# **F**<sub>D</sub>-Term Hybrid Inflation

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- Standard Big-Bang Cosmology and WMAP
- *F*<sub>*D*</sub>-Term Hybrid Inflation
- Solution to Gravitino Overabundance Problem
- Cosmological and Particle-Physics Implications
- Conclusions and Future Directions

\*Talk based on

- B. Garbrecht and A.P., PLB636 (2006) 154 [hep-ph/0601080];
- B. Garbrecht, C. Pallis and A.P, hep-ph/0605264, to appear in JHEP

• Standard Big-Bang Cosmology and WMAP

## **Density perturbations** as observed by WMAP



$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \sim 10^{-5}$$

#### - Evolution of the Early Universe

Friedman–Robertson–Walker Equation:

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$$H^2 - \frac{\rho}{3 m_{\rm Pl}^2} = -\frac{K}{a^2}$$
, where  $H = \frac{\dot{a}}{a}$ ,  $m_{\rm Pl} = 2.4 \times 10^{18} \,{\rm GeV}$ 

$$\rightarrow \qquad \Omega - 1 = \frac{K}{a^2 H^2}, \qquad \text{where} \quad \Omega = \frac{\rho}{\rho_c}, \qquad \rho_c = 3H^2 m_{\text{Pl}}^2$$

 $\ddot{a} > 0 \rightarrow$  Inflation: accelerated expansion of the Universe.

Quantity	Inflation	Radiation	Matter	
$w = P/\rho$	$-1 \le w \ll -1/3$	$w = \frac{1}{3}$	w = 0	
a(t)	$a_i e^{Ht}$	$a_f \left(t/t_f\right)^{1/2}$	$a_f  (t/t_f)^{2/3}$	
H(t)	const.	1/(2t)	2/(3t)	
ho(t)	const.	$ ho_f a^{-4}$	$ ho_f a^{-3}$	
$d_{ m H}(t)$	$H^{-1}$	$\propto~t^{1/2}$	$\propto~t^{1/3}$	
$D_{ m H}(t)$	$H^{-1}e^{Ht}$	$\propto~2t$	$\propto~3t$	
$L_{ m EH}(t)$	$H^{-1}e^{-Ht}$	$\infty$	$\infty$	

#### - Flatness Problem

SBBM prediction for a radiation dominated epoch:

$$\frac{|\Omega_0 - 1|}{|\Omega_f - 1|} = \frac{t_0}{t_f} = \frac{10^{17} \text{ sec}}{10^{-43} \text{ sec}} = 10^{60}$$

Degree of tuning required: 1 part in  $10^{60}$ !

Solution to the Flatness Problem through Inflation:

$$\frac{|\Omega_f - 1|}{|\Omega_i - 1|} \approx \frac{a_i^2}{a_f^2} \approx e^{-2H(t_f - t_i)} \lesssim 10^{-60}$$

$$\implies \mathcal{N}_e \approx H(t_f - t_i) \gtrsim 60$$

One needs a sufficient long period of inflation of  $\sim 60~e\text{-folds}.$ 

Other problems solved by inflation: horizon and homogeneity problems, dilution of unwanted relics and defects etc.

#### – Inflation Dynamics

Number of *e*-folds:

$$\mathcal{N}_e = \int_{t_{\mathcal{N}}}^{t_{\text{end}}} dt \ H(t) \approx \frac{1}{m_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\mathcal{N}}} d\phi \ \frac{V}{V_{\phi}} \approx 50 - 60$$

Power spectrum of curvature perturbations:

$$P_{\mathcal{R}}^{1/2} = \frac{1}{2\sqrt{3}\pi m_{\rm Pl}^3} \frac{V^{3/2}}{|V_{\phi}|} \approx 4.86 \times 10^{-5} \qquad (k_0 = 0.002 \ {\rm Mpc}^{-1})$$

Spectral index:

$$n_s - 1 = rac{d \ln P_{\mathcal{R}}^{1/2}}{d \ln k} = 2\eta - 6 arepsilon \, pprox \, -0.049 \, {}^{+0.015}_{-0.019}$$
 (WMAP 3 years data)

Slow-roll parameters:

$$\varepsilon = \frac{1}{2} m_{\mathrm{Pl}}^2 \left(\frac{V_{\phi}}{V}\right)^2 \ll 1 , \qquad \eta = m_{\mathrm{Pl}}^2 \frac{V_{\phi\phi}}{V} \ll 1$$

## • **F**<sub>D</sub>-**Term** Hybrid Inflation

## - Hybrid Inflation

[A.D. Linde, PLB259 (1991) 38]



$$V = \frac{\lambda}{4} (|\chi|^2 - M^2)^2 + \frac{1}{2} g |\chi|^2 |\phi|^2 + \frac{1}{2} m^2 |\phi|^2$$

Inflation starts, when  $\phi \gg \phi_c \sim M$ ,  $\chi = 0 \rightarrow V \simeq \frac{\lambda}{4} M^4 + \frac{1}{2} m^2 |\phi|^2$ 

Inflation ends with the so-called waterfall mechanism

#### - **F**-Term Hybrid Inflation

[ E. Copeland, A. Liddle, D. Lyth, E. Stewart, D. Wands, PRD49 (1994) 6410; G. Dvali, Q. Shafi, R. Schaefer, PRL73 (1994) 1886 ]

Superpotential:

$$W = \kappa \,\widehat{S} \,(\widehat{X}_1 \,\widehat{X}_2 - M^2)$$

Real Potential determined from F terms:

$$V = |\partial W/\partial S|^2 + |\partial W/\partial X_1|^2 + |\partial W/\partial X_2|^2$$
  
=  $\kappa^2 |X_1 X_2 - M^2|^2 + \kappa^2 S^2 (|X_1|^2 + |X_2|^2)$ 

Start of inflation:  $S^{\text{in}} > M$ ,  $X_{1,2}^{\text{in}} = 0$ , with  $V = \kappa^2 M^4$ .  $X_{1,2}$ -Mass Matrix:

$$M_{X_{1,2}}^2 = \begin{pmatrix} |\kappa|^2 |S|^2 & -\kappa^2 M^2 \\ -\kappa^{*2} M^2 & |\kappa|^2 |S|^2 \end{pmatrix}$$

End of inflation:  $S < M \rightarrow \det M^2_{X_{1,2}} < 0 \rightarrow \text{waterfall mechanism}.$ 

#### - Slope of the Potential

#### Potential is too flat! $\partial V / \partial S = 0$ .

Radiative lifting of the S-flat direction:

$$V_{1-\text{loop}} = \frac{\kappa^4 M^4}{16\pi^2} \ln\left(\frac{|S|^2}{M^2}\right)$$

SUGRA corrections: 
$$V_{SUGRA} = -c_H^2 H^2 |S|^2 + \kappa^2 M^4 \frac{|S|^4}{2 m_{Pl}^4} + \dots$$

Number of *e*-folds:

$$\mathcal{N}_e = \frac{4\pi^2}{\kappa^2} \frac{(S^{\rm in})^2}{m_{\rm Pl}^2} \approx 55$$

For  $10^{-3} \lesssim \kappa \lesssim 10^{-2} \longrightarrow S^{\text{in}} \lesssim 10^{-1} m_{\text{Pl}} \rightarrow \text{predictive scenario}$ 

Power spectrum:  $P_{\mathcal{R}}^{1/2} = \sqrt{\frac{4\mathcal{N}_e}{3}} \left(\frac{M}{m_{\text{Pl}}}\right)^2 = 5 \times 10^{-5} \rightarrow M \sim 10^{16} \text{ GeV}.$ 

M close to the GUT or gauge-coupling unification scale!

Spectral index:  $n_s - 1 = -\frac{1}{N_e} \approx -0.02$  (mSUGRA).

- **F**<sub>D</sub>-Term Hybrid Inflation

$$W = \kappa \widehat{S} \left( \widehat{X}_1 \widehat{X}_2 - M^2 \right) + \lambda \widehat{S} \widehat{H}_u \widehat{H}_d + \frac{\rho}{2} \widehat{S} \widehat{N}_i \widehat{N}_i + h_{ij}^{\nu} \widehat{L}_i \widehat{H}_u \widehat{N}_j + W_{\text{MSSM}}^{(\mu=0)}$$

+ Subdominant FI *D*-term of U(1)<sub>X</sub>:  $-\frac{g_X}{2}m_{FI}^2D_X$ 

#### Remarks:

- Mass scales:  $m_{\rm Pl}$ , M,  $m_{\rm FI}$  and  $M_{\rm SUSY}$ . Gauge-coupling unification scale may reduce the number of scales, e.g.  $M = M_X \approx 10^{16}$  GeV.
- $\langle S \rangle \sim \frac{1}{\kappa} M_{SUSY}$  sets the Electroweak and the Singlet Majorana scale:

$$\mu = \lambda \langle S \rangle, \quad m_N = \rho \langle S \rangle$$

• Lepton Number Violation mediated by right-handed neutrinos  $N_i$  occurs at the EW scale  $\mu \sim m_N$ .  $\rightarrow$  BAU may be explained by thermal EW-scale resonant leptogenesis.







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## **Related considerations:**

L. Boubekeur and D.H. Lyth, JCAP **0507** (2005) 010.

M. Bastero-Gil, S.F. King and Q. Shafi, hep-ph/0604198.



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## M (in units of $10^{16}$ GeV), $\phi_{ m min}$ , $\phi_{ m max}$ , $\phi_{ m exit}$ (in units of $\sqrt{2}M$ )

$\kappa$	$^{c}H$	M	$\phi_{\min}$	$\phi_{ m max}$	$\phi_{ m exit}$	$\Delta_{ ext{exit}}$	$^{c}H$	M	$\phi_{\min}$	$\phi_{ m max}$	$\phi_{ ext{exit}}$	$\Delta_{\mathrm{exit}}$	
	$n_{ m S}=0.913$			$n_{ m S}=0.951$									
	$\lambda= ho=\kappa$												
0.01	0.179	0.34	73.6	11.9	11.3	0.050	0.130	0.53	32.0	10.8	8.75	0.19	
0.005	0.176	0.34	73.1	6.0	5.7	0.053	0.120	0.53	32.2	6.2	4.48	0.18	
0.001	0.173	0.25	95.6	1.64	1.6	0.028	0.120	0.38	45.0	1.55	1.42	0.09	
0.0005	0.165	0.19	121	1.23	1.21	0.014	0.116	0.28	58.8	1.19	1.15	0.04	
	$\lambda =  ho = 4\kappa$												
0.01	0.216	0.56	49	23.0	22.0	0.046	0.190	0.83	23.0	21.9	17.0	0.22	
0.005	0.188	0.61	41	11.4	10.8	0.050	0.146	0.96	26.0	9.1	8.30	0.19	
0.001	0.177	0.57	43	2.48	2.38	0.043	0.125	0.89	24.6	2.28	1.96	0.14	
0.0005	0.178	0.46	54	1.53	1.49	0.028	0.129	0.68	26	1.45	1.33	0.08	

Fine-tuning parameter:

$$\Delta_{\text{exit}} = \frac{\phi_{\text{max}} - \phi_{\text{exit}}}{\phi_{\text{max}}}$$

## – Post-inflationary Dynamics

$$X_{\pm} = \frac{1}{\sqrt{2}} (X_1 \pm X_2) = \langle X_{\pm} \rangle + \frac{1}{\sqrt{2}} (R_{\pm} + iI_{\pm}),$$
  
with  $\langle X_{\pm} \rangle = \sqrt{2}M$  and  $\langle X_{\pm} \rangle = \frac{v}{\sqrt{2}} = \frac{m_{\rm FI}^2}{2\sqrt{2}M}$ 

Sector	Boson	Fermion	Mass	D-parity
Inflaton $(\kappa$ -sector)	$S$ , $R_+$ , $I_+$	$\psi_{\kappa} = \left( \begin{array}{c} \psi_{X_{+}} \\ \psi_{S}^{\dagger} \end{array} \right)$	$\sqrt{2}\kappa M$	+
${\sf U}(1)_X$ Gauge $(g ext{-sector})$	$V_{\mu}~[I_{-}]$ , $R_{-}$	$\psi_g = \left( egin{array}{c} \psi_{X} \ -\mathrm{i}\lambda^\dagger \end{array}  ight)$	gM	_

$$\Gamma_{\kappa} = \frac{1}{32\pi} (4\lambda^2 + 3\rho^2) m_{\kappa}, \qquad \Gamma_g = \frac{g^2}{128\pi} \frac{m_{\rm FI}^4}{M^4} m_g.$$

#### - Reheat Temperature and Gravitino Constraint

The decays of the inflaton field end its coherent oscillations. The Universe then becomes radiation dominated, when  $\Gamma_{\kappa} \gtrsim H(T_{\kappa}^{\text{reh}})$ :

$$T_{\kappa}^{\rm reh} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_{\kappa} m_{\rm Pl}}$$

Gravitino constraint ( $T_{\kappa}^{\rm reh} \lesssim 10^9$  GeV) implies

$$\kappa \left(\lambda^2 + \frac{3}{4}\rho^2\right) \lesssim 3 \cdot 10^{-15} \times \left(\frac{T_{\kappa}^{\text{reh}}}{10^9 \text{ GeV}}\right)^2 \left(\frac{10^{16} \text{ GeV}}{M}\right)$$

For  $\kappa \approx \lambda \approx \rho \quad \rightarrow \quad \kappa, \ \lambda, \ \rho \ \stackrel{<}{{}_\sim} \ 10^{-5}$ 

This is a bit unnatural, since all couplings must be suppressed.

– The Waterfall *X*-Sector

$$V(X_{\pm}) = \frac{\kappa^2}{4} |X_{\pm}^2 - X_{\pm}^2 - 2M^2|^2 + \frac{g^2}{8} (X_{\pm}^*X_{\pm} + X_{\pm}^*X_{\pm} - m_{\rm FI}^2)^2$$

If  $m_{\rm FI}^2 = 0 \rightarrow V$  is invariant under the discrete symmetry (*D*-parity):

$$X_{\pm} \to \pm X_{\pm}$$

This is true, even after the SSB of the U(1)<sub>X</sub>, because  $\langle X_{-} \rangle = 0$ .

#### **Disasterous Consequence:**

Without a FI *D*-term ( $m_{\rm FI}^2 = 0$ ), the *g*-sector particles would be stable, i.e.  $\Gamma_g = 0$ .

The *g*-sector particles can be produced via preheating.

If they were produced abundantly, they could overclose our Universe!

#### - Thermal History of the Universe



#### - Preheating

[J. Garcia-Bellido, E. Ruiz Morales, PLB536 (2002) 193; LATICEEASY: G. Felder and I.I. Tkachev, hep-ph/0011159]







- SU(2)<sub>X</sub> Waterfall Gauge Sector

$$\mathcal{L}_{\mathrm{kin}} = \int d^{4}\theta \left[ \frac{1}{2} \operatorname{Tr} \left( W^{\alpha} W_{\alpha} \right) \delta^{(2)}(\bar{\theta}) + \frac{1}{2} \operatorname{Tr} \left( \overline{W}_{\dot{\alpha}} \overline{W}^{\dot{\alpha}} \right) \delta^{(2)}(\theta) \right. \\ \left. + \widehat{X}_{1}^{\dagger} \mathrm{e}^{2g \widehat{V}_{X}} \widehat{X}_{1} + \widehat{X}_{2}^{\dagger} \mathrm{e}^{-2g \widehat{V}_{X}^{T}} \widehat{X}_{2} \right].$$

*D*-parities:

#### **Consequences:**

 $\langle X_1 \rangle = \langle X_2 \rangle = (M, 0)^T$  invariant under  $D_{1,2} \longrightarrow D$ -parities survive after the SSB of SU(2)<sub>X</sub>.

*D*-parity conservation  $\longrightarrow$  all *g*-sector particles remain stable, i.e.  $\Gamma_g = 0$ . SU(2)<sub>X</sub>  $\rightarrow$  I  $\longrightarrow$  No cosmic strings and monopoles!

## - Particle Spectrum of the SU(2)<sub>X</sub> Waterfall Gauge Sector

Sector	Boson	Fermion	Mass	$D_1$ -parity	$D_2$ -parity
Inflaton $(\kappa$ -sector)	$S$ , ${}^+\!R_+$ , ${}^+I_+$	$\psi_{\kappa} = \begin{pmatrix} \psi_{+X_{+}} \\ \psi_{S}^{\dagger} \end{pmatrix}$	$\sqrt{2}\kappa M$	+	+
${ m SU(2)}_X$ Waterfall Gauge $(g$ -sector)	$V^1_\mu[{}^-\!I],\ {}^-\!R$ ;	$\psi_g^1 = \left( egin{array}{c} \psi_{-X} \ -\mathrm{i}\lambda^{1^\dagger} \end{array}  ight)$	gM		_
	$V^2_\mu [{}^-\!R_+],\ {}^-\!I_+;$	$\psi_g^2 = \left( egin{array}{c} \mathrm{i}\psi_{-X_+} \ -\mathrm{i}\lambda^{2^\dagger} \end{array}  ight)$	gM	+	_
	$V^3_\mu [^+\!I] \ , \ ^+\!R$	$\psi_g^3 = \left( egin{array}{c} \psi_{+X} \ -\mathrm{i}\lambda^{3^\dagger} \end{array}  ight)$	gM	_	+

**Convention**:

$$Z = \begin{pmatrix} +Z \\ -Z \end{pmatrix}$$

#### - How to Get Small *D*-Parity Violation in a U(1) Waterfall Sector

Introduce a pair of Planck-scale superfields  $\hat{\overline{X}}_{1,2}$  with  $Q(\hat{\overline{X}}_{2(1)}) = +(-)1$ :

$$W_{\mathrm{IW}}^{\mathrm{ext}} = \kappa \widehat{S} \left( \widehat{X}_1 \widehat{X}_2 - M^2 \right) + \xi \, m_{\mathrm{Pl}} \widehat{\overline{X}}_1 \widehat{\overline{X}}_2 + \xi_1 \frac{\left( \widehat{\overline{X}}_1 \widehat{X}_1 \right)^2}{2 \, m_{\mathrm{Pl}}} + \xi_2 \frac{\left( \widehat{\overline{X}}_2 \widehat{X}_2 \right)^2}{2 \, m_{\mathrm{Pl}}} \dots$$
$$\overline{X}_1 \left( \overline{X}_2 \right) \left( \begin{array}{c} \mathcal{F} \\ \mathcal{F}$$

$$\mathcal{D}_{\overline{X}} = -\frac{g}{2} \left( |\overline{X}_1|^2 - |\overline{X}_2|^2 \right) \,, \qquad \mathcal{F} = \left(\xi_1^2 - \xi_2^2\right) \,\frac{M^4}{2 \, m_{\rm Pl}^2} \, \left( |\overline{X}_1|^2 - |\overline{X}_2|^2 \right)$$

$$m_{\rm FI}^2 \approx \frac{\xi_1^2 - \xi_2^2}{8\pi^2} \frac{M^4}{m_{\rm Pl}^2} \ln\left(\frac{m_{\rm Pl}}{M}\right)$$

For  $\xi_{1,2} \lesssim 10^{-3}$  and  $M = 10^{16} \text{ GeV} \implies m_{\text{FI}}/M \lesssim 10^{-6} \longrightarrow T_g \sim 1 \text{ TeV}.$ 

- *D*-Parity Violation in the  $SU(2)_X$  Waterfall Sector





$$\mathcal{D}_{\overline{X}}^{a} = -\frac{g}{2} \left( \overline{X}_{1}^{T} \tau^{a} \overline{X}_{1}^{*} - \overline{X}_{2}^{\dagger} \tau^{a} \overline{X}_{2} \right)$$

$$(m_{\mathrm{FI}}^{1,2,3})^{2} = -\frac{\left[\operatorname{Re}\left(\theta_{-}\zeta_{-}^{*}\right), \operatorname{Im}\left(\theta_{-}\zeta_{+}^{*}\right), \operatorname{Re}\left(\zeta_{+}\zeta_{-}^{*}\right)\right]}{4\pi^{2}} \frac{M^{4}}{m_{\mathrm{Pl}}^{2}} \ln\left(\frac{m_{\mathrm{P}}}{M}\right)$$

For  $\theta_{\pm}, \zeta_{\pm} \sim 10^{-3} \longrightarrow m_{\rm FI}^{1,2,3}/M \lesssim 10^{-6} \longrightarrow T_g \lesssim 1 {\rm ~TeV}$ 

#### • Further Cosmological Implications

## - Can Thermal Right-Handed Sneutrinos be the CDM?

 $\Delta(B-L) = 0 \text{ or } 2 \longrightarrow F_D$ -Term Hybrid Model conserves *R*-parity.

Right-handed sneutrino mass matrix:

$$\mathcal{M}_{\widetilde{N}}^2 = \begin{pmatrix} \rho^2 v_S^2 + M_{\widetilde{N}}^2 & \rho A_\rho v_S + \rho \lambda v_u v_d \\ \rho A_\rho^* v_S + \rho \lambda v_u v_d & \rho^2 v_S^2 + M_{\widetilde{N}}^2 \end{pmatrix}$$

$$\longrightarrow m_{\widetilde{N}_{\text{LSP}}}^2 = \rho^2 v_S^2 + M_{\widetilde{N}}^2 - (\rho A_\rho v_S + \rho \lambda v_u v_d).$$

New LSP interaction:

$$\mathcal{L}_{\text{int}}^{\text{LSP}} = \frac{1}{2} \lambda \rho \, \widetilde{N}_i^* \widetilde{N}_i^* H_u H_d \quad + \quad \text{H.c.}$$

Process:  $\widetilde{N}_{\text{LSP}}\widetilde{N}_{\text{LSP}} \to \langle H_u \rangle H_d \to W^+W^- (m_{\widetilde{N}_{\text{LSP}}} > M_W)$ 

$$\Omega_{\rm DM} h^2 \sim \left(\frac{10^{-4}}{\rho^2 \lambda^2}\right) \left(\frac{\tan\beta M_H}{g_w M_W}\right)^2 \longrightarrow \lambda, \rho \gtrsim 0.1$$

**Process:**  $\widetilde{N}_{\text{LSP}}\widetilde{N}_{\text{LSP}} \to \langle H_u \rangle H_d \to b\bar{b} \quad (M_{H_d} \approx 2m_{\widetilde{N}_{\text{LSP}}} < 2M_W)$ 

$$\Omega_{\rm DM} \, h^2 \, \sim \, 10^{-4} \times B^{-1}(H_d \to \tilde{N}_{\rm LSP} \tilde{N}_{\rm LSP}) \times \left(\frac{M_H}{100 \ {\rm GeV}}\right)^2 \quad \longrightarrow \quad \lambda \,, \rho \, \gtrsim \, 10^{-2}$$

## - Matter-AntiMatter Asymmetry

$$\eta_B^{\text{CMB}} = \frac{n_B}{n_\gamma} = 6.1^{+0.3}_{-0.2} \times 10^{-10}$$
  $(\eta_B^{\text{BBN}} = 3.4 - 6.9 \times 10^{-10}, \text{ at 95\% CL})$ 

Sakharov's conditions for generating the BAU:

- B-violating interactions
- C and CP violation
- Out-of-equilibrium dynamics

**Quality Factor:** Degree of (in)dependence of the observed BAU on the initial conditions:  $Q = \eta_B^{\text{in}}/\eta_B^{\text{fin}}$ .

#### - Resonant Flavour-Leptogenesis at the Electroweak Scale



### LeptoGen



#### Conclusions

- F<sub>D</sub>-Term Hybrid Inflation provides an interesting framework for building a Minimal Particle-Physics and Cosmology Model.
- The  $\mu$ -parameter of the MSSM is tied to an universal Majorana mass  $m_N$ , via the VEV of the inflaton field.
- The entropy release from the late *D*-tadpole-induced decays of the *g*-sector particles offers a simple solution to the gravitino problem.
- Waterfall Sector:  $SU(2)_X \rightarrow I \implies No$  cosmic strings and monopoles.
- Baryon Asymmetry in the Universe can be explained by thermal Electroweak-Scale Resonant Leptogenesis, in a way independent of any pre-existing lepton or baryon-number abundance.

- Further Particle-Physics Implications:
  - Higgs phenomenology of large  $\mu$ -scenarios, such as CPX, where  $\mu = 4M_{SUSY}$  and  $A_t = 2M_{SUSY}$ .

[M. Carena, J. Ellis, A.P., C. Wagner, PLB495 (2000) 155;

D.K. Ghosh, R.M. Godbole, D.P. Roy, PLB628 (2005) 131.]

- Observable Signatures:  $B(\mu \to e\gamma) \sim 10^{-13}$ ,  $B(\mu \to eee) \sim 10^{-14}$ ,  $B(\mu \to e) \sim 10^{-13}$ , LNV/LFV at the ILC.

#### • Future Directions

- Further improvements in the theory of the (pre-inflationary), inflationary and post-inflationary dynamics.
- Study of Cold Dark Matter (CDM) abundances, e.g. of  $\widetilde{N}_{\text{LSP}}$ .
- Further connections between inflation, leptogenesis, CDM, neutrino-mass parameters, Higgs physics and other laboratory observables in constrained minimal versions of the  $F_D$ -Term Hybrid Model.

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- Possible realizations of the  $F_D$ -Term Hybrid Model in GUTs. [e.g.  $E(7) \rightarrow SU(2)_X \otimes SO(12) \rightarrow SU(2)_X \otimes SO(10) \otimes U(1)$ ]
- Model-building constraints from a natural solution to the cosmological constant problem.