Workshop and Minicourse

"Conceptual Aspects of Turbulence: Mean Fields vs. Fluctuations"

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Rotating convection with anisotropic diffusivities

Introduction. In our model of rotating convection (Chandrasekhar 1961), as well as in many other models of convection (Brestenský et al 2004, 2005ab, 2006ab; Šoltis et al 2004, 2005abc, 2006, 2007abcd) we understand the turbulence as an important background providing anisotropic diffusive coefficients – viscosity and thermal diffusivity.

This anisotropy of diffusive coefficients in realistic fluids is related to the magnetic field, rotation and stratification of the fluid. Eddy diffusivities can be many orders of magnitude greater in the direction of magnetic field as well as in the axis of rotation in comparison with perpendicular directions (Braginsky and Meytlis 1990). Yet, stably or unstably stratified layer significantly decreases or increases eddy diffusivities in the direction perpendicular to the layer in comparison with the directions to the layer.

The motivation of presented model as well as of our other models and the studies of other authors (Phillips and Ivers 2001, Donald and Roberts 2004) is in the geodynamo paradox. We investigate how two various anisotropies, caused e.g. by stratification, influence the onset of marginal convection in the horizontal fluid layer rotating about vertical axis of rotation.

Introducing of anisotropic diffusive coefficients

We suppose that anisotropy of the layer is caused by turbulent state of the fluid.

Dimensionless viscous force, $\boldsymbol{F}_n = E_z[(1-a_n)\partial_{zz} + a_n\nabla^2]\boldsymbol{u},$ related to the anisotropic viscosity and the anisotropic thermal diffusion of temperature perturbation, ϑ,

$$\nabla\cdot\left(k\cdot\nabla\vartheta\right) = \nabla\cdot\left(k_{zz}\partial_z\vartheta\hat{\boldsymbol{z}} + k_{xx}\partial_x\vartheta\hat{\boldsymbol{x}} + k_{yy}\partial_y\vartheta\hat{\boldsymbol{y}}\right)$$

{the dimensionless thermal diffusion is

$k_{zz}^{-1}\nabla\cdot\left(k\cdot\nabla\vartheta\right) = \left[\left(1-a_J\right)\partial_{zz} + a_J\nabla^2\right]\vartheta$}

are chosen in the way that in cartesian geometry, the solutions of governing equations are in separable form.

Investigated cases of viscosity and thermal diffusivity anisotropies

g anisotropic cases (1, 2) $n_{zz} \neq n_{xx} = n_{yy}$

(1) n_{zz} **=** $n_{xx} = n_{yy}$ analogy to upper waters of oceans, o-anisotropy

(2) n_{zz} **?** $n_{xx} = n_{yy}$ rough analogy to Braginsky and Meytlis (1990) model of the Earth's core turbulence ($n_{zz} \approx n_{jj}$ **?** n_{ss}), analogy to the lower atmosphere, a-anisotropy

g isotropic case (3) $n_{zz} = n_{xx} = n_{yy} = n$

The convection with rotation

Linearized dimensionless equations for velocity, vorticity and temperature perturbation in *z*-direction have following form

$$E_z\left(\frac{\partial}{\partial t}\nabla^2 u_z-\nabla_a^2\nabla^2 u_z\right)+\frac{\partial w_z}{\partial z}=R_a\left(\frac{\partial^2 J}{\partial x^2}+\frac{\partial^2 J}{\partial y^2}\right),$$

$$E_z\left(\frac{\partial w_z}{\partial t}-\nabla_a^2 w_z\right)-\frac{\partial u_z}{\partial z}=0$$

$$p\frac{\partial J}{\partial t}-u_z=\nabla_a^2 J,$$

where $R_a=\dfrac{a_T b\, d^2 g}{2\Omega k_{zz}}$, $p=\dfrac{n_{zz}}{k_{zz}}$, $E_z=\dfrac{n_{zz}}{2\Omega d^2}$ is Ekman number,

$$\nabla_a^2=(1-a)\frac{\partial^2}{\partial z^2}+a\nabla^2,\ a=\frac{n_{xx}}{n_{zz}}=\frac{k_{xx}}{k_{zz}}.$$

Results

– anisotropic diffusive coefficients influence the arising convection in all investigated problems in the sense, that ocean (atmospheric) anisotropy type $\left[a > 1 \ \left(a < 1\right)\right]$ inhibit (facilitate) convection and make the horizontal sizes of convecting cells greater (smaller) in comparison with isotropic case $\left(a = 1\right)$

– too strong atmospheric anisotropy type (a **=** 1) effectivelly suppresses inhibiting role of rotation for rotating convection (and magnetic field for magnetoconvection)

– oceanic type of anisotropic diffusivities (a **?** 1) supports the preference of overstability while, stationary convection is preferred at the atmospheric type of anisotropy (a **=** 1)

Conclusions. We would like to indicate that relatively simple experiments could confirm (or not ?) the presented results due to strong influence of anisotropic diffusive coefficients on arising instabilities in the model of rotating convection in planar layer as well as in other models of convection (Benard convection, magnetoconvection and rotating magnetoconvection with various kinds of magnetic field).

References

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Presentation on conferences

Brestenský et al (2004, 2005ab, 2006ab)

– 7th MHD Days Ilmenau 2004, Riga Pamir 2005a, 6th Slovak geophysical conference Bratislava 2005b, EGU Vienna 2006a, SEDI Prague 2006b,

Šoltis et al (2004, 2005abc, 2006ab, 2007abcd)

– SEDI GaPa 2004, EGU Vienna 2005a, IAGA Toulouse 2005b, 8th MHD Days Potsdam 2005c, EGU Vienna 2006a, SEDI Prague 2006b, EGU Vienna 2007a, 7th Slovak geophysical conference 2007b, IUGG Perugia 2007c, Les Houches Summer school 2007d