Quantum Field Theory Michaelmas Term 2012 Prof. J. Cardy

Problem Set 2

Although everyone in the class is welcome to try these problems, only the papers of students in Theoretical Physics (or any others taking the course for credit who have notified me) will be marked. They should be handed in by **Thursday November 29 at 2pm**, either in the lecture that day or to Curt von Keyserlingk in Theoretical Physics. They will be reviewed at a Problems Class on **Wednesday December 5** from 2–4pm in the Seminar Room, DWB, which everyone in the class is welcome to attend. Any questions before the due date should be directed to me at `j.cardy1@physics.ox.ac.uk`

1. In QCD, the renormalisation group functions have the form $\beta(g) = -bg^3 + O(g^4)$ and $\gamma(g) = cg^2 + O(g^3)$, where b and c are positive constants.

   (a) QCD is believed to exhibit dynamical mass generation: there is no mass term in the lagrangian, nevertheless the physical particles have mass. By observing that the mass of, say, the proton must have the form $M = \mu f(g)$, but that M cannot in fact depend on the renormalisation scale, deduce how M must depend on g for small g.

   (b) What is the asymptotic behaviour of $\Gamma^{(2)}(p)$ in this theory for $p \to \infty$? [By this I mean not just 0 or ∞ but how it gets there. You will need to use the full solution of the C-S equation, given in the notes.]

2. In massless ϕ^4 field theory, write down the Callan-Symanzik equation for the Fourier transform $G^{0,2}(p)$ of the 2-point function of ϕ^2

   $$G^{0,2}(p) = \int e^{ip\cdot x}\langle\phi^2(x)\phi^2(0)\rangle d^dx$$

   and hence deduce how this depends on p at the fixed point, in terms of γ_ϕ^* and $\gamma_{\phi^2}^*$.

   Use scaling arguments to deduce how $\int G^{0,2}(p)d^dp$ depends on the renormalised mass m in the *massive* theory. This is proportional to the specific heat C (why?) Deduce the power dependence of this as a function of $|T - T_c|$. Express the exponent in terms of ν.

3. Consider the theory of a massive real scalar field ϕ with interaction $\frac{1}{6}\lambda\phi^{\mathbf{3}}$.

   (a) what is the critical dimension d_c in which this theory is exactly renormalisable?

   (b) in d_c, which of the $\Gamma^{(N)}$ contain primitive divergences, and how should these be made finite? [Note that $\Gamma^{(1)}$ and related 'tadpole' diagrams can be removed by a constant shift of the field.]

   (c) in the *massless* theory, using dimensional regularisation and minimal subtraction, work out the renormalised coupling constant at one loop in $d = d_c - \epsilon$. [This involves computing two one-loop diagrams, one for $\Gamma^{(3)}$, and also the field renormalisation from $\Gamma^{(2)}$ which in this theory has a contribution at one loop order.]

   (d) If you got this far, you can work out the beta-function to one loop.