Jonas Probst

HT 2014, Week 8, Holography Journal Club

D - BRANES

References:

- Zwiebach
- Tong: String Theory lecture notes
- Johnson 9606196 review
- Thorlacius 9708078 — " —
- Polchinski 9510017 D-Branes & RR Charges
- Fradkin, Tseytlin (85): Effective FT from Quantised Strings
  — " — (85): Non-linear ED — " —

Outline

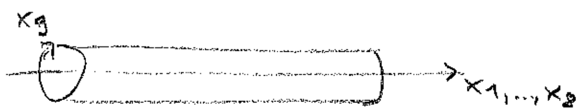
1) T-Duality & D-Branes
2) P-Branes: Solitons in Sugra
3) D-Branes: R-R p-Branes of ST
4) Applications of Non-Perturbative Properties
5) Low Energy Effective Action

1) T-Duality & D-Branes

Johnson: "D-Branes are [..] a more precise language with which we can construct perturbative string backgrounds.
[...] certain properties of D-branes allow us to make powerful statements about such perturbative backgrounds which remain true beyond perturbation theory."

Closed Strings

Compactify X^9 dim on S^1 of radius R



For observer in remaining large dim's (Kaluza-Klein):

$$M^2_{8+1} = (p^9)^2 + (2\pi m R \cdot T)^2 + \frac{2}{\alpha'}(N_L^\perp + N_R^\perp - 2)$$

$$= \frac{n^2}{R^2} + m^2\left(\frac{R}{\alpha'}\right)^2 + \frac{2}{\alpha'}(N_L^\perp + N_R^\perp - 2)$$

$\rightarrow$ invariant under $(\underbrace{n}_{\text{mom.}}, \underbrace{m}_{\text{winding}}, R) \leftrightarrow (m, n, \frac{\alpha'}{R})$

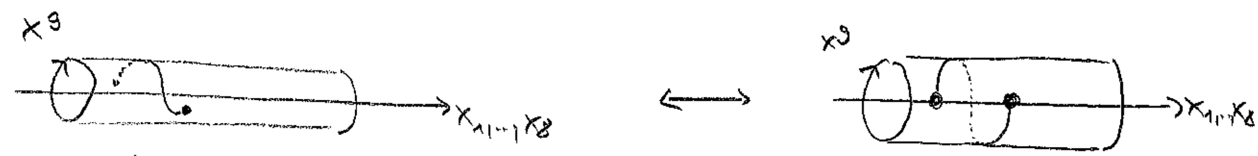
<u>Note</u> $\exists$ also precise def. as a certain trafo of background fields a string moves in in the presence of a Killing vector field

Open Strings

Consider type I, ordinary Neumann (N) bdy cdts in all (9+1)-dim ("D9 brane").

Need winding for T-duality!

$\hookrightarrow$



can be shrunken to 0

$p = \frac{n}{R}$ quantised, no winding

cannot be shrunken

$\to$ winding!

Explicitly:

$$X^9(\tau,\sigma) = X_L^9(\tau+\sigma) + X_R^9(\tau-\sigma), \quad p^9 = \int d\sigma\, \partial_\tau X^9 = \frac{n}{R}$$

Define $\tilde{X}^9(\tau,\sigma) = X_L^9(\tau+\sigma) - X_R^9(\tau-\sigma)$

$$\Rightarrow \quad \partial_\sigma \tilde{X}^9 = \partial_\tau X^9 \quad \Rightarrow \quad \tilde{X}(\tau,\sigma+\pi) - \tilde{X}(\tau,\sigma) = 2\pi n \frac{\alpha'}{R}$$

$$\partial_\tau \tilde{X}^9 = \partial_\sigma X^9 \quad ; \quad \tilde{p}^9 = m \frac{R}{\alpha'}$$

- $\tilde{X}^9$ satisfies Dirichlet (D) bdy cdt $\Rightarrow$ ends lie on hypersurface =: D-brane
- H for the D-type $\tilde{X}$ is the same as for the N-type X

$\longrightarrow$ (D9, S^1 with R) $\overset{T}{\longleftrightarrow}$ (D8, $\frac{\alpha'}{R}$)

Generally (Dp, $q \leq p \times S^1$ with R_i) $\overset{T}{\longleftrightarrow}$ (D$(p-q)$, $q \leq p$ S^1 with $\frac{\alpha'}{R_i}$)

$\Rightarrow$ $\boxed{\text{Dp-brane} = (p+1)\text{-dim. hypersurface on which open strings end}}$

2) P-Branes: Solitons in Sugra

II Sugra: $S_{II} = \underbrace{S_{NS-NS}}_{g_{\mu\nu},\, B_{\mu\nu},\, \phi} + \underbrace{S_{R-R}}_{} + S_{\text{fermionic}}$

IIA: $a^{(1)}, a^{(3)}$ potentials

IIB: $a^{(0)}, a^{(2)}, a^{(4)}$ potentials

$\rightarrow$ $\exists$ classical sol^n, where p-branes (p+1 dim. hypersurface

$\Rightarrow SO(1,9) \rightarrow SO(1,p) \times SO(9-p)$) source

the field strength $F_{p+2} = d \wedge a^{(p+1)}$

$\rightarrow$ p-brane = (p+1) dim. hypersurface charged under $a^{(p+1)}$, soliton

Sol^n characterised by p, ADM mass M & charge $Q = \int_{S^{8-p}} * F_{p+2}$

SUSY $\Rightarrow M \geq Q$

$M > Q$: black p-brane

$M = Q$: extremal p-brane $\rightarrow$ preserves half of SUSY

"BPS state"

BPS state : saturate lower mass bound

⇓

break / preserve $\frac{1}{2}$ SUSY,

sit in short multiplets

→ ◦ zero force between each other

◦ spectrum of masses, charges non-renormalised!

3) D-Branes: R-R p-Branes of ST

Sugra = low energy effective theory of II ST

↳ what are RR charged p-branes in ST ?

Polchinski : D-branes ⊂ p-branes
[open strings!] [closed strings!]

p-brane = classical soln of II

ST : classical soln described by (super-)conformal 2d
worldsheet (WS) action!

⇝ let D-branes be described by open strings ending on them!

Evidence

(i) D-Branes break $\frac{1}{2}$ SUSY

bosons | fermions

N $\quad n^\alpha \partial_\alpha X^m = 0$, $\sigma = 0, \pi$ $\quad \parallel$ brane $\quad \xrightarrow{\text{SUSY}}$ $\quad \psi_L^m = \psi_R^m$ at $\sigma = {}^0_\pi$

↑ normal

D $\quad X^i = \text{const}$, $\sigma = 0, \pi$ $\quad \perp$ brane $\quad$ $\psi_L^i = \pm \psi_R^i$ at $\sigma = {}^0_\pi$

only one linear combi of Q_L & Q_R is unbroken!

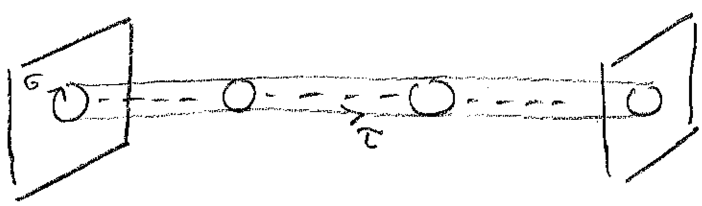
$\Rightarrow \frac{1}{2}$ SUSY

(ii) D-Brane Tension & Charge

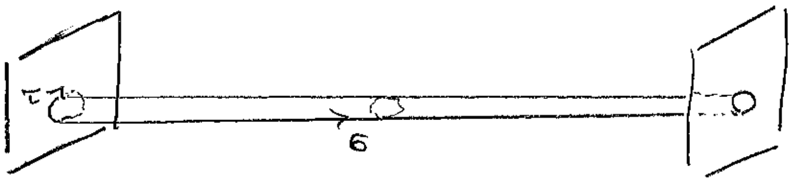
$$\text{tension} = \frac{\text{mass}}{\text{volume}} \quad , \quad \text{mass} \to \text{ADM} : \text{grav. field at infinite transverse distance}$$

II closed strings $\to$ gravity : everything gravitates

$\Rightarrow$ Consider two branes exchanging closed string :



$\updownarrow$ WS duality $\sigma \leftrightarrow \tau$



open string vacuum loop

Amplitude:

$$A = V_{p+1} \underbrace{\int \frac{d^{p+1}k}{(2\pi)^{p+1}}}_{\text{zero modes}} \underbrace{\int_0^\infty \frac{dt}{2t} \sum_{\substack{\text{open string}\\ \text{spectrum } n}} e^{-2\pi t (k^2 + M_n^2)}}_{\substack{\text{renormalized log}\\ \text{[Schwinger rep]}}}$$

$\sim \operatorname{tr} \log (-\partial^2 + M^2)$

$\sim \log \det (-\partial^2 + M^2)^{-1/2}$ ✓

↑
single particle

$= 0$ ✓ SUSY: vac. energy $= 0$

↑
"abstruse Jacobi"

→ closed string picture:

grav. attraction cancels R-R charge repulsion!

⇒ vanishing force between BPS states ✓

Extraction of M & Q:

isolate leading IR behaviour ($\hat{=}$ graviton & R-R boson)

in the modulus integral of the closed string exchange

and compare with FT calculation for effective FT

of $a^{(p+1)}$ potential coupled to p-brane source:

$$S_{\text{eff}} = \frac{1}{2(p+2)!} \int d^{10}x \, (F_{p+2})^2 + i\mu_p \int_{V_{p+1}} a^{(p+1)}$$

$$\hat{=} \; S_{\text{particle}} = \frac{1}{4} \int d^4x \; F^2 + ie \int_{\text{worldline}} A_\mu dx^\mu$$

$\Rightarrow$ p-brane charge $\mu_p^2 = \pm\sqrt{2\pi} \, (2\pi)^{3-p}$

tension $M = \frac{\sqrt{\pi}}{g_s} (2\pi)^{3-p} \sim \frac{1}{g_s} \rightarrow$ soliton!

Note: coupling to gravity $\sim \kappa^2 \cdot M$

$$\sim g_s^2 \frac{1}{g_s}$$

Polchinski: Maldacena conj. $\sim$ adiabatic variation of g_s

Non-trivial check: Dirac quantisation

F_{p+2} coupled to Dp

$\updownarrow$ Hodge

F_{8-p} coupled to $D(6-p)$

Dirac : $\mu_p \cdot \mu_{6-p} = 2\pi n$, $n \in \mathbb{Z}$

D branes : $n = \pm 1$ $\Rightarrow$ D brane = p-brane carrying fundamental unit of R-R charge

Note: D brane $\leftrightarrow$ p brane

rearrangement of degrees of freedom

4) Applications of Non-Perturbative Properties

(i) String / String Duality:

weak	$\longleftrightarrow$	strong
fund. string		soliton string (D1 brane)
("fund. charge")		("monopole")
mass $\sim g_s$		mass $\sim \frac{1}{g_s}$

$\rightsquigarrow$ excitations of D1 brane $\hat{=}$ spectrum of fundamental d.o.f. in dual theory

e.g. II B at g $\longleftrightarrow$ II B at $1/g_s$

string charged under NS-NS $\longleftrightarrow$ soliton string charged under R-R

I at g_s $\longleftrightarrow$ heterotic at $1/g_s$

(ii) Entropy of extremal BH

Strong coupling:
Embedd BH as BPS in string context

$\updownarrow$

Weak coupling:
$\exists$ unique BPS state with same quantum #'s : bound D-branes

$\hookrightarrow$ calculate degeneracy!

5) Low Energy Effective Action

Recall:

Bosonic ST:

Step 1: Spectrum,

quantisation $\to$ massless modes $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

Step 2: Worldsheet Theory,

write down effective action of probe string in background formed by coherent excitations of massless modes,

i.e. $e^{-S} \to e^{-S} e^{-V}$ with V vertex op

[e.g. $V_{grav} = \frac{1}{4\pi\alpha'} \int \sqrt{\gamma}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu\, h_{\mu\nu}\, e^{ip\cdot X}$] :

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{\gamma} \Big\{ G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \gamma^{\alpha\beta} + i B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \varepsilon^{\alpha\beta}$$
$$+ \alpha' \Phi(X) R \Big\}$$

Step 3, Conformal Invariance

Require conformal symm. of quantum theory ($\to$ no ghosts)

$\Leftrightarrow$ vanishing β-functions

$\hookrightarrow$ e.o.m. for background fields

e.g. $\alpha' \left\{ R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} \right\}$

$+ \frac{1}{2}\alpha'^2 \left\{ R_{\mu\lambda\rho\sigma} R_\nu{}^{\lambda\rho\sigma} + \dots \right\} = 0$

Step 4 Target Space Action

$$S = \frac{1}{2\kappa_0^2} \int d^{26}X \sqrt{-G}\, e^{-2\phi} \left(R - \frac{1}{2}|H_3|^2 + 4(\partial\phi)^2 \right)$$

with $G_N \sim \kappa^2 = \kappa_0^2 \, e^{2\Phi_0} \sim l_s^8 \, g_s^2$

reproduces e.o.m.

Now for Dp-brane:

1) $(p+1)$ dim gauge field A_m & $(9-p)$ scalars Φ^i

e.g. $V_A \sim \int_{\partial\Sigma} ds \, \xi_m \partial_\sigma X^m e^{ip\cdot X}$

2) Open Dp-brane string in closed string background:

$$S = S_{IIB} + \int_{\partial\Sigma} ds \left[A_m (x^0, .., x^p) \partial_{\parallel} X^m + \phi_i (x^0, .., x^p) \partial_{\perp} X^i \right]$$

Note: open string bdy $\rightarrow$ only combi $B_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}$ is gauge invariant!

3) (*)

<u>Method</u>: calculate effective action as gen. functional

$$\Gamma = \langle e^{-\int_{\partial\Sigma} ds \, i A_m \partial_s X^m} \rangle$$

$$= \sum_{\chi} g_s^{-2\chi} \int D\gamma_{\alpha\beta} \, DX^m \, e^{-S_{Poly} - \int_{\partial\Sigma} ds \, i A_m \partial_s X^m}$$

$$= V \frac{1}{g_0^2} \sqrt{\det (\gamma_{mn} + 2\pi\alpha' F_{mn})} + O(2 \text{ loops})$$

explicit calculation for $F_{\alpha\beta} = \text{const}$

($\rightarrow \mathcal{L}$ Gaussian)

$\rightsquigarrow$ Natural generalisation of Nambu-Goto $S = -T_p \int d^{p+1}\xi \sqrt{-\det \gamma}$

Method 2

For simplicity: rigid brane in trivial background

(a) Expand X^m around classical sol^n $\bar{X}^m$:

$$X^m = \bar{X} + \sqrt{\alpha'}\, Y^m$$

(b) modified N bdy: $\partial_\sigma \bar{X}^m + 2\pi\alpha' \, i \, F^{mn} \partial_\tau \bar{X}_n = 0$ at $\sigma = 0$

modified propagator (= Green's fct with N bdy cdt)

$$\langle Y^m (z, \bar{z} = z)\, Y^n (w = z + \epsilon, \bar{w} = w) \rangle$$

$$= -\frac{2}{\epsilon} \left(\frac{1}{1 - 4\pi\alpha'^2 F^2} \right)^{mn}$$

(c) $S[X^m = \bar{X}^m + \sqrt{\alpha'}\, Y^m]$

$$= S[\bar{X}] + \frac{1}{4\pi} \int_\Sigma d^2\xi \, \partial Y^m \partial Y^n \delta_{mn} + \frac{i\alpha'}{2} \int_{\partial\Sigma} d\tau \Big\{ F_{mn}(\bar{X}) Y^m \dot{Y}^n$$

$$+ \partial_p F_{mn}(\bar{X}) Y^m \dot{Y}^n \dot{\bar{X}}^p \Big\} + \dots$$

$\rightarrow$ need counterterm

$$\lim_{\epsilon \to 0} \left\{ -\frac{i\alpha'}{2} \int_{\partial\Sigma} d\tau \, \partial_p F_{mn} \langle Y^m(z)\, Y^n(z+\epsilon) \rangle \dot{\bar{X}}^p \right\}$$

for β-fct to vanish

$$\Longrightarrow \quad \partial_p F_{mn} \left(\frac{1}{1 - 4\pi^2\alpha'^2 F^2} \right)^{mn} = 0$$

✓ reproduced by DBI

(*) DBI action

$$S_p = -T_p \int d^{p+1}\xi \sqrt{-\gamma}\, e^{-(\phi-\phi_0)} \sqrt{\det(\gamma_{mn} + B_{mn} + 2\pi\alpha' F_{mn})}$$

$$+ S_{CS} + S_{\text{fermions}}$$

$$\xi^m = X^m , \quad X^i = 2\pi\alpha' \phi^i$$

$$\gamma_{mn} = \partial_m X^\mu \partial_n X^\nu G_{\mu\nu}(X)$$

Low energy: $B_{mn} = \phi = \phi_0 = 0$, $G_{\mu\nu} = \eta_{\mu\nu}$

write $\gamma_{mn} = \eta_{mn} + 2\pi\alpha' \partial_m \phi^i \partial_n \phi^j \delta_{ij}$

and use $\det[1 + 2\pi\alpha'(F + 2\pi\alpha' \partial\phi\partial\phi)]$

$$= \det\left[e^{2\pi\alpha'(F + 2\pi\alpha'\partial\phi\partial\phi)} \left(1 - \tfrac{1}{2}(2\pi\alpha')^2 FF\right)\right] + \mathcal{O}(\alpha'^3)$$

$$\simeq \exp\left[2\pi\alpha' \,\mathrm{tr}(F + 2\pi\alpha'\partial\phi\partial\phi)\right]\left[1 - \tfrac{1}{2}(2\pi\alpha')^2 \,\mathrm{tr}\, FF\right]$$

$\det \exp A = \exp \mathrm{tr} A$

$$= 1 + (2\pi\alpha')^2 \left[\partial_m\phi\, \partial^m\phi - \tfrac{1}{2} F_{mn} F^{nm}\right]$$

$$\rightarrow \quad S_p \simeq -T_p \int d^{p+1}\xi\, (2\pi\alpha')^2 \left\{ \tfrac{1}{4} F_{mn}F^{mn} + \tfrac{1}{2}(\partial\phi)^2 \right\}$$