Quantum Field Theory Michaelmas Term 2010 Prof. J. Cardy

Problem Set 2

Although everyone in the class is welcome to try these problems, only the papers of students in Theoretical Physics will be marked. They should be handed in to Dr. Stefan Zohren in Theoretical Physics (either to his room 4.5 or his pigeon hole) by **Thursday November 25 at 2pm.** They will be reviewed at a Problems Class on **Monday November 29** from 2–4pm in the Seminar Room, DWB, which everyone in the class is welcome to attend. Any questions before the due date should be directed to Stefan at `s.zohren1@physics.ox.ac.uk`

Parts of questions marked † may be found slightly harder. However anyone seriously interested in learning QFT should try to tackle them.

1. In QCD, the renormalisation group functions have the form $\beta(g) = -bg^3 + O(g^4)$ and $\gamma(g) = cg^2 + O(g^3)$, where b and c are positive constants.

   (a) QCD is believed to exhibit dynamical mass generation: there is no mass term in the lagrangian, nevertheless the physical particles have mass. By observing that the mass of, say, the proton must have the form $M = \mu f(g)$, but that M cannot in fact depend on the renormalisation scale, deduce how M must depend on g for small g.

   (b) †What is the asymptotic behaviour of $\Gamma^{(2)}(p)$ in this theory for $p \to \infty$? [By this I mean not just 0 or ∞ but how it gets there. You will need to use the full solution of the C-S equation, given in the notes.]

2. Consider a a theory with a *complex* scalar field ϕ and a bare lagrangian density (in euclidean space)

$$(\partial\phi_0^*)(\partial\phi_0) + m_0^2\phi_0^*\phi_0 + \tfrac{1}{4}\lambda_0(\phi_0^*\phi_0)^2 \,.$$

   Assume that the bare mass m_0 is adjusted so that the renormalised mass vanishes.

   (a) Calculate, within minimal subtraction or any other suitable scheme, the renormalised coupling λ to 1-loop order in λ_0. [Note that the Feynman integral is the same as that computed in the notes.]

(b) Hence work out the beta-function $\beta(g)$ in $4-\epsilon$ dimensions, and show there is an IR-stable zero $O(\epsilon)$.

(c) draw the diagrams which contribute to the field renormalisation (to 2 loops) and the renormalisation of $\phi^*\phi$ (to 1 loop), and their symmetry factors, and hence, by comparison with the notes, calculate the critical exponents η and ν to $O(\epsilon^2)$ and $O(\epsilon)$ respectively.[1]

3. In the lecture we computed the anomalous dimension $\gamma^*_{\phi^2}$ of ϕ^2 at the fixed point of a real scalar $\lambda\phi^4$ theory, to $O(\epsilon)$.

   (a) analogously, compute the anomalous dimension of ϕ^n for general n, to $O(\epsilon)$.

   (b) †does anything special happen for $n=3$? If so, why?

4. In the lectures we considered O(N) ϕ^4 theory at large N and derived an equation for how the renormalised mass m depends on the difference $m_0^2 - m_{0c}^2$. We omitted the interesting case when $d=4$. What is the correct answer in this case?

5. Consider the theory of a real scalar particle with lagrangian density (in Minkowski space)

$$\tfrac{1}{2}((\partial_\mu\phi)(\partial^\mu\phi) - m^2\phi^2) - \tfrac{1}{6}\lambda\phi^3\,.$$

   (a) Draw the diagrams contributing to the $\mathcal{T}$-matrix for $\phi\phi \to \phi\phi$ scattering to lowest non-trivial order in λ, and hence (using the results in the notes) calculate the differential cross-section for this process in the CM frame, showing explicitly how it depends on energy E and scattering angle θ.

   (b) †is the interaction between the particles attractive or repulsive?

[1] There is a typo in the current notes on the web. The correct formula for ν is $1/(2-\gamma^*_{\phi^2})$.