich
1309.2850

\[ j^z_f = \frac{1}{2N^3} \sum_{\sigma=1}^N \langle \bar{c}_{\sigma} \gamma^5 u_{\sigma} \rangle \]

\[ j^z_f = \frac{1}{2N^3} \sum_{\sigma=1}^N \langle \bar{c}_{\sigma} \gamma^5 u_{\sigma} \rangle \approx 0.9 \]

\[ j^z_f = \frac{1}{2N^3} \sum_{\sigma=1}^N \langle \bar{c}_{\sigma} \gamma^5 u_{\sigma} \rangle \nonumber \]

\[ j^z_f = \frac{1}{2N^3} \sum_{\sigma=1}^N \langle \bar{c}_{\sigma} \gamma^5 u_{\sigma} \rangle \approx 0.9 \]

\[ j^z_f = \frac{1}{2N^3} \sum_{\sigma=1}^N \langle \bar{c}_{\sigma} \gamma^5 u_{\sigma} \rangle = 0.\]
For theorists

Can we COMPUTE $\sigma_\chi$ and $\Gamma$

For QCD at $T \approx 2 - 4 \times T_c$?

CHANGE THE PROBLEM
Can we find **ANY** **STRONGLY-COUPLED** **NON-ABELIAN** gauge theory for which we **CAN** compute $\sigma_\chi$ and $\Gamma$?
Can we find ANY STRONGLY-COUPLED NON-ABELIAN gauge theory for which we CAN compute $\sigma_\chi$ and $\Gamma$? 

YES!
Can we find **ANY** STRONGLY-COUPLED NON-ABELIAN gauge theory for which we **CAN** compute \( \sigma \chi \) and \( \Gamma \) ?

Use the AdS/CFT Correspondence
Outline:

• The Chiral Magnetic Effect (CME)
• The CME and Heavy-Ion Collisions
• The CME and Holography
Chiral Magnetic Conductivity

$U(1)_V \times U(1)_A$ gauge fields
Anomaly: (4+1)-dim. Chern-Simons term

Sakai-Sugimoto

Soft-wall AdS/QCD

$AdS_5$

Fluid-gravity Models

D3/D7

D3/D7 + $\alpha'$ corrections

Yee
0908.4189

Rebhan, Schmitt, Stricker
0909.4782

Gorsky, Kopnin, Zayakin
1003.2293

Landsteiner, Rebhan, et al.
1005.2587, 1102.4577

Gahramanov, Kalaydzhyan, Kirsch
1102.4334, 1203.4259, 1301.6558

Hoyos, Nishioka, O’Bannon
1106.4030

Ali-Akbari and Taghavi
1209.5900
called “black hole” embeddings. In Euclidean signature, the category is called “Minkowski” embeddings while the solution intersects the AdS-Schwartzschild horizon. In the current context, solutions in the first compact time direction of the background, corresponding to an AdS-Schwarzschild horizon, are monotonically and eventually reach zero, while the general lesson is that chiral symmetry breaking, when it occurs, acts against the CME in our system. Before doing so, let us briefly review what occurs when different temperatures are considered.

Our main result in this section is fig. 2. The green solid curve, namely D7-branes for which the different temperatures are 1 and 2, respectively. (b.) The pseudo-scalar condensate is shown in the same graph. Increasing $\langle \phi \rangle$ and $\langle J^z V \rangle$ versus $V z \mu$ for different temperatures as in (a.).

At $m = 0$ and $m / \mu_A = 1$, for the same temperatures as in (a.). The pseudo-scalar condensate and the CME eventually vanish, summarizing refs. [60–65]. The main difference is that at these temperatures, the CME does not describe a CME and so are of less interest. The figure shows the behavior of the system at different temperatures and how it evolves with changing parameters. The graph is labeled with $T / \mu_A$, with different lines representing $B / \mu_A = 2$, $4 / \pi$, $2 / \pi$, $1 / \pi$, and $0$.
Maxwell gauge fields in AdS, fully back-reacted
(solid = real part)  (dotted = imaginary part)
Figure 5: Time-dependent chiral magnetic conductivity \( \sigma(\omega) \) for various axial chemical potentials in the Sakai-Sugimoto model with \( T = 200 \text{ MeV} \). The solid line is the real part while the dashed line is the imaginary part.

Figure 6: More results in the Sakai-Sugimoto model with \( T = 200 \text{ MeV} \) for other values of axial chemical potentials.

Summary of results

We set-up a couple of holographic frameworks for computing time-dependent chiral magnetic conductivity. In the first model, we consider a full back-reacted Reisner-Nordstrom black-hole solution with only an axial chemical potential turned on, to study the induced electromagnetic current in response to a small, time-dependent magnetic field perturbation. Our second model is based on the more realistic model of Sakai and Sugimoto in its deconfined and chiral symmetry restored phase, but with a quenched/perturbative approximation. Both models give us qualitatively similar results for the frequency dependent chiral magnetic conductivity, which may be a useful complementary computation for the strongly coupled regime to the existing recent weak-coupling computation in perturbative QCD [4]. Our numerical results are presented in Figures 2, 3, 5, and 6 for an illustrative purpose. As the results show, the real part of chiral magnetic conductivity stays to the value at \( \omega = 0 \) for small \( \omega \), contrary to other results in weak-coupling where it drops to 1 as soon as \( \omega \neq 0 \) [4].

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Sphaleron Transition Rate

\[ \mathcal{O}(x) \equiv \frac{g^2}{32\pi^2} \text{tr} F^{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) \]

\[ \Gamma \equiv \int d^4x \left< \mathcal{O}(x)\mathcal{O}(0) \right>_{W} \]

Kubo formula

\[ \Gamma = - \lim_{\omega \to 0} \frac{2T}{\omega} \text{Im} G_R(\omega, \vec{k} = 0) \]
Sphaleron Transition Rate

Axion is dual to \( \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \)

Solve linearized equation of motion for axion

In-going boundary conditions

Take zero frequency and momentum limit

In many backgrounds, just a free, massless scalar

Calculation similar to shear viscosity!
Sphaleron Transition Rate

Son and Starinets
hep-th/0205051

\[ \mathcal{N} = 4 \text{ SYM} \quad N_c, \lambda \to \infty \]

\[ \Gamma = \frac{\lambda^2}{256\pi^3} T^4 \]

Kharzeev 0911.3715

“...which shows that the topological transitions become more frequent at strong coupling, even though the dependence on the coupling is weaker than suggested by”

\[ \Gamma \approx 10^2 \alpha_s^5 T^4 \]
FIG. 1: The Chern-Simons diffusion rate \( \hat{\Omega} = (B, T) / 0(T) \), normalized by the zero magnetic flux value as a function of the dimensionless magnetic field \( B/T^2 \). The diffusion rate is monotonously increasing with the magnetic field strength and has the asymptotic behavior \( \sim B^2/T^4 \) for \( B >> T^2 \).

From the linearized (in \( B^2 \)) Einstein equations we obtain the following relations:

\[
\begin{align*}
\Gamma(B) &= \frac{2}{9} r^2 (1 + 3 \log(r/r_p)) \\
\Gamma(B = 0) &= \frac{2}{9} r^2 \frac{1}{\lambda^2} \\
\end{align*}
\]

These corrections should vanish as \( r \to \infty \) since we should have asymptotically AdS behavior. This condition sets the constant \( \Gamma(B) + \Gamma(B = 0) = 0 \) leading to:

\[
\begin{align*}
\frac{g}{r_p} &= \frac{r^3}{\lambda^2} H \left( 1 + \frac{1}{6} \pi^2 B^2 r^4 H \right) = \frac{\pi^3 T^2}{5} H \left( 1 + \frac{1}{6} \pi^2 B^2 T^4 H \right)
\end{align*}
\]

As a result, to leading order in \( B^2 \) we get:

\[
\begin{align*}
\mathcal{N} &= 4 \text{ SYM} \quad N_c, \lambda \to \infty \\
+ U(1)_R \text{ magnetic field } B
\end{align*}
\]

Comparing with the exact numerical result (36) we have found that this approximation differs from the exact result only by 2.5% for \( B \sim 10 T^2 \). This suggests that a proper expansion parameter is probably \( B^2 / (\lambda^2 T^4) \).

B. Strong magnetic field (low T) limit and BTZ black hole

For strong magnetic field \( B >> T^2 \), \( \Gamma \) and \( \omega \) have the limits:

\[
\lim_{B \to \infty} \frac{\Gamma(B)}{\Gamma(B = 0)} = \frac{B^6}{\pi^2 T^2} \]

In the last step we plugged in the physical magnetic field strength \( B = \sqrt{3} B/v \). Therefore in the presence of a strong magnetic field the diffusion rate is:

\[
\begin{align*}
\mathcal{N} &= 4 \text{ SYM} \quad N_c, \lambda \to \infty \\
+ U(1)_R \text{ magnetic field } B
\end{align*}
\]

It is possible to obtain the same result in a way where the physics is more transparent. In [29], it was shown that there is another solution to Einstein-Maxwell configuration which is a product of a (2+1)-dimensional BTZ black hole.
Sphaleron Transition Rate

Craps, Hoyos, Surowka, Taels
1209.2352

D4-branes wrapped on a circle of radius $R$
anti-periodic boundary conditions for fermions
in high-$T$, deconfined phase

$$\Gamma = \frac{1}{2\pi} \frac{\lambda^3}{3^6 \pi^2} (2\pi R)^2 T^6$$

Fixed by conformal invariance of M5-brane theory
Improved Holographic QCD

\[ S_\alpha = -\frac{1}{2} M_p^3 \int d^5 x \sqrt{-g} \ Z(\lambda) (\partial \alpha)^2 \]

\[ \frac{\Gamma/(Z(\lambda_h)/(2\pi))}{sT/N^2} \]

\[ \frac{\Gamma/(Z(\lambda_h)/(2\pi))}{sT/N^2} \] vs \( T/T_c \)
Sphaleron Transition Rate

Bu, submitted to PRD

\[ \mathcal{N} = 4 \text{ SYM} \quad N_c, \lambda \to \infty \]

\[ \theta = a \tau \]

Anisotropic background of

Mateos and Trancanelli
1105.3472, 1106.1637

\[ \Gamma = \frac{\lambda^2}{256\pi^3} T^4 \left( 1 - \gamma \frac{a^2}{T^2} \right) \]

\[ \gamma \equiv \frac{4 \ln 2 - 1}{8\pi^2} \]
Sphaleron Transition Rate

\[ \mathcal{N} = 4 \text{ SYM} \quad N_c, \lambda \to \infty \]

+1/\lambda \text{ corrections}

AdS$_5$ + Gauss-Bonnet

\[ \Gamma = \frac{\lambda^2}{256\pi^3} T^4 \left( \frac{1 - \sqrt{1 - 4\alpha}}{2\alpha} \right)^{3/2} \]
Open Questions for CME in Holography

Other models?

Chiral magnetic conductivity (full frequency dependence)

Sphaleron transition rate
Open Questions for CME in Holography

What does a sphaleron look like holographically?

Drukker, Gross, Itzhaki
hep-th/0004131

Unstable D-particle

type IIB SUGRA on $AdS_5 \times S^5$

D0-brane
(couples only to metric and dilaton)

If we introduce a sphaleron in a magnetic field, do we see a chiral magnetic current?
Open Questions for CME in Holography

- Time-dependent magnetic field!
- Thermalization time?
- Sphaleron transition rate?
- Chiral magnetic current?

Lin and Yee
1305.3949
Open Questions for CME in Holography

Time-dependent magnetic field!

Photon spectrum? Yee 1303.3571

One striking recent result from PHENIX [7] and ALICE [8] is the large elliptic flow of direct photons at relatively large transverse momentum $p_T > 1$ GeV. At $p_T = 2$ GeV, it is as large as $v_2 = 0.25$. Contrary to hadronic elliptic flow which builds up its magnitude through collective hydrodynamic evolution, high frequency photons coming from early stage are not expected to be sensitive to this late time collective evolution. (The low frequency spectra do get some effects from the Doppler shifts.) Given this, it is natural to seek a more direct source of azimuthal asymmetry relevant for the photon emission at the early stage of the heavy-ion collision.

An interesting candidate is the magnetic field created by two fast moving colliding heavy-charged nuclei [11]. Its magnitude was estimated to be as large as $eB = 10^{17}$ Gauss $\sim m_\pi^2$. Possible experimental signatures of the effects of this magnetic field have been suggested previously in conjunction with triangle anomaly; chiral magnetic/separation effects [11, 12, 13] and chiral magnetic waves [14, 15], leading to charge dependent two-particle correlations [16] and charge-dependent elliptic flows of pions [17, 18]. See also Ref.[19]. These predictions are indeed observed experimentally at STAR/PHENIX [20, 21, 22, 23] and LHC [24], and more experimental results are on the way.
Thank You.