Field Theory in Condensed Matter Hilary Term 2012 Prof. J. Cardy

Problem Set 1

Although everyone in the class is welcome to try these problems, only the papers of students in Theoretical Physics will be marked. They should be put in the pigeon hole of Adam Nahum in Theoretical Physics by **Friday February 17 at 4 pm.** They will be reviewed at a Problems Class the following week on **Wednesday February 22 2-4 pm** in the **Fisher Room**, which everyone in the class is welcome to attend. Any questions before the due date should be directed to me at `j.cardy1@physics.ox.ac.uk`
Parts of questions marked † may be found slightly harder. However anyone seriously interested in learning QFT should try to tackle them.

1. A free scalar field and a free Dirac fermion have a kinetic term in the Lagrangian density respectively $\propto (\partial\phi)^2$ and $\overline{\psi}(\gamma\cdot\partial)\psi$. Using power-counting, work out what interactions between these fields are permitted so that the theory is exactly renormalisable in $d = 4$. Which irreducible vertex parts contain primitive divergences, and what subtractions or multiplicative renormalisations should be required to make the theory finite? Draw the appropriate 1-loop diagrams. [There's no need to actually calculate any Feynman integrals.]

2. In QCD, the renormalisation group functions have the form $\beta(g) = -bg^3 + O(g^4)$ and $\gamma(g) = cg^2 + O(g^3)$, where b and c are positive constants.

   (a) QCD is believed to exhibit dynamical mass generation: there is no mass term in the lagrangian, nevertheless the physical particles have mass. By observing that the mass of, say, the proton must have the form $M = \mu f(g)$, but that M cannot in fact depend on the renormalisation scale, deduce how M must depend on g for small g.

   (b) †What is the asymptotic behaviour of $\Gamma^{(2)}(p)$ in this theory for $p \to \infty$? [By this I mean not just 0 or ∞ but how it gets there. You will need to use the full solution of the C-S equation, given in the notes.]

3. In the lecture we computed the anomalous dimension $\gamma^*_{\phi^2}$ of ϕ^2 at the fixed point of a real scalar $\lambda\phi^4$ theory, to $O(\epsilon)$.

   (a) analogously, compute the anomalous dimension of ϕ^n for general n, to $O(\epsilon)$.

   (b) †does anything special happen for $n = 3$? If so, why?

4. Consider the theory of a massive real scalar field ϕ with interaction $\frac{1}{6}\lambda\phi^{\mathbf{3}}$.

   (a) what is the critical dimension d_c in which this theory is exactly renormalisable?

   (b) in d_c, which of the $\Gamma^{(N)}$ contain primitive divergences, and how should these be made finite? [Note that $\Gamma^{(1)}$ and related 'tadpole' diagrams can be removed by a constant shift of the field.]

   (c) †in the *massless* theory, using dimensional regularisation and minimal subtraction, work out the renormalised coupling constant at one loop in $d = d_c - \epsilon$. [This involves computing two one-loop diagrams, one for $\Gamma^{(3)}$, and also the field renormalisation from $\Gamma^{(2)}$ which in this theory has a contribution at one loop order.]

   (d) † If you got this far, you can work out the beta-function to one loop.