Second Year Theory Option TT 2001

Quantum Mechanics:- Prof. I.J.R. Aitchison (10 lectures, weeks 4 - 8)

Aim: to impart a deeper understanding of the structure of the theory, and to develop skills in problem-solving techniques at this level.

Objectives: Students should acquire understanding of topics 1 - 10 in the synopsis listed below (which corresponds to the syllabus printed on page 31 of the 2000-2001 edition of the Physics Course Handbook), and by working through the Problem Sets issued with this course (or other similar ones) become proficient in the relevant techniques.

Books: The book which seems closest in level and approach to the course is Mandl *Quantum Mechanics*, which includes the A-paper material as well in a compact form. At the next level up, there are several well-established texts, but they tend to contain a good deal more than we need - for example, Shankar *Principles of Quantum Mechanics* (**2nd edition**), Merzbacher *Quantum Mechanics* (**3rd edition**), Gasiorowicz *Quantum Physics* (**2nd edition**), or Schiff *Quantum Mechanics* (**3rd edition**). Standard reference books are Dirac *Principles of Quantum Mechanics* (**4th edition**), Messiah *Quantum Mechanics* (two volumes), and Landau and Lifshitz *Quantum Mechanics (Non-relativistic Theory)* (**3rd edition**).
Mention must also be made of Feynman's *Lectures on Physics, Vol.III Quantum Mechanics.* Although this tends not to work very well as a course textbook, students may find chapters 8-12 particularly helpful to consult as regards the material in lectures 1-4. In any case, the volume is wonderful for additional reading.

Synopsis: The following breakdown of the syllabus indicates the order in which it will be covered, and the approximate division of the material into lectures. References are also given to appropriate reading from Mandl (Ma), Shankar (Sh), Merzbacher (Me), and Feynman (Fe). It is important to **note** that the lectures will generally avoid repeating standard derivations which are contained in books, even though these derivations are required in the syllabus. Rather, emphasis will be placed on ideas and examples. *Items in italics below indicate material not in the syllabus.*

1,2. State vectors, bra and ket (Dirac) notation. Simple ideas of representations, change of basis.

3,4. Quantum mechanics of finite-state systems. Solution of problems in coordinate and matrix representations. Operator method for the simple harmonic oscillator.

[Ma chptr 12; Sh chptr 1 and section 7.4; Me chptrs 9, 10; Fe chptrs 8-12]

5. The Hamiltonian for a non-relativistic charged particle in an external classical electromagnetic field; physical identification of terms. *Gauge invariance.*

[Sh sections 14.4, 18.4; Me section 4.6]

6. Operator methods for angular momentum. Matrix representation of angular momentum. Pauli spin matrix formalism for spin-$\frac{1}{2}$ particles.

[Ma chptr 5; Sh chptrs 12, 14; Me chptrs 11, 16]

7. Wave functions for two identical particles of spin-0 and of spin-$\frac{1}{2}$.

[Ma sections 4.4, 5.7; Sh chptr 10]

8, 9, 10. First and second order time-independent perturbation theory including the degenerate case. First order time-dependent perturbation theory.

[Ma chptrs 7, 9; Sh chptrs 17, 18; Me chptrs 18, 19]

Pre-requisites: (i) Familiarity with the second year M.T. course on Mathematical Methods, in particular with vector spaces , matrix theory, and Fourier Transforms. *In addition*, students should make sure that they have some prior knowledge of the Dirac delta function (see for example Boas *Mathematical Methods in the Physical Sciences* (**second edition**), section 15.7, Mandl section 2.6.2, Shankar section 1.10, or Merzbacher Appendix section 1). (ii) Familiarity with the second year M.T./H.T. course on Quantum Mechanics.

You will be in a well-prepared state if you find all of the following revision/warm-up exercises easy.

Revision/warm-up exercises

1. Two vectors $\mathbf{a}', \mathbf{b}'$ are related to $\mathbf{a}, \mathbf{b}$ by
$$\mathbf{a}' = R\,\mathbf{a}, \quad \mathbf{b}' = R\,\mathbf{b}$$
where R is a real 3 x 3 orthogonal matrix. Show that $\mathbf{a}'.\mathbf{b}' = \mathbf{a}.\mathbf{b}$.

2. Show that the eigenvalues of a Hermitian matrix are real, and that the eigenvectors of a Hermitian matrix belonging to different eigenvalues are orthogonal.

3. The vectors
$$u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, u_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$
form an orthonormal basis for a 2-dimensional complex vector space. Find the components of the vector $\begin{pmatrix} 1 \\ 2i \end{pmatrix}$ in the u_1, u_2 basis.

4. Use the relation
$$\int_{-\infty}^{\infty} e^{i(k'-k)x}\,dx = 2\pi\delta(k'-k)$$
to show that the Fourier transform of the convolution $f * g$ of two functions f and g, defined by
$$f * g = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)g(x-u)du,$$
is $\bar{f}(k)\bar{g}(k)$, where $\bar{f}$ and $\bar{g}$ are the Fourier transforms of f and g.

5. A two-state quantum system has Hamiltonian H and energy eigenfunctions ϕ_1 and ϕ_2 such that $H\phi_1 = (E_0+\delta)\phi_1$ and $H\phi_2 = (E_0-\delta)\phi_2$. At a certain instant $t=0$, the system is in the state described by the wavefunction $\phi_+ = \frac{1}{\sqrt{2}}(\phi_1+\phi_2)$. What is the probability that at a later time t the system will be found, on measurement, to be in the state described by ϕ_+?

6. A particle in one dimension has wavefunction
$$N e^{ik_0x} e^{-x^2/2a^2}$$
where N is a normalisation constant that need not be calculated. Sketch

(a) the position-space probability distribution

(b) the momentum-space probability distribution.

(For (b) you may if you wish obtain an analytical answer, but a qualitatively correct sketch is sufficient).

Problem sets for the course: Three problem sets will be issued as follows.
Set 1: accompanies lectures 1 - 3
Set 2: accompanies lectures 4 - 6
Set 3: accompanies lectures 7 - 10.
These may be viewed (as ps files) at
http://www-thphys.physics.ox.ac.uk/ users/IanAitchison/home.html .

The total number of problems provided in these sets is likely to be greater than can be covered in three tutorials. Further examples may be found in Section B of the Part A Theoretical Physics examination papers from 1996 onwards.

Queries/feedback: Any queries or feedback arising from the lectures can be e-mailed to me at i.aitchison1@physics.ox.ac.uk. Queries/feedback about the questions in the problem sets should be discussed first with students' college tutors.

I.J.R. Aitchison