# Bifurcation phenomena in pipe flow through a sudden expansion

James Seddon[1], Tom Mullin[1], Mick Mantle[2] and Andy Sederman[2]

[1] Manchester Centre for Nonlinear Dynamics, University of Manchester, Oxford Road, Manchester, M13 9PL.

[2] Magnetic Resonance Research Centre, University of Cambridge, Pembroke Street, Cambridge, CB2 3RA.

## 1. Introduction

Flows through pipes with sudden expansions are of both fundamental and practical interest. They occur in several industrial devices,[1] including heat exchangers, combustion chambers, mixing vessels and nuclear reactors, as well as in biological problems such as flow through stenoses.[2]

We are interested in the 3-dimensional problem of flow through a sudden expansion in a circular pipe. A schematic diagram is shown in Figure 1. The flow separates at the point of expansion to create a central jet. This jet reattaches to the walls at a distance downstream which is linearly proportional to Re and a recirculating region is formed in the corners of the expansion.

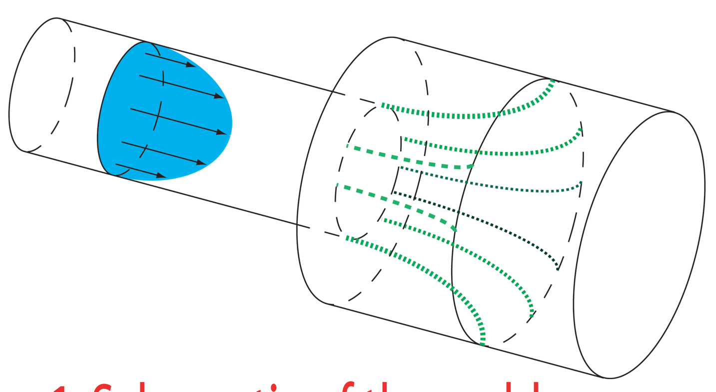

Figure 1: Schematic of the sudden expansion.

## 2. Experiment

The experimental system consisted of two circular pipes, connected together by a 90° step. The upstream pipe had length 1.23m and diameter 8mm, such that the flow of water was fully developed for the full range of Re investigated here (where Re is based on the upstream diameter). The downstream pipe was 1.09m long with diameter 16mm, leading to an expansion ratio of 1:2.

Two flow control mechanisms have been used: constant mass flux and constant pressure head. The results from both methods were comparable to very high accuracy. The constant mass flux system used a high-precision, feedback controlled, constant mass-flux needle valve to control the outflow to an absolute accuracy in Re of 6 at Re=1000.

Magnetic Resonance Imaging was used to visualise the flow field across the entire pipe cross-section in a thin (0.5, 1 or 2mm deep) fluid layer downstream of the expansion. The proton spins of the water molecules were magnetically excited at time t=0. At time t=200ms, the same cross-section of pipe was scanned to detect any remaining excited protons. Hence, spins in rapid flow would move out of the field of view before the scan and spins in zero flow would remain in the field of view and be detected.

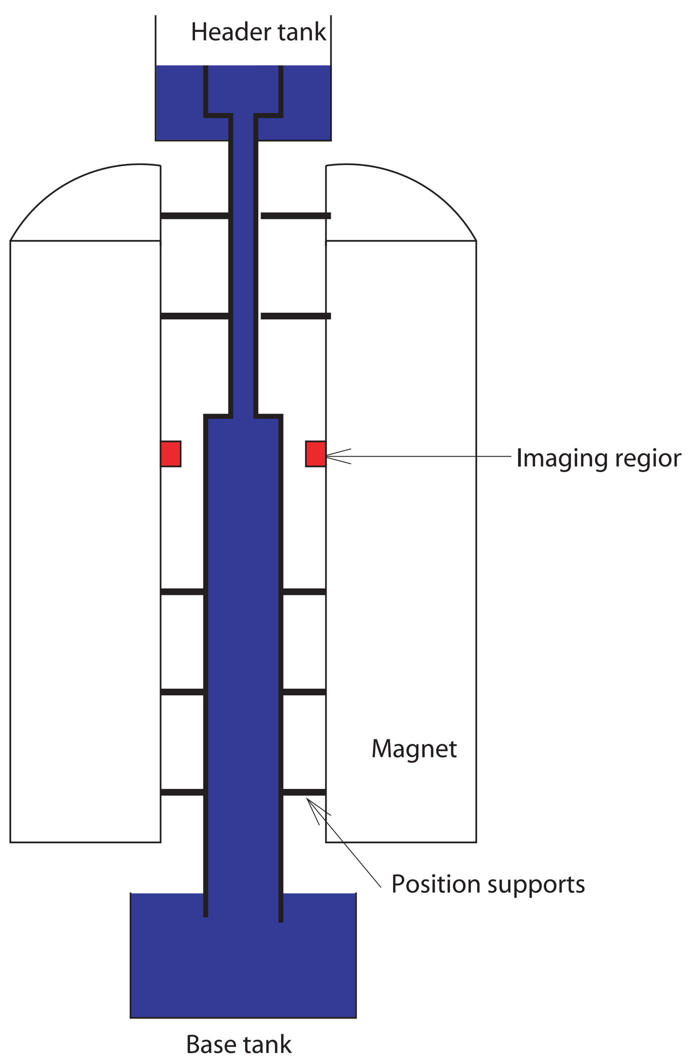


Figure 2: Schematic of the MRI visualisation system.

## 3. Results

Typical MRI images of flow 13.9cm downstream of the expansion are shown in Figure 3. White corresponds to areas of zero flow and black corresponds to areas of faster flow. In the left image, Re=636 and we can see a bright ring corresponding to the pipe walls and a second bright ring corresponding to the shear layer within the recirculating eddy. However, as Re is increased, the shear layer steadily moves towards a wall, becoming asymmetric within the pipe (right image).

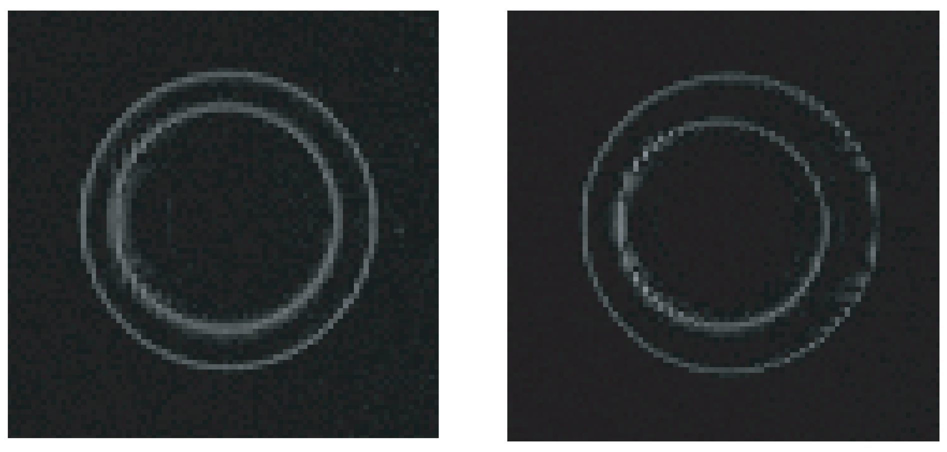

Figure 3: MRI image of pipe cross section. Re=636 (left), Re=1750 (right).

The distance between the centroid of the shear layer and that of the pipe walls provides a useful measure of the amount of asymmetry. Figure 4 is a graph of this measure as a function of Re, revealing a supercritical symmetry breaking bifurcation at Re=1139(10).

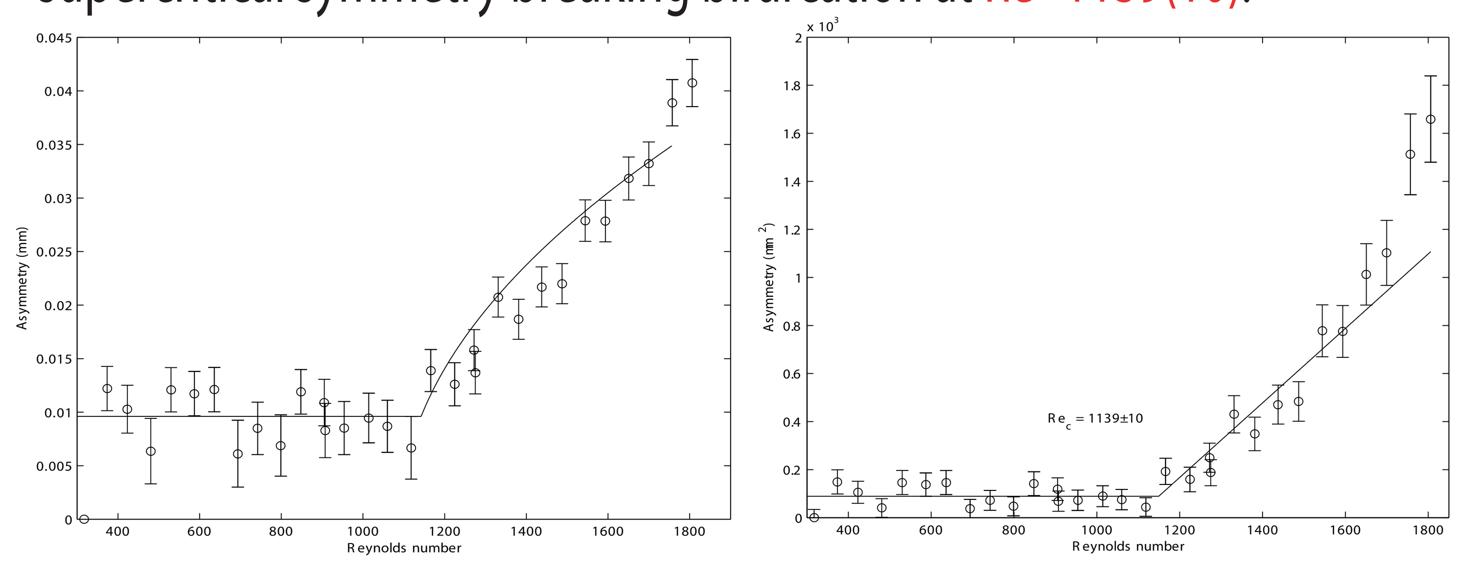


Figure 4: Symmetry breaking bifurcation at Re=1139(10).

Further increase in Re leads to time dependent effects, such as waves in the shear layer, shown in Figure 5.

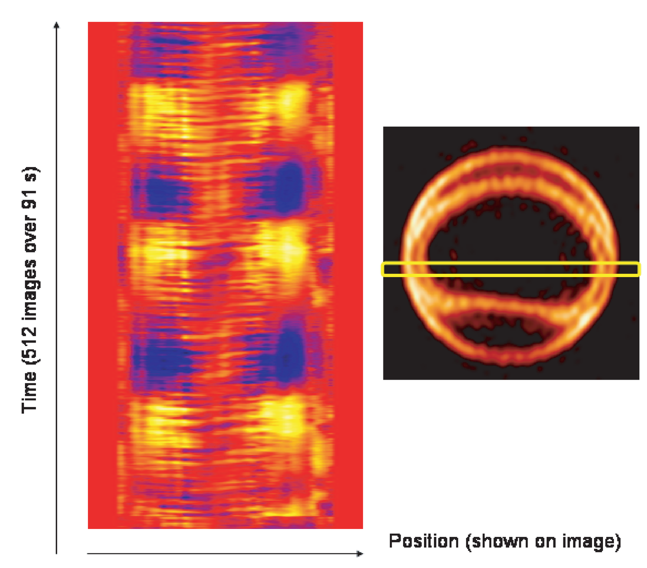


Figure 5: (left) Time progression of the shear layer at Re~1800. (right) waves in the shear layer.

## 4. Conclusions

Flow through a sudden expansion in a 3D pipe loses stability via a supercritical symmetry breaking bifurcation at Re=1139(10). A pitchfork of revolution exists but the flow breaks symmetry in a predominant direction due to imperfections within the system.

At higher Re, the flow becomes time-dependent, possibly via a Hopf bifurcation.

We gratefully acknowledge the EPSRC for funding.

### References

1. Latornell and Pollard, Phys. Fluids 29, 2828-2835 (1986).
2. Sherwin and Blackburn, J. Fluid Mech. 533, 297-327 (2005).