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Kinetic Theory - MT12
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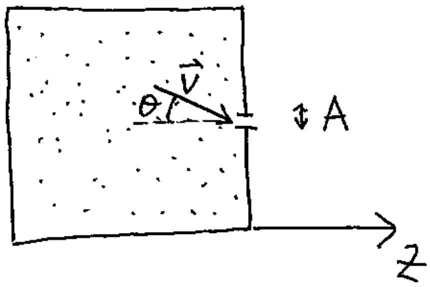
§3. Effusion.

Let's practice our newly acquired knowledge of particle distributions on a simple, but interesting problem.



Make a hole in a container.
If size of the hole

$d \ll \lambda_{mfp}$ (distance particles travel between collisions - TBD in the next section)

then macroscopically the gas does not know about it - so this is a way to abduct particles w/o changing their distribution

(this can be a way to find out what the distribution is inside the container if we have a way of measuring velocities of the escaping particles
— or we might be interested in this problem because we are concerned about small leaks)

So, two interesting questions:

1) Given the distribution inside, what will be the distribution of the emerging particles?
2) Given area of the hole, how many particles escape per unit time? (flux)

We already know the answer! This is just like the calculation of pressure: the escaping particles are the ones that hit the hole.

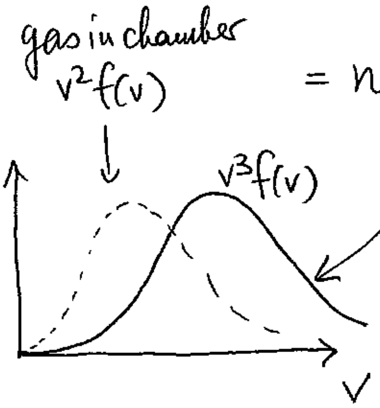
Their # per unit time per unit area, with $[\vec{v}, \vec{v}+d^3\vec{v}]$:

$$d\Phi(\vec{v}) = n v_z f(\vec{v})\, d^3\vec{v} \qquad [\text{see eq. (8) §1}]$$

$$= n v \cos\theta \underset{\substack{\uparrow \\ \text{isotropic}}}{f(v)}\, v^2 \sin\theta\, dv\, d\theta\, d\varphi$$

NB: provided $v_z > 0$

$$= n \underbrace{v^3 f(v)\, dv}_{\text{speed distr.}} \underbrace{\cos\theta \sin\theta\, d\theta\, d\varphi}_{\text{angular distribution}} \qquad (1)$$



So, it's neither isotropic nor Maxwellian

particles travelling at ~90° to the wall escape with greater probability

faster particles get out with greater probability (like smarter students getting in through narrow admissions filter to Oxford)

If we are only interested in the distribution of speeds, integrate out angular dependence:

$$d\Phi(v) = n v^3 f(v)\, dv \int_0^{\pi/2 \,\leftarrow\, \text{NB! } v_z>0!} d\theta \cos\theta \sin\theta \int_0^{2\pi} d\varphi$$

$$= \pi n v^3 f(v)\, dv \qquad (2)$$

$$= \frac{1}{4} n v \tilde{f}(v)\, dv \quad \left(\text{see eq. (15) §1} \quad \tilde{f}(v) = 4\pi v^2 f(v)\right)$$

So, total flux of effusing particles:

$$\Phi = \int_0^\infty dv\, \frac{1}{4} n v \tilde{f}(v) = \frac{1}{4} n \underset{\substack{\uparrow \\ \text{average speed}}}{\langle v \rangle} \qquad (3)$$

Since we know $\tilde{f}(v)$ (Maxwellian),

$$\boxed{\Phi = \frac{1}{4} n \sqrt{\frac{8k_BT}{\pi m}} = \frac{p}{\sqrt{2\pi m k_B T}}} \qquad (4)$$

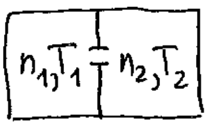
calculate $\langle v \rangle$ for a Maxwellian

use $p = nk_BT$

NB: also need to know what kind of gas! (m)

So, we can predict Φ from macroscopic measurable quantities: p and T. (total flux through hole $= \Phi A$)

Ex. Show that the condition of no mass current between 2 insulated chambers connected by a hole



is $$\frac{p_1}{\sqrt{T_1}} = \frac{p_2}{\sqrt{T_2}} \qquad (5)$$

Ex. Convince yourself that you can obtain (4) from dimensional analysis ($\Phi \propto p/\sqrt{Tm}$, but not the prefactor, of course).

Ex. What is the energy flux through the hole? (i.e. how much energy is the container losing per unit time)