Mathematical Methods (Second Year) MT 2002

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ORGANISATION, CONTENT and READING

The course is scheduled to occupy 20 *lecture slots*, 3 per week in weeks 1-6, 1 in week 7 and 1 in week 8. These 20 slots will be divided into 5 *blocks* of 4 slots each. Each block of 4 will consist of three lectures followed by one 'consolidation period', during which the previous 3 lectures will be briefly summarised, some illustrative examples worked out in detail, and interaction encouraged with the audience. Each block of 4 will be supported with either a class problem set or a tutorial problem set, with the intention that four of these problem sets will be done in MT02, leaving the fifth for vacation work. In addition, a Collection Paper on Mathematical Methods (for week 0 of HT03) will be available to tutors; and also a Supplementary Problem set mostly containing problems of a more advanced and/or off-syllabus nature. The break-down of the material into the 5 blocks will be as follows.

BLOCK ONE

Coordinate transformations and symmetries

Week 1 (W.)	Lecture 1	Week 2 (W.)	Consolidation 1
Week 1 (Th.)	Lecture 2		
Week 1 (F.)	Lecture 3		

<u>Content</u> Translations; translation invariance and momentum conservation. Rotations. Rotation matrices. What is a vector? Angular momentum conservation. Parity. Galilean transformations. Accelerating frames. Rotating frames. Lorentz transformations.
<u>Reading</u> Feynman Lectures I, chs. 11, 12, 18, 17. Boas ch 3, sections 4, 6 (revision); ch 10 sections 1-3.

<u>Class Problems Set No.1</u> Exercises on Block 1.

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BLOCK TWO

Eigenvalues, eigenvectors and diagonalisation

Week 2 (Th.)	Lecture 4	Week 3 (W.)	Lecture 6
Week 2 (F.)	Lecture 5	Week 3 (Th.)	Consolidation 2

<u>Content</u> 2×2 motivation. Eigenvalues and eigenvectors of real symmetric 3×3 and $N \times N$ matrices; eigenvalues (assumed distinct) are real; eigenvectors are orthogonal. Diagonalisation of real symmetric matrices. Applications to: quadratic forms, coupled oscillations, eigenvector expansions. Eigenvalues, eigenvectors and diagonalisation of Hermitian matrices.
<u>Reading</u> Feynman Lectures II, ch 31. Boas ch 10, sections 4, 5, 10.

<u>Tutorial Problems Set No.1</u> Problems on Block 1, exercises on Block 2.

BLOCK THREE

Linear partial differential equations

Week 3 (F.) Lecture 7 Week 4 (W.) Lecture 8
Week 4 (Th.) Lecture 9
Week 4 (F.) Consolidation 3

Content Meaning and significance of 'linear' and 'homogeneous' in pde context. Analogy between 'linear differential operator' and 'matrix', and between 'function' and 'vector'. Types of second order linear homogeneous pde's. Physical examples. General orientation to solutions of pde's: initial and boundary conditions; some general solutions. Method of separation of variables in 2, 3 and 4 dimensions; use of Cartesian, spherical polar and cylindrical polar coordinates (the forms of ∇^2 in the latter two systems are assumed). Simple separable solutions.
Reading Stephenson 'Partial Differential Equations for Science Students' (3rd edtn, revised) chs 1, 2, 3, 4, 5 (for Blocks 3 and 4); Boas ch 13 (for Blocks 3 and 4 also).

Class Problems Set No.2 Exercises on Block 3.

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BLOCK FOUR

Eigenvalues and eigenfunctions of second-order ordinary linear de's of S-L type; eigenfunction expansions; use in solving pde's

Week 5 (W.) Lecture 10 Week 6 (W.) Consolidation 4
Week 5 (Th.) Lecture 11
Week 5 (F.) Lecture 12

Content Eigenvalues and boundary conditions. The example of trigonometric functions. General second-order ordinary differential equation of Sturm-Liouville type. Orthogonality of eigenfunctions. Normalisation. Legendre's equation. Hermite's equation. Eigenfunction expansions (analogy with eigenvector expansion). Fourier series. Use of Fourier series to solve initial and boundary value problems in pde's. Use of other eigenfunction expansions.
Reading Stephenson chs 3, 4, 5. Boas ch 12 sections 1, 2, 6, 7, 8, 9, 10, 22; ch 13.

Tutorial Problems Set No.2 Problems on Block 3, exercises on Block 4.

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BLOCK FIVE

Fourier transforms

Week 6 (Th.) Lecture 13 Week 7 (W.) Lecture 15
Week 6 (F.) Lecture 14 Week 8 (W.) Consolidation 5

Content The ideas of 'repetition period $\to \infty$' and 'discrete eigenvalue $\to$ continuous eigenvalue'. Fourier transform as limit of Fourier series. Inverse transform. The expansion functions e^{ikx}. The δ-function. Simple applications (single slit diffraction, momentum probability amplitude in QM, decaying state in QM). Parseval's theorem. Convolution; simple applications.
Reading Boas ch 15 sections 4, 5, 7; Stephenson ch 6 sections 1, 2, 3.

Class Problems Set No.3 Exercises on Block 5.