Two Lectures on Simple Susy and the MSSM

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1.1 Dirac

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix} \tag{1}$$

ψ, χ each have 2 complex components $\longrightarrow$ 8 real (field) degrees of freedom. Equation of motion is $\not{p}\Psi = m\Psi$, or

$$(E - \boldsymbol{\sigma}.\boldsymbol{p})\psi = m\chi \tag{2}$$

$$(E + \boldsymbol{\sigma}.\boldsymbol{p})\chi = m\psi \tag{3}$$

where $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ etc. and $\hbar = c = 1$.Consistency requires $E^2 = m^2 + \boldsymbol{p}^2$. Only one of ψ, χ is independent if equation of motion is obeyed: 4 real degrees of freedom (e.g. $\mathrm{f}_\uparrow, \mathrm{f}_\downarrow, \bar{\mathrm{f}}_\uparrow, \bar{\mathrm{f}}_\downarrow$).

ψ and χ are "Weyl fermion" fields.

1.2 Massless

$E = |\boldsymbol{p}|$

$$\boldsymbol{\sigma}.\boldsymbol{p}\,\psi = E\psi \rightarrow \boldsymbol{\sigma}.\hat{\boldsymbol{p}}\,\psi = \psi \qquad h = +1/2 \ \ \mathrm{R} \tag{4}$$

Similarly

$$\boldsymbol{\sigma}.\hat{\boldsymbol{p}}\,\chi = -\chi \quad h = -1/2 \ \ \mathrm{L} \tag{5}$$

$$\begin{pmatrix} p_z & p_x - \mathrm{i}p_y \\ p_x + \mathrm{i}p_y & -p_z \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = E \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$p_z\psi_1 + (p_x - \mathrm{i}p_y)\psi_2 = E\psi_1 \tag{6}$$

$$(p_x + \mathrm{i}p_y)\psi_1 - p_z\psi_2 = E\psi_2 \tag{7}$$

Consistency requires $E^2 = \boldsymbol{p}^2$. Only one of ψ_1, ψ_2 is independent if equation of motion is obeyed: 2 real degrees of freedom. Same for χ_1, χ_2.

1.3 Majorana

Can manufacture a 4-component fermion field with still only 2 degrees of freedom by using the charge conjugate of χ (or ψ):

$$\Psi_{\mathrm{M}} = \begin{pmatrix} \mathrm{i}\sigma_2\chi^* \\ \chi \end{pmatrix} \equiv \begin{pmatrix} \chi^{\mathrm{c}} \\ \chi \end{pmatrix} \tag{8}$$

which is a Majorana spinor field and satisfies $\mathbf{C}\Psi_{\mathrm{M}} = \Psi_{\mathrm{M}}$, i.e. the field is even under $\mathbf{C}$. So antiparticle = particle: this halves the number of degrees of freedom in Ψ_{M} ($\mathrm{f}_\uparrow, \mathrm{f}_\downarrow, \mathrm{f} = \bar{\mathrm{f}}$). Note that χ^{c} behaves as an R field.

In this kind of notation, we can write the Dirac field as

$$\Psi = \begin{pmatrix} \chi_{\bar{\mathrm{f}}}^{\mathrm{c}} \\ \chi_{\mathrm{f}} \end{pmatrix} \tag{9}$$

which is built of L fields for f and $\bar{\mathrm{f}}$, instead of L and R for f alone.

1.4 3-D rotations

$$\psi' = \mathrm{e}^{-\mathrm{i}\boldsymbol{\alpha}.\boldsymbol{\sigma}/2}\psi \tag{10}$$

3 real parameters $\boldsymbol{\alpha} \leftrightarrow$ angle, axis of rotation. For an infinitesimal rotation, this is

$$\psi' = (1 - \mathrm{i}\boldsymbol{\epsilon}.\boldsymbol{\sigma}/2)\psi \tag{11}$$

or

$$\delta_{\boldsymbol{\epsilon}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = -\mathrm{i}\boldsymbol{\epsilon}.\boldsymbol{\sigma}/2\ \psi = (\text{2x2matrix}) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{12}$$

This is a symmetry: different components of the spin-1/2 field get mixed under the rotation. Similarly isospin, $\mathrm{SU}(3)_{\mathrm{c}}$, etc. None of these change the *spin* of the field!

1.5 Simple SUSY

A simple SUSY transformation will "rotate" a spin-0 field into a spin-1/2 field, and v.v.!! Something like

$$\delta_\xi \phi(x) = (?)\ \xi\ \chi(x) \tag{13}$$

and

$$\delta_\xi \chi(x) = (?)\ \xi\ \phi(x) \tag{14}$$

How can we make sense of these? Take (13) first.

1. Field degrees of freedom must match on both sides of (13); can't have more d.o.f.s on one side than on other. If ϕ is a real scalar field, $\phi = \phi^\dagger$, only 1 d.o.f. No fermion field has 1 d.o.f. Must have at least 2. So need complex $\phi, \phi^\dagger \neq \phi$, 2 d.o.f.s. This could balance a Weyl ψ or χ field with 2 d.o.f.s. By convention, we choose χ. (We could, equivalently, use a Majorana field.)

But χ only has 2 d.o.f.s when equation of motion is obeyed. In general will need χ "off-shell" i.e. not obeying equation of motion (e.g. recall propagator $(\not{p} - m)^{-1}$, $\not{p} \neq m$), so really need 4 scalar d.o.f.s $\Rightarrow \phi$ and another complex scalar field F ("auxiliary field"). We will *ignore* this complication, and deal with $\{\phi,\ \chi\}$ rather than $\{\phi,\ \chi,\ F\}$.

2. Parameter ξ must be a spin-1/2 object, which combines with the spin-1/2 χ on RHS to make spin-0 ϕ on LHS.

3. ξ is independent of x: "global" SUSY ($\xi(x)$ is local SUSY $\longrightarrow$ supergravity). ξ is a constant spinor, $\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$ not a field, but ξ_1, ξ_2 are anticommuting objects.

4. Consider dimensions of ξ. Dimensions must match on each side of (13).

   Recall that with $\hbar = c = 1$ there is only one independent dimension which is taken to be that of mass, M. We denote the dimension of something by "[....]". For instance, [Action]=M^0, [x]=M^{-1} (like $\hbar/Mc$). A typical spin-0 mass term in an Action is $\int \mathrm{d}^4x\, m^2\, \phi^\dagger \phi$, from which it follows that [ϕ]=M. A fermion mass term is $m\bar{\Psi}\Psi$, whence $[\Psi] = M^{3/2}$; similarly for χ. We deduce that [ξ]= $M^{-1/2}$ for dimensions to match in (13).

5. RHS of (13) must be spin-0 combination of ξ, χ. Take ξ to be L spinor, like χ. Then we know how to couple two spin-1/2 objects to spin-0 in q.m. : $\xi \cdot \chi = \xi_1 \chi_2 - \xi_2 \chi_1$ ("up-down - down-up"). Actually this is Lorentz invariant coupling! (boosts as well). So take

$$\delta_\xi \, \phi(x) = \xi \cdot \chi(x). \tag{15}$$

Note by the way that $\chi \cdot \chi = \chi_1 \chi_2 - \chi_2 \chi_1$ is not zero.

6. Now consider (14). Checking dimensions we see LHS is $M^{3/2}$, RHS is $M^{1/2}$; need M^1 from somewhere. In massless theory, only possibility is ∂_μ. Then we need to get rid of the 'μ' index. This is done by dotting with $\sigma^\mu = (1, \boldsymbol{\sigma})$. Correct transformation has the form

$$\delta_\xi \, \chi(x) = A \sigma^\mu \sigma_2 \xi^* \partial_\mu \phi(x). \tag{16}$$

The rather complicated-looking expression on the RHS does in fact behave like a 'χ'.

7. We need a Lagrangian. Consider free case: $\mathcal{L}$ for massless complex scalar field ϕ is $\partial_\mu \phi^\dagger \partial^\mu \phi$, and for massless Dirac is

$$\bar{\Psi} \mathrm{i} \gamma^\mu \partial_\mu \Psi = \mathrm{i} \psi^\dagger \sigma^\mu \partial_\mu \psi + \mathrm{i} \chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi \quad (17)$$

where $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$. We want the χ part. So our candidate (free) Lagrangian is

$$\mathcal{L}_{\text{free}} = \partial_\mu \phi^\dagger \partial^\mu \phi + \mathrm{i} \chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi. \quad (18)$$

This is invariant under the SUSY transformations (15) and (16) if A=1!

1.6 SUSY generators and SUSY algebra

Recall for rotations

$$\chi' = \exp \mathrm{i}\boldsymbol{\alpha}.\boldsymbol{J}\ \chi\ \exp -\mathrm{i}\boldsymbol{\alpha}.\boldsymbol{J} \tag{19}$$

$\boldsymbol{J} = (J_1, J_2, J_3)$ are the *generators* of rotations, and also the angular momentum operators. They obey the *algebra* $[J_1, J_2] = \mathrm{i}J_3$. In a similar way, for a SUSY transformation we write

$$\chi' = \exp \mathrm{i}\xi \cdot Q\ \chi \exp -\mathrm{i}\xi \cdot Q. \tag{20}$$

Q must be a two-component spinor operator $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$.

The algebra of the SUSY generators Q_1, Q_2 is

$$\{Q_a, Q_b\} = (\sigma^\mu)_{ab} P_\mu \quad a, b = 1, 2 \tag{21}$$

The fermionic operators Q_a obey anticommutation relations rather than commutation relations. The P_μ has its origin in the ∂_μ of (16).

Equation (21) is very remarkable: it tells us that the Q's have a status on a par with the space-time translation operators P_μ. Indeed, the Q's are a kind of "square root of a translation". The familiar space-time transformations of the Lorentz group and space-time translations (which together make up the Poincaré group) are extended to include the anticommuting Q's. It seems that this is the *only* possible extension of the Poincaré group. Does Nature make use of it?

1.7 SUSY multiplet structure

The SUSY algebra (21) determines the structure of SUSY multiplets. The answer is that massless SUSY multiplets have the form

$\{|h = -j>, |h = -j + 1/2>\}$ + TCP conjugate .

So taking $j = 1/2$ we have the multiplet $\{|h = -1/2>, |h = 0>\}$, which is our "L χ + scalar" case, called the "(left) chiral supermultiplet". Taking $j = 1$, we have the multiplet $\{|h = -1>, |h = -1/2>\}$, which consists of a massless spin-1 (gauge) field - which has 2 on-shell d.o.f.s - and a massless spin-1/2 L field. This is the "gauge supermultiplet". This is all we need for the MSSM. [The case $j = 2$ gives us the graviton partnered by the spin-3/2 gravitino.]

Just as the angular momentum operators move us between the different m_j states, so the Q's move us between the states of a supermultiplet. For instance

$$Q_1|-j>= 0, \;\; Q_2|-j>\propto |-j+1/2> \; . \quad (22)$$

Note that Q_2 *does* change the spin of the state! That's because it itself is a spinor operator, carrying 1/2 unit of angular momentum. Note also that *the Q's do not change the values of any internal quantum numbers* - i.e. all members of a SUSY multiplet must have the same charge, colour, electroweak quantum numbers, etc.

Technical aside: Actually the algebra (21) is not quite the only possible mathematical extension of the Poincaré group. More general SUSY algebras also exist, in which the single generator Q_a is replaced by N generators Q_a^A $(A = 1, 2, \ldots N)$, and the anticommutation relations (21) are appropriately generalized. However, for all $N \neq 1$, every multiplet contains both R and L states, for given j. Such states, being in the same supermultiplet, must have the same gauge quantum numbers. But we know that the $SU(2)_L$ gauge symmetry of the SM treats the L and R components of quark and lepton fields differently. So, if we want to make a SUSY extension of the SM, it can only use the simple $N = 1$ case, where we are free to treat L and R fields differently.

Note that although TCP symmetry requires the existence of the TCP-conjugate multiplets, these are not in the same SUSY supermultiplet, and don't need to have the same gauge quantum numbers.

1.8 The fields of the MSSM

In Table 1 we list the chiral supermultiplets appearing in the MSSM, and in Table 2 the gauge supermultiplets of the MSSM. For every SM field there is a corresponding SUSY partner, carrying a $\tilde{}$. Note that the SUSY partners of the quarks and leptons are spinless bosons, those of the Higgs fields are L-type fermions, and those of the gauge fields are L-type fermions.

Two $\mathrm{SU(2)_L}$ Higgs spin-0 doublets are shown in Table 1, whereas the SM uses only one. In the SM, Yukawa interactions involving the Higgs doublet $\phi = (\phi^+, \phi^0)$ give masses to the $t_3 = -1/2$ components of fermion $\mathrm{SU(2)_L}$ doublets when ϕ^0 gets a vacuum expectation value, $<\phi^0>= v$. The $t_3 = +1/2$ components of quark $\mathrm{SU(2)_L}$ doublets are given mass via the corresponding Yukawa interaction with the charge-conjugate field $\phi_\mathrm{C} = \mathrm{i}\tau_2\phi^{\dagger\mathrm{T}} = (\bar{\phi}^0 \quad -\phi^-)$. But in the SUSY version, Yukawa couplings cannot involve both ϕ and $\phi^\dagger$ (see next lecture). The MSSM therefore requires two separate Higgs scalar doublets, one for the $t_3 = +1/2$ fermion masses, and one for the $t_3 = -1/2$ fermion masses. There are two corresponding superpartner doublets.

The field content of the MSSM $\longrightarrow$ gauge couplings unify accurately at $M_\mathrm{U} \sim 2 \times 10^{16}$GeV.

Names		spin 0	spin 1/2	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
squarks, quarks (× 3 families)	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L) or (χ_u, χ_d)	$\mathbf{3}$, $\mathbf{2}$, 1/3
	$\bar{u}$	$\tilde{\bar{u}}_L = \tilde{u}_R^\dagger$	$\bar{u}_L = (u_R)^c$ or $\chi_{\bar{u}} = \psi_u^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, -4/3
	$\bar{d}$	$\tilde{\bar{d}}_L = \tilde{d}_R^\dagger$	$\bar{d}_L = (d_R)^c$ or $\chi_{\bar{d}} = \psi_d^c$	$\bar{\mathbf{3}}$, $\mathbf{1}$, 2/3
sleptons, leptons (× 3 families)	L	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L) or (χ_{ν_e}, χ_e)	$\mathbf{1}$, $\mathbf{2}$, -1
	$\bar{e}$	$\tilde{\bar{e}}_L = \tilde{e}_R^\dagger$	$\bar{e}_L = (e_R)^c$ or $\chi_{\bar{e}} = \psi_e^c$	$\mathbf{1}$, $\mathbf{1}$, 2
higgs, higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1}$, $\mathbf{2}$, 1
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1}$, $\mathbf{2}$, -1

Table 1: Chiral supermultiplet fields in the MSSM.

Names	spin 1/2	spin 1	$SU(3)_c$, $SU(2)_L$, $U(1)_y$
gluinos, gluons	$\tilde{g}$	g	$\mathbf{8}$, $\mathbf{1}$, 0
winos, W bosons	$\widetilde{W}^\pm, \widetilde{W}^0$	$W^\pm$, W^0	$\mathbf{1}$, $\mathbf{3}$, 0
bino, B boson	$\tilde{B}$	B	$\mathbf{1}$, $\mathbf{1}$, 0

Table 2: Gauge supermultiplet fields in the MSSM.

2.1 Interactions

ϕ, χ model.

$$\mathcal{L}_{\text{free}} = \partial_\mu \phi^\dagger \partial^\mu \phi + \chi^\dagger \mathrm{i} \bar{\sigma}^\mu \partial_\mu \chi = T \tag{23}$$

$$\mathcal{L} = T - V \tag{24}$$

Renormalizability $[V] \leq M^4$. e.g. $e\, \bar{\Psi} \gamma^\mu A_\mu \Psi$ $[e] = M^0$.

Example: $Y(\phi, \phi^\dagger)\chi \cdot \chi$. Y can only be linear in $\phi, \phi^\dagger$: Yukawa-type interaction. Consider SUSY transform of $\phi\chi \cdot \chi$:

$$\delta_\xi(\phi\chi \cdot \chi) = (\xi \cdot \chi)\chi \cdot \chi + \phi\delta_\xi(\chi \cdot \chi) \tag{25}$$

Term with 3 χs can't be cancelled by anything else. However,

$$(\xi_1\chi_2 - \xi_2\chi_1)(\chi_1\chi_2 - \chi_2\chi_1) = 2(\xi_1\chi_2 - \xi_2\chi_1)\chi_1\chi_2 \tag{26}$$

$$= 2\xi_1\chi_2\chi_1\chi_2 \quad \text{because} \quad (\chi_1)^2 = 0 \tag{27}$$

$$= -2\xi_1\chi_1\chi_2\chi_2 = 0. \tag{28}$$

So actually this term is SUSY-invariant. But this will *not* work for $\phi^\dagger\chi \cdot \chi$! SUSY-invariant Yukawa interactions can only involve $\phi\chi \cdot \chi$, not $\phi^\dagger\chi \cdot \chi$. So can't use $\phi_{\mathrm{C}} = \mathrm{i}\tau_2\phi^\dagger$ as in SM. Must have two separate Higgs scalar $\mathrm{SU(2)_L}$ doublet fields for $t_3 = +1/2$ and $t_3 = -1/2$ fermion mass generation.

General solution (Wess-Zumino model)

$$V = |M\phi + \frac{1}{2}y\phi^2|^2 + \frac{1}{2}(M + y\phi)\,\chi\cdot\chi + \text{h.c.} \quad (29)$$

or

$$V = \left|\frac{\partial W(\phi)}{\partial\phi}\right|^2 + \frac{1}{2}\frac{\partial^2 W}{\partial\phi^2}\chi\cdot\chi + \text{h.c.} \quad (30)$$

W is "superpotential; here $W = \frac{1}{2}M\phi^2 + \frac{1}{6}y\phi^3$. We note the following terms in (29). (i) Mass for spin-0 $|M|^2\phi^\dagger\phi$; (ii) mass for spin-1/2 $\frac{1}{2}M\chi\cdot\chi$+h.c.; (iii) cubic scalar self-interactions $\frac{1}{2}yM^*\phi^2\phi^\dagger$+h.c.; (iv) quartic scalar self-interactions $\frac{1}{4}y^2\phi^2\phi^{2\dagger}$; (v) Yukawa interactions $\frac{1}{2}y\phi\chi\cdot\chi$ + h.c..

These interactions are highly constrained. For example, the same coupling parameter enters into the cubic and quartic scalar self-interactions, as well as the Yukawa-like boson-fermion interaction. In particular, the quartic coupling $\frac{1}{4}y^2$ is exactly the square of the Yukawa coupling. This results in a cancellation between the quadratic divergences encountered in one-loop corrections to the Higgs mass parameter, leaving only logarithmic divergences (a boson loop can cancel a fermion loop because the latter carries a relative minus sign).

This property means that the "SM fine-tuning problem" can be resolved in a SUSY-invariant theory.

In general, with many chiral supermultiplets as in the MSSM, the Yukawa term is

$$\frac{1}{2}\Sigma_{i,j}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\,\chi_i\cdot\chi_j. \tag{31}$$

2.2 The Yukawa interactions of the MSSM

In the SM, the Yukawa interactions of the Higgs fields with the fermions generate their (Dirac) masses when the neutral Higgs develops a vev (SSB). Dirac mass is

$$\begin{aligned} m_{\rm f}\bar{\Psi}_{\rm f}\Psi_{\rm f} &= m_{\rm f}(\psi_{\rm f}^{\dagger}\chi_{\rm f} + \chi_{\rm f}^{\dagger}\psi_{\rm f}) && (32)\\ &= m_{\rm f}((\mathrm{i}\sigma_2\chi_{\bar{\rm f}}^{\dagger\mathrm{T}})^{\dagger}\chi_{\rm f} + \text{h.c.} && (33)\\ &= m_{\rm f}\chi_{\bar{\rm f}}\cdot\chi_{\rm f} + \text{h.c.}. && (34)\end{aligned}$$

Here $\mathrm{i}\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Want similar mechanism in MSSM.

Consider as an example a single W-Z superpotential term of the form

$$W_1 = y_u \tilde{\bar{u}}_{\rm L}(\tilde{u}_{\rm L}H_u^0 - \tilde{d}_{\rm L}H_u^+) \qquad (35)$$

which is an $\mathrm{SU}(2)_{\rm L}$ singlet. This will generate the Yukawa interactions

$$\frac{1}{2}\frac{\partial^2 W}{\partial\tilde{\bar{u}}_{\rm L}\partial\tilde{u}_{\rm L}}\,\chi_{\bar{u}_{\rm L}}\cdot\chi_{u_{\rm L}} + \frac{1}{2}\frac{\partial^2 W}{\partial\tilde{u}_{\rm L}\partial\tilde{\bar{u}}_{\rm L}}\chi_{u_{\rm L}}\cdot\chi_{\bar{u}_{\rm L}} + \text{h.c.} \qquad (36)$$

$$= \frac{1}{2}y_u H_u^0(\chi_{\bar{u}_{\rm L}}\cdot\chi_{u_{\rm L}}+\chi_{u_{\rm L}}\cdot\chi_{\bar{u}_{\rm L}}) = y_u H_u^0\chi_{\bar{u}_{\rm L}}\cdot\chi_{u_{\rm L}} \;+\text{h.c.}. \qquad (37)$$

When H_u^0 develops a vev $< H_u^0 >= v_u$, we find a u-mass term of just the form (34) with $m_u = y_u v_u$.

In the realistic case, we need to include flavour mixing via a 3×3 matrix $\mathbf{y}$.

Two similar superpotentials give mass to "d" and "e" states (νs massless) exactly as in SM with SM Yukawa couplings, along with $< H_d^0 >= v_d$.

There is also the possible new term

$$W_H = \mu(H_u^+ H_d^- - H_u^0 H_d^0). \qquad (38)$$

This "μ" is the *only* new parameter required, so far.

2.3 R-parity

Unfortunately, however, there seems nothing to stop us from having a term of the form

$$\lambda \tilde{\bar{\tau}}_{\mathrm{L}}(\tilde{\nu}_{\mathrm{eL}}\tilde{\mu}_{\mathrm{L}} - \tilde{e}_{\mathrm{L}}\tilde{\nu}_{\mu\mathrm{L}}). \tag{39}$$

The problem with (39) is that it carries net lepton number, $\Delta L = 1$. Also B-violating couplings are possible. In the SM, it is remarkable that there are *no* possible gauge-invariant renormalizable terms which violate B or L. These are in fact violated by non-perturbative electroweak effects, so we cannot simply impose B and L conservation as a fundamental symmetry. Instead we impose "R-parity", where $R = +1$ for all SM particles, and $R = -1$ for sparticles. The results of this imposition are (i) the lightest sparticle is stable, and if neutral is a candidate for dark matter; (ii) sparticle decays contain an odd number of LSPs; (iii) sparticles are only produced in pairs at the LHC, $\Delta E \geq 2m_{\tilde{\chi}_1^0}$.

2.4 Gauge interactions

U(1) case

$$\mathcal{L}_{\text{free}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathrm{i}\tilde{A}^\dagger\bar{\sigma}^\mu\partial_\mu\tilde{A}. \tag{40}$$

The first term is the usual Maxwell Lagrangian involving $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ where A_μ is the photon field; $\tilde{A}$ is the neutral L-fermion partner of the photon, the photino. A_μ and $\tilde{A}$ each have 2 d.o.f.s when their equations of motion are satisfied. (40) by itself is SUSY-invariant. Need to include chiral supermultiplets carrying the gauge group quantum numbers. Simple U(1) case: add in a chiral supermultiplet $\{\phi, \chi\}$ carrying U(1) charge q. The gauge interactions are as usual taken care of by the replacements $\partial^\mu\phi^\dagger\partial_\mu\phi \rightarrow D^\mu\phi^\dagger D_\mu\phi \quad D_\mu = \partial_\mu + \mathrm{i}qA_\mu$ and $\mathrm{i}\chi^\dagger\bar{\sigma}^\mu\partial_\mu\chi \rightarrow \mathrm{i}\chi^\dagger\bar{\sigma}^\mu D_\mu\chi$. To make the whole thing SUSY-invariant additional interactions are required, namely

$$\sqrt{2}q[(\phi^\dagger\chi)\cdot\tilde{A} + \text{h.c.}] + \tfrac{1}{2}q^2(\phi^\dagger\phi)^2 \tag{41}$$

$(\phi^\dagger\chi)$ is U(1) neutral, and the $\cdot$ is the spinor dot product between the L-spinors χ and $\tilde{A}$.

(43) generalizes in the non-Abelian case to

$$\sqrt{2}g[(\phi^\dagger T^\alpha \chi)\cdot \tilde{g}^\alpha + \text{h.c.}] + \frac{1}{2}g^2(\phi^\dagger T^\alpha \phi)(\phi^\dagger T^\alpha \phi). \tag{42}$$

In the $\mathrm{SU}(2)_\mathrm{L}$ case, the first term will involve terms of the form

$$(H^\dagger \boldsymbol{\tau} \tilde{H}) \cdot \tilde{\boldsymbol{W}}. \tag{43}$$

There will also be a similar term involving $\tilde{B}$. After electroweak symmetry-breaking, the neutral wino, bino and Higgsino fields will be mixed by these interactions, leading to the physical neutralinos $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$. Similarly the physical charginos will emerge from mixing of the charged components. An important point to note is that the second term in (42) is a Higgs-like quartic scalar self-coupling, but with a prescribed coefficient; this has to be added to the W-Z superpotential term $|W|^2$. It is this prescribed quartic scalar coupling that is responsible for the bound on the lightest Higgs mass in the MSSM (see section 2.6).

All these gauge-related interactions are fully determined - the only new parameter up to this point (beyond those of the SM) is still 'μ' of (38).

2.5 SUSY breaking

Clearly the SUSY of the MSSM cannot be an exact symmetry. There are two ways to break it: SSB, or explicitly. Two points:

(a) The "$V(\phi)$" for the scalar field in our simple $\{\phi - \chi - \tilde{A}\}$ model is

$$\left|\frac{\partial W}{\partial \phi}\right|^2 + \frac{1}{2}q^2(\phi^\dagger \phi)^2 \tag{44}$$

which is necessarily ≥ 0. Usually we picture SSB via a potential of the form

$$V_{\mathrm{H}} = -\mu_{\mathrm{H}}^2 \phi^\dagger \phi + \lambda_{\mathrm{H}}(\phi^\dagger \phi)^2 \tag{45}$$

which has a minimum at a non-zero value of ϕ^0 (the vev), at which point V_{H} is negative. In a SUSY theory, the vacuum energy is exactly zero.

It has proved very difficult to construct any useful model of spontaneous SUSY breaking. Furthermore the terms quadratic in the Higgs fields in $|\partial W/\partial\phi|^2$, which arise from the "$\mu$" term $\mu H_u \cdot H_d$ in W, are

$$|\mu|^2(|H_u^+|^2 + |H_u^0|^2 + |H_d^-|^2 + |H_d^0|^2) \quad (46)$$

which don't have any minus signs, as would be needed to trigger electroweak SSB. So as SUSY-invariant theory can't accommodate e-w symmetry breaking (might the two breakings be related?).

(b) Explicit SUSY-breaking terms. They have to be "soft" (i.e. mass terms or cubic scalar ϕ^3 couplings) or else the cancellation of those quadratic divergences will be spoiled; and they have to be gauge invariant. There are no possible gauge-invariant mass terms in the SM - that's why gauge boson as well as fermion masses are generated by SSB. But superpartner fields *can* have gauge invariant mass terms:

1. gaugino masses
$$\frac{1}{2}(M_3\tilde{g}^\alpha \cdot \tilde{g}^\alpha + M_2\tilde{W}^a \cdot \tilde{W}^a + M_3\tilde{B}\cdot\tilde{B} + \text{h.c.}) \tag{47}$$
where α runs from 1 to 8 and a runs from 1 to 3.

2. squark $(\text{mass})^2$ matrices
$$m^2_{\tilde{Q}_{ij}}\tilde{Q}_i^\dagger.\tilde{Q}_j \quad \text{etc.} \tag{48}$$

3. and slepton $(\text{mass})^2$ matrices.

There are also

Triple scalar couplings

and

Higgs (mass)^2 terms which are SUSY breaking because they involve only the scalar fields and not their~partners:

$$m^2_{H_u} H_u^\dagger . H_u + m^2_{H_d} H_d^\dagger . H_d + b(H_u . H_d + \text{h.c.}) \tag{49}$$

where the parameters, despite appearances, are not necessarily positive!

Altogether these possible explicit SUSY-breaking terms introduce very many parameters - more than 100. Furthermore, the transformation that diagonalizes the quark (lepton) mass matrix of the SM does not in general diagonalize the squark (slepton) mass matrix $\rightarrow$ danger of large FNC processes. Also, complex mass matrices $\rightarrow$CP violation.

One framework which can lead naturally to suppression of dangerous off-diagonal terms is 'minimal supergravity theory' (mSUGRA), in which the masses take the simple form (at the GUT scale)

$$M_1 = M_2 = M_3 = m_{1/2} \tag{50}$$

$$m^2_{\tilde{Q}} = m^2_{\tilde{\bar{u}}} = m^2_{\tilde{\bar{d}}} = m^2_{\tilde{L}} = m^2_{\tilde{\bar{e}}} = m_0^2 \mathbf{1}. \tag{51}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2. \tag{52}$$

The values of the parameters in (49) depend on the mass scale, as usual. In particular, the dominant 1-loop shift in $m^2_{H_u}$ in running down from a high scale $\Lambda_{\rm U}$ to the weak scale is given approximately by

$$\delta m^2_{H_u} \approx -\frac{3y_t^2}{4\pi^2} m_S^2 \ln(\Lambda_{\rm U}/m_S), \qquad (53)$$

where $y_t \approx 1$ is the top quark Yukawa coupling, and m_S is the average of the two top squark squared masses. This contribution is capable of pulling $m^2_{H_u}$ down to a negative value at the weak scale, and hence potentially triggering electroweak symmetry breaking.

2.6 Higgs sector and e-w symmetry breaking

The doublets (H_u^+, H_u^0) and (H_d^0, H_d^-) contain 4 complex fields = 8 real d.o.f.s As in the SM, 3 of these combine with the gauge fields to give the massive $W^{\pm}$, W^0 and Z^0 after SSB. Here there are two vevs, v_u and v_d. We have

$$m_W^2 = \frac{1}{2} g^2 (v_u^2 + v_d^2) \tag{54}$$

for example. This leaves 5 remaining d.o.f.s in the MSSM Higgs sector, with masses as follows.

$$m_{A^0} = (2b/\sin 2\beta)^{1/2} \tag{55}$$

where

$$\tan\beta = v_u/v_d. \tag{56}$$

This is actually a pseudoscalar boson.

$$m_{H^{\pm}} = (m_W^2 + m_{A^0}^2)^{1/2}. \tag{57}$$

$$m_{H^0}^2 = \frac{1}{2}\{m_{A^0}^2 + m_Z^2 + [(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta]^{1/2}\} \tag{58}$$

$$m_{h^0}^2 = \frac{1}{2}\{m_{A^0}^2 + m_Z^2 - [(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta]^{1/2}\} \tag{59}$$

There is no constraint on the masses of $m_{A^0}, m_{H^\pm}, m_{H^0}$, but quite remarkably

$$m_{h^0} \leq m_Z |\cos 2\beta| \leq m_Z \,, \qquad (60)$$

the limit being reached as $m_{A^0} \to \infty$.

In the SM there is *no* such bound on the Higgs mass - here it has arisen essentially from the fact that the "ϕ^4" part of the scalar potential has the *given* (not a free parameter) form

$$\frac{(g^2 + g'^2)}{4}[|H_u^0|^2 - |H_d^0|^2]^2 \qquad (61)$$

(see (44)). Of course, this limit $m_{h^0} \leq m_Z$ has already been passed experimentally. The theory is still alive because one-loop corrections to these tree-level masses can be quite large. Such contributions grow as $\ln(m_S/m_t)$ where m_S is an average top squark mass. By pushing m_S up to say a few TeV one can increase the bound on m_{h^0} to about 130 GeV.

But fine-tuning will begin to be a worry once more if the mass scale of sparticles is much higher than this. The reason is that the mass-shift (53) enters into the minimization conditions on the Higgs potential. All terms in these conditions should be not be greatly larger than the weak scale, if fine-tuning is to be avoided. But the shift (53) grows quadratically with m_S, whereas the Higgs mass shift grows only logarithmically. In order to produce a large shift in the Higgs mass we need a correspondingly large value of m_S, which threatens to push (53) to an uncomfortably high value.