Hot Physics at Large N : a lattice overview

Michael Teper (Oxford) – Perimeter Workshop, May '07

- prelude
- $T = T_c$
- $T < T_c$
- $T > T_c$
- conclusions

Lattice – Preamble

Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4

$x_\mu \bullet - \bullet x_\mu + \widehat{\mu}\delta x \ : \ A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

$x_\mu \bullet - - - \bullet x'_m u \ : \ P\left\{ e^{\int_x^{x'} A.dx} \right\} \in$ SU(N) group

$x_\mu \underrightarrow{=} a n_\mu$

$a n_\mu \bullet - - - \bullet a n_\mu + a\widehat{\mu} \quad : \quad U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

gauge transformation: $U_\mu(n) \to g(n)U_\mu(n)g^\dagger(n+\hat{\mu})$

$\longrightarrow$ gauge invariant action?

$\mathrm{Tr}\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$ so

$Z = \int \prod_l dU_l e^{-\beta S}$ where $S = \sum_p \left\{1 - \frac{1}{N}\mathrm{ReTr}u_p\right\}$
where u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

$\longrightarrow$ symmetries ensure that:

$$\int \prod_l dU_l e^{-\beta S} \quad \stackrel{a\to 0}{\longrightarrow} \quad \int DA e^{-\frac{4}{g^2}\int d^4x Tr F_{\mu\nu}F_{\mu\nu}}$$
with
$\beta = \frac{2N}{g^2(a)} \stackrel{a\to 0}{\longrightarrow} \infty$
and we vary the parameter β in order to vary the lattice spacing a

$\longrightarrow$ Monte Carlo:

$$\int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n}\sum_{I=1}^{n} \Phi(U^I) + O\left(\frac{1}{\sqrt{n}}\right)$$

- calculating masses from Euclidean correlators:

$\Phi(t)$ a gauge invariant operator

$$\langle \Phi^{\dagger}(t = an_t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \stackrel{t\to\infty}{\simeq} |c|^2 e^{-man_t}$$

where am is lightest mass with quantum numbers of Φ in lattice units

- continuum limit :

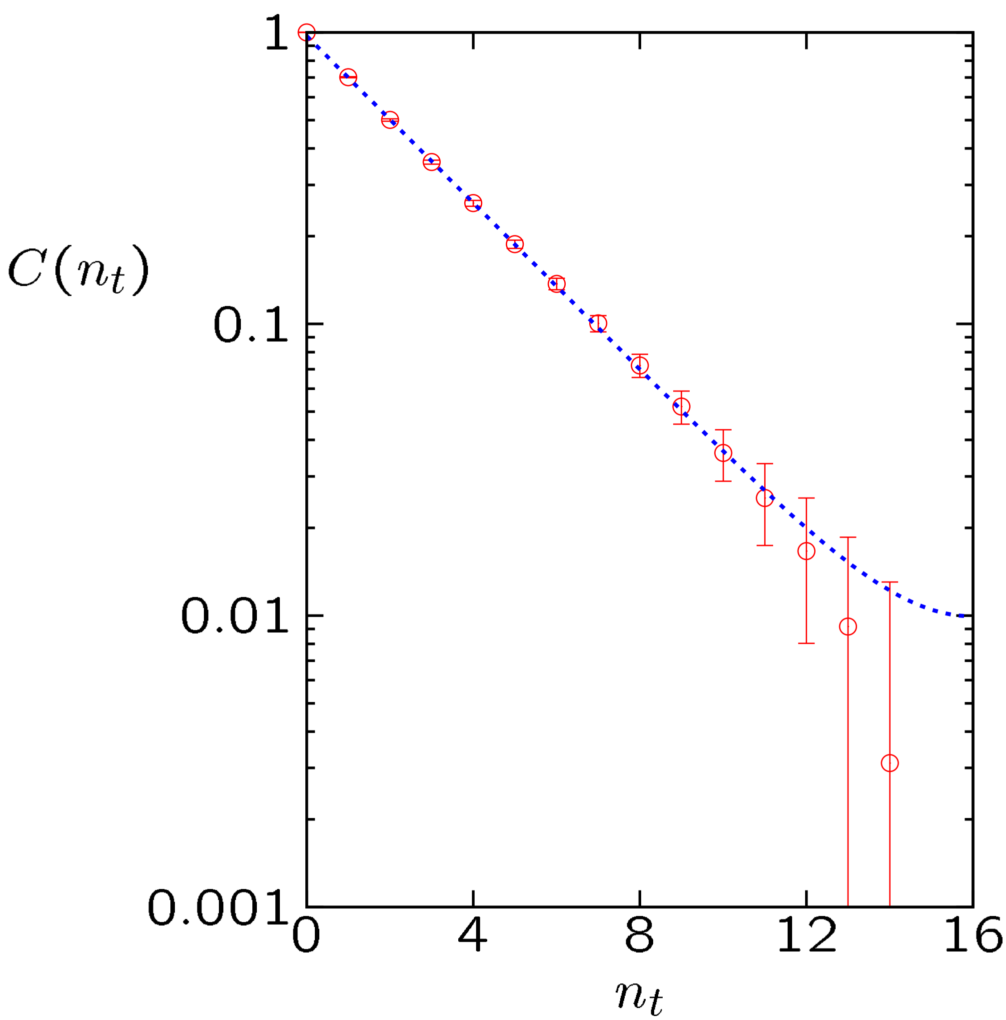
$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2 \sigma + O(a^4)$$

- large N limit :

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Can we do accurate calculations?

SU(3), 32^4, $a \simeq 0.046$ 'fm'

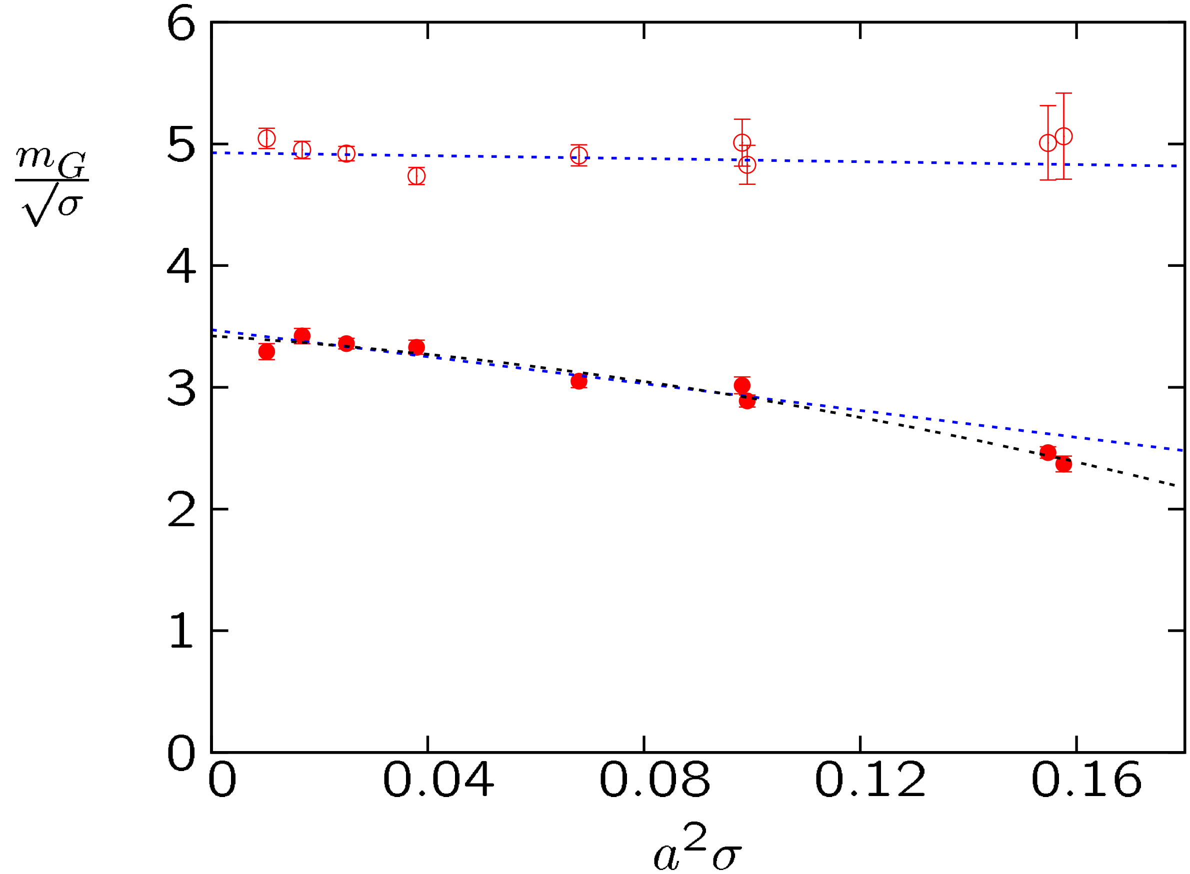


$$C(t = an_t) \overset{t\uparrow}{\simeq} |c|^2 e^{-man_t}$$

$\Rightarrow$

fit : $am_{0^{++}} = 0.330(7)$

Continuum limit mass spectrum: SU(3)



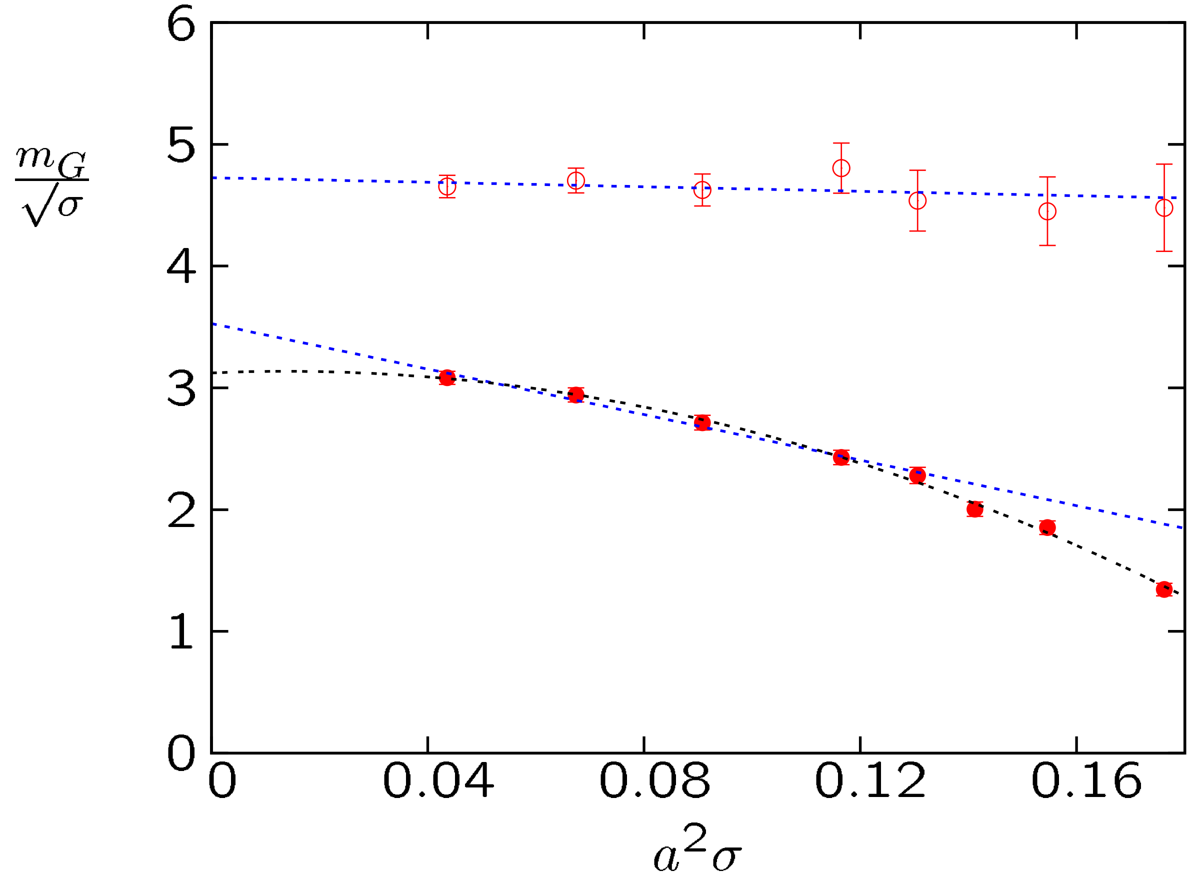
$O(a^2)$ continuum extrapolations:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Continuum limit mass spectrum: SU(8)



$O(a^2)$ continuum extrapolation:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.53(8) - 9.3(1.0)a^2\sigma$$

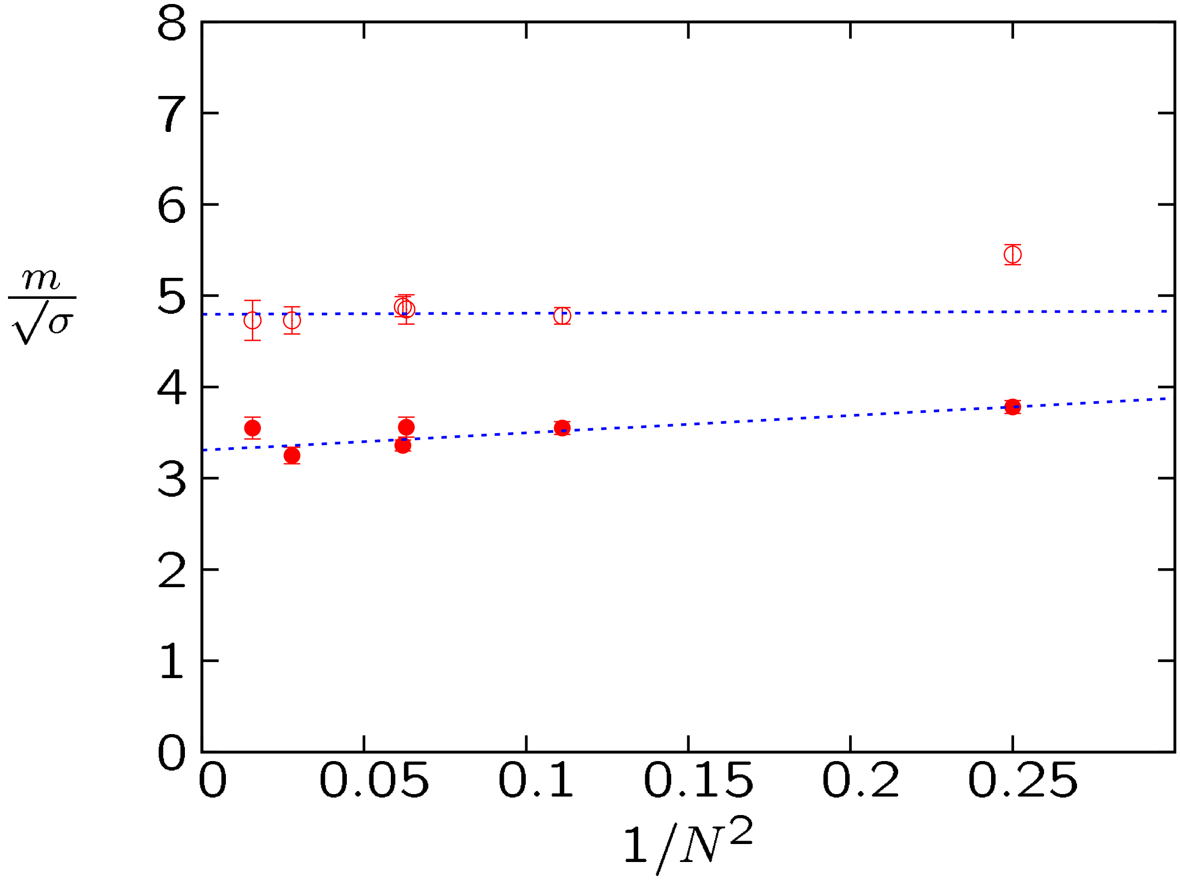
$O(a^4)$ extrapolation

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.13(25) + 1.66a^2\sigma - 66.0(a^2\sigma)^2$$

systematic errors!

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



$O(1/N^2)$ extrapolations to $N = \infty$:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}}|_N = 3.31 + \frac{1.90}{N^2}$$

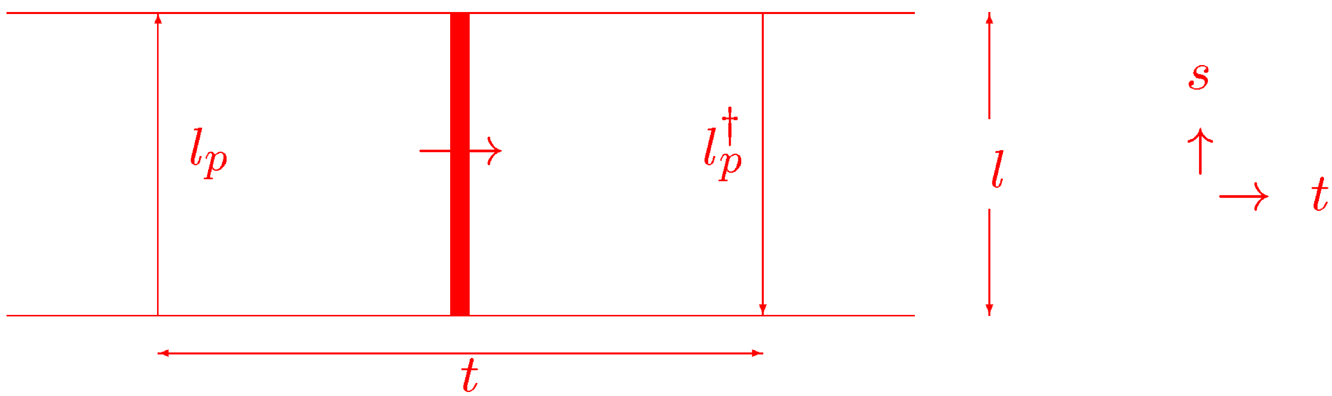
$$\frac{m_{2^{++}}}{\sqrt{\sigma}}|_N = 4.80 + \frac{0.11}{N^2}$$

Linear confinement in SU($N \to \infty$)?

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0) \rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



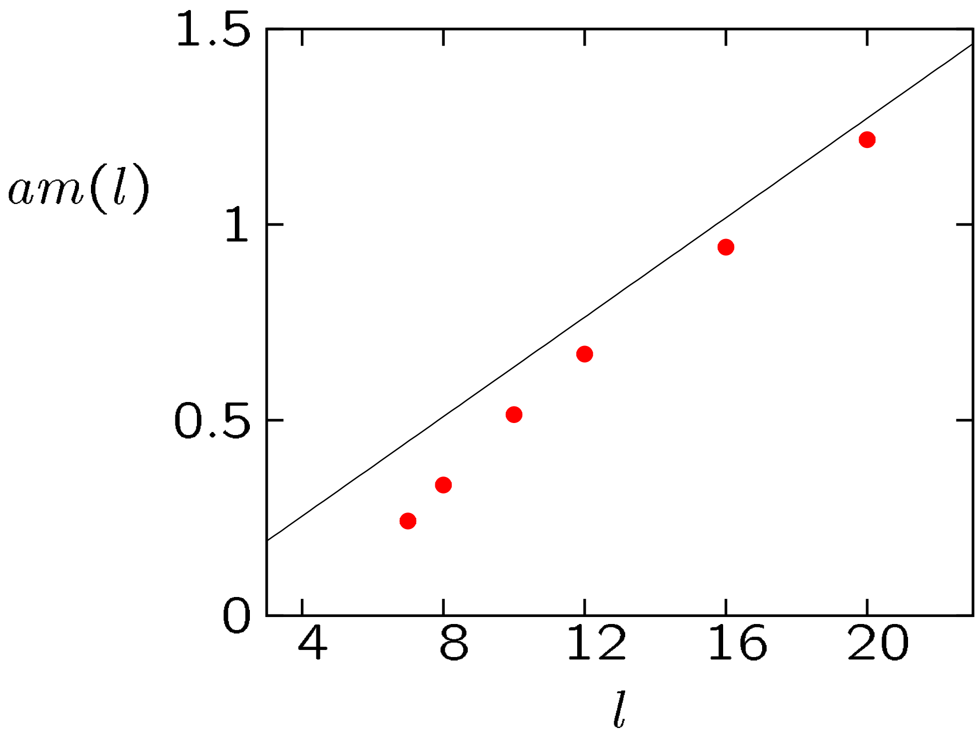
where we expect, for linear confinement,

$$m_p(l) = \sigma l - \frac{\pi(D-2)}{6l^2} + O\left(\frac{1}{l^4}\right)$$

- no sources, no Coulomb terms

SU(6)

H. Meyer, M. Teper: hep-lat/0411039



indeed we find

$$am(l) \simeq \sigma l$$

over a range of ‘string’ lengths up to

$$l \simeq 5.0 \times \frac{1}{\sqrt{\sigma}}$$

surely large enough to be asymptotic ...

So :

- accurate lattice calculations possible

- accurate continuum extrapolations possible

- SU(3) $\sim$ SU(∞) for many quantities

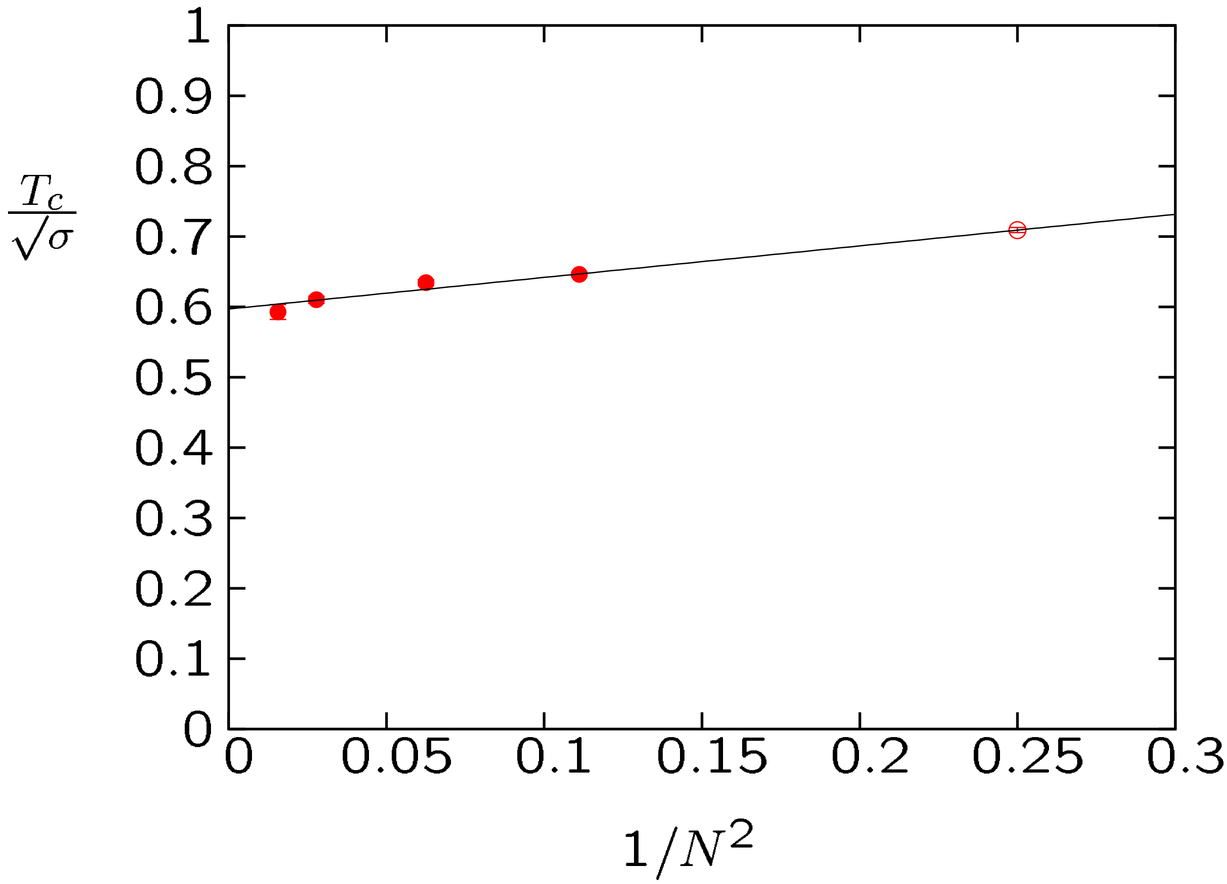
- linear confinement persists at large N

Motivated by the apparent phenomenological relevance of large-N, let us turn to the detailed properties of SU(N) gauge theories at finite T ...

Deconfining temperature in D=3+1

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta)L_t}$ if $L_s \gg L_t$



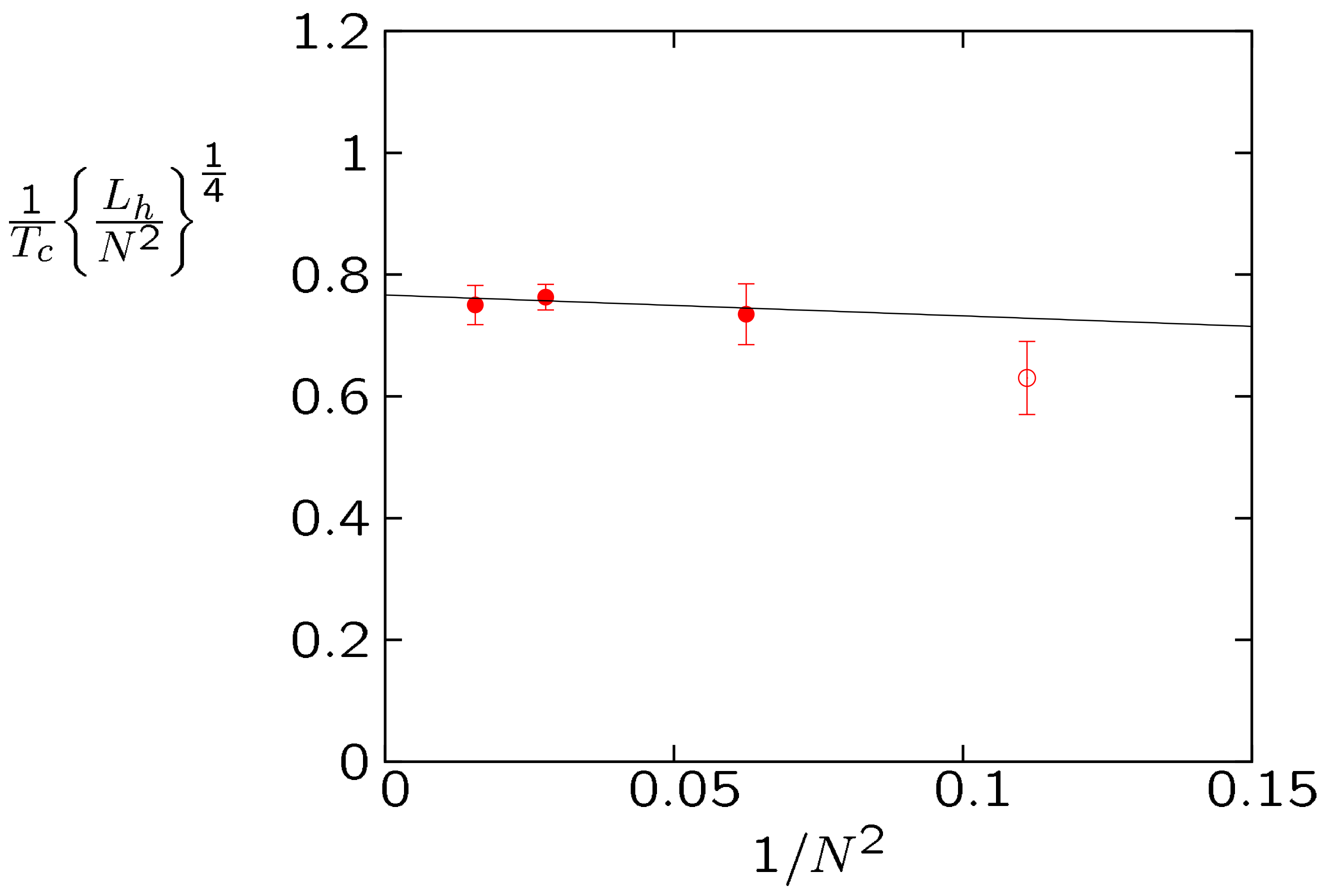
2nd order ∘ ; 1st order •

$\Rightarrow$

$$\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$$

Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003



$\Rightarrow$

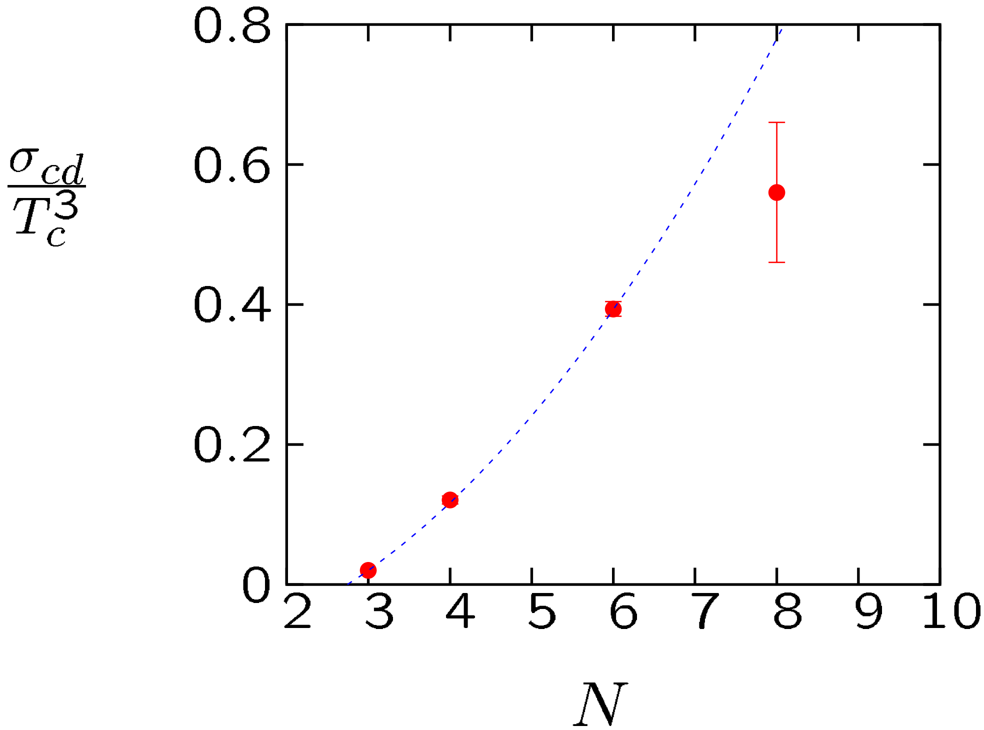
large-N deconfinement is 'normal' first order

$N = 3$ 'weakly' first order

Confinement-deconfinement wall tension

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

aT=0.2 :



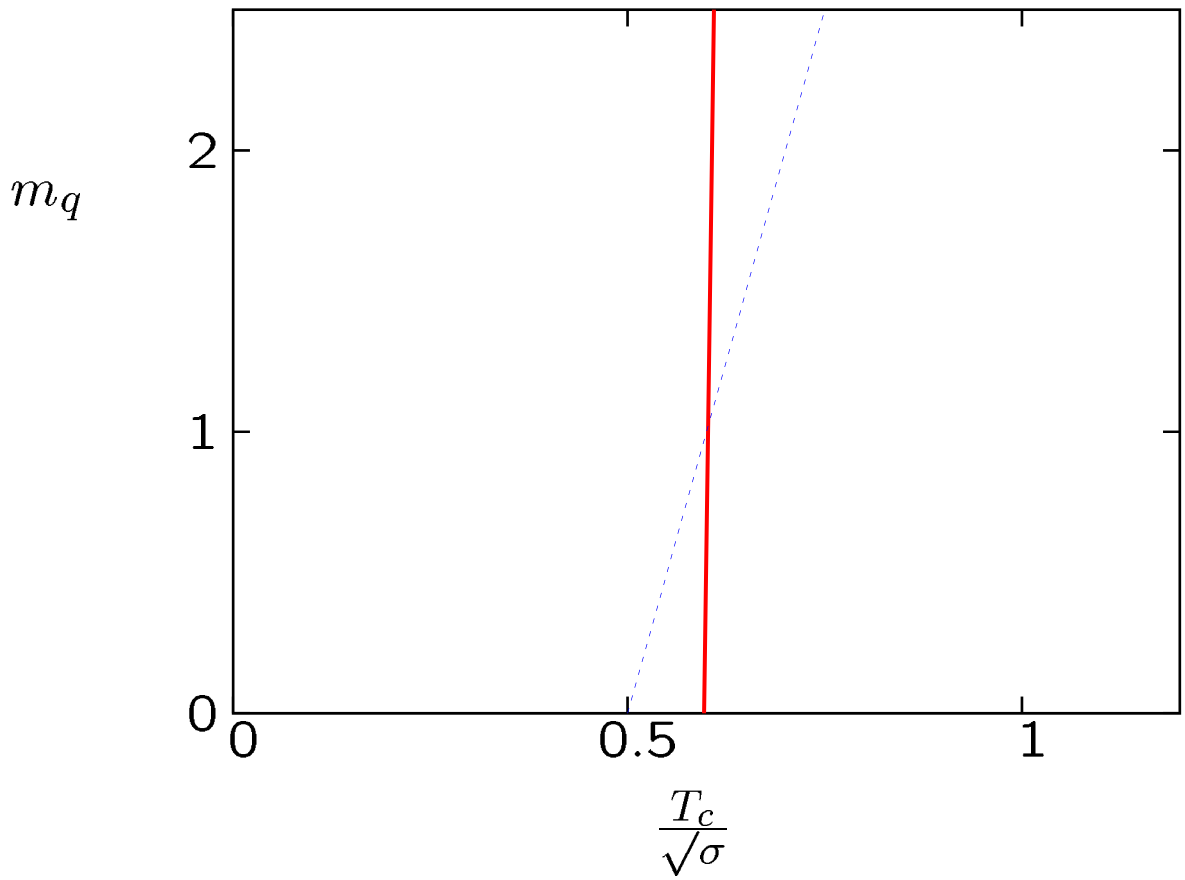
fit :

$$\frac{\sigma_{cd}}{T_c^3} = 0.0138N^2 - 0.104$$

$\Rightarrow$

interface tension small

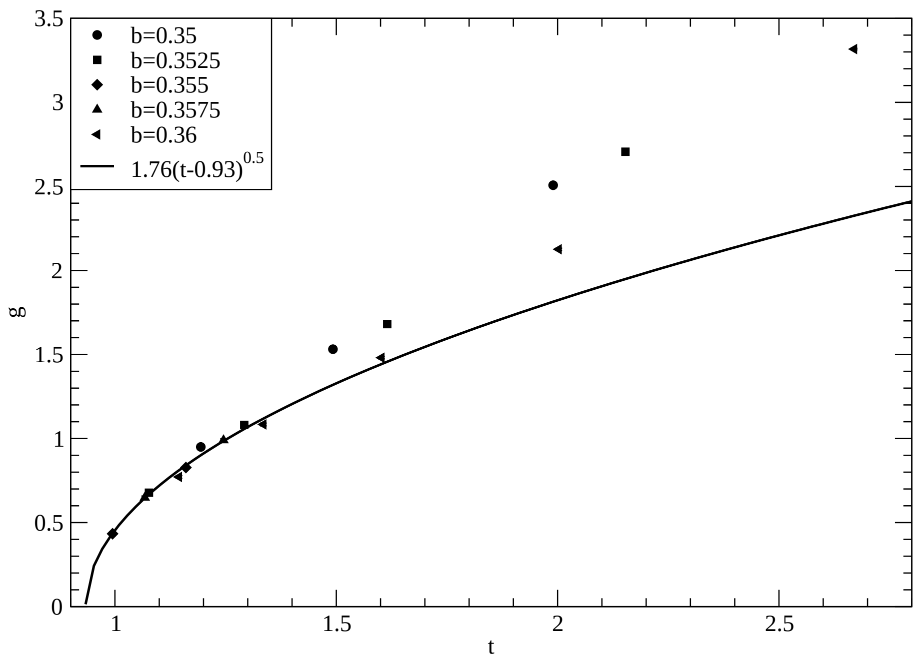
Confinement-deconfinement in QCD_∞?



—— $T = T_c$ ind of m_q at $N = \infty$

Chiral symmetry restoration as $T \to T_c$ at large N

R. Narayanan, H. Neuberger: hep-th/0605173

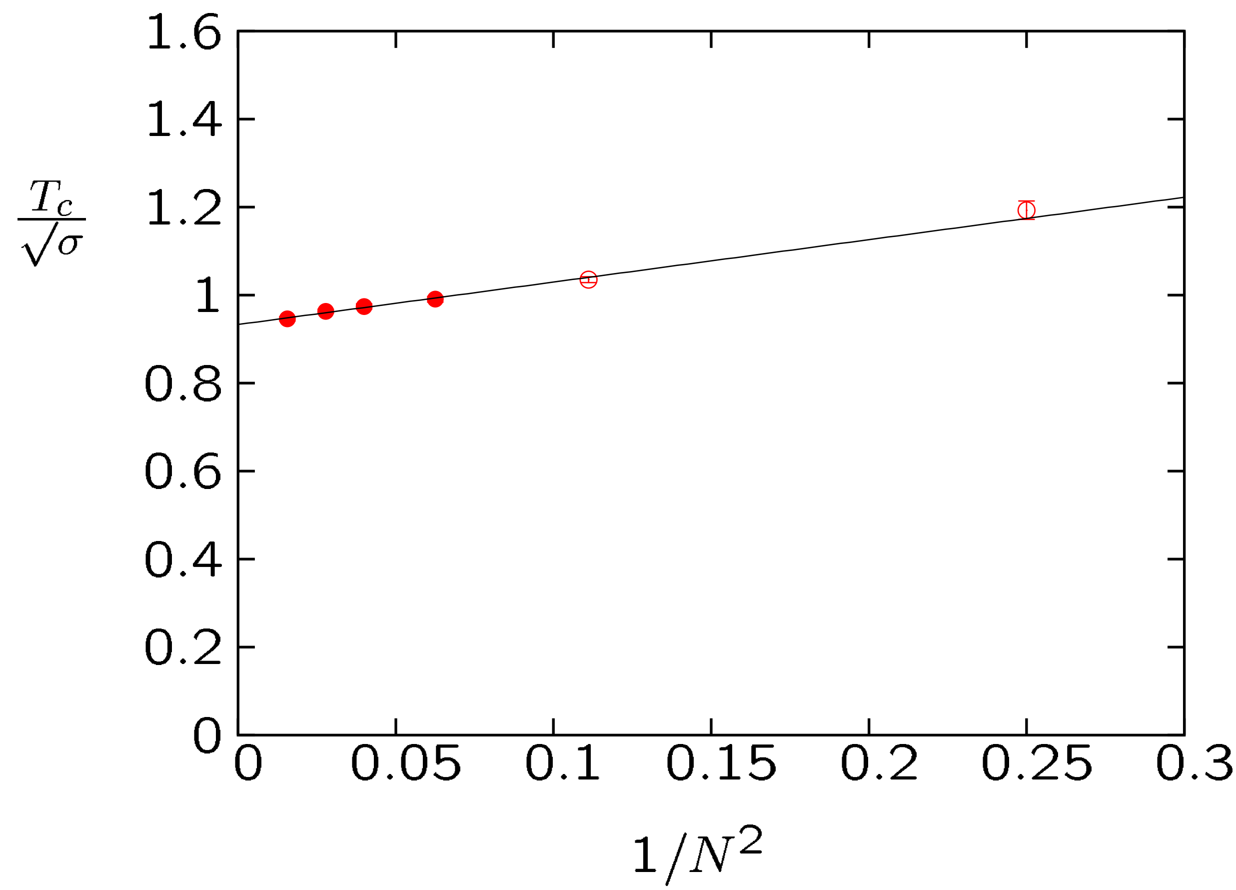


the gap in the eigenvalue spectrum of the Dirac operator, D, at $\lambda \simeq 0$ for $N = 23$ to $N = 53$.

D=3+1 $\longrightarrow$ D=2+1

$\frac{T_c}{g^2 N}$ J. Liddle, M.Teper : hep-lat/0509082; in preparation

$\frac{\sqrt{\sigma}}{g^2 N}$ B. Bringoltz, M.Teper : hep-th/0611286



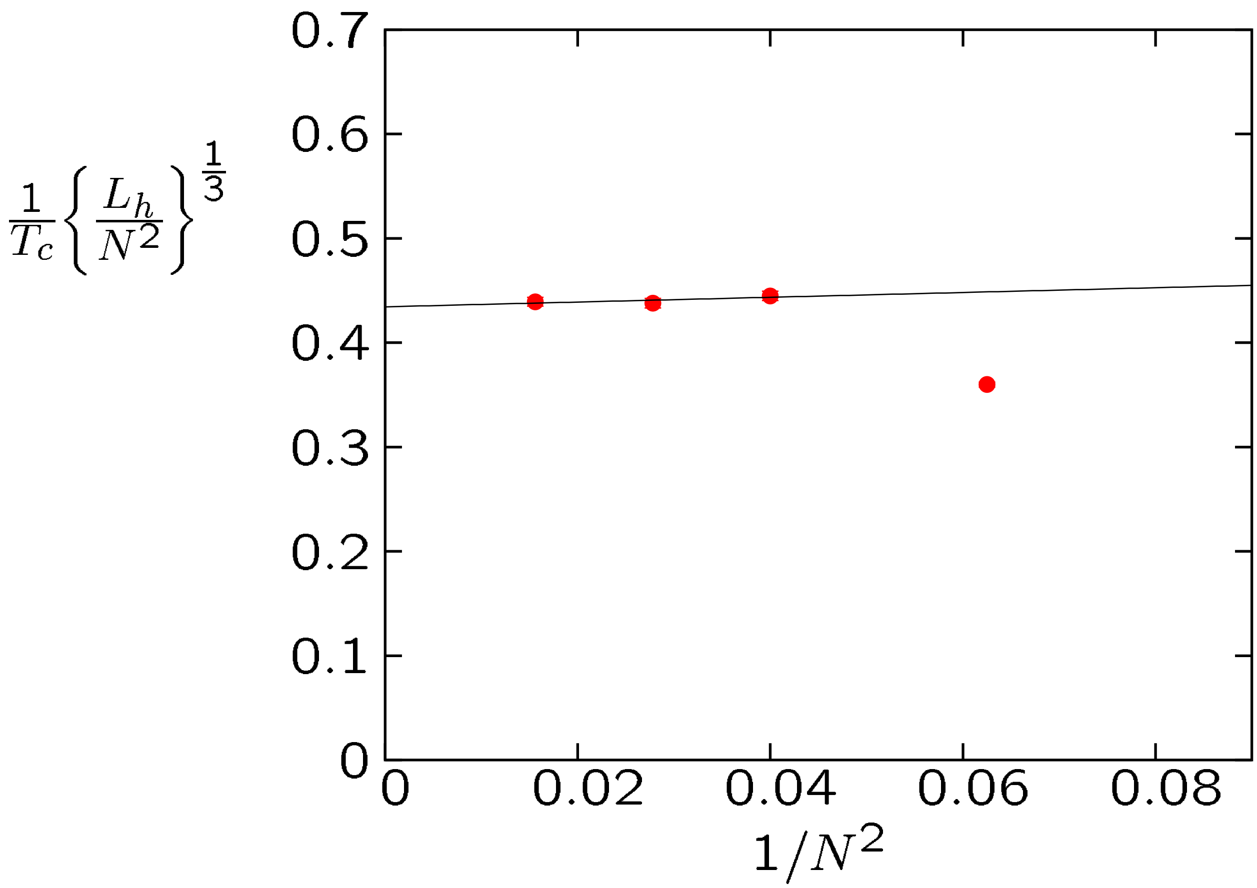
2nd order ○ ; 1st order ●

$\Rightarrow$

fit : $\frac{T_c}{\sqrt{\sigma}} = 0.933(4) + \frac{0.96(8)}{N^2}$ preliminary

D=2+1 latent heat

J. Liddle, M.Teper : in preparation



$\Rightarrow$

large-N deconfinement is ‘normal’ first order

$N = 4$ ‘weakly’ first order

Single or multiple deconfining transitions?

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

let l_p be the Polyakov loop (fund repn), then

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined

i.e. deconfinement $\leftrightarrow$ Z_N ssb

$\Rightarrow$

is there one transition or several?

e.g.

$$SU(4): \qquad Z_4 \stackrel{T=T_c}{\longrightarrow} Z_2 \stackrel{T=T_{c'}}{\longrightarrow} 1$$

corresponding to

$T = T_c$: k=2 strings break – but not k=1

$T = T_{c'}$: k=1 strings break

NO : there is only one transition

Polyakov loop order parameter - a problem

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined ssb $V = \infty$

BUT
if V is small enough to see CD tunnellings, which you need to identify T_c accurately, then

$$\sigma_{DD'} \overset{T=T_c}{\leq} \sigma_{CD}/2 \quad \rightarrow \quad \langle l_p \rangle \overset{T=T_c+\epsilon}{=} 0$$

due to DD' tunnellings just above T_c

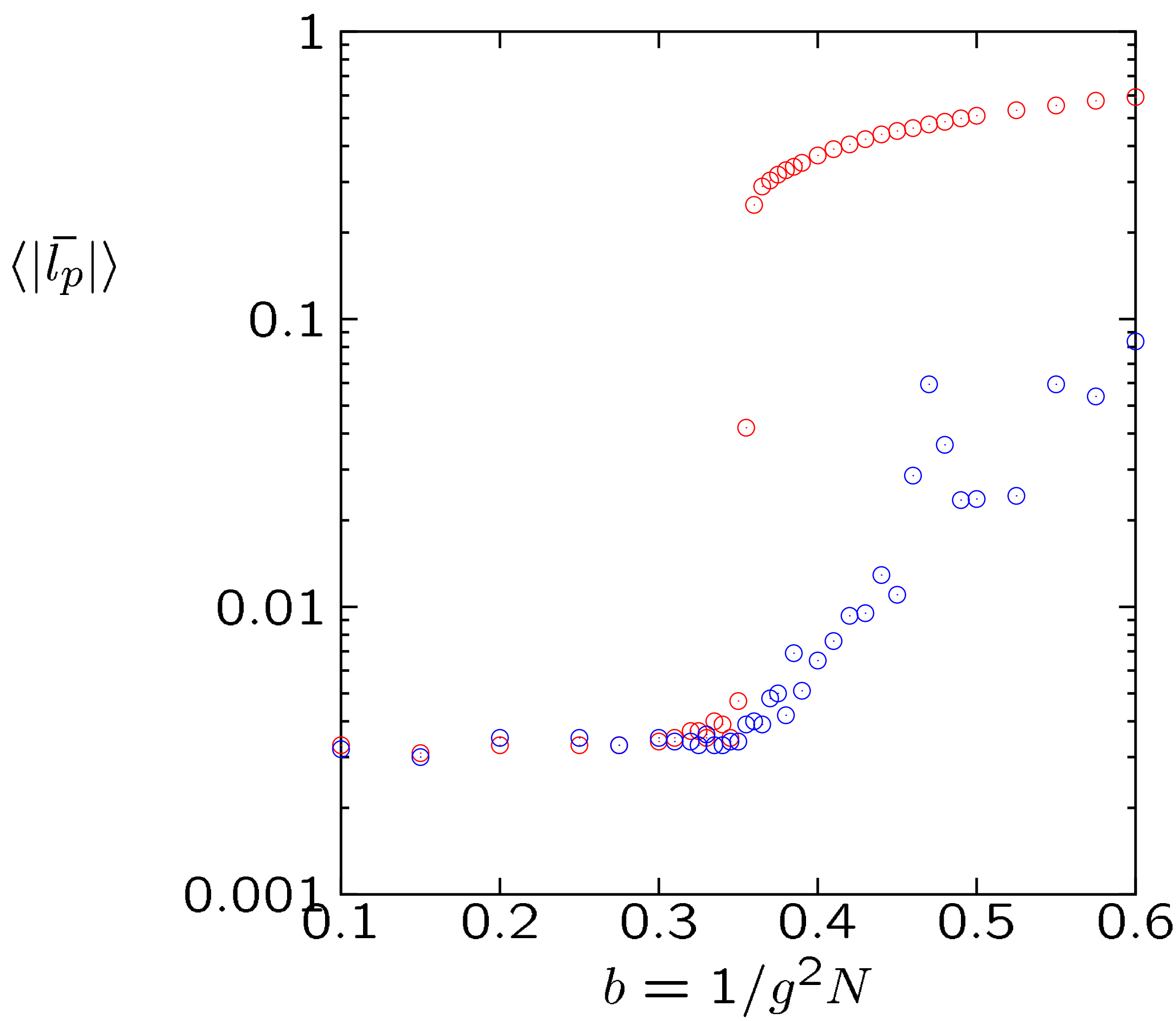
$\rightarrow$

usually modify the Polyakov loop operator:

$$\langle l_p \rangle \quad \rightarrow \quad \langle |\bar{l}_p| \rangle$$

(usual) Polyakov loop order parameter

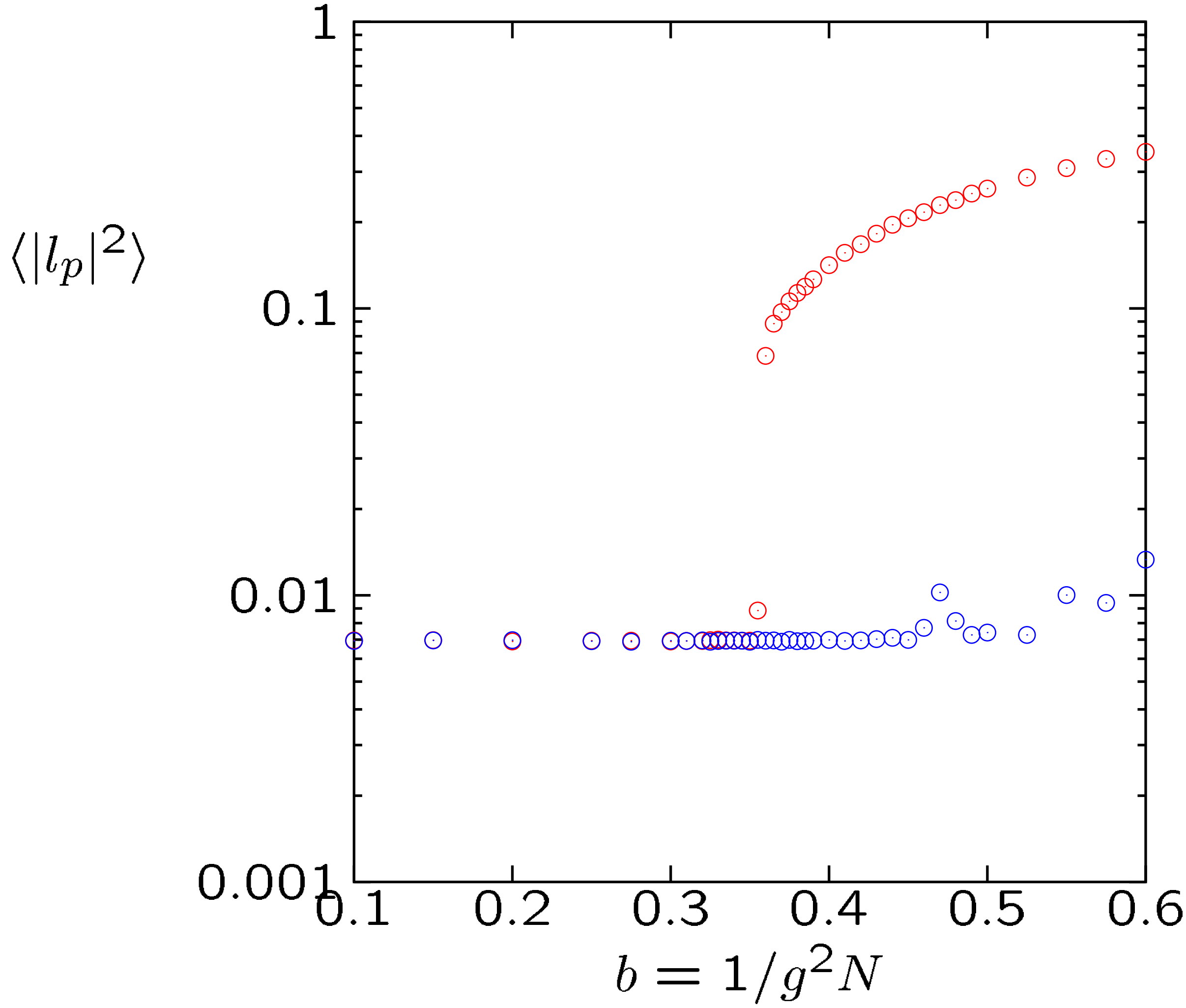
SU(12)



time-like Polyakov loop, ○

space-like Polyakov loop, ○

adjoint Polyakov loop



time-like adjoint Polyakov loop, ○
space-like adjoint Polyakov loop, ○

adjoint Polyakov loop

$\langle |l_p|^2 \rangle = 1/N^2$
$=$ value random matrix (Haar measure)
in both strong and weak coupling confined phases

now

our normal Polyakov loop is a normalised trace in the fundamental representation: $l_p = \frac{1}{N}\mathrm{Tr}_f l$

and so the loop in the adjoint representation is:
$\mathrm{Tr}_a l = N^2 |l_p|^2 - 1$

$\rightarrow$

$\langle \mathrm{Tr}_a l \rangle = 0$ in the confining phase
$\langle \mathrm{Tr}_a l \rangle = O(1)$ in the deconfined phase

$\leftrightarrow$

confined phase : adjoint string does not (easily) break
deconfined phase : adjoint string easily breaks

good order parameter even if we use an adjoint action

N counting of free energies (heuristic)

$$Z = e^{-\frac{F}{T}} = \sum_n e^{-\frac{E_n}{T}} \quad (1)$$

$$= \sum_{c=singlet} e^{-\frac{E_c}{T}} + \sum_{g=gluons} e^{-\frac{E_g}{T}} \quad (2)$$

$$= e^{-\frac{F_c}{T}} + e^{-\frac{F_g}{T}} \quad (3)$$

and at $T = T_c$ we have

$$F_c = F_g$$

but

$$F_g \sim N^2 \qquad \text{colour singlet entropy} \sim N^0$$

so reason that $T_c \not\to 0$ as $N \to \infty$ is that

$$E_c = hadron\, masses + E_{vac}$$

and

$$E_{vac} \sim -N^2 \sim gluon\, condensate$$

so

$$F_g = -E_{vac} \text{ at } \; T = T_c$$

T-dependence of the string tension?

extra free energy of 2 sources a distance r apart

$$\Delta F(r) = V_{eff}(r,T) \stackrel{r\to\infty}{\simeq} \sigma_{eff}(T) r$$

but

$$\exp\{-\Delta F(r)/T\} \stackrel{r\to\infty}{\simeq} \exp\{-m_l(l) r\}$$

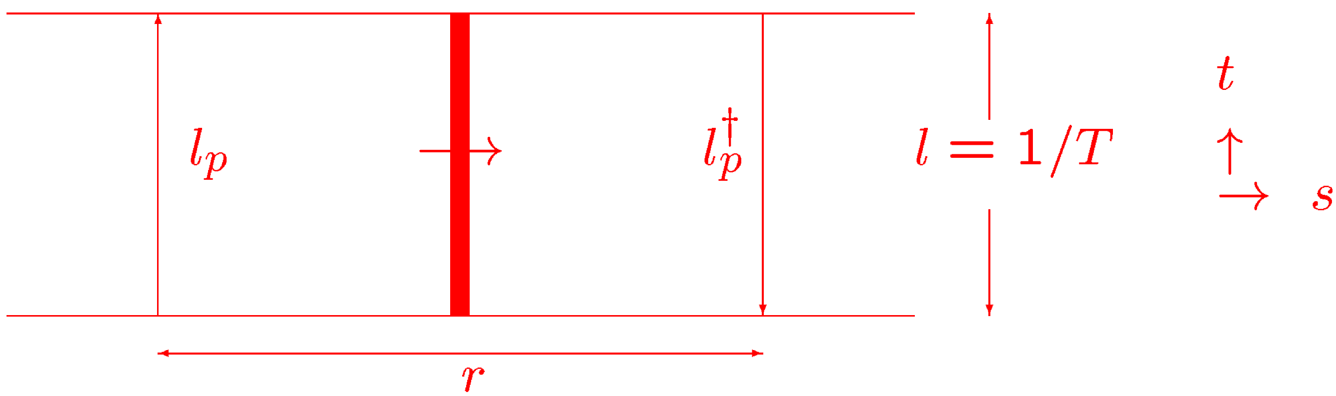
where $m_l(l = 1/aT)$ is the lightest mass coupling to the Polyakov loop that represents the world-line of the source, i.e. it is the mass of the lightest flux loop that winds around the time-torus, so

$$\sigma_{eff}(T) = m_l(l = 1/T).T$$

calculate from space-like correlations of time-like Polyakov loops

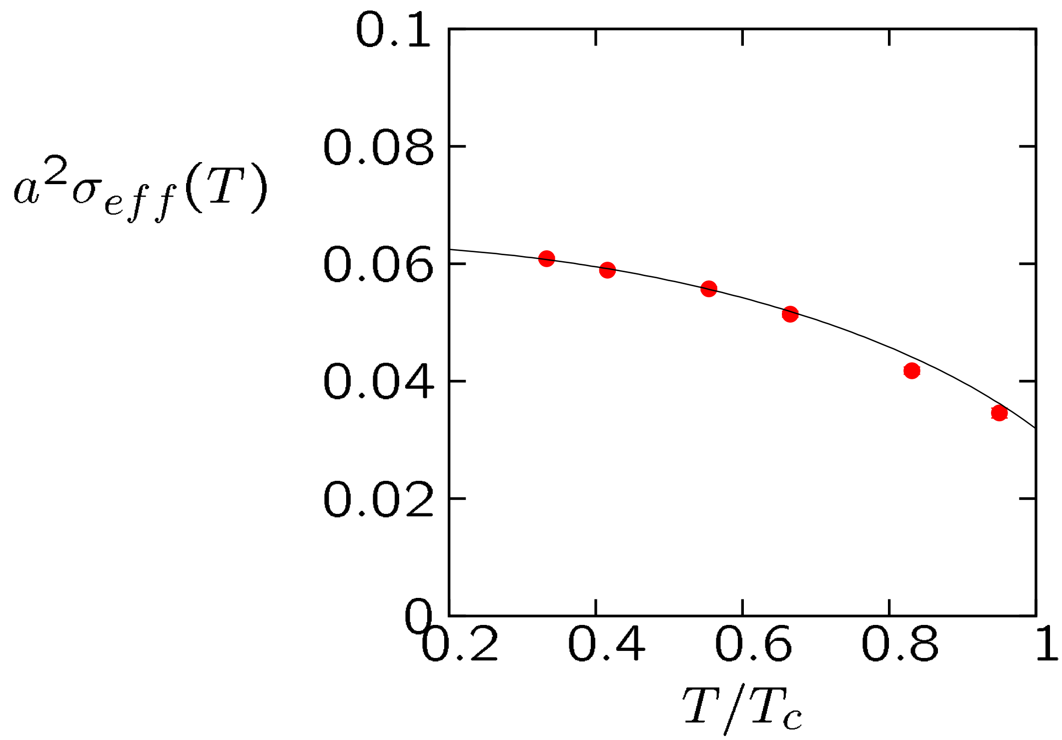
$$\langle l_p^\dagger(r) l_p(0)\rangle \stackrel{t\to\infty}{\propto} \exp\{-m_p(l=1/T)r\}$$

in pictures



D=3+1 SU(6)

H.Meyer, M.Teper: hep-lat/0411039



Nambu-Goto fit shown:

$$a^2\sigma_{eff}(l = 1/aT) = \frac{am_l(l)}{l} = a^2\sigma\left(1 - \frac{2\pi}{3}\frac{1}{\sigma l^2}\right)^{\frac{1}{2}}$$

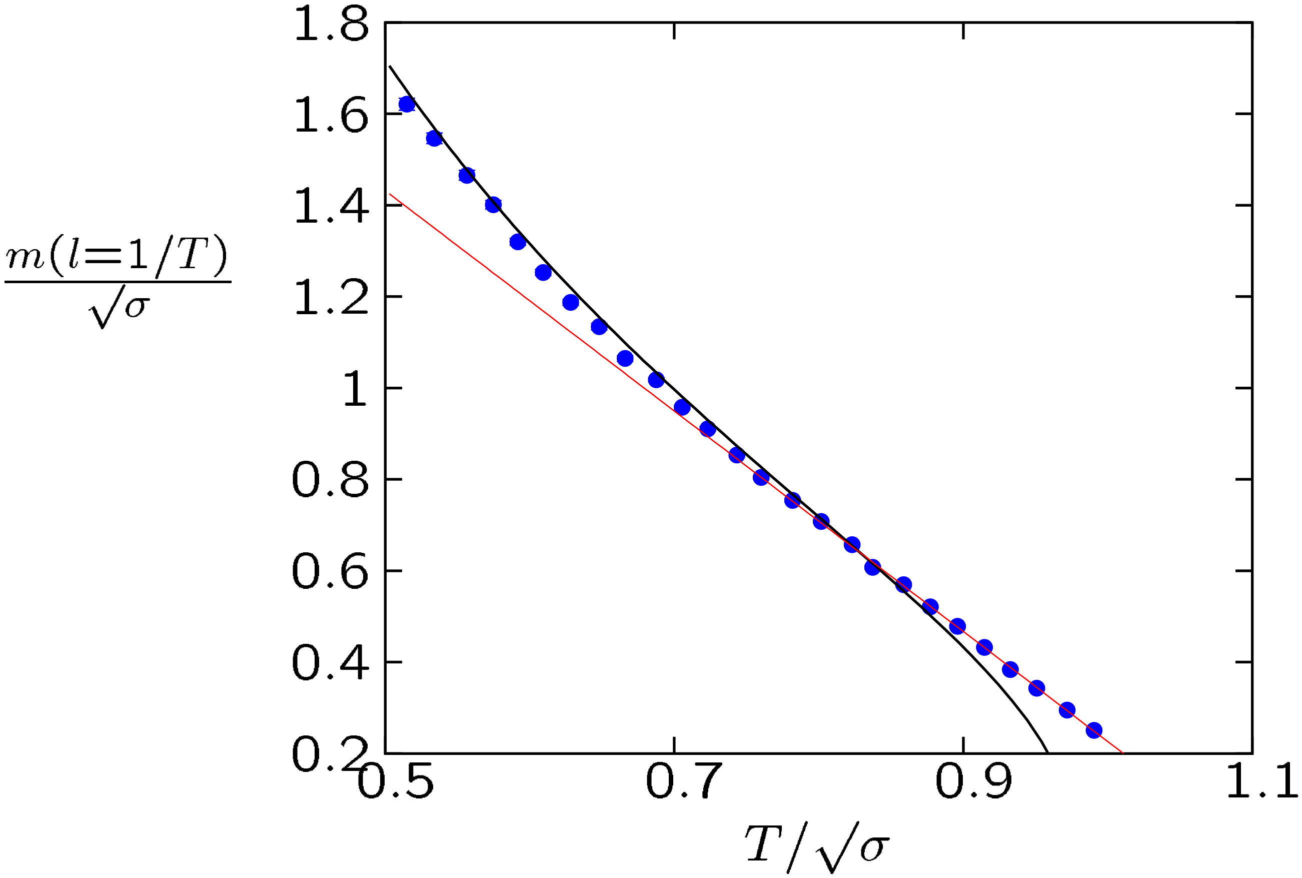
$\xrightarrow{T\to 0}$

$$\sigma_{eff}(T) = \sigma - \frac{\pi}{3T^2} + O(\frac{1}{T^4})$$

what of 2nd order transitions?

NG $\rightarrow$ critical behaviour?

e.g. SU(2), D=2+1:



- critical: $\propto (T - T_c)^{\nu=1}$
- NG: $\frac{\sqrt{\sigma}}{T}\left(1 - \frac{\pi}{3}\frac{T^2}{\sigma}\right)^{1/2}$

‘Hagedorn’ string condensation – heuristics

probability flux tube of length l
$\propto$ number paths length l $\times$ Boltzmann suppression
$\propto \quad \exp\{+cl\}\exp\{-\sigma l/T\} \quad \propto \quad \exp\{-(\sigma - cT)l/T\}$

$\rightarrow$

$\sigma_{eff}(T) \rightarrow 0$ as $T \rightarrow T_H = \sigma/c$

$\rightarrow$

a second-order deconfining transition
BUT neglects interactions (e.g. excluded volume effects) so argument best for $N = \infty$

although Nambu-Goto values match well on to SU(2) in D=3+1 and SU(3) in $D = 2 + 1$:
$T_c/\sqrt{\sigma} = 0.7091(36)$ vs 0.691 in SU(2) D=3+1
$T_c/\sqrt{\sigma} = 1.007(4)$ vs 0.977 in SU(3) D=2+1

Can we hope to see it through the metastable window provided by the strongly first order deconfining transition?

Searching for the Hagedorn transition : SU(12)

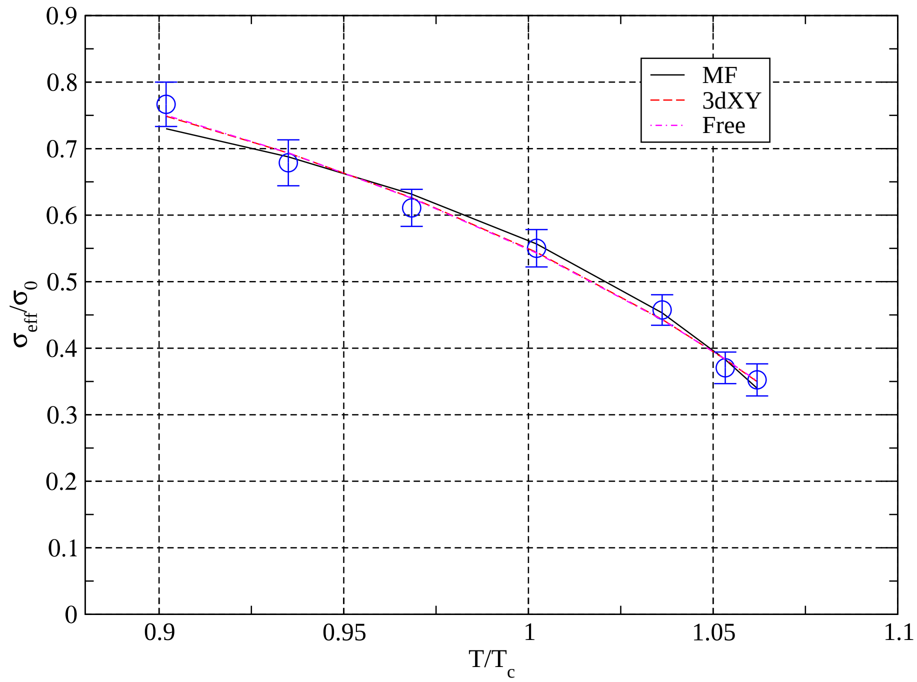
B. Bringoltz, M.Teper: hep-lat/0508021

use strong metastability of the 1st order deconfining transition to stay in the confining phase for

$$T > T_c$$

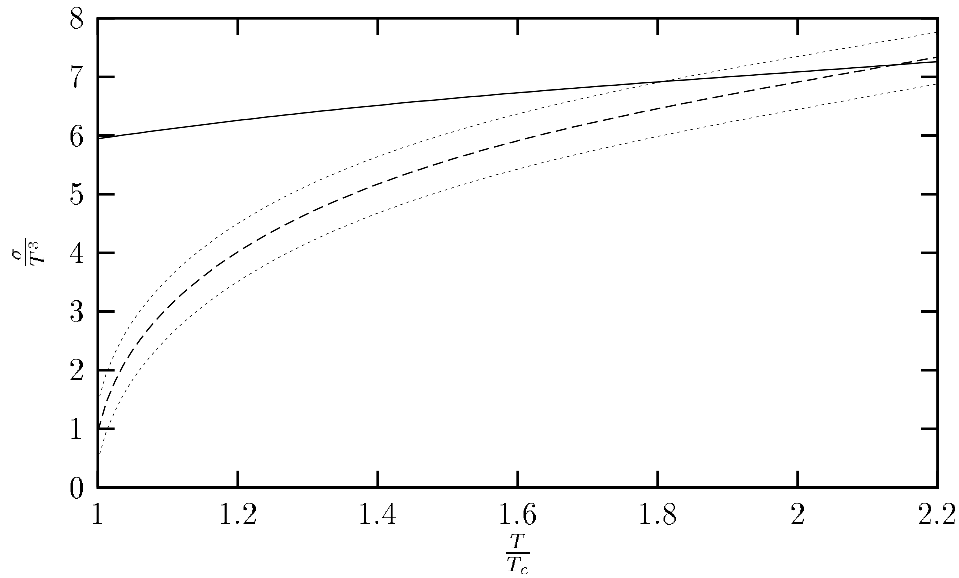
and try to extrapolate to

$$\lim_{T \to T_H} \sigma_{eff}(T) = 0$$



The 't Hooft string tension ...

F. Bursa, M. Teper: hep-lat/0505025



SU(4) 't Hooft string tension in units of T (with 2-loop perturbative result using $g^2(T) \simeq g^2_{MFI}(a)$).

$\Rightarrow$

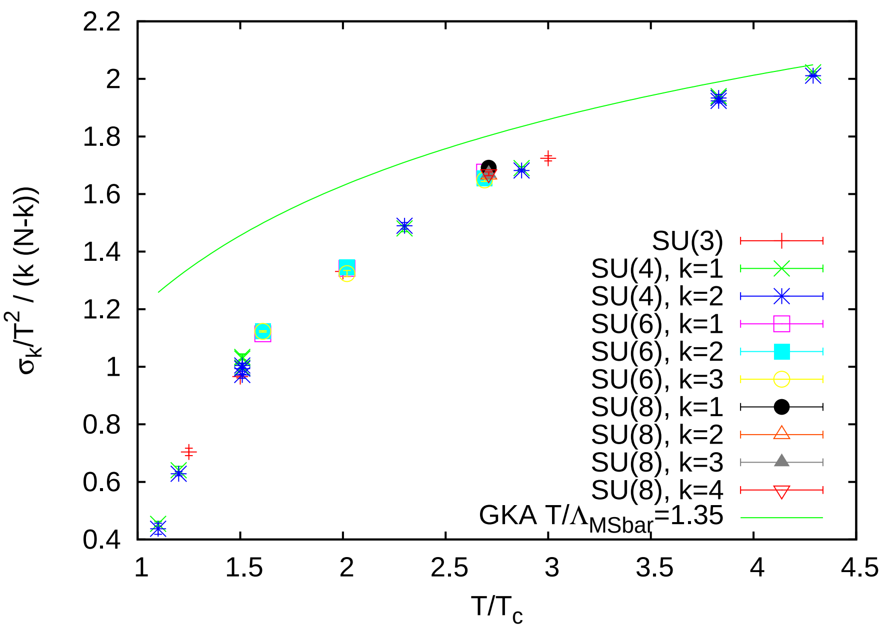
hint of approximate duality

$T \leftrightarrow \frac{1}{T}$

between the confining and 't Hooft string tensions

also ...

de Forcrand, Lucini, Noth: hep-lat/0510081



$\Rightarrow$

tension very small at $T = T_c$

small deconf-deconf wall tensions at $T = T_c$

$\Rightarrow$

$\sigma_{cd} \overset{T=T_c}{=} 2\sigma_{k=N/2}$

$\Leftrightarrow$

small conf-deconf wall tension at $T = T_c$

Strongly Coupled Gluon Plasma - at large N?

B. Bringoltz, M.Teper: hep-lat/0506034

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp\left(-\beta S_{\mathrm{W}}\right).$$

now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W \rangle = N_p \langle u_p \rangle$

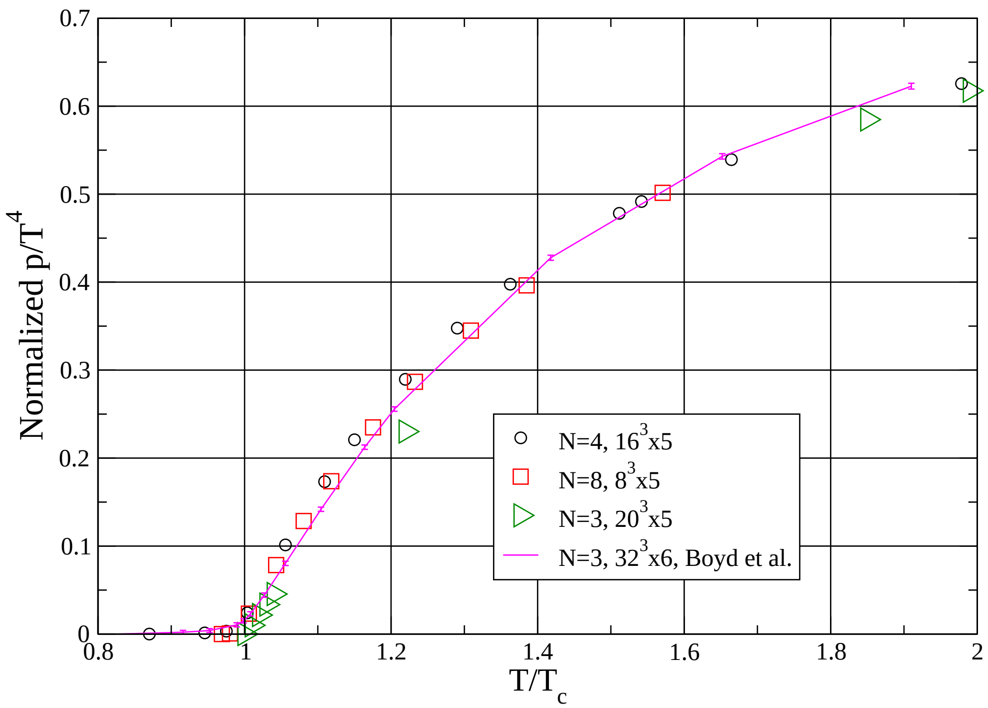
so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0).$

i.e. $\frac{p(T)}{T^4} = 6L_t^4 \int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0).$

similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial \beta}{\partial \log(a(\beta))}.$

Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

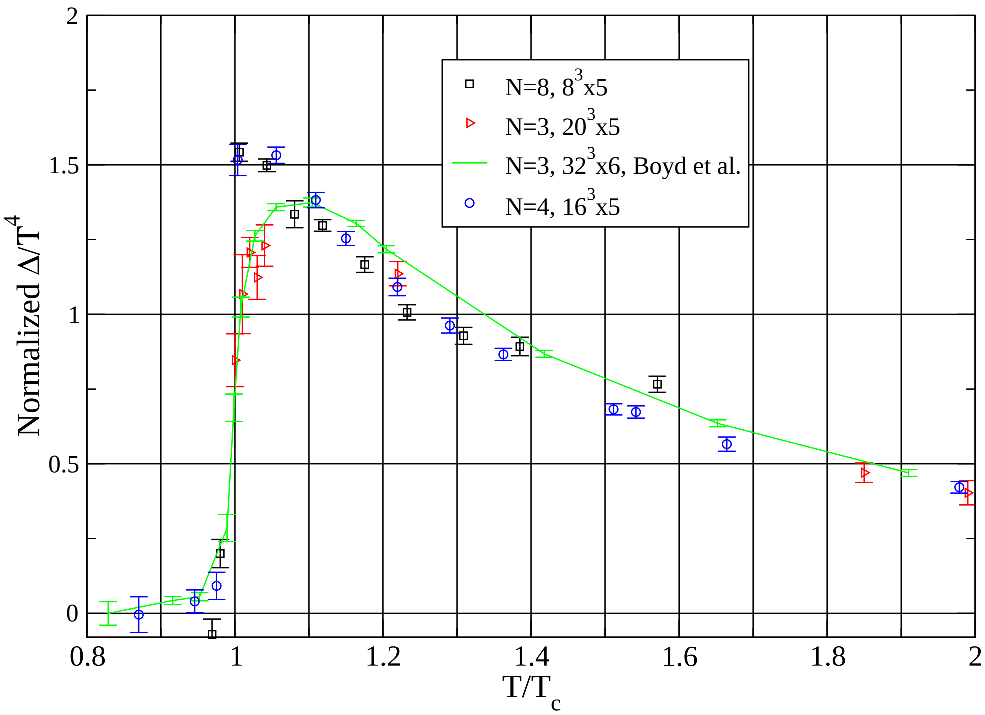
$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)

$$\Delta \equiv \epsilon - 3p$$

B. Bringoltz, M.Teper: hep-lat/0506034

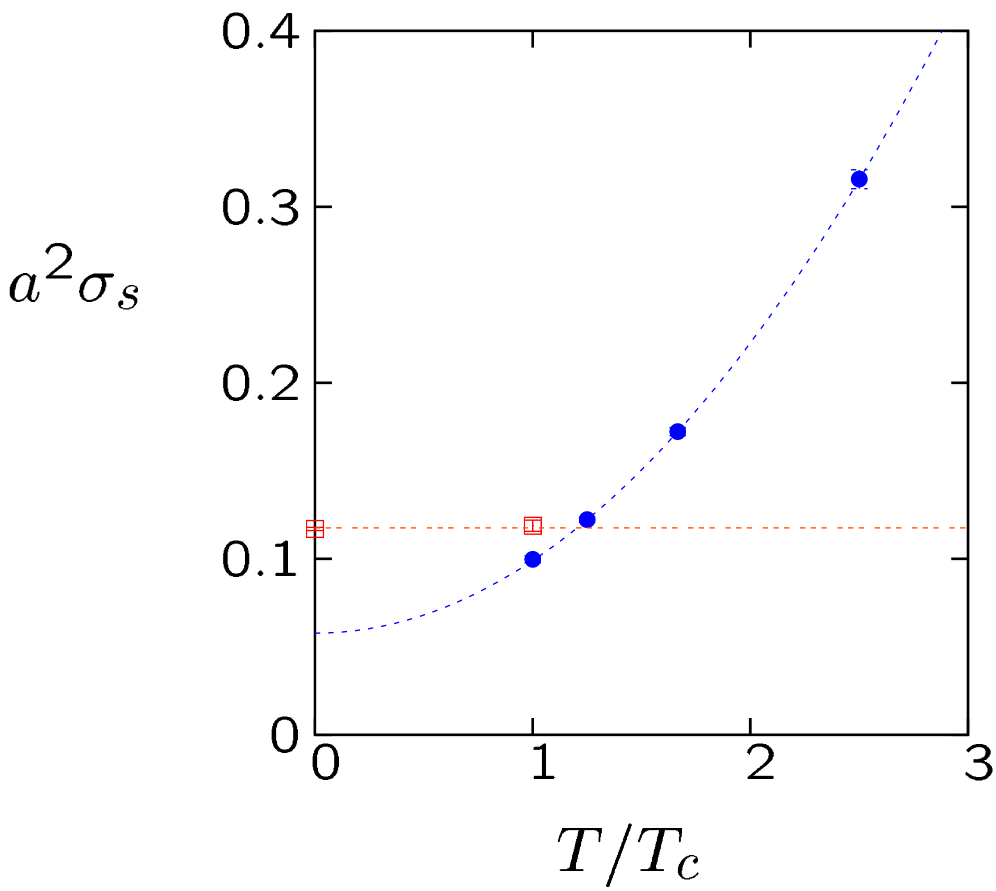
$\Delta = 0$ in Stefan-Boltzman gas



$N = 8$ spatial string tension

Lucini,Teper,Wenger: hep-lat/0502003

$a = 1/5T_c$:

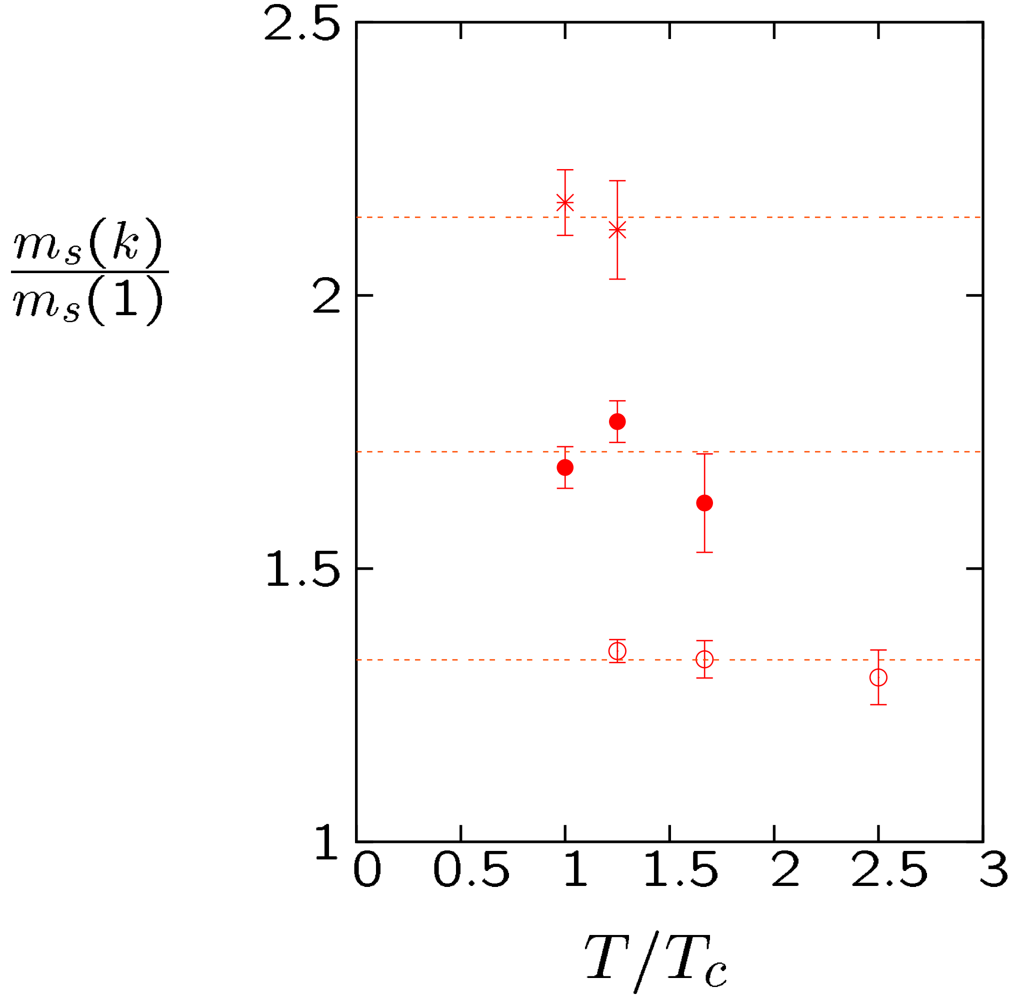


quadratic and constant fits shown

spatial k-string tensions

Lucini,Teper,Wenger: hep-lat/0502003

$a = 1/5T_c$:



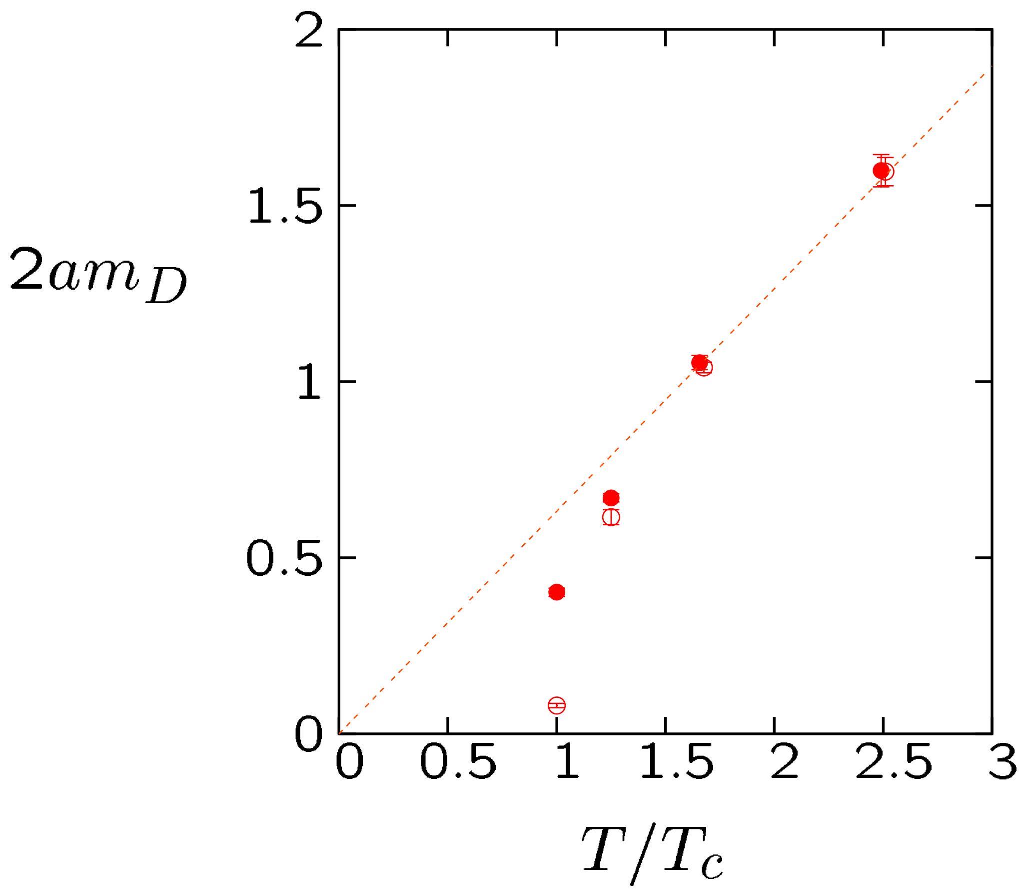
$m_s(k=2)/m_s(k=1)$ in SU(4),$\circ$, and SU(8),$\bullet$, and $m_s(k=3)/m_s(k=1)$ in SU(8),$\star$. Casimir scaling shown.

Debye electric screening mass

Lucini, Teper, Wenger: hep-lat/0502003

from the lightest mass coupling strongly to the vacuum-subtracted time-like Polyakov loop

$a = 1/5T_c$:



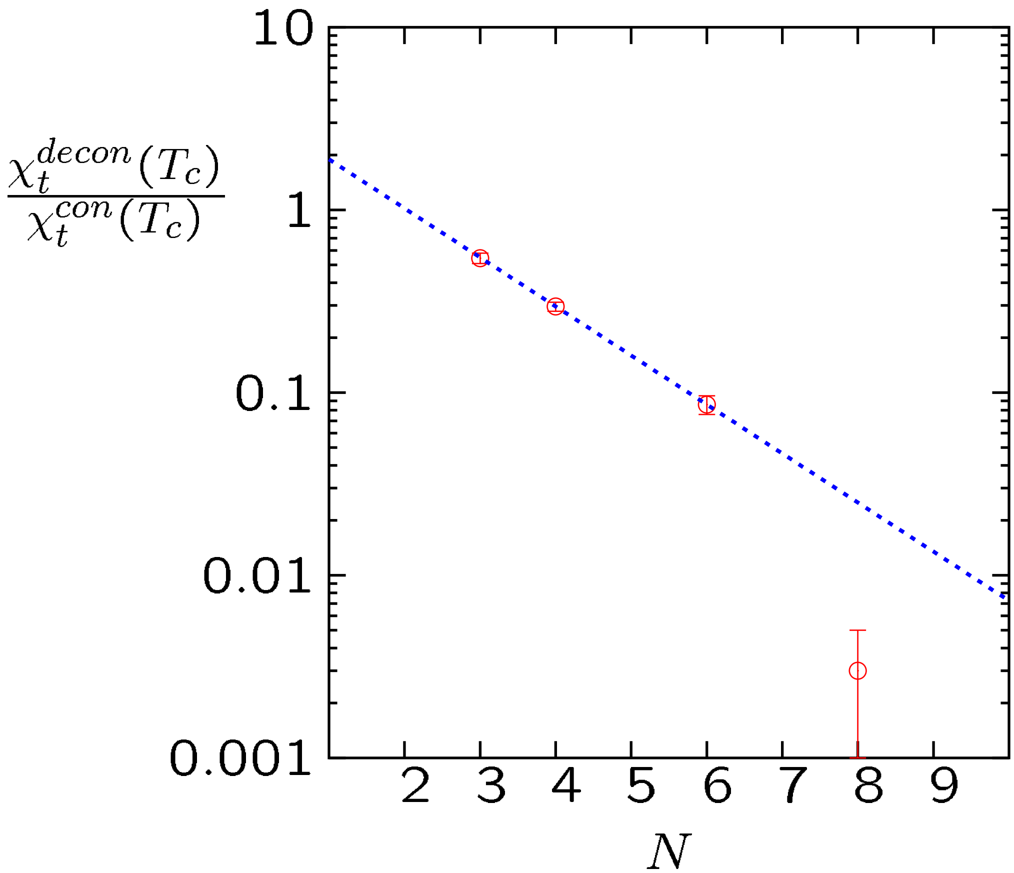
$N = 8$, $\bullet$, $N = 3$, $\circ$.

no topological fluctuations in deconfined phase ...

Lucini,Teper,Wenger: hep-lat/0401028

(Del Debbio,Panagopoulos,Vicari: hep-lat/0407068)

$\chi_t \equiv \langle Q^2 \rangle / V$ in confining/deconfining phases at $T = T_c$



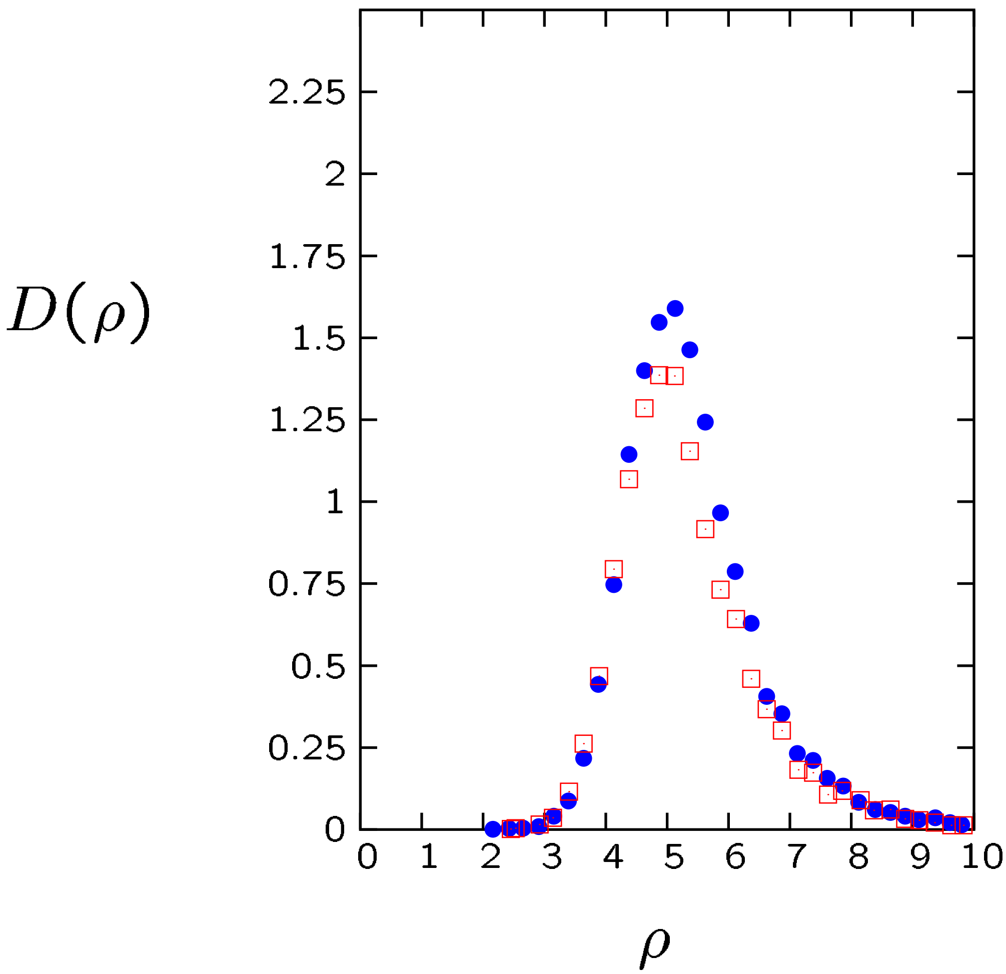
$\Rightarrow$

deconfined topological fluctuations
vanish with N exponentially fast

instanton density at $T = T_c$ in confined phase

Lucini,Teper,Wenger: hep-lat/0401028

SU(8) at $a = 1/5T_c$ for $T = 0$, $\bullet$,and $T = T_c$, $\square$,in confined phase.

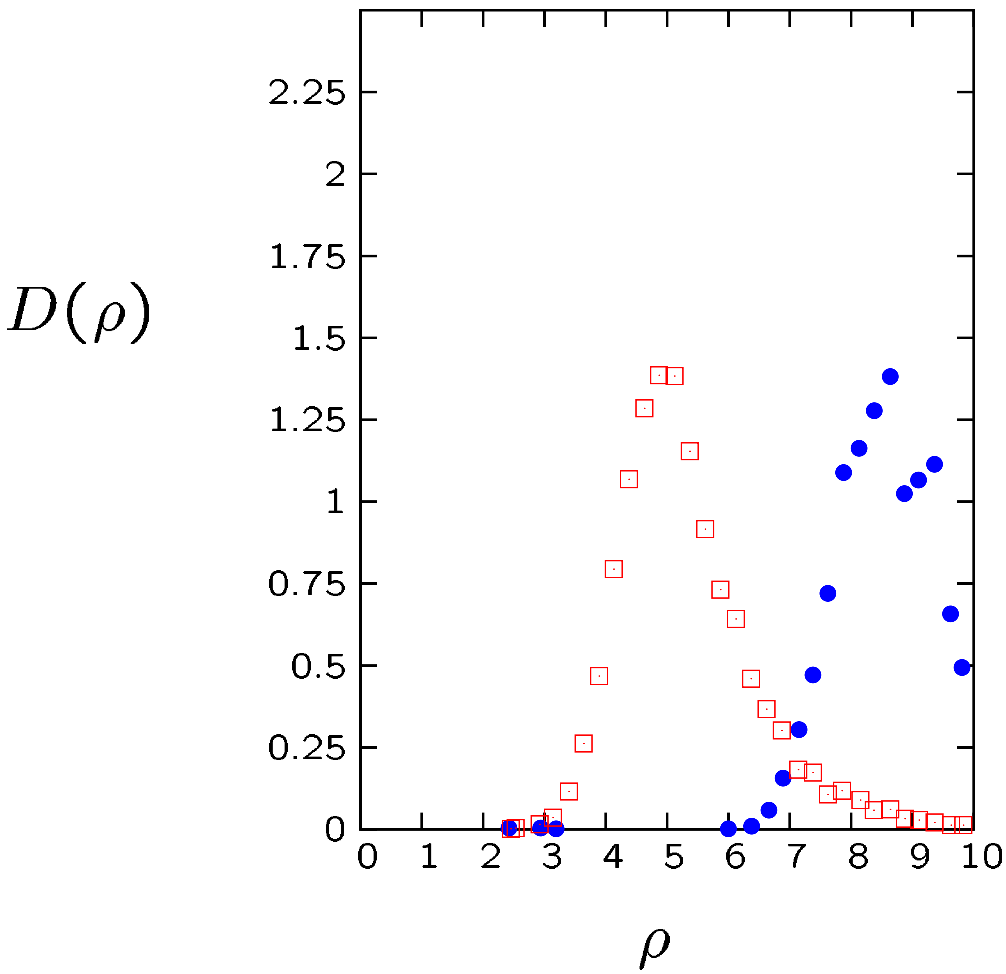


$\Rightarrow$
$D(\rho)$ is independent of T in confined phase as $N \to \infty$

instanton density at $T = T_c$ in deconfined phase

Lucini,Teper,Wenger: hep-lat/0401028

SU(8) at $a = 1/5T_c$ for deconfined • ,and confined, □ ,phases.



$\Rightarrow$

$D(\rho) \equiv 0$ in deconfined phase as $N \to \infty$

not so surprising ...

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- for small instantons, $\rho \ll 1/\sqrt{\sigma}$,

$$D(\rho) \sim \exp\{-8\pi^2/g^2(\rho)\} \sim \exp\{-c(\rho)N\}$$

with $c(\rho) = 8\pi^2/g^2(\rho)N$

- for T high enough for perturbation theory in $g^2(T)$ to be good, Gross,Pisarski,Yaffe RMP 53(19981)43

$$D(\rho, T) \sim D(\rho, T=0)\exp(-\tfrac{2N}{3}\{\pi\rho T\}^2 - \gamma(\rho T))$$

and instantons with $\rho T > 1$ again vanish exponentially in N

So

the lattice results suggest that these two regions of exponential suppression overlap for any T in the deconfined phase, so that all instantons vanish $\propto \exp\{-cN\}$

dimensional cascade at $N = \infty$ ...

F. Bursa, M. Teper: hep-lat/0511081

take a space-time volume $l_0 l_1 l_2 l_3$ with $l_0 \ll l_1 \ll l_2 \ll l_3$ and rescale all lengths by some common factor ξ

$\longrightarrow$

decrease ξ so as to increase $T = 1/al_0 \Rightarrow$ a deconfining phase transition at $T = T_c$ such that $\langle l_p(\mu = 0)\rangle \not= 0$ for$T > T_c$

$\longrightarrow$

increase $T = 1/al_0$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a $l_1 l_2 l_3$ gauge-adjoint Higgs theory

$\longrightarrow$

increase $T = 1/al_1 \Rightarrow$ a deconfining phase transition at $T = T_c^{D=3}$ such that $\langle l_p(\mu = 1)\rangle \not= 0$ for$T > T_c^{D=3}$

$\longrightarrow$

increase $T = 1/al_1$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a $l_2 l_3$ gauge-adjoint Higgs theory

$\longrightarrow$

increase $T = 1/al_2 \Rightarrow$ a deconfining phase transition at $T = T_c^{D=2}$ such that $\langle l_p(\mu = 2)\rangle \not= 0$ for$T > T_c^{D=2}$

$\longrightarrow$

increase $T = 1/al_2$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a l_3 gauge-adjoint Higgs theory

.....

$\updownarrow$

sequence of $N = \infty$ finite volume transitions on an l^4 space-time volume, as l is decreased

R. Narayanan, H. Neuberger: hep-lat/0704.2591,0509014,0303023

In Summary:

- $SU(\infty)$ is linearly confining in both $D = 3+1$ and $D = 2+1$ and for many quantities $SU(3) \simeq SU(\infty)$.

- in particular, the 'strong coupling gluon plasma' (e.g. pressure anomaly) is a large-N phenomenon

some quantities that have been calculated at finite T and for 'all' N:

- $T_c/\sqrt{\sigma}$ in $D = 3+1, 2+1$;
- L_h/T_c^D latent heat in $D = 3+1, 2+1$;
- σ_{CD} surface tension in $D = 3+1$;
- $\sigma_{DD'}$ 't Hooft tension in $D = 3+1$;
- effective string tension for $T < T_c$
- spatial k-string tensions for $T > T_c$
- electric screening masses for $T > T_c$

some observations:

• the transition is first order for $N \geq 3$ in $D = 3 + 1$, and for $N \geq 4$ in $D = 2 + 1$

• at this point the $O(N^2)$ gluon plasma free energy is entirely balanced by the $O(N^2)$ confining vacuum energy (gluon condensate)

• we have gone beyond T_c, about halfway to 'T_H', in the confined metastable phase

• the 't Hooft tension decreases rapidly as $T \to T_c^+$ suggesting some approximate duality with the (Wilson) string tension, with a dual Hagedorn transition just below T_c

• topological fluctuations vanish as $\exp\{-cN\}$ at all T in the deconfined phase (calorons?)

• a single phase transition at larger N where the whole Z_N group is broken

• a cascade of finite volume transitions at $N \to \infty$ that are essentially deconfining transitions on ever more dimensionally reduced gauge + adjoint scalar theories

• adjoint Polyakov loops make an elegant order parameter at larger N