Thermalization and Revivals after a Quantum Quench in a Finite System

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(Global) Quantum Quench

- prepare an extended system (in thermodynamic limit) at time $t = 0$ in a (translationally invariant) pure state $|\psi_0\rangle$ – e.g. the ground state of some hamiltonian H_0
- evolve unitarily with a hamiltonian H for which $|\psi_0\rangle$ is not an eigenstate and has extensive energy above the ground state of H
- how do correlation functions of local observables, and quantum entanglement of subsystems, evolve as a function of t?
- for a subsystem do they become stationary?
- if so, what is the stationary state? Is it thermal?

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- how do correlation functions of local observables, and quantum entanglement of subsystems, evolve as a function of t?
- for a subsystem do they become stationary?
- if so, what is the stationary state? Is it thermal?
- for a large but finite system, does the initial state ever revive?

Quantum quench in a 1+1-dimensional CFT

- P. Calabrese + JC [2006,2007] studied this problem in an infinite stystem in 1+1 dimensions when $H = H_{\rm CFT}$ and $|\psi_0\rangle$ is a state with short-range correlations and entanglement
- $H_{\rm CFT}$ describes the universal low-energy, large-distance properties of many gapless 1d systems
- 1+1-dimensional CFT is exactly solvable, so we can get analytic results for interacting systems – however, these turn out to depend only on general properties of any CFT, and they can be interpreted in terms of a simple physical picture of locally entangled quasiparticles which then propagate semiclassically

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- in this talk I will describe the extension of this work to finite systems

Results: Thermalization

- suppose the energy density in the initial state $\equiv$ effective temperature β^{-1} for $H_{\rm CFT}$
- let $\rho_t(\ell)$ be the reduced density matrix of a subinterval of length ℓ with $\beta \ll \ell \ll L$ at time t
- let $\tilde\rho_\beta(\ell)$ be the reduced density matrix of the subinterval if whole system is in a thermal ensemble at temperature β^{-1}
- then for $\ell/{\rm v} < t \ll L/{\rm v}$, the overlap

$$\frac{\mathrm{Tr}\,\big(\rho_t(\ell)\tilde\rho_\beta(\ell)\big)}{\big(\mathrm{Tr}\,\rho_t(\ell)^2\big)^{\frac{1}{2}}\big(\mathrm{Tr}\,\tilde\rho_\beta(\ell)^2\big)^{\frac{1}{2}}} = 1 - O\big(e^{-4\pi\Delta(t-\ell/{\rm v})/\beta}\big)$$

Results: Revivals

- define return amplitude $\mathcal{F}(t) \equiv |\langle\psi_0|e^{-iHt}|\psi_0\rangle|$
- this has a universal initial decay for any CFT

$$\mathcal{F} \sim \exp\left(-\frac{\pi c L t^2}{3\mathrm{v}\beta(\beta^2+4t^2)}\right) \quad \text{for} \quad t \ll (L\beta)^{1/2}$$

- for a rational CFT there are partial revivals with $\mathcal{F}(t) = O(1)$ when $2\mathrm{v}t/L =$ integer (for periodic b.c.; $\mathrm{v}t/L =$ integer for open b.c.), and eventually a complete revival with $\mathcal{F} = 1$
- in general $\mathcal{F}$ is exponentially small as $L \to \infty$, but

$$\lim_{L\to\infty} (1/L) \log \mathcal{F}(t)$$

has structure near every rational value of $\mathrm{v}t/L$

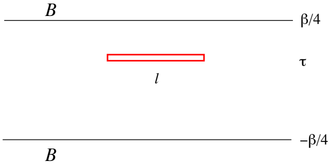
Initial state

- CC assumed $|\psi_0\rangle \propto e^{-(\beta/4)H_{\rm CFT}}|\mathcal{B}\rangle$ where $|\mathcal{B}\rangle$ is a conformally invariant boundary state, e.g. a product state
- $\beta/4$ chosen so that $\langle\psi_0|H|\psi_0\rangle = {\rm Tr}\, He^{-\beta H}/{\rm Tr}\, e^{-\beta H}$
- this state has short-range ($\sim \beta{\rm v}$) correlations and entanglement
- more generally one may take

$$|\psi_0\rangle \propto e^{-\sum_j \beta_j \int \Phi_j(x)dx}|\mathcal{B}\rangle$$

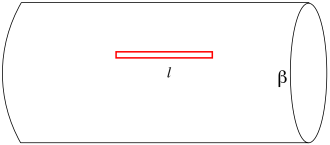
  where the Φ_j are all possible boundary operators acting on $\mathcal{B}$: this leads to a generalised Gibbs ensemble for $\ell < {\rm v}t \ll L$
- we shall focus on the CC initial state although the results are qualitatively correct in the more general case

Path integral formulation: thermalisation overlap



$$\rho_\tau(\ell) \propto \mathrm{Tr}_{L-\ell}\, e^{-(\beta/4+\tau)H}|\mathcal{B}\rangle\langle\mathcal{B}|e^{-(\beta/4-\tau)H}$$

is given by the partition function on $\mathcal{S}_\ell$, a strip slit along (ℓ, τ)



$$\tilde\rho_\beta(\ell) \propto \mathrm{Tr}_{L-\ell}\, e^{-\beta H}$$

is given by the partition function on $\mathcal{C}_\ell$, a cylinder slit along ℓ

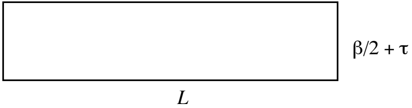
$$\frac{\mathrm{Tr}\left(\rho_t(\ell)\tilde\rho_\beta(\ell)\right)}{\left(\mathrm{Tr}\,\rho_t(\ell)^2\right)^{\frac{1}{2}}\left(\mathrm{Tr}\,\tilde\rho_\beta(\ell)^2\right)^{\frac{1}{2}}} = \frac{Z(\mathcal{S}_\ell \oplus \mathcal{C}_\ell)}{Z(\mathcal{S}_\ell \oplus \mathcal{S}_\ell)^{1/2}\, Z(\mathcal{C}_\ell \oplus \mathcal{C}_\ell)^{1/2}}$$

- continue result to $\tau = it$ and use OPE of twist operators

Path integral formulation: return amplitude

Finite interval, length L, periodic boundary conditions:

$$\mathcal{F}(\tau) = \langle\mathcal{B}|e^{-(\beta/4)H}e^{-\tau H}e^{-(\beta/4)H}|\mathcal{B}\rangle = \frac{Z_{\text{annulus}}(\beta/2+\tau, L)}{Z_{\text{annulus}}(\beta/2, L)}$$



For a CFT a great deal is known about Z_{annulus}:

$$Z_{\text{annulus}}(W, L) = \sum_{\Delta} |\mathcal{B}_\Delta|^2 \chi_\Delta(q) = \sum_{\tilde{\Delta}} n_{\Delta}^{\mathcal{B}}\, \chi_\Delta(\tilde{q})$$

where the sums are over scaling dimensions of (primary) operators and

$$q = e^{-4\pi W/L}, \quad \tilde{q} = e^{-\pi L/W}, \qquad \chi_\Delta(q) = q^{-c/24+\Delta}\sum_{N=0}^{\infty} d_N q^N$$

We need to continue these formulae to $W = \beta/2 + it$.

Initial decay and revivals

Initial decay

- $\tilde{q} = e^{-2\pi L(\beta - 2it)/(\beta^2+4t^2)}$
- so for $t \ll (L\beta)^{1/2}$, $|\tilde{q}| \ll 1$ and $Z \sim \tilde{q}^{-c/24}$
- normalising by the $t = 0$ amplitude gives the result

$$\mathcal{F} \sim \exp\left(-\frac{\pi c L t^2}{3\beta(\beta^2+4t^2)}\right) \underset{t \gg \beta}{\rightarrow} \exp\left(-\frac{\pi c L}{12\beta}\right)$$

- initial gaussian decay to a plateau value

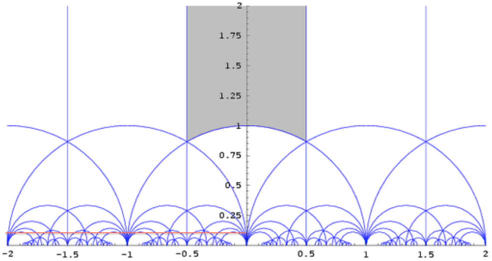
Revivals

- spectrum of H is $4\pi(\Delta + N)/L$
- for a rational CFT all the Δs are rational $= N/M$
- so we get partial revivals at $2t/L = M/N$, complete revival at $2t/L = M$

Structure at rational t/L

- it is conventional to write $q = e^{2\pi i\tau}$ (τ is not imaginary time!)
- in a rational CFT the characters $\chi_\Delta(q)$ transform linearly under the modular group, generated by

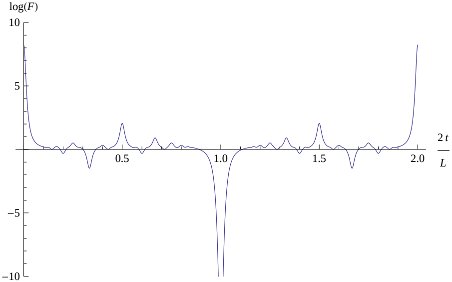
$$S : \tau \to -1/\tau \quad (q \to \tilde{q}); \qquad T : \tau \to \tau + 1 \quad (q \to e^{2\pi i} q)$$



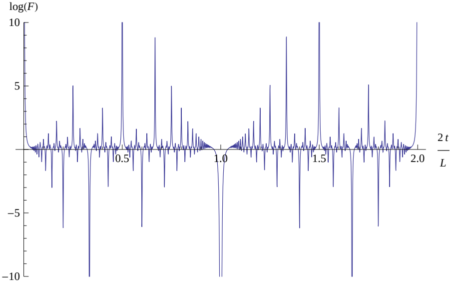
Any curvilinear triangle is mapped back into the principal region (grey) by the modular group

- in our case $\tau = -2t/L + i\beta/L$
- as t increases we pass close to every rational point on the real line which gets mapped back to $\tilde{q} \ll 1$

Example: Ising model



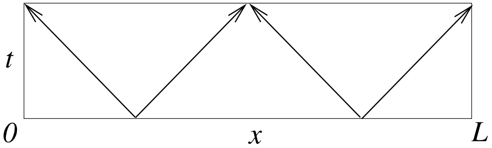
Quench from disordered phase with $\pi\beta/L = 0.1$



Quench from disordered phase with $\pi\beta/L = 0.01$

Quasiparticle picture

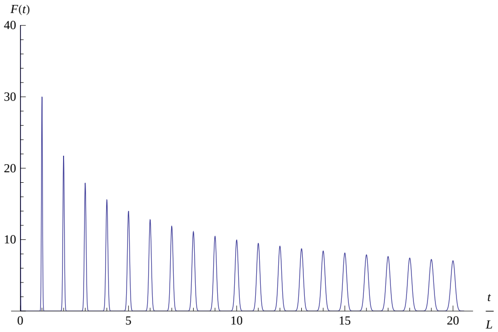
Quasiparticle configuration leading to the feature at $t = L/4$:



- exponentially suppressed because the two pairs of particles a distance $O(L)$ apart must be entangled with each other
- if $2t/L = p/q$ a similar construction can be made with q pairs

Non-integrable perturbation of CFT

- exact revivals depend on integer spacing $\propto 1/L$ in CFT: do nonintegrable but irrelevant deformations change this?
- example: $H = H_{\text{CFT}} + \lambda \int T\overline{T} dx$
- to first order an eigenvalue at $4\pi(\Delta + N)/L$ is shifted by an amount $\sim \lambda(\Delta + N)^2/L^3$ (but the degeneracies remain)
- result: peak in $\mathcal{F}$ at $t = nL/2$ is broadened by an amount $\sim (\lambda n)^{1/2}$



Summary

- in a quench from a short-range entangled state in a 1+1-dimensional CFT, a region of length ℓ thermalises (more generally goes to a GGE) after a time $\ell/2\mathrm{v}$, in the sense that its reduced density matrix becomes exponentially close to that of a thermal system (GGE)
- however, it also becomes entangled with the rest of the system, and after a time $(L-\ell)/2\mathrm{v}$ it starts to dethermalize, leading to a (in general, partial) revival of the initial state at time $L/2\mathrm{v}$ (for periodic bc)

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- so was it ever thermalized in the first place?