Dynamics of a 2D quantum spin liquid: signatures of emergent excitations in Kitaev's honeycomb model

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Outline

Spin liquids

— characteristics and candidates

— dynamics: ordered vs. fractionalised magnets

Kitaev's honeycomb lattice model

— from spins to fluxes and free fermions

Dynamics in the Kitaev model

— signatures of emergent excitations in $S(Q, \omega)$

— relation to quantum quench and x-ray edge problems

Spin liquids

- **Many types**

  e.g. gapped vs gapless

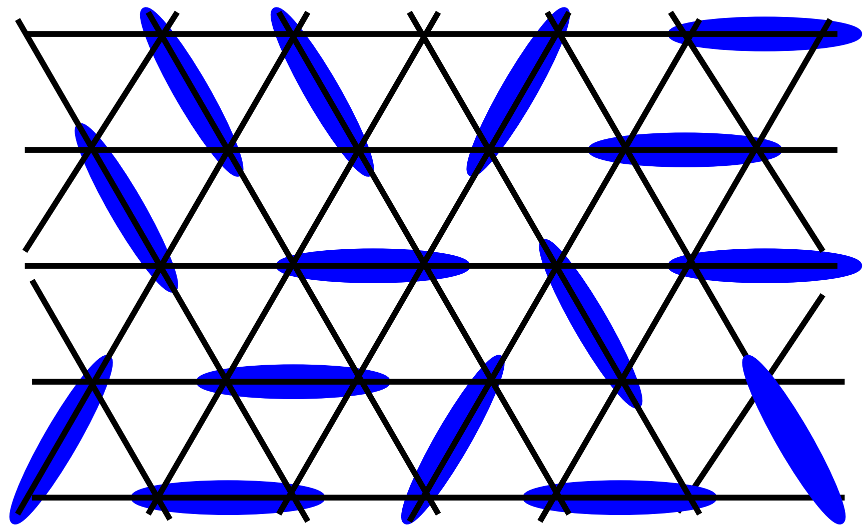
- **Unusual quasiparticles**

  e.g. gauge fields

  & fractionalised excitations

- **Absence of spin order**

  poor diagnostic

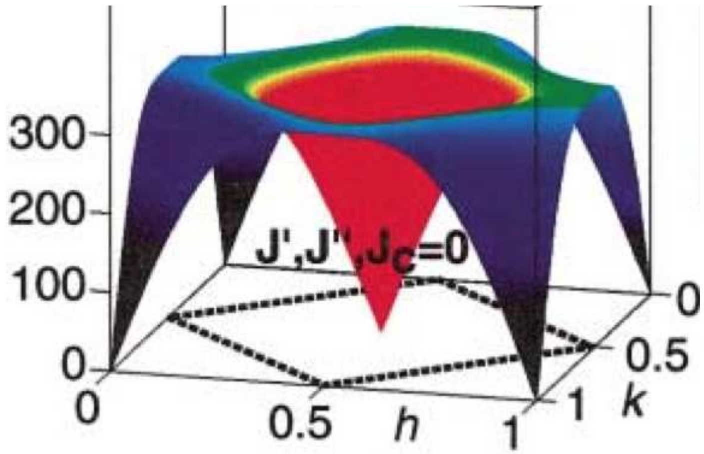


RVB state — Anderson (1973)

Dynamics: ordered vs. fractionalised magnets

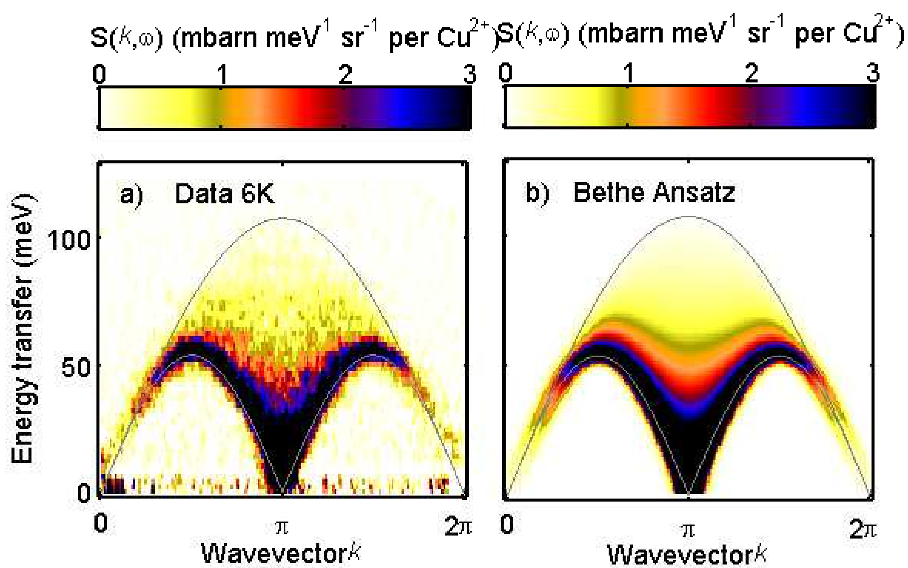
Compare dynamic structure factors

Magnon dispersion in Néel state



undoped cuprate, Coldea *et al.* (2001)

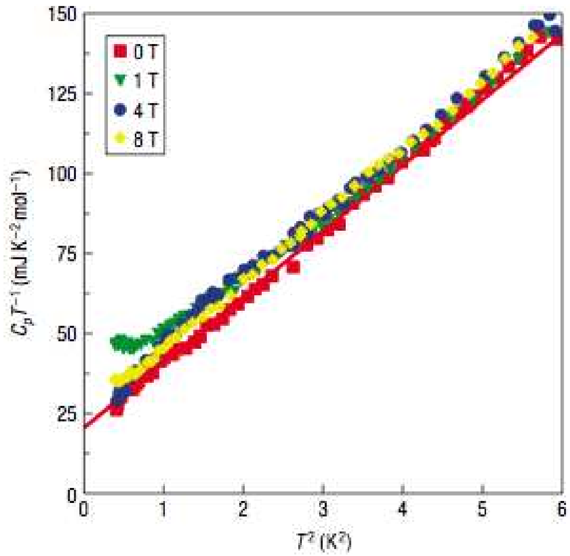
Spinon continuum in Heisenberg chain



$KCuF_3$, Caux *et al.* & Lake *et al.* (2013)

2D spin liquid candidates

κ-(ET)$_2$Cu$_2$(CN)$_3$

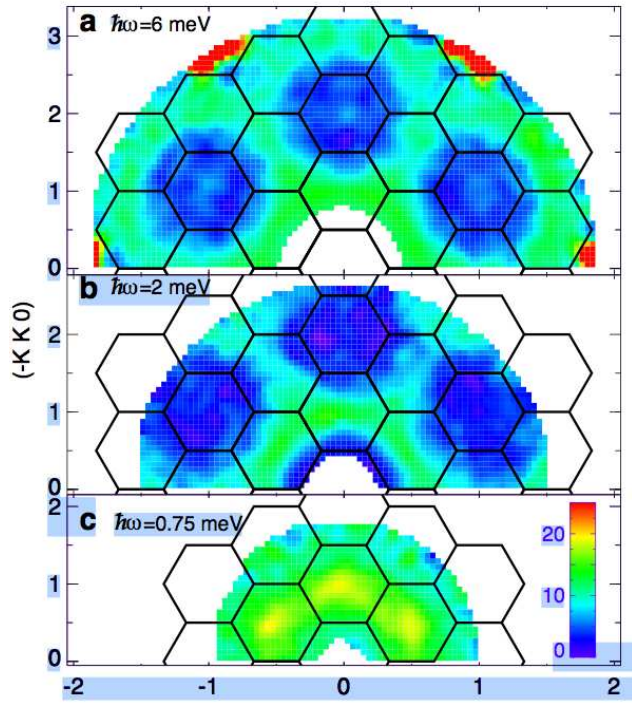


Heat capacity $\sim aT + bT^3$

Kanoda *et al.* (2008)

Herbertsmithite

ZnCu$_3$(OH)$_6$Cl$_2$



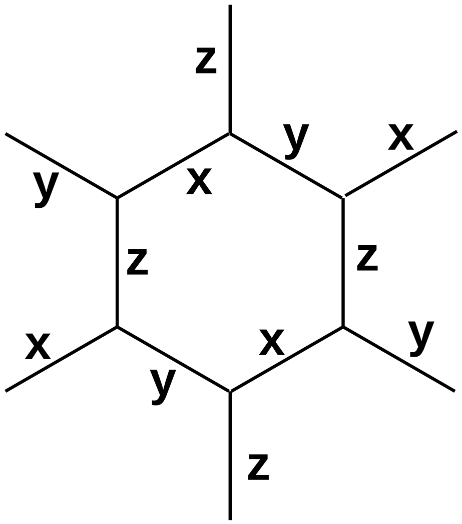
$S(Q, \omega)$ **broad**

Young Lee *et al.* (2012)

Kitaev's honeycomb model

Spin $S = 1/2$ quantum magnet

with strong 'spin-orbit' anisotropy



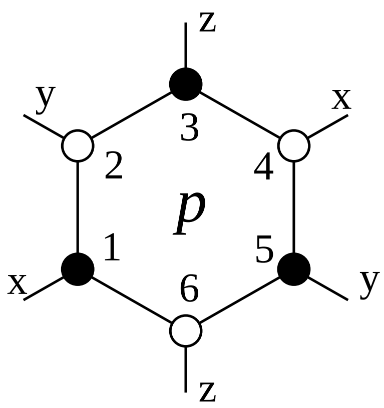
$$\mathcal{H} = -J_x \sum_{\mathrm{x-bonds}} \sigma_i^x \sigma_j^x - J_y \sum_{\mathrm{y-bonds}} \sigma_i^y \sigma_j^y - J_z \sum_{\mathrm{z-bonds}} \sigma_i^z \sigma_j^z$$

A. Kitaev, Ann. Phys. 321, 2 (2006)

Suggested realisation: G. Jackeli and G. Khaliullin, PRL 102, 017205 (2009)

Emergent degrees of freedom

Static fluxes



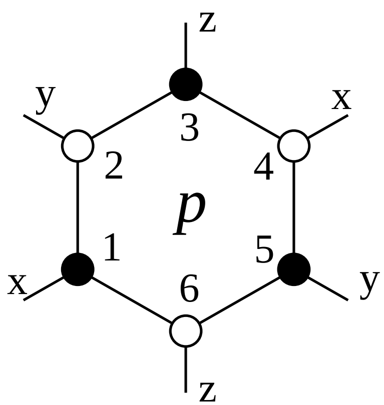
$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

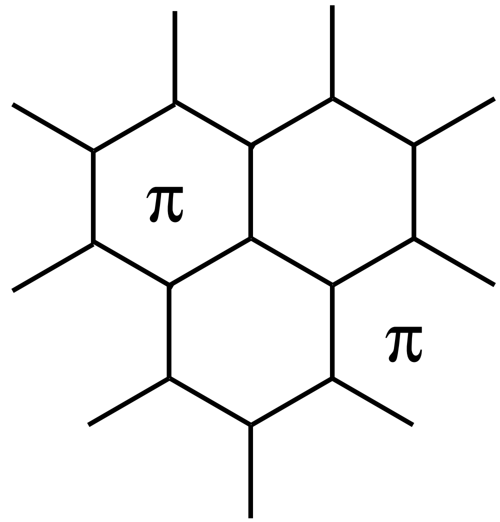
$$[W_p, H] = 0$$

$$[W_p, W_q] = 0$$

Emergent degrees of freedom

Static fluxes . . . and . . . free fermions





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$$[W_p, H] = 0$$

$$[W_p, W_q] = 0$$

Tight binding model

hopping magnitudes J_x, J_y & J_z

signs set by Z_2 fluxes

Spin correlations ultra-short-range: $\langle \sigma_j^\alpha \sigma_k^\alpha \rangle = 0$ **for** $|\mathbf{r}_j - \mathbf{r}_k| > 1$

Baskaran *et al.* **(2007)**

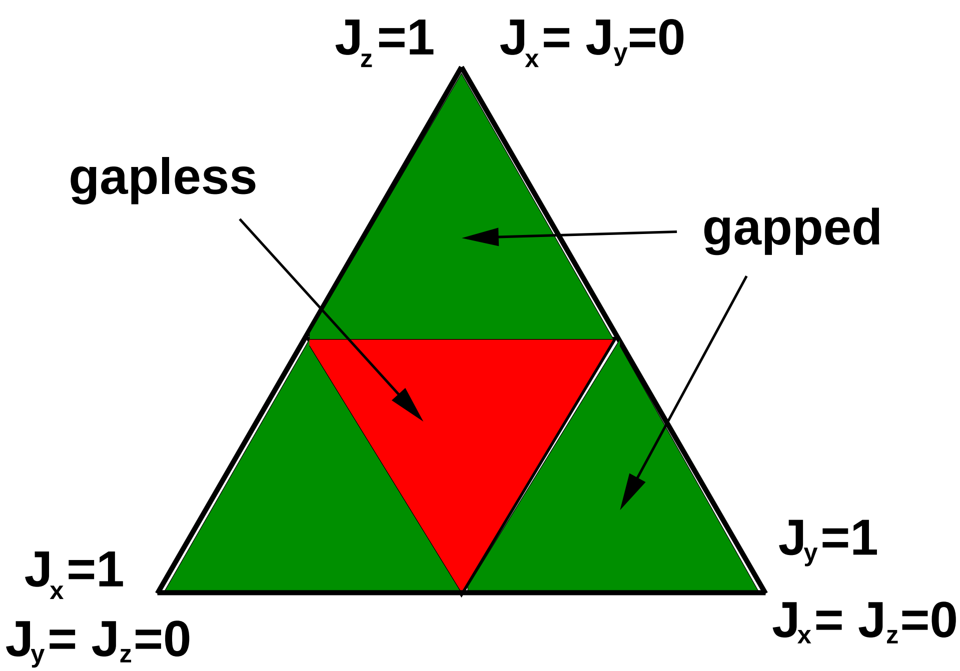
Ground state phase diagram

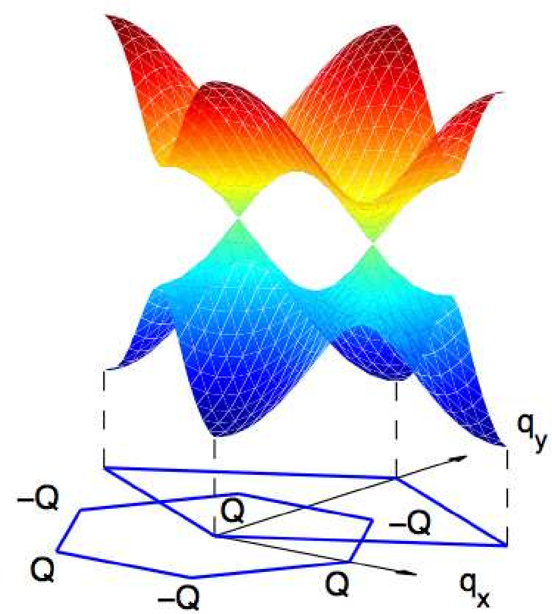
- Gapped liquid phases for $J_z \gg J_x,\ J_y$ and permutations

  Weakly coupled dimers – both sectors gapped

- Gapless liquid phase around $J_x = J_y = J_z \equiv J$

  Dirac cones in fermion spectrum – flux sector gapped



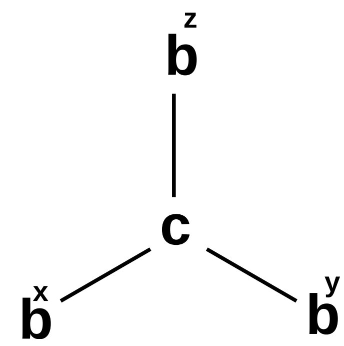


From spins to fermions

— sketch of Kitaev's solution

Represent each spin using 4 Majorana fermions $(bc = -cb,\ c^\dagger = c)$

$\vec{\sigma} \to \{c, b^x, b^y, b^z\}$ with $\sigma_k^\alpha = \mathrm{i}\, b_k^\alpha c_k$ so $\sigma_j^\alpha \sigma_k^\alpha = b_j^\alpha b_k^\alpha c_j c_k$

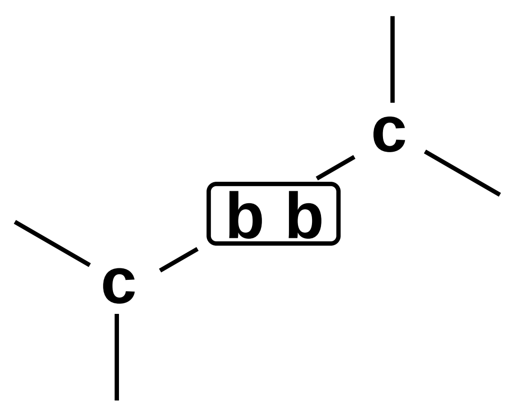


- Resulting $\mathcal{H}$ is quadratic in c_k's
- $[\mathcal{H}, \hat{u}_{jk}] = 0$ with $\hat{u}_{jk} = \mathrm{i} b_j^{\alpha_{jk}} b_k^{\alpha_{jk}}$

$$\mathcal{H} = \frac{i}{4} \sum_{jk} \hat{A}_{jk} c_j c_k$$

$$\hat{A}_{jk} = \begin{cases} 2J_{\alpha_{jk}} \hat{u}_{jk} & j, k \text{ neighbours} \\ 0 & \text{otherwise} \end{cases}$$

– honeycomb tight binding model

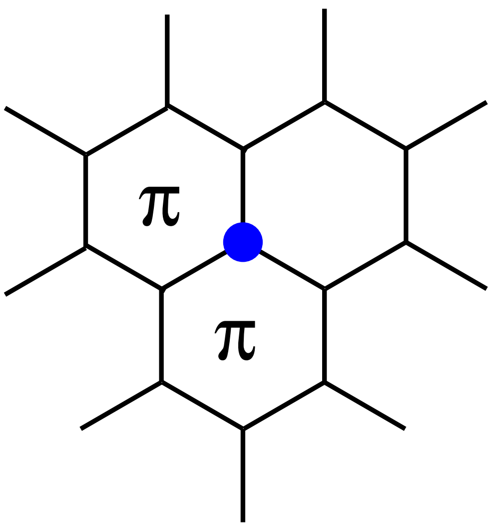


Project to get physical states

Computing the dynamic response

$$S^{\alpha\beta}(\mathbf{r},t) = \langle 0| e^{i\mathcal{H}t} \sigma_{\mathbf{r}}^{\alpha} e^{-i\mathcal{H}t} \sigma_0^{\beta} |0\rangle$$

$\sigma_0^{\beta} \equiv i c_0 b_0^{\beta}$ **adds two fluxes and fermion to** $|0\rangle$



'just' free fermion time evolution

in presence of added fluxes

Baskaran ***et al.*** **(2007)**

Gross features of $S(Q,\omega)$

- **Fractionalisation**

  $\Rightarrow$ **broad response**

  — **correlations short range**

- **Energy cost for flux addition**

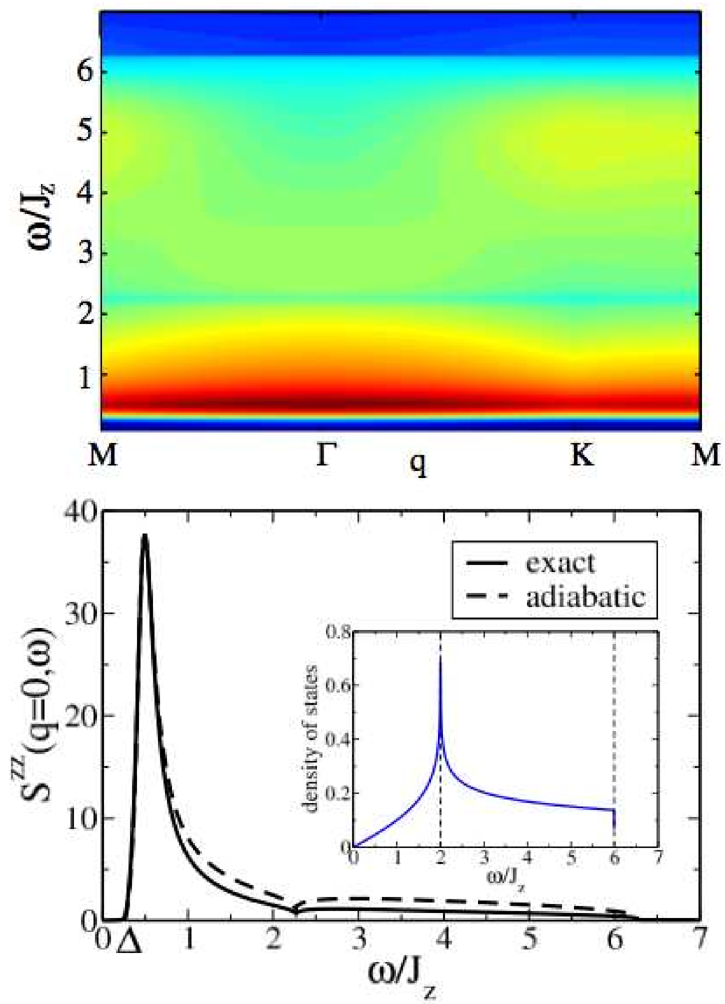
  $\Rightarrow$ **gapped response**

- $S(Q,\omega)$ **is imperfect image of fermion density of states**

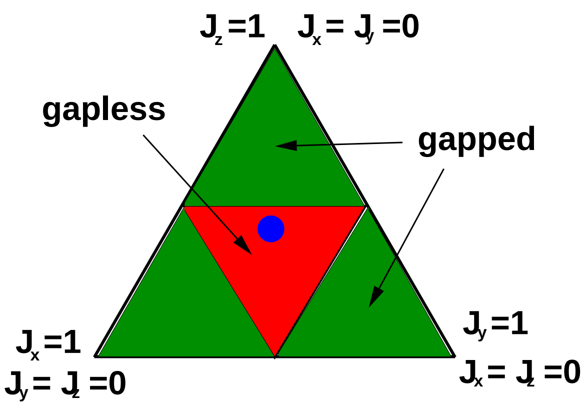
  — **influence of fluxes on dynamics**

  but $\sim$ **98% of wt single pcle**

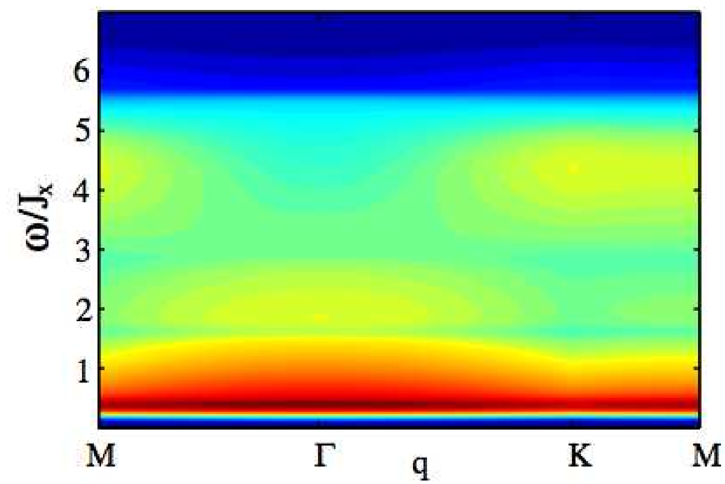
Gapless phase: $J_x = J_y = J_z$

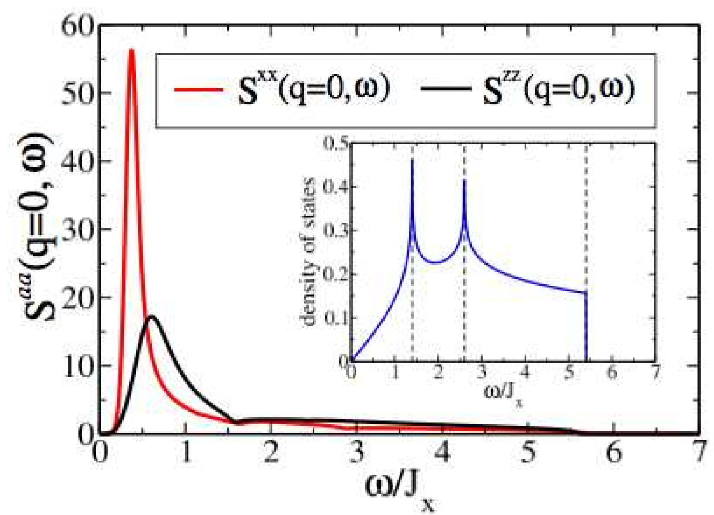


Adding spatial anisotropy



Gapless phase: $J_z/2 < J_x, J_y < J_z$





- **Lower symmetry**

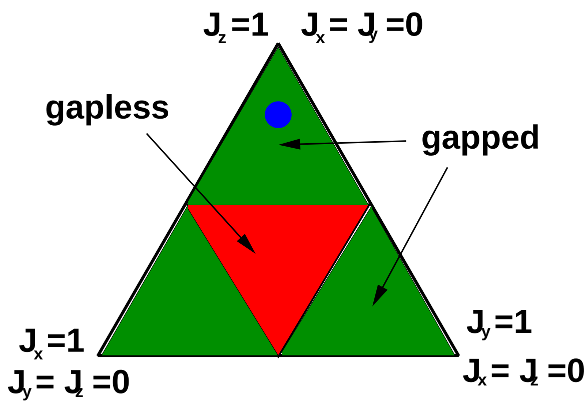
  distinct responses

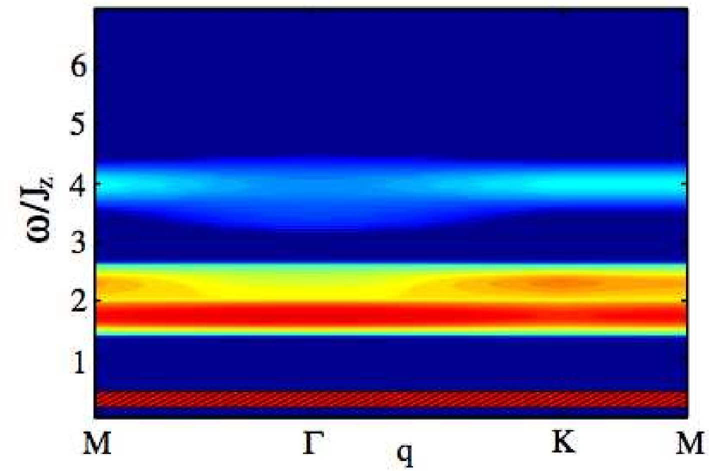
  $S^{xx}(Q,\omega)$ **and** $S^{zz}(Q,\omega)$

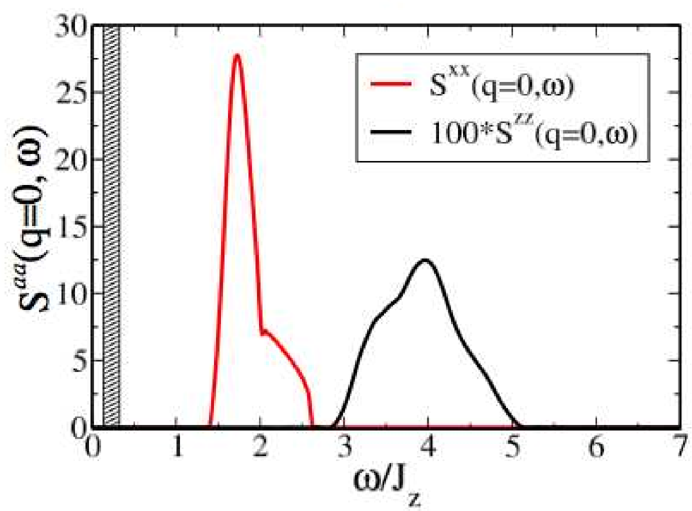
- **Smaller flux gap**

Gapped phase

$J_x = J_y < J_z/2$







- **Small flux gap but large fermion gap**

  flux gap $\Delta \propto (J_x/J_z)^4$

- δ**-function response at flux gap**

  appears at dynamical transition

Dynamical transition

onset of sharp response at flux gap

Matter fermion Hamiltonian includes

pair creation & annihilation terms

— but fermion parity well-defined

Use Lehmann representation

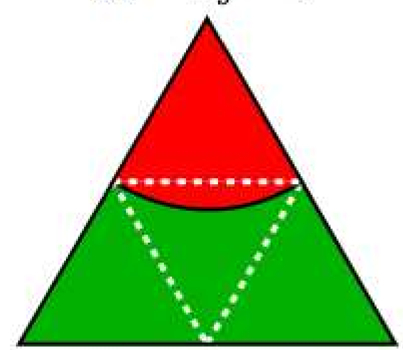
$$S(\mathbf{r},\omega) = \sum_n \langle 0|\sigma_{\mathbf{r}}|n\rangle\langle n|\sigma_0|0\rangle\delta([E_n - E_0] - \omega)$$

$|0\rangle$ **is ground state (flux-free)**

σ_0 **adds two fluxes & fermion**

$|n\rangle$ **are eigenstates in presence of flux pair**

Dynamical phase diagram



Relative parity of ground states in two flux sectors matters:

$|n\rangle$**'s restricted —** ***either*** **to odd** ***or*** **to even fermion excitation numbers**

Sharp response from ***ground-state*** **to** ***ground state*** **contribution**

Relation to quantum quench and x-ray edge

Dynamic response $S(\mathbf{r}, t_{\rm f} - t_{\rm i}) = \langle 0|\sigma_{\mathbf{r}}(t_{\rm f})\sigma_0(t_{\rm i})|0\rangle$

Equivalent quench protocol

add fluxes at $t_{\rm i}$ $\Rightarrow$ evolve $\Rightarrow$ remove fluxes at $t_{\rm f}$

Cf x-ray edge problem

evolve Fermi sea in presence of core hole

Anderson orthogonality catastrophe?

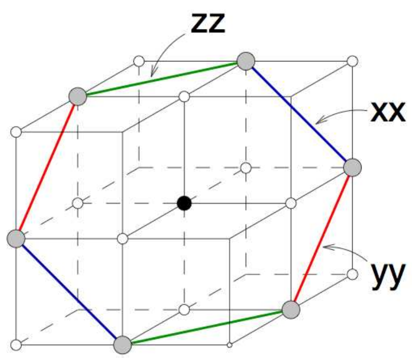
Distinctive features of Kitaev problem:

- **Dirac/gapped DoS**
- **Dynamical transition & parity effects**

Away from integrability

Search for a realisation

Materials: spin-orbit coupling



— layered iridates?

Jackeli & Khalliulin (2009)

Cold atoms: quantum simulator

— optical lattice + spin-dept tunnelling

Duan, Demler & Lukin (2003)

Consequences of departures from Kitaev

E.g. Heisenberg exchange

— fluxes acquire dynamics

— further neighbour correlns develop

— sharp response broadened

— response gap softened

Spin liquid has window of stability

— evolution of response smooth inside window

Summary

Exact calculation of dynamic structure factor

— in gapped & gapless phases

— signatures of emergent fluxes and fermions

Unusual features

— response gap in gapless phase

— sharp response despite fractionalisation

X-ray edge & quantum quench

— no orthogonality catastrophe