YEAR 2: ELECTRICITY AND MAGNETISM

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PROBLEM SET 2: STEADY CURRENTS AND MAGNETISM

† Trickier problems, for people who have finished all the others.

A. Revision (repeated from set 0)

1. A particle moves in a region of space where there is a magnetic field B in the x-direction and an electric field E in the z-direction. At time $t = 0$ it is at the origin with velocity
(a) $\vec{v} = (0, E/B, 0)$,
(b) $\vec{v} = (0, E/2B, 0)$,
(c) $\vec{v} = (0, E/B, E/B)$.
Find, and sketch, its trajectory for each initial condition.

2. A long thin wire carries a current I_1 in the positive z-direction along the axis of a cylindrical co-ordinate system. A thin, rectangular loop of wire lies in a plane containing the axis. The loop contains the region $0 \le z \le b$, $R - a \le r \le R + a$ and carries a current I_2 which has the direction of I_1 on the side nearer the axis. Find the vector force on each side of the loop which results from the current I_1 and the resultant force on the loop.

3. A loop of wire is formed by two semicircles, $r = a$, $0 < \theta < \pi$ and $r = b$, $0 < \theta < \pi$, joined by radial line segments at $\theta = 0$ and $\theta = \pi$. Find the magnetic field at the origin when the wire carries a current I anticlockwise.

4. (a) Show that the magnitude of the magnetic field on the axis of a solenoid closely wound with n turns per unit length and carrying a current I is

$$B = \frac{1}{2}\mu_0 n I(\cos\phi_1 - \cos\phi_2).$$

Use a diagram to define ϕ_1 and ϕ_2 and to show the direction of the field.
(b) Hence show that the magnetic field on the axis of a long solenoid at the ends is half the value at any point within an infinite solenoid with the same n and I.

5. Consider a round straight wire carrying a current density J throughout, except for a round cylindrical hole parallel to the wire axis of constant cross section. Call the radius of the wire R, the radius of the hole R_H and the distance of the centre of the hole from the centre of the wire a. Take $R_H < a < R$ and $R_H < R - a$. Calculate and sketch the magnetic field $\vec{B}$ as a function of position along a radial line through the centre of the hole.

B. <u>Magnetic dipoles</u>

6. A magnetic dipole $\vec{m} = -m_0\hat{z}$ is situated at the origin in an otherwise uniform magnetic field $\vec{B} = B_0\hat{z}$. Show that there exists a spherical surface, centred at the origin, through which no magnetic field lines pass. Find the radius of this sphere, and sketch the field lines, inside and out.

7.† A uniformly charged sphere of radius R carries a constant charge density ρ. It is set spinning with angular velocity ω about the z-axis.
(a) What is the magnetic dipole moment of the sphere?
(b) Find the approximate vector potential at a point (r, θ) where $r \gg R$.
(c) Calculate explicitly the vector potential at a point (r, θ) outside the sphere, and check that it is consistent with (c).
(d) Find the magnetic field at a point (r, θ) both inside and outside the sphere and check that the field obeys the correct boundary condition on the surface of the sphere.

C. <u>Magnetizable materials</u>

8. An infinitely long cylinder of radius R carries a 'frozen-in' magnetisation parallel to the axis

$$\vec{M} = kr\hat{\vec{z}}$$

where k is a constant and r is the distance from the axis. There is no free current anywhere. Find the magnetic field inside and outside the cylinder by
(a) locating *all* bound currents and calculating the field they produce.
(b) finding $\vec{H}$ and using the relationship between $\vec{B}$, $\vec{H}$ and $\vec{M}$.

9. (a) Derive the boundary conditions for $\vec{B}$ and $\vec{H}$ at the surface between two magnetic media.

(b) Under what conditions is it possible to describe the fields in terms of a scalar potential ϕ?

(c) A sphere of radius R of a linear magnetic material with relative permeability μ is placed in a previously uniform magnetic field $\vec{B}$. Find expressions for the scalar potential, both inside and outside the sphere, together with the values of $\vec{B}$, $\vec{H}$ and $\vec{M}$.

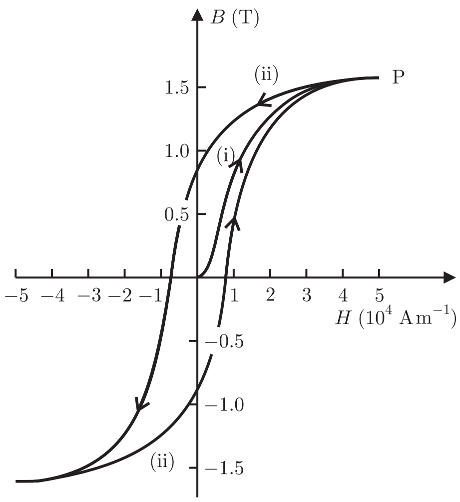
(d) Deduce the value of the magnetic dipole moment of the magnetised sphere from (i) $\vec{M}$ inside the sphere, (ii) the form of $\vec{B}$ (or $\vec{H}$) outside the sphere, and make sure that your answers are consistent.

10. Estimate the magnetic field needed to levitate a frog (see http://www.hfml.ru.nl/froglev.html).

11. (a) A bar of soft iron of cross sectional area A and relative permeability μ is bent into a ring of radius R. The bar is wound with n turns of a wire carrying a current I. Deduce a value for the magnetic field in the iron, explaining your assumptions.

(b) A narrow gap of width w is sawn into the ring. Show that in the gap the magnetic field is

$$B_{gap} = \frac{nI\mu\mu_0}{w\mu + 2\pi R}.$$



12. The soft iron of problem 11 is replaced by a ferromagnet with the $B - H$ hysteresis loop shown above.

(a) Explain what is meant by *hysteresis*, *remanence field* and *coercive force*. How do the magnitudes of these quantities determine the suitability of a ferromagnetic material for application as a (i) permanent magnet, (ii) transformer core?

(b) Show that the magnetic field in the gap may be deduced by finding the intersection of the hysteresis loop with the straight line $Bw + 2\pi R\mu_0 H = \mu_0 nI$.

(c) Sketch B and H in the ring and in the gap.

(d) Show that the work expended as heat in taking the magnet once around the hysteresis loop is approximately $10^4\pi RA$ Joules.

E. Electrodynamics

13. A current I flows in the y-direction through a rectangular bar of conducting material in the presence of a uniform magnetic field $\vec{B}$ along z. The bar has dimensions $L_x \times L_y \times L_z$.

(a) If the moving charges are negative, draw a diagram showing the direction in which they are deflected.

This deflection results in an accumulation of charge on the surfaces of the bar perpendicular to x, and hence a voltage across x, known as the Hall voltage.

(b) Obtain an expression for the Hall voltage for a material with n charge carriers per unit volume and calculate its value for a semiconductor with $n = 10^{21}\text{m}^{-3}$ for a bar of dimensions $L_x = 2 \times 10^{-2}$m, $L_y = 10^{-1}$m, $L_z = 10^{-2}$m carrying a current of 0.16A in a field of 1T.

14. (from Finals 1994) (a) Explain the terms *self-inductance* and *mutual inductance*.

(b) Show that the mutual inductance of two coaxial, single-turn, circular loops of wire of radii a and b ($b \ll a$), when there centres are a distance z apart is

$$M = \frac{\mu_0 \pi a^2 b^2}{2(z^2 + a^2)^{3/2}}.$$

(c) A magnetic dipole of moment $\vec{m}$ aligned along the z-axis is allowed to fall from rest under the action of gravity from the centre of a loop of radius a and high resistance R, and which lies in the (x, y) plane. Calculate the maximum current induced in the loop and the position of the dipole when this occurs.

15. In a *perfect conductor* the conductivity is infinite. Show that

(i) $\vec{E}$=0.

(ii) the magnetic field inside the conductor is independent of time.

(iii) the magnetic flux through a perfectly conducting loop is constant in time.

A *superconductor* is a perfect conductor with the additional property that $\vec{B} = 0$ inside (the Meissner effect). Show that

(iv) the current in a superconductor is confined to the surface.

(v) A sphere of a normal metal of radius R is held in a uniform magnetic field $B_0\hat{\vec{z}}$ and cooled below the critical temperature at which it becomes superconducting. Find the surface current density as a function of the polar angle θ.