Field Theory in Condensed Matter Hilary 2009 Prof. J. Cardy

Problem Set 1

These problems will not be marked, but will be discussed in a Problems Class, TBA, probably in 9th week. Any questions or clarifications before then can be addressed to me at j.cardy1@physics.ox.ac.uk

- 1. [The purpose of this question is to test your understanding of simple scaling/RG arguments. It should not involve any complicated explicit calculations.] The Ising model in d dimensions, away from its critical temperature, is described by the fixed point action perturbed by $t\int \phi_2 d^dr$, where $\phi_2 =: \phi^2 :$ and $t \propto T - T_c$. Assume that ϕ_2 has scaling dimension x_2. What is the relation between x_2 and the critical exponent ν describing the behaviour of the correlation length $\xi \sim t^{-\nu}$? Write down a scaling form for the correlation function $\langle \phi_2(r)\phi_2(0)\rangle$ away from the critical point as a function of r and ξ. The integral of this correlation function over r is proportional to the specific heat C [why?], which is supposed to behave like $t^{-\alpha}$. Hence work out a relation between ν and α. Summarise what were the crucial assumptions involved in your argument, and try to explain why they go wrong for dimensions $d > 4$.

The remaining questions should be answered using the 'poor man's' perturbative RG:

- 2. The tricritical Ising model is described by a field theory with action

$$S = \int [(\nabla\phi)^2 + g_2\phi_2 + g_4\phi_4 + g_6\phi_6] d^dr$$

  where $\phi_n =: \phi^n :$. Work out the OPE coefficients at the gaussian fixed point and hence write down the RG equations to second order in the g's. Show that there is a fixed point in $d = 3 - \epsilon$ dimensions at which $g_6 = O(\epsilon)$ and the other g's are smaller. Calculate the RG eigenvalues of g_2' and g_4' to first order in ϵ.

- 3. Consider a system made of two layers coupled together. Each layer is a two-dimensional system described by a reduced hamiltonian (action) $S_j = S_j^* - g_j a^{x_j-2}\int \phi_j(r) d^2r$ where $j = 1, 2$ labels the two layers. For simplicity, each fixed point action S_j^* is perturbed by only one scaling operator ϕ_j, with scaling dimension x_j. Initially the layers are decoupled, and the OPEs have the form

$$\phi_j \cdot \phi_j = \mathbf{1} + b_j\phi_j + \cdots$$

  where b_j is an OPE coefficient. The two layers are now coupled by adding a term $-\Delta a^{x_1+x_2-2}\int \phi_1(r)\phi_2(r) d^2r$ to the action. Work out the RG equations to second order in g_2, g_2 and Δ, and explore all the possible fixed points. Assume that $\epsilon \equiv 2 - x_1 - x_2$ is small.

- 4. Consider the sine-Gordon theory in $2d$ with action

$$S = \int [\tfrac{1}{2}K(\partial\theta)^2 - \lambda_p \cos(p\theta)]d^2x\,.$$

Work out the RG equations for K and λ_p to $O(\lambda_p^2)$ and sketch the RG flows, discussing their physical interpretation.

Now suppose that $d = 2 + \epsilon$. How does the RG equation for K get modified, in the absence of the interaction. By assuming that the rest of the RG equations are unchanged even when $\lambda_p \neq 0$ [when is this justified?] discuss the RG flows, fixed point structure and the physical interpretation (for both signs of ϵ.)