Lattice gauge theories, large N and QCD strings

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INI 07

Preamble

Is $N = 3$ close to $N = \infty$? Is large-N confining?

Hot SU(N) gauge theory

The closed string spectrum in $D = 3$ and $D = 4$

also – given time

Topology and interlacing θ-vacua

$D = 3$: comparing with Karabali-Nair

Twisted Eguchi-Kawai : space-time reduction

- ‘Oxford’ group’

Bringoltz, Bursa, Liddle, Lucini, Meyer, Teper, Vairinhos, Wenger, ...

some other groups:

- ‘Pisa’

Del Debbio, Panagopoulos, Rossi, Vicari, ...
Campostrini, ...

- ‘Rutgers’

Narayanan, Neuberger, ...

- ‘Torino’

D’Adda, Caselle, Gliozzi, Hasenbusch, Panero, Rago, ...

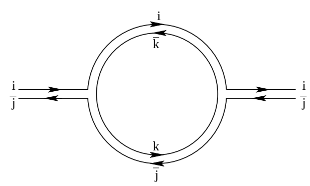
Large N – Preamble

- QCD : $g^2 \leftrightarrow$ scale, so no small expansion parameter

$\overset{'tHooft}{\Rightarrow}$

try

$$\mathrm{SU}(N) \simeq \mathrm{SU}(\infty) + \mathrm{O}(1/N^2)$$

- 

$$\Rightarrow \quad g^2 N \quad \text{constant at large } N$$

- $N \to \infty$ colour singlet phenomenology

't Hooft, Witten, Manohar, ...

zero decay widths; no mixing; exact OZI, η'; SU(6) for baryons; ...

• no scattering of colour singlets – integrability?
but strongly interacting bound states

• factorisation colour singlet operators: e.g.

$$\langle \Phi_1(x_1)\Phi_2(x_2)\rangle = \langle \Phi_1\rangle\langle \Phi_2\rangle \left\{1 + O\left(\frac{1}{N^2}\right)\right\}$$

$\Rightarrow$ Witten's Master Field $\rightarrow$ translation invariant $\rightarrow$
Eguchi-Kawai single point reduction

• Feynman diagrams on 2D surfaces :
$g^2 N \rightarrow \infty \rightarrow$ vertices dense $\rightarrow$ stringy sheets
$\Rightarrow$
$N = \infty$ gauge theory $\sim$ a string theory 't Hooft
$N = \infty$ gauge theory $\sim$ dual to a string theory
Maldacena

Lattice – Preamble

Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4

$x_\mu \bullet - \bullet\, x_\mu + \widehat{\mu}\delta x \;:\; A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

$x_\mu \bullet - - - \bullet\, x'_m u \;:\; P\left\{e^{\int_x^{x'} A.dx}\right\} \in$ SU(N) group

$x_\mu \underrightarrow{=} a n_\mu$

$a n_\mu \bullet - - - \bullet\, a n_\mu + a\widehat{\mu} \quad : \quad U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

gauge transformation: $U_\mu(n) \to g(n)U_\mu(n)g^\dagger(n+\widehat{\mu})$

$\longrightarrow$ gauge invariant action?

$\mathrm{Tr}\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$ so

$Z = \int \prod_l dU_l e^{-\beta S}$ where $S = \sum_p \left\{1 - \frac{1}{N}\mathrm{ReTr}u_p\right\}$
where u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

$\longrightarrow$ symmetries ensure that:

$$\int \prod_l dU_l e^{-\beta S} \quad \stackrel{a\to 0}{\longrightarrow} \quad \int DAe^{-\frac{4}{g^2}\int d^4x TrF_{\mu\nu}F_{\mu\nu}}$$

with

$$\beta = \frac{2N}{g^2(a)} \stackrel{a\to 0}{\longrightarrow} \infty$$

and we vary the parameter β in order to vary the lattice spacing a

$\longrightarrow$ Monte Carlo:

$$\int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n}\sum_{I=1}^{n} \Phi(U^I) + O(\frac{1}{\sqrt{n}})$$

• calculating masses from Euclidean correlators:

$\Phi(t)$ a gauge invariant operator

$$\langle \Phi^\dagger(t = an_t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \overset{t\to\infty}{\simeq} |c|^2 e^{-man_t}$$

where am is lightest mass with quantum numbers of Φ in lattice units

• continuum limit :

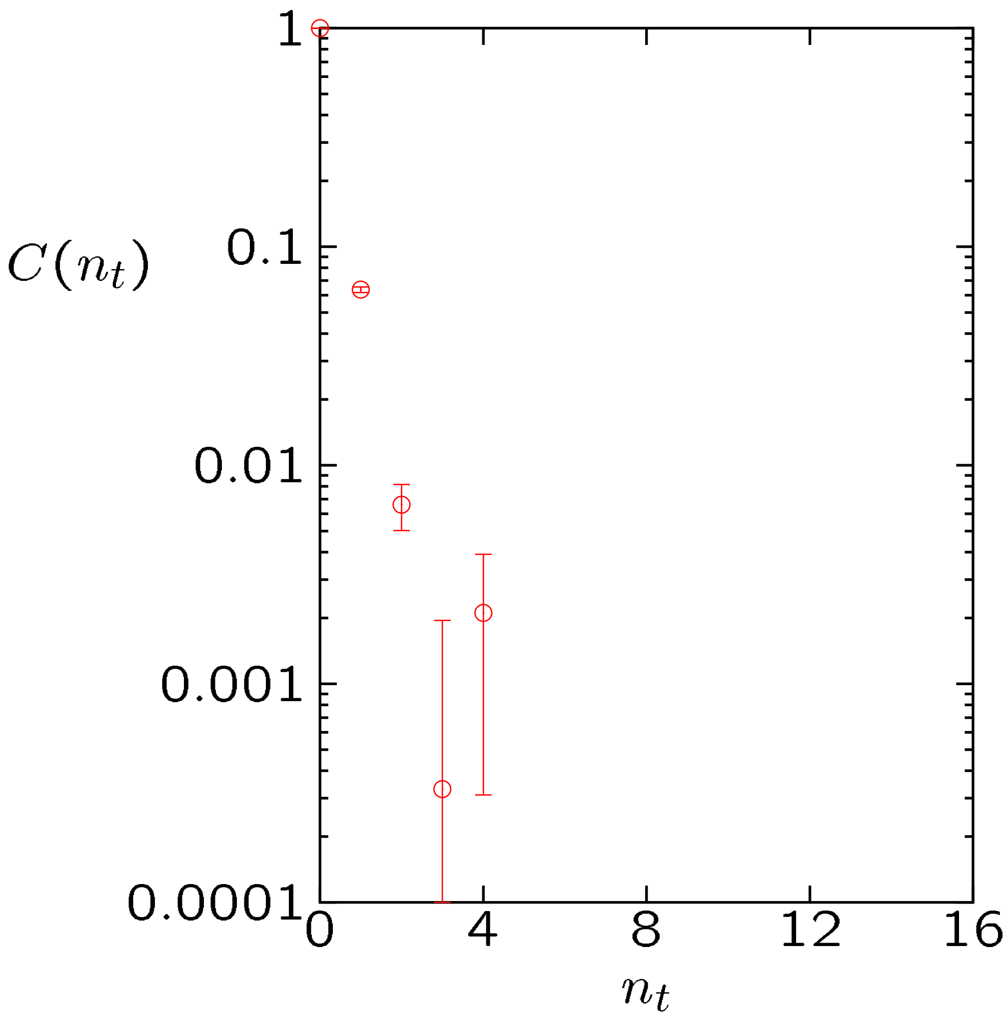
$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2\sigma + O(a^4)$$

• large N limit :

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

using the simple plaquette for the glueball operator:

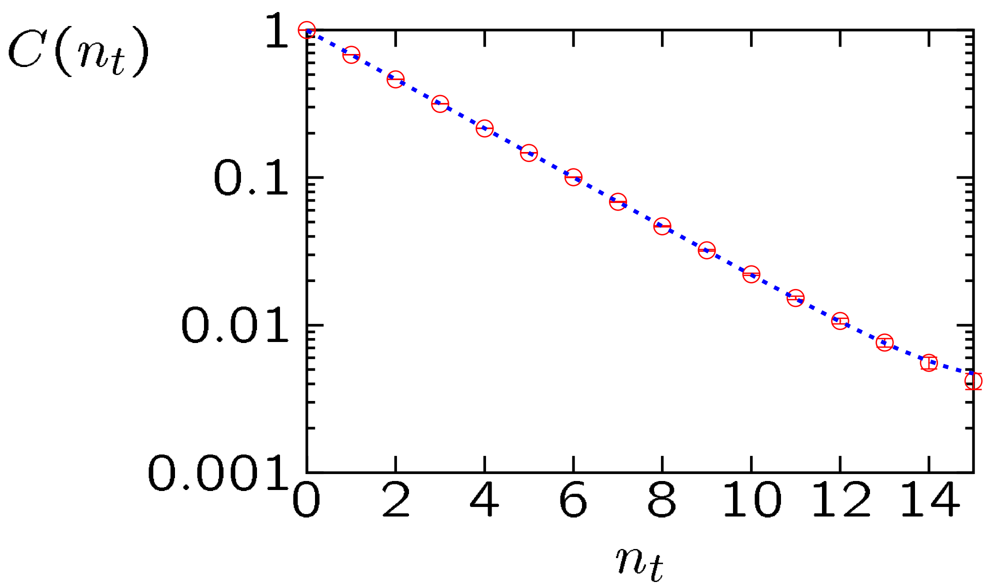
SU(3), 32^4, $a \simeq 0.046$ 'fm'



Can we do accurate calculations?

SU(3), 32^4, $a \simeq 0.046$ 'fm'

best blocked/smeared glueball operator

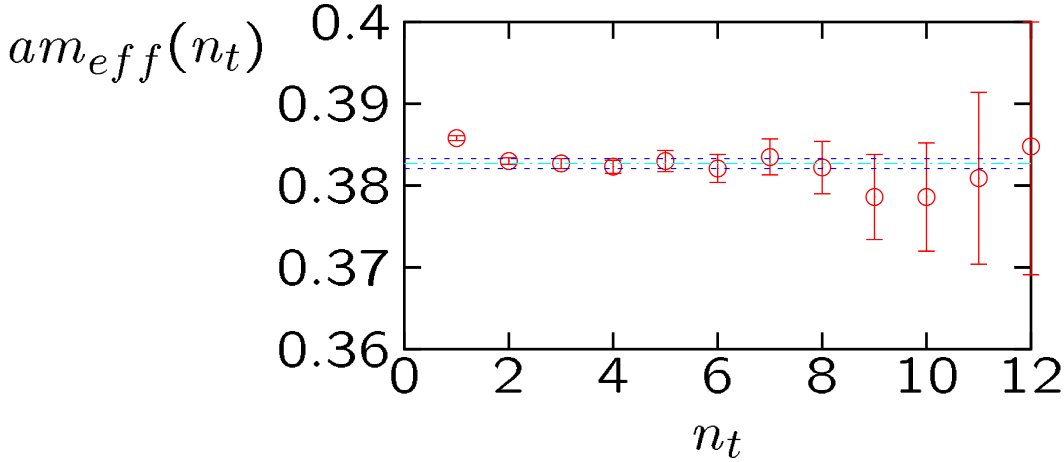


$$C(t = an_t) \overset{t\uparrow}{\simeq} |c|^2 e^{-man_t}$$

$\Rightarrow$

fit : $am_{0^{++}} = 0.330(7)$

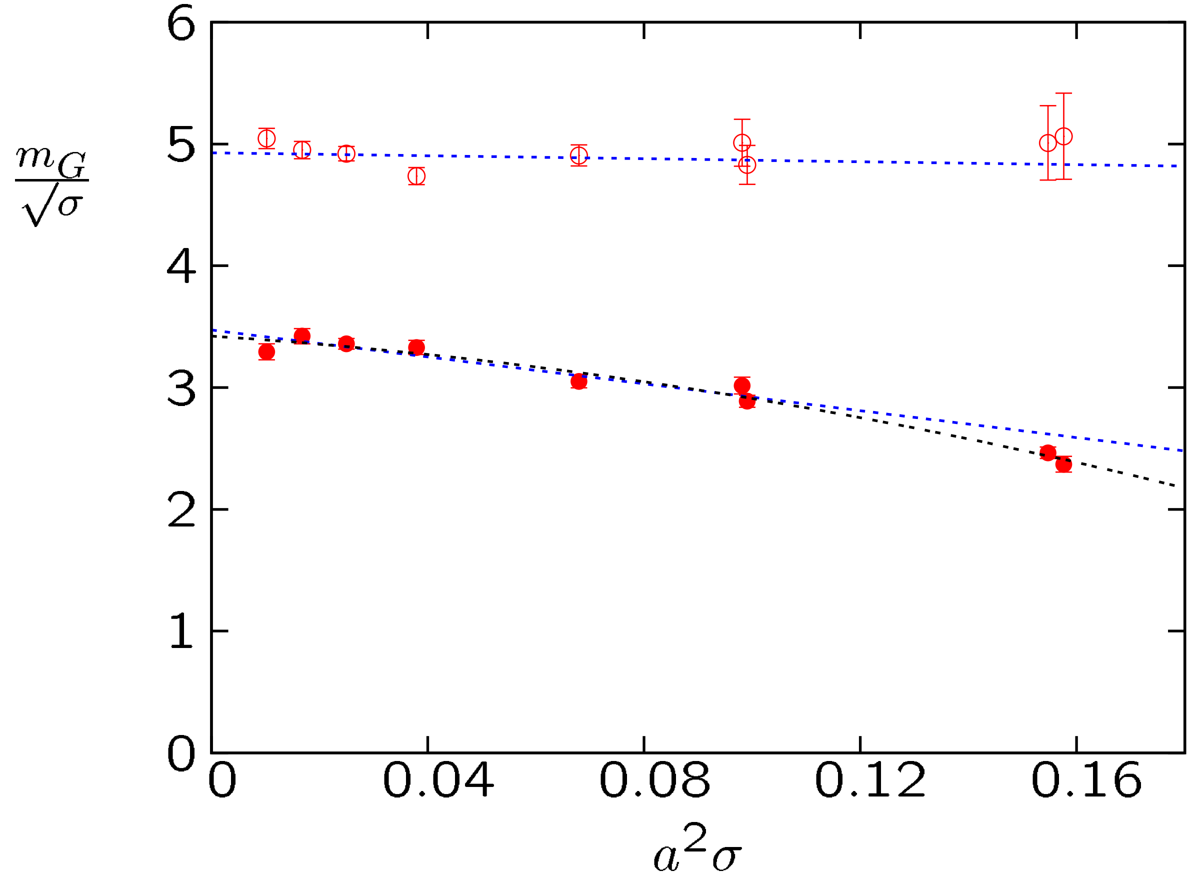
$$am_{eff}(n_t) = -\ln \frac{C(n_t)}{C(n_t-1)} \xrightarrow{n_t \to \infty} am_{0^{++}}$$



$\Rightarrow$

fit : $am_{0^{++}} = 0.3312(67)$

Continuum limit mass spectrum: SU(3)



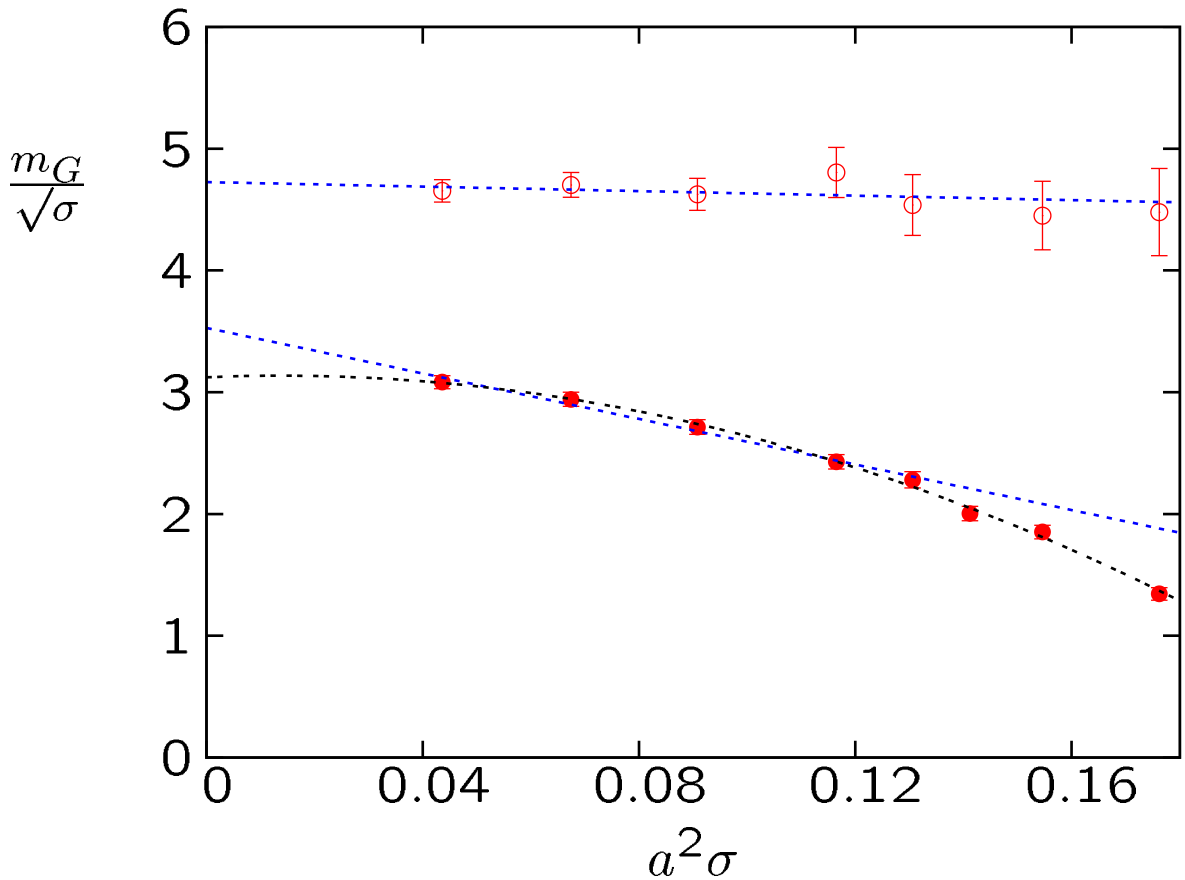
$O(a^2)$ continuum extrapolations:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Continuum limit mass spectrum: SU(8)



$O(a^2)$ continuum extrapolation:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.53(8) - 9.3(1.0)a^2\sigma$$

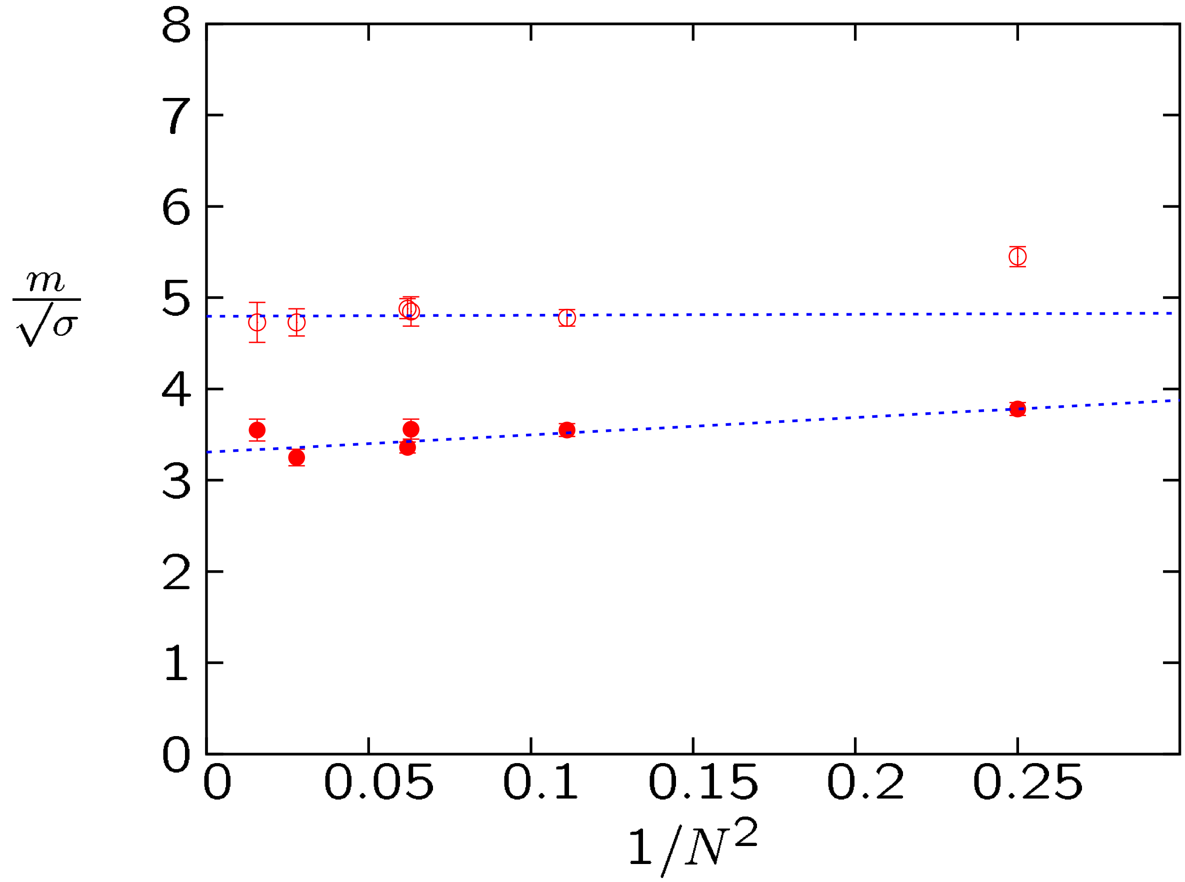
$O(a^4)$ extrapolation

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.13(25) + 1.66a^2\sigma - 66.0(a^2\sigma)^2$$

this systematic error $\sim 5 \pm 3\times$ naive $O(a^2)$ statistical error !

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



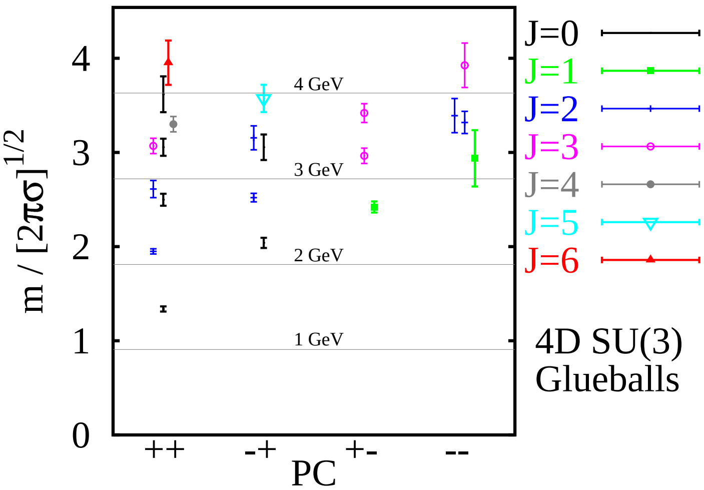
$O(1/N^2)$ extrapolations to $N = \infty$:

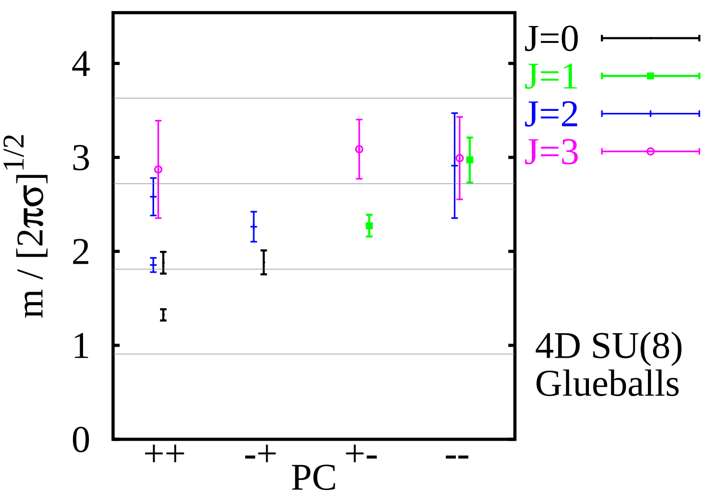
$$\frac{m_{0^{++}}}{\sqrt{\sigma}}\Big|_N = 3.31 + \frac{1.90}{N^2}$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}}\Big|_N = 4.80 + \frac{0.11}{N^2}$$

spectrum : SU(3) vs SU(8)

H. Meyer, M. Teper: hep-th/0409183



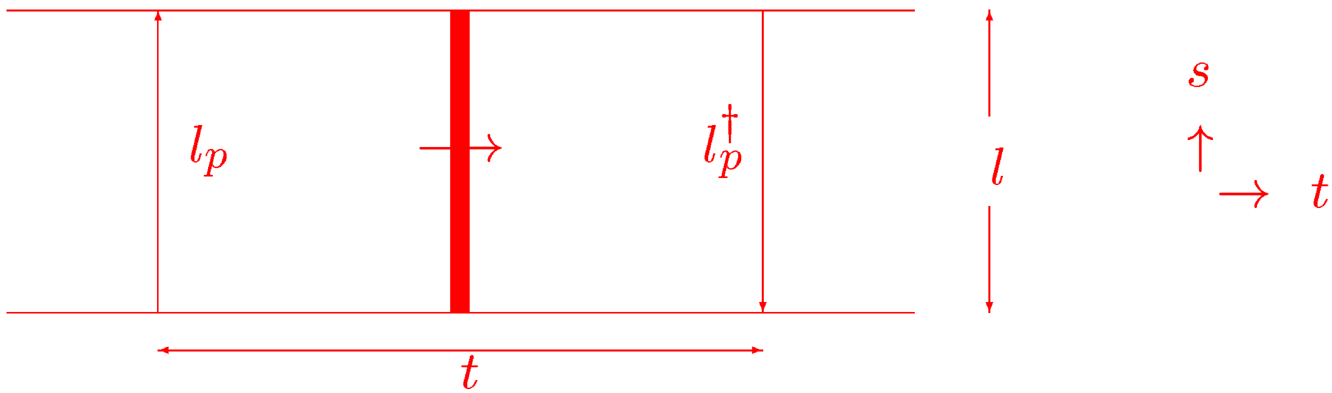


Linear confinement in SU($N \to \infty$)?

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0)\rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



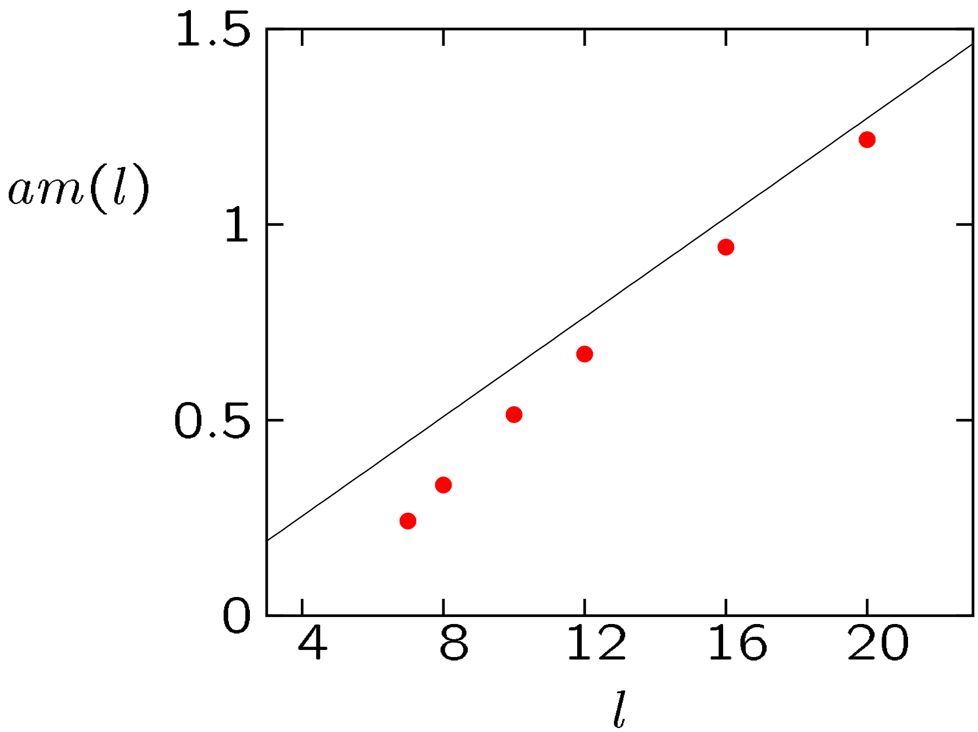
where we expect, for linear confinement,

$$m_p(l) = \sigma l - \frac{\pi(D-2)}{6l^2} + O\left(\frac{1}{l^4}\right)$$

- no sources, no Coulomb terms, flux tubes for $l \geq 1/T_c$

SU(6)

H. Meyer, M. Teper: hep-lat/0411039



indeed we find

$$am(l) \simeq \sigma l$$

over a range of 'string' lengths up to

$$l \simeq 5.0 \times \frac{1}{\sqrt{\sigma}}$$

surely large enough to be asymptotic ...

So :

- SU(3) $\sim$ SU(∞) for many quantities

- linear confinement persists at large N

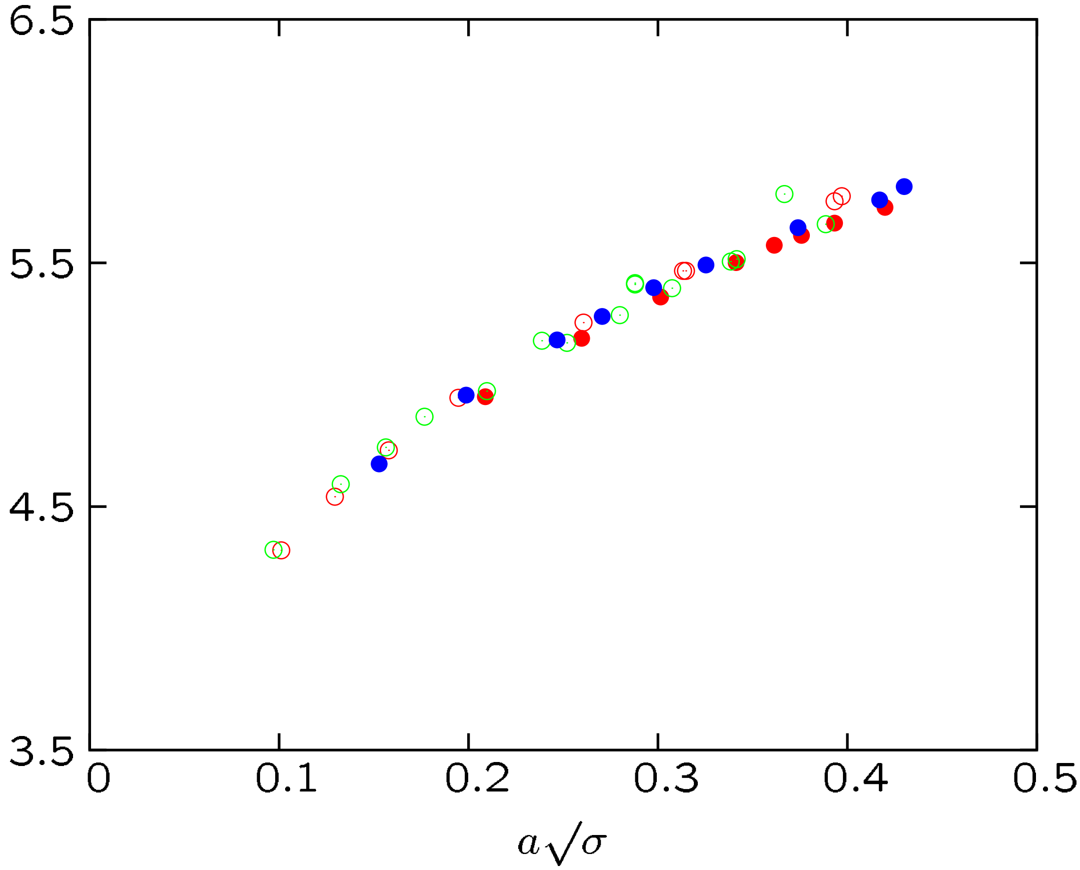
the apparent phenomenological relevance of large-N, provides the motivation for pursueing further the properties of this theory ...

$g^2 N$ fixed as $N \to \infty$?

Lucini, Teper, Wenger: hep-lat/0502003

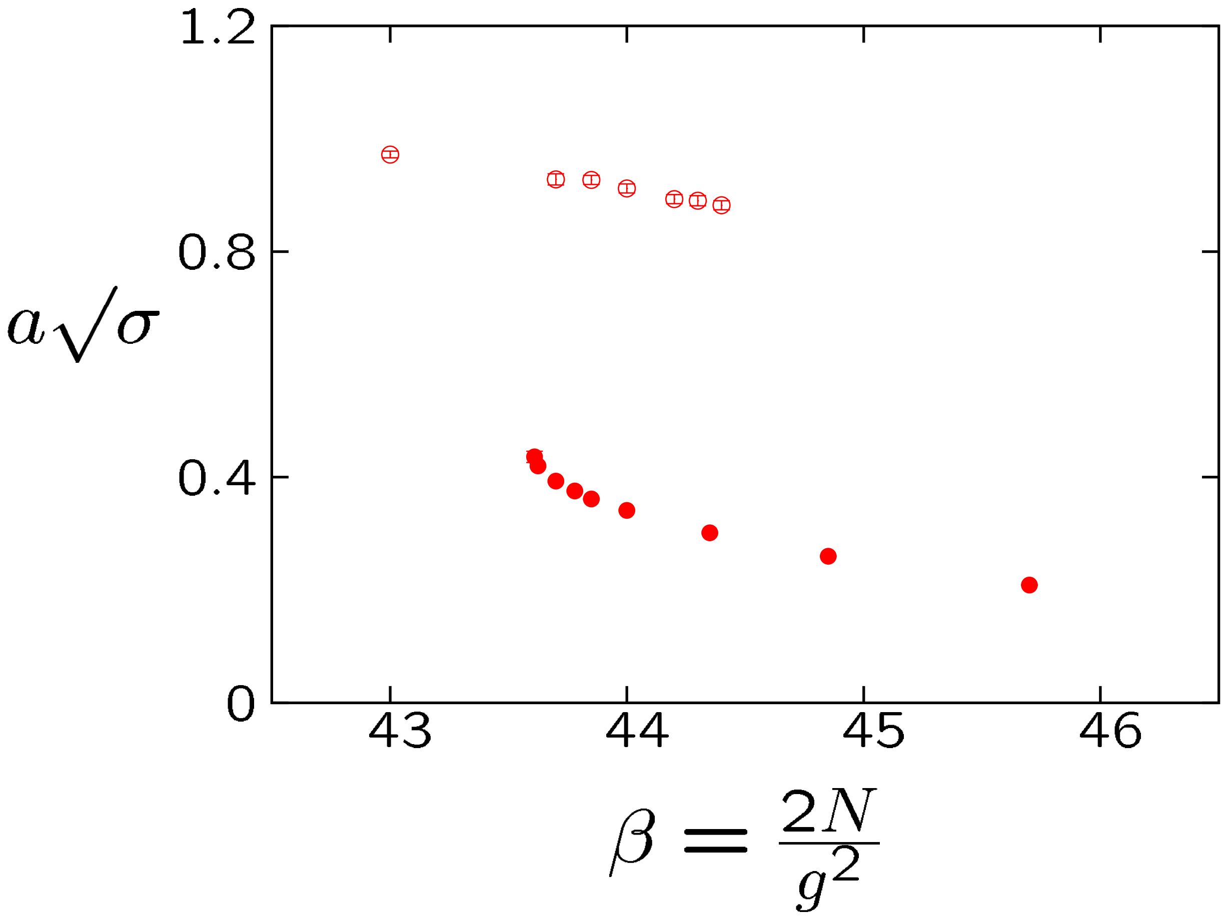
- $g^2(l)$ $g^2(l)$ versus $\frac{l}{\xi}$ with $\xi = \frac{1}{\sqrt{\sigma}}$, $l = a$

and using $\beta = 2N/g_L^2(a)$ $\quad g_I^2 = g_L^2/u_p$



SU(2) ○ ; SU(3) ○; SU(4) ● ;
SU(6) ○ ; SU(8) ●

SU(8) in 3+1 dimensions



1st order bulk transition for $N \geq 5$ ensures clean weak-coupling physics on weak-coupling branch
$\Rightarrow$

Is there a 'physical' lattice strong coupling regime?

- Why ask? AdS/CFT addresses $g^2N \to \infty$.

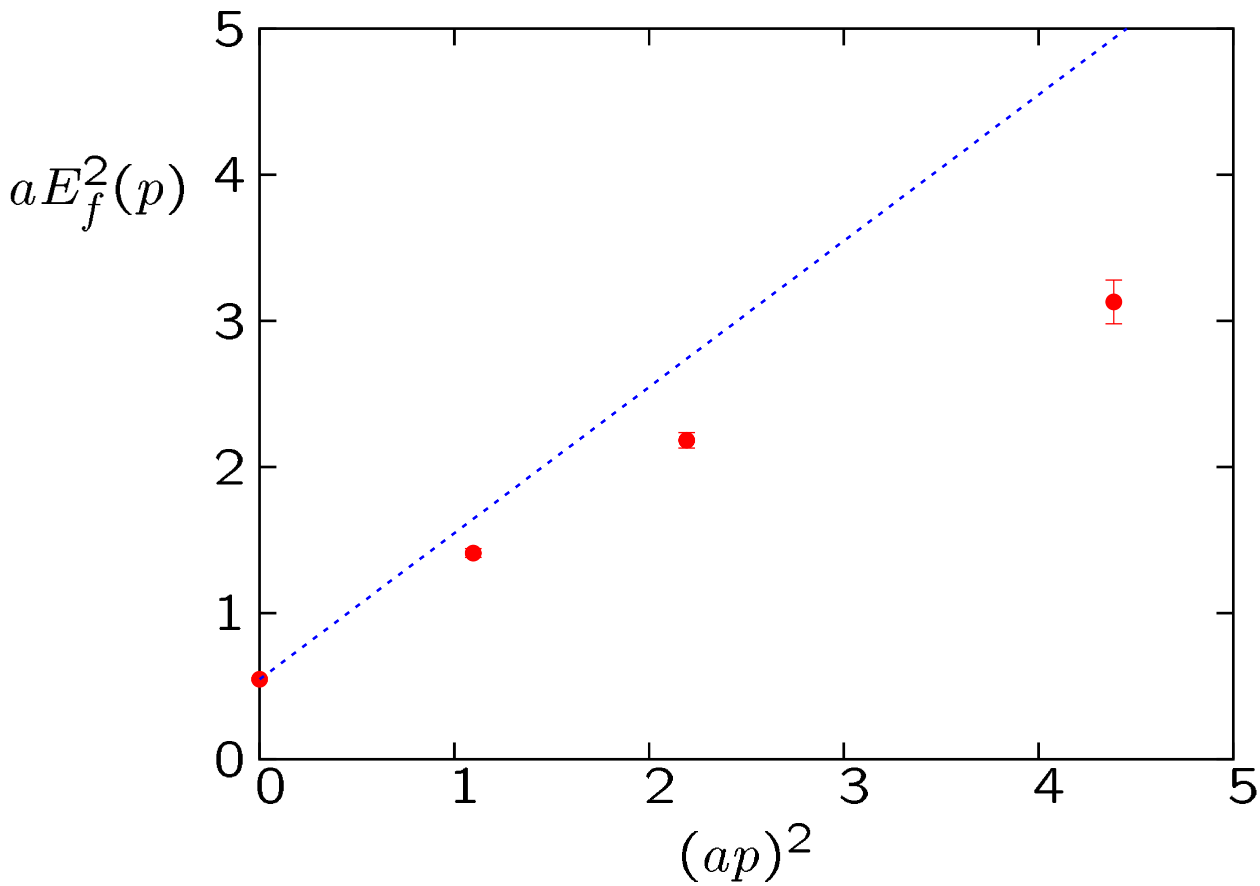
- usual lattice
$g^2N \to \infty \quad \leftrightarrow \quad \beta \to 0$
is presumably not relevant as essential space-time symmetries badly broken in that limit

- but at large N we can use the metastability of the strong-to-weak coupling bulk transition to go to smaller β while remaining still in a strong coupling phase

- if we are past the 'roughening transition' then the space-time symmetries will begin to be restored and we might be in a strong coupling phase that has some physical features – even if it does not have an asymptotically free UV completion

there is some numerical evidence for this:

SU(8) , $\beta = 44.3$, $2 \times 6^2 \times 8$



$- - -$ $E^2 = m^2 + p^2$

with

$p^2 \rightarrow 4\sin^2 p/2$

better

Calculating masses as $N \to \infty$

We calculate masses from connected correlators i.e. correlations between fluctuations

but

as $N \to \infty$ all fluctuations vanish

$\Rightarrow$

mass calculations become impossible as $N \to \infty$?

NO

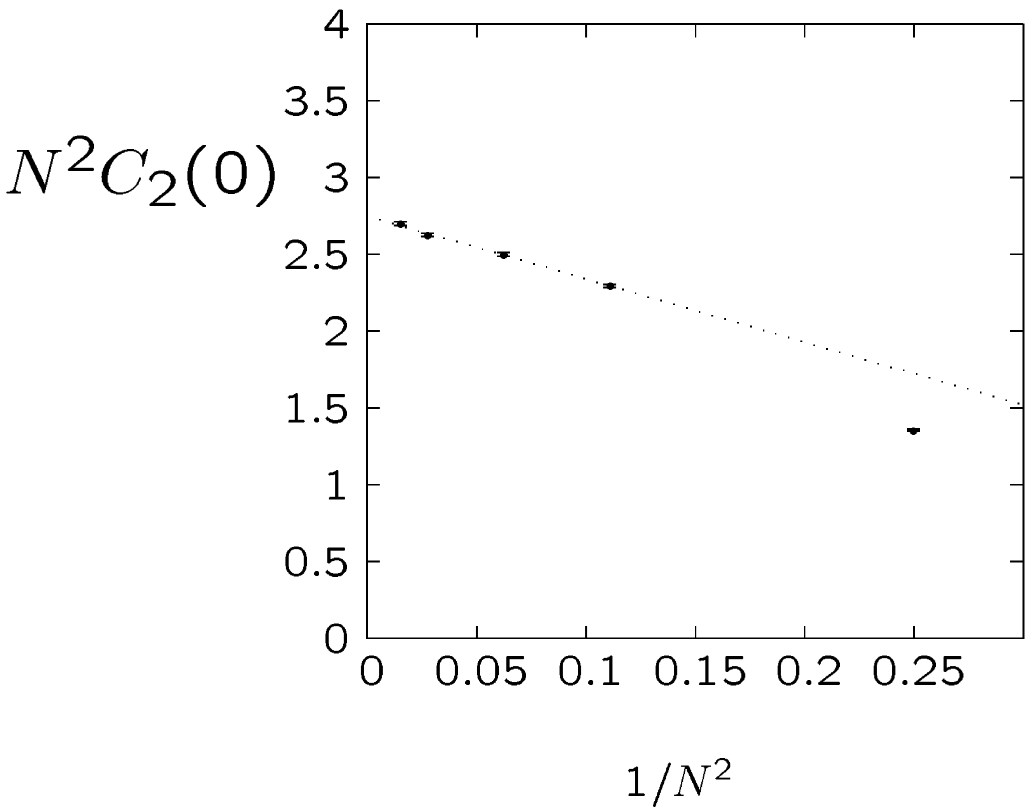
the errors on the fluctuations are themselves determined by higher order correlators, which genericall vanish at the same rate

$\Rightarrow$

as $N \to \infty$ calculations get no harder – apart from the naive N^3 factor coming from matrix multiplication, which in principle can be partly offset by smaller finite volume corrections at large N.

consider the ratio of connected to disconnected pieces of a typical gluball operator

$$C_2(t) = \frac{\langle \Phi(t)\Phi(0) \rangle - \langle \Phi \rangle^2}{\langle \Phi \rangle^2}$$

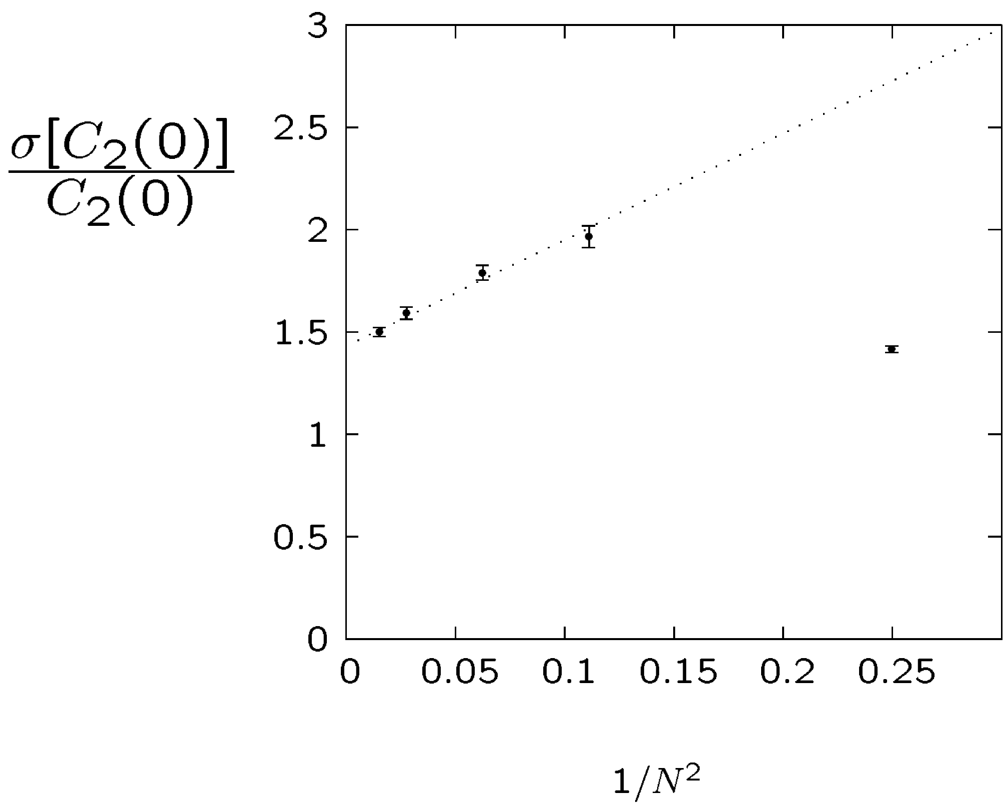


Indeed: $C_2(t=0) = \frac{2.75(1)}{N^2} - \frac{4.1(2)}{N^4}$

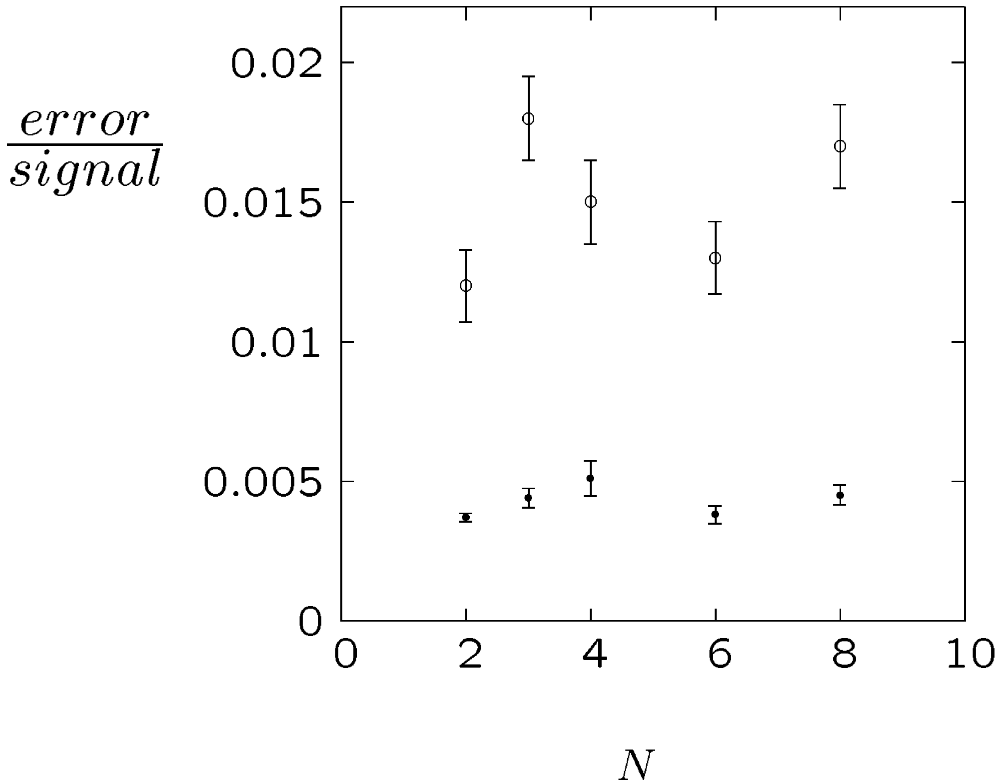
fluctuations: $\sigma[C_2(t)] = \frac{\sigma_c(t)}{\langle\Phi\rangle^2}$

where

$$\begin{aligned} \sigma_c^2(t) &= \langle\{\Phi_v(t)\Phi_v(0) - \langle\Phi_v(t)\Phi_v(0)\rangle\}^2\rangle \\ &= \langle\Phi_v(t)\Phi_v(0)\Phi_v(t)\Phi_v(0)\rangle - \langle\Phi_v(t)\Phi_v(0)\rangle^2 \end{aligned} \quad (1)$$



and the observed ratio of error to signal for the same number of Monte Carlo field configurations is roughly independent of N

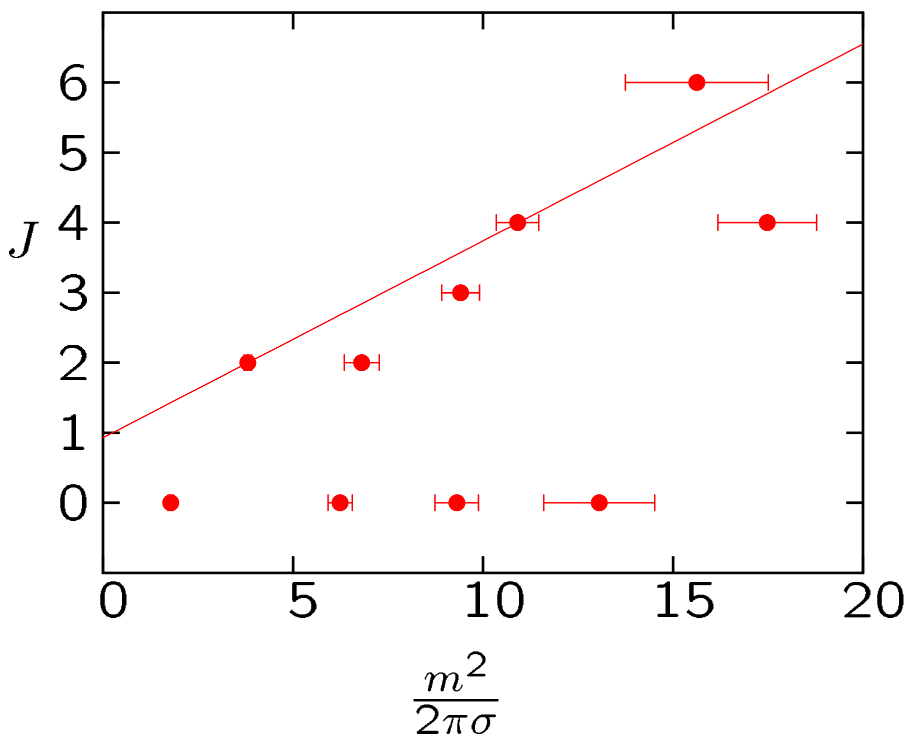


Error to signal ratio for $C_2(t)$ after 10^5 sweeps on 10^4 lattices at fixed lattice spacing, $a \simeq 1/5T_c$, and for $t = 0$ ($\bullet$ and $t = a$ ($\circ$).

Pomeron: the leading glueball Regge trajectory?

H. Meyer, M. Teper: hep-th/0409183

Chew-Frautschi plot: $PC = ++$ states in SU(3) gauge theory

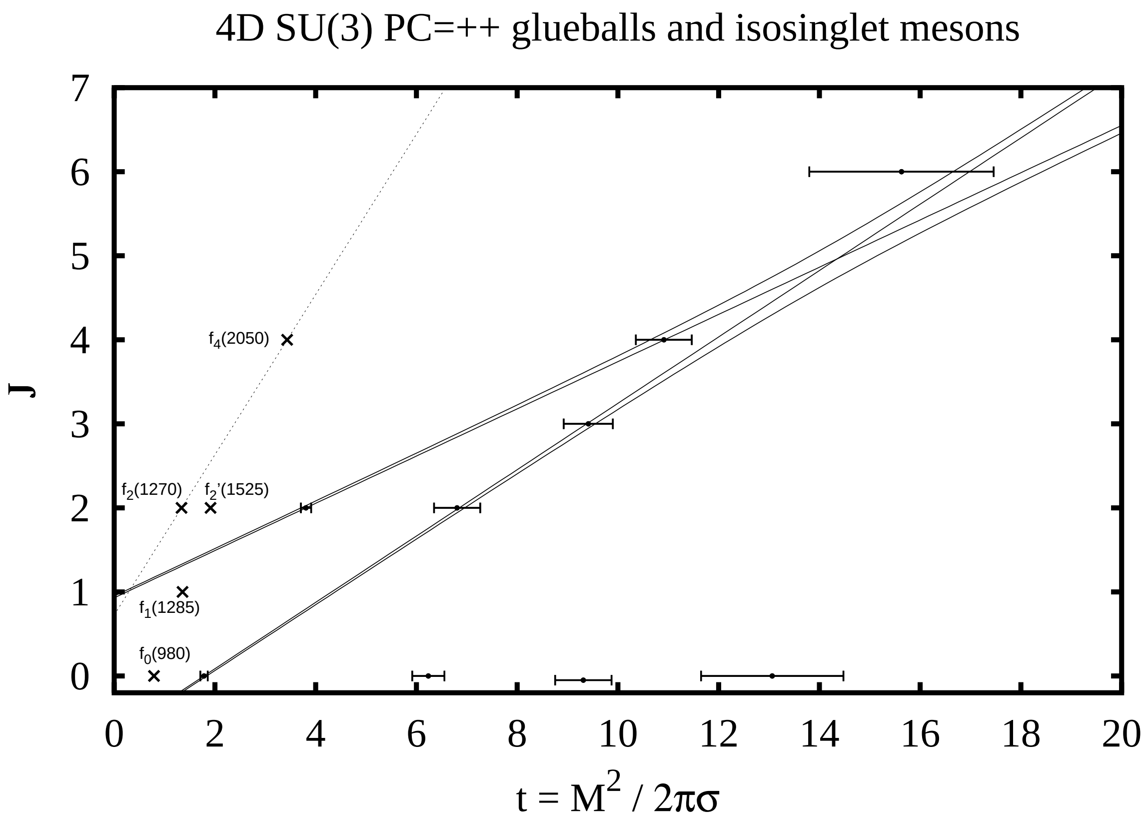


$$\alpha(t) = 0.93(24) + 0.28\alpha'_R t$$

$$\alpha'_R = \frac{1}{2\pi\sigma} \simeq 0.9\text{GeV}^{-2}$$

Pomeron in QCD?

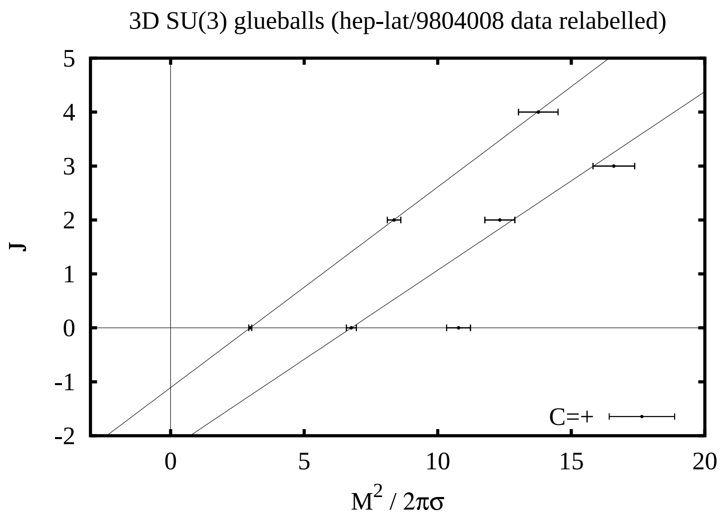
H. Meyer, M. Teper: hep-th/0409183



Pomeron in D=2+1

H. Meyer, M. Teper: hep-th/0409183

Chew-Frautschi plot: $C = +$ states in D=2+1 SU(3) gauge theory



$$\alpha(t) = -1.14(7) + 0.38\alpha'_R t$$

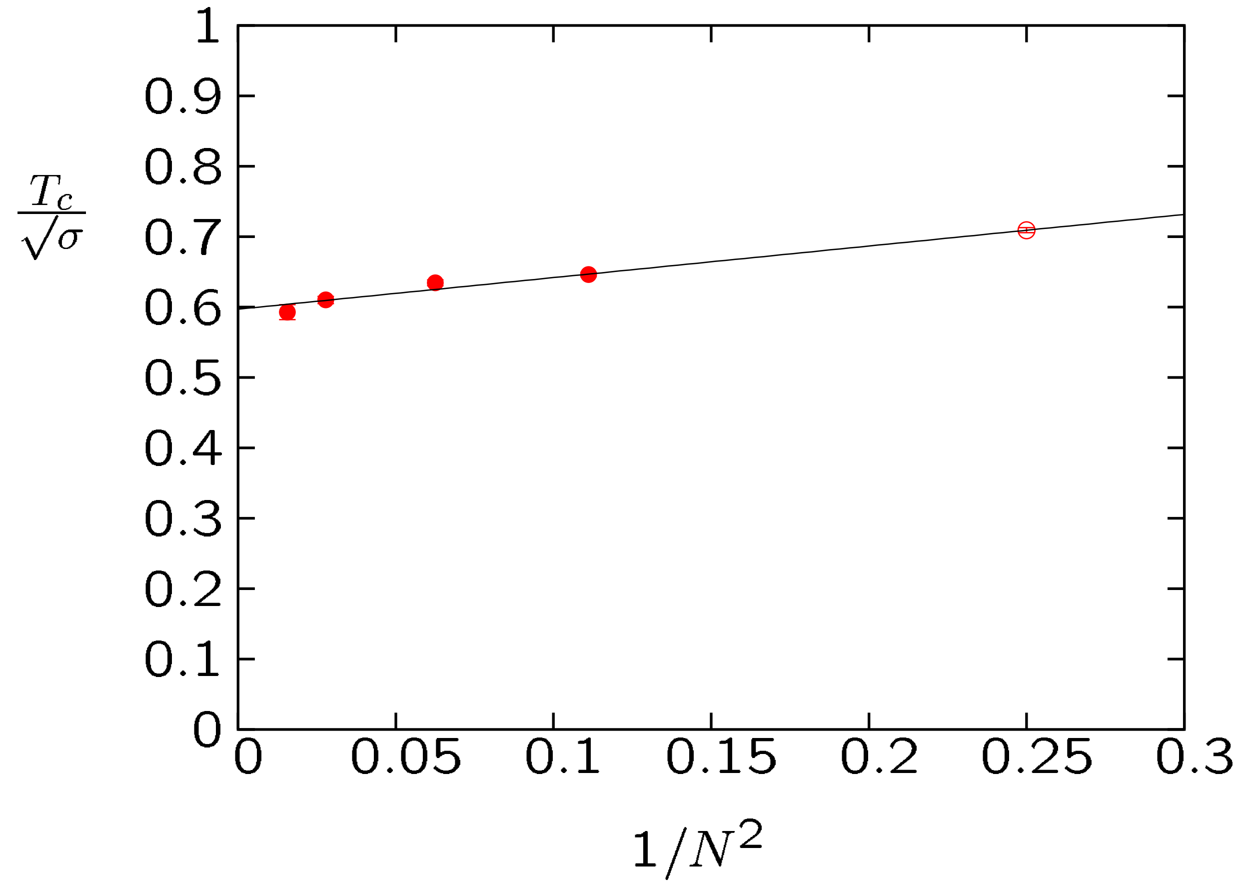
Hot Physics at Large N

- prelude
- $T = T_c$
- $T < T_c$
- $T > T_c$
- conclusions

Deconfining temperature in D=3+1

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta)L_t}$ if $L_s \gg L_t$



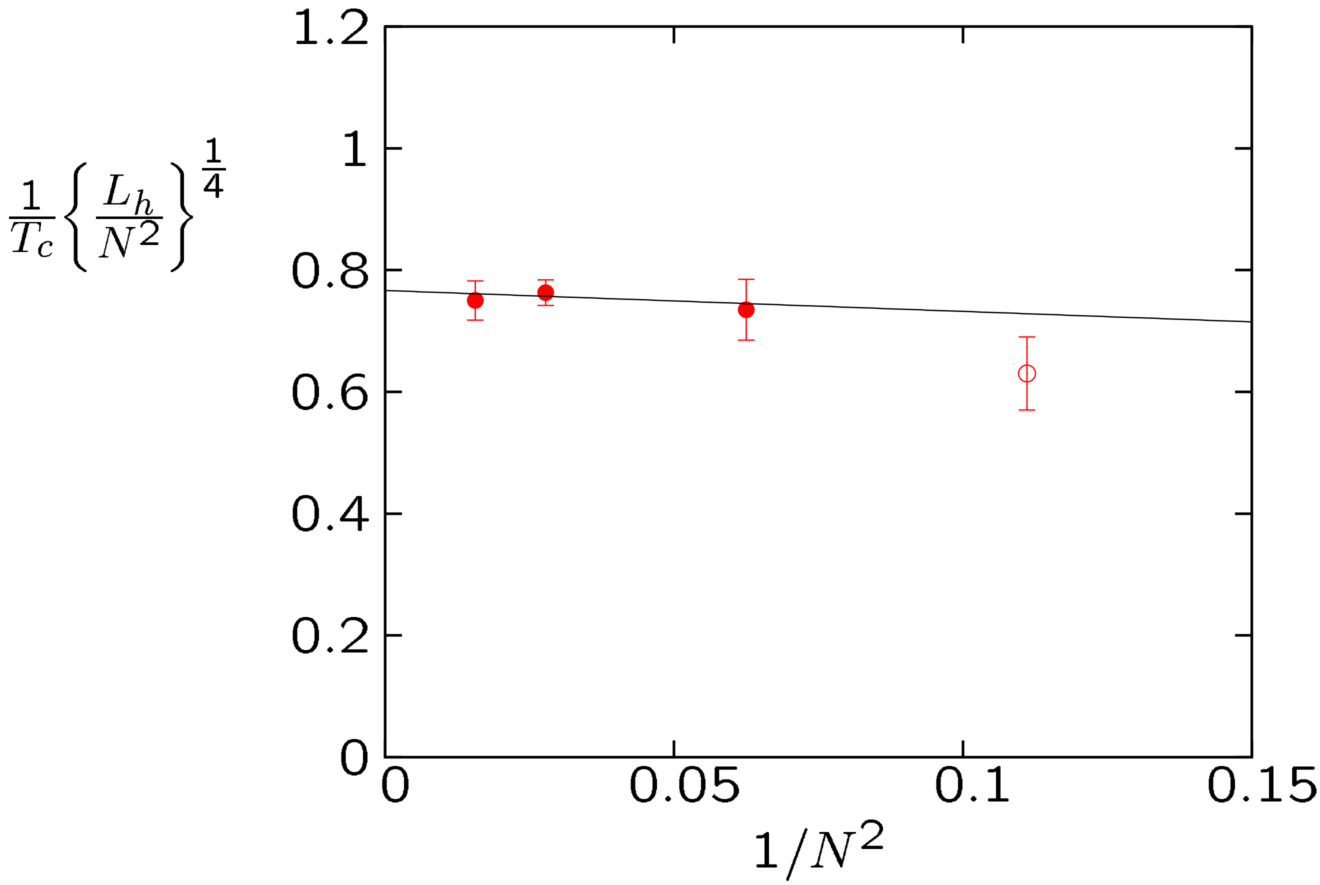
2nd order ∘ ; 1st order •

$\Rightarrow$

$$\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$$

Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003



$\Rightarrow$

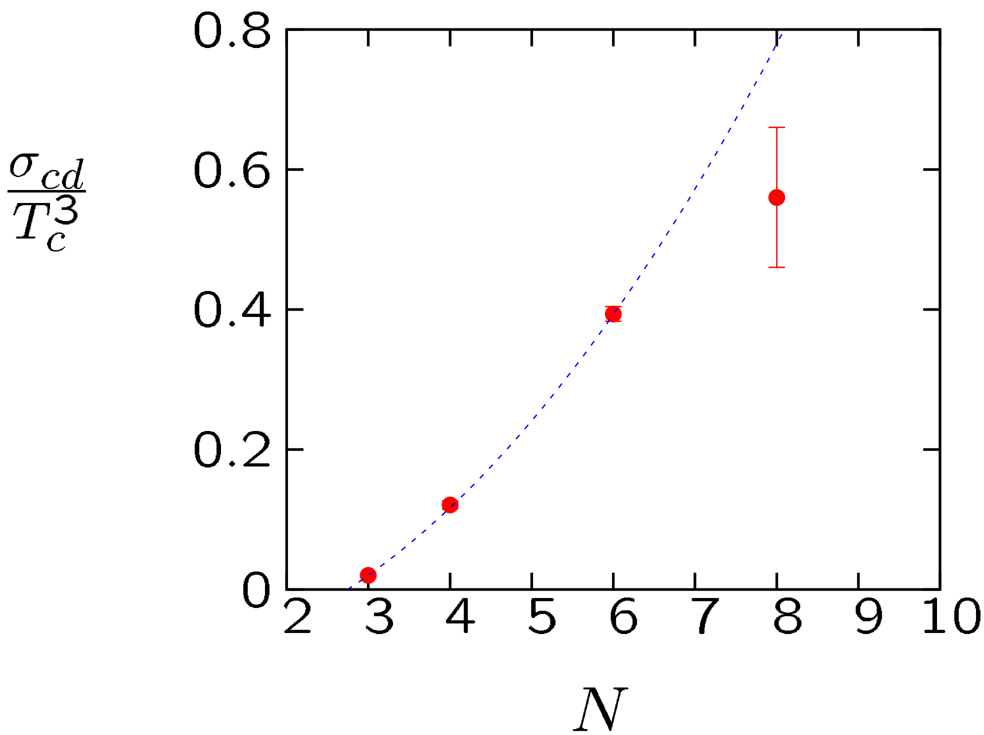
large-N deconfinement is 'normal' first order

$N = 3$ 'weakly' first order

Confinement-deconfinement wall tension

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

aT=0.2 :



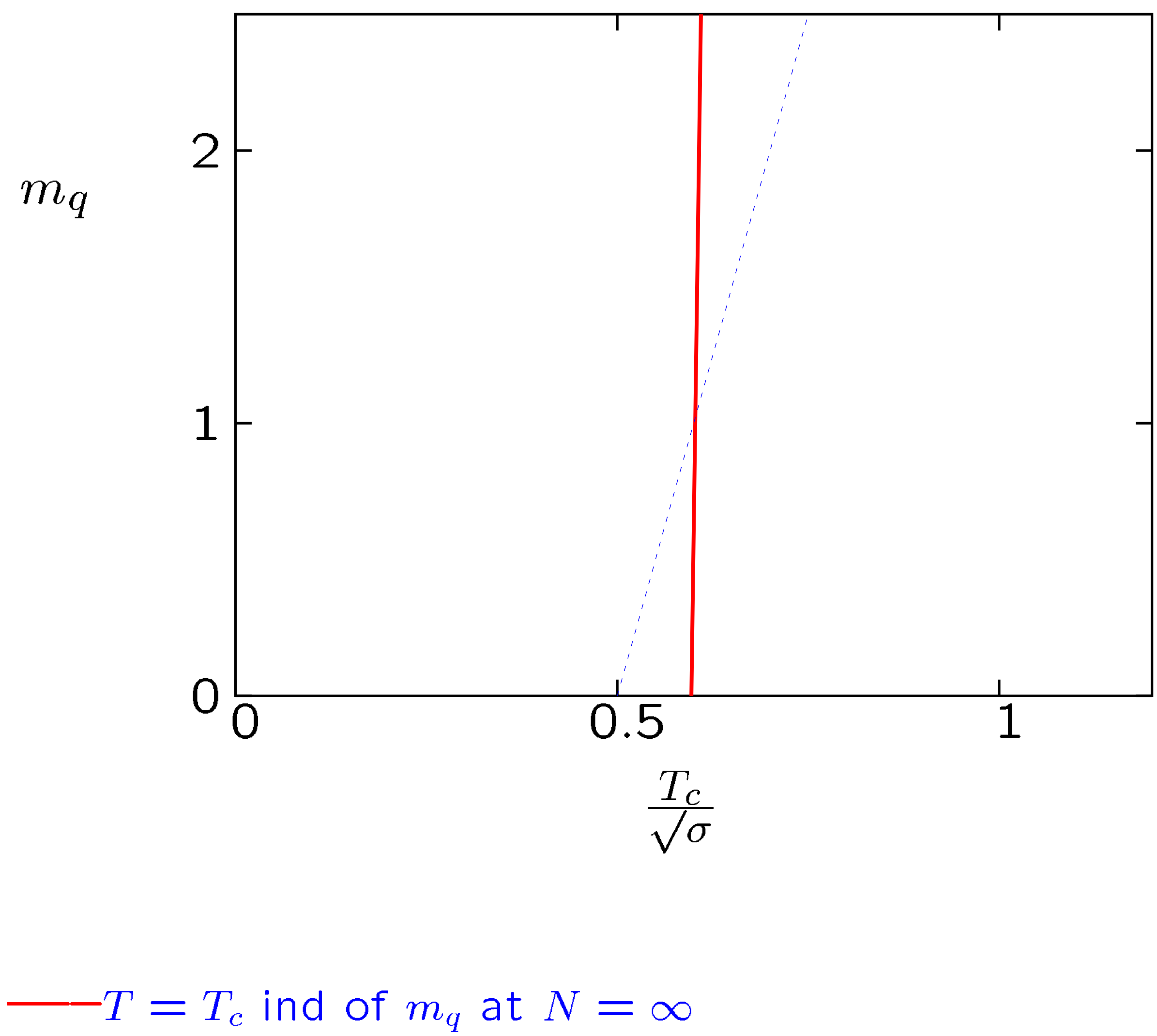
fit :

$$\frac{\sigma_{cd}}{T_c^3} = 0.0138 N^2 - 0.104$$

$\Rightarrow$

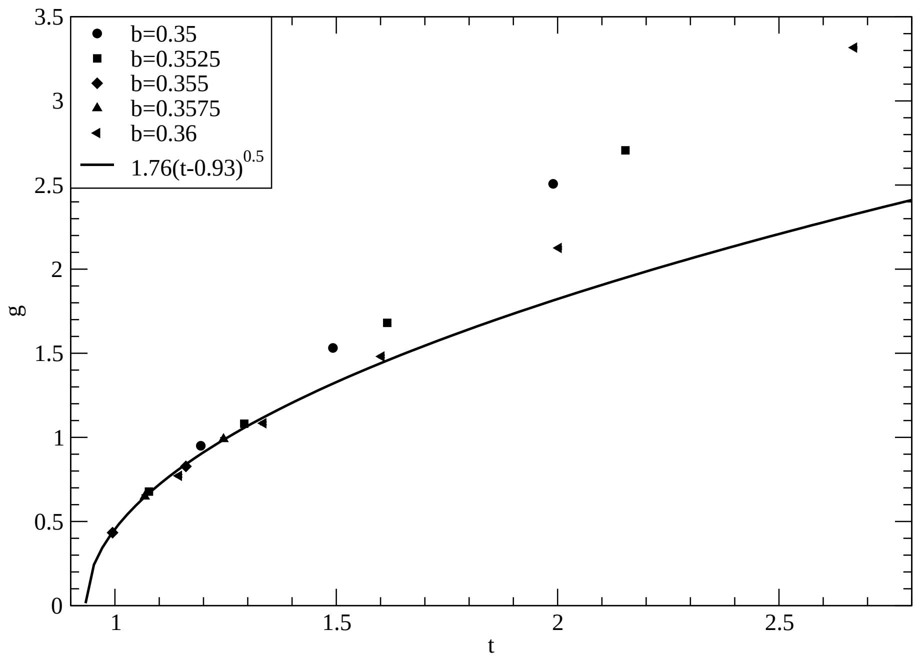
interface tension small

Confinement-deconfinement in QCD_∞?



Chiral symmetry restoration as $T \to T_c$ at large N

R. Narayanan, H. Neuberger: hep-th/0605173

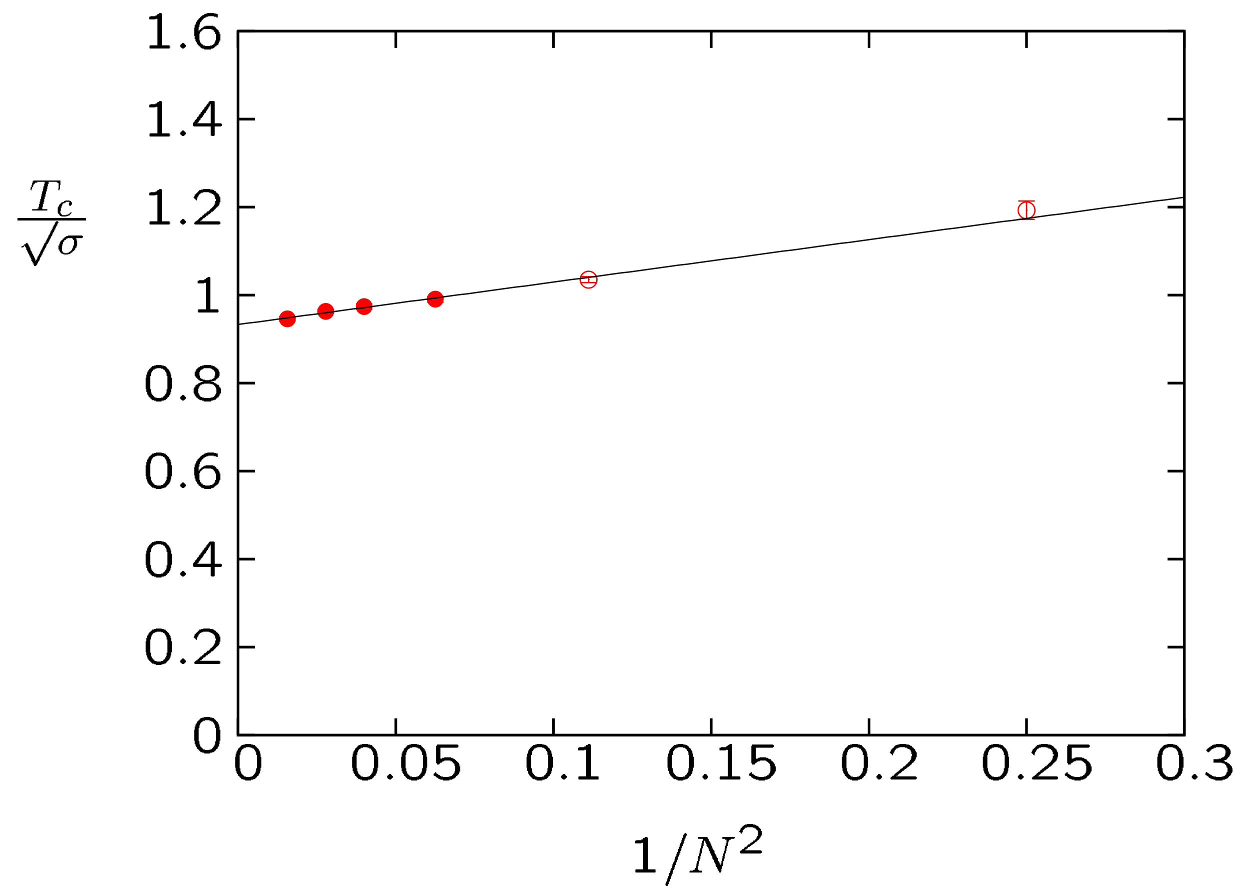


the gap in the eigenvalue spectrum of the Dirac operator, D, at $\lambda \simeq 0$ for $N = 23$ to $N = 53$.

D=3+1 $\longrightarrow$ D=2+1

$\frac{T_c}{g^2 N}$ J. Liddle, M.Teper : hep-lat/0509082; in preparation

$\frac{\sqrt{\sigma}}{g^2 N}$ B. Bringoltz, M.Teper : hep-th/0611286



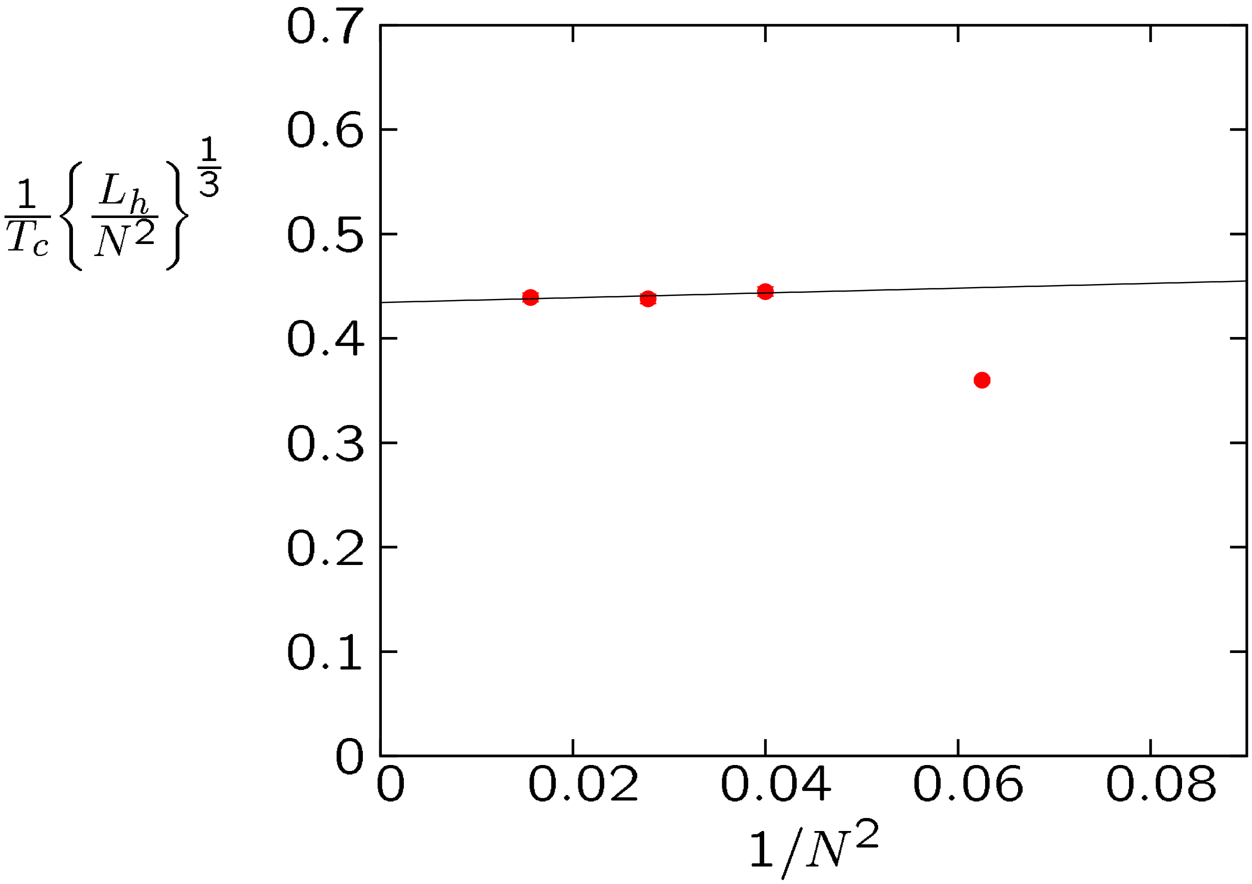
2nd order ○ ; 1st order ($N = 4$?) ●

$\Rightarrow$

fit : $\frac{T_c}{\sqrt{\sigma}} = 0.933(4) + \frac{0.96(8)}{N^2}$ preliminary

D=2+1 latent heat

J. Liddle, M.Teper : in preparation



$\Rightarrow$

large-N deconfinement is 'normal' first order

$N = 4$ 'weakly' first order

Single or multiple deconfining transitions?

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

let l_p be the Polyakov loop (fund repn), then

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined

i.e. deconfinement $\leftrightarrow Z_N$ ssb

$\Rightarrow$

is there one transition or several?

e.g.

$$SU(4): \qquad Z_4 \stackrel{T=T_c}{\longrightarrow} Z_2 \stackrel{T=T_{c'}}{\longrightarrow} 1$$

corresponding to

$T = T_c$: k=2 strings break – but not k=1

$T = T_{c'}$: k=1 strings break

NO : there is only one transition

Polyakov loop order parameter - a problem

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined ssb $V = \infty$

BUT
if V is small enough to see CD tunnellings, which you need to identify T_c accurately, then

$$\sigma_{DD'} \overset{T=T_c}{\leq} \sigma_{CD}/2 \quad \rightarrow \quad \langle l_p \rangle \overset{T=T_c+\epsilon}{=} 0$$

due to DD' tunnellings just above T_c

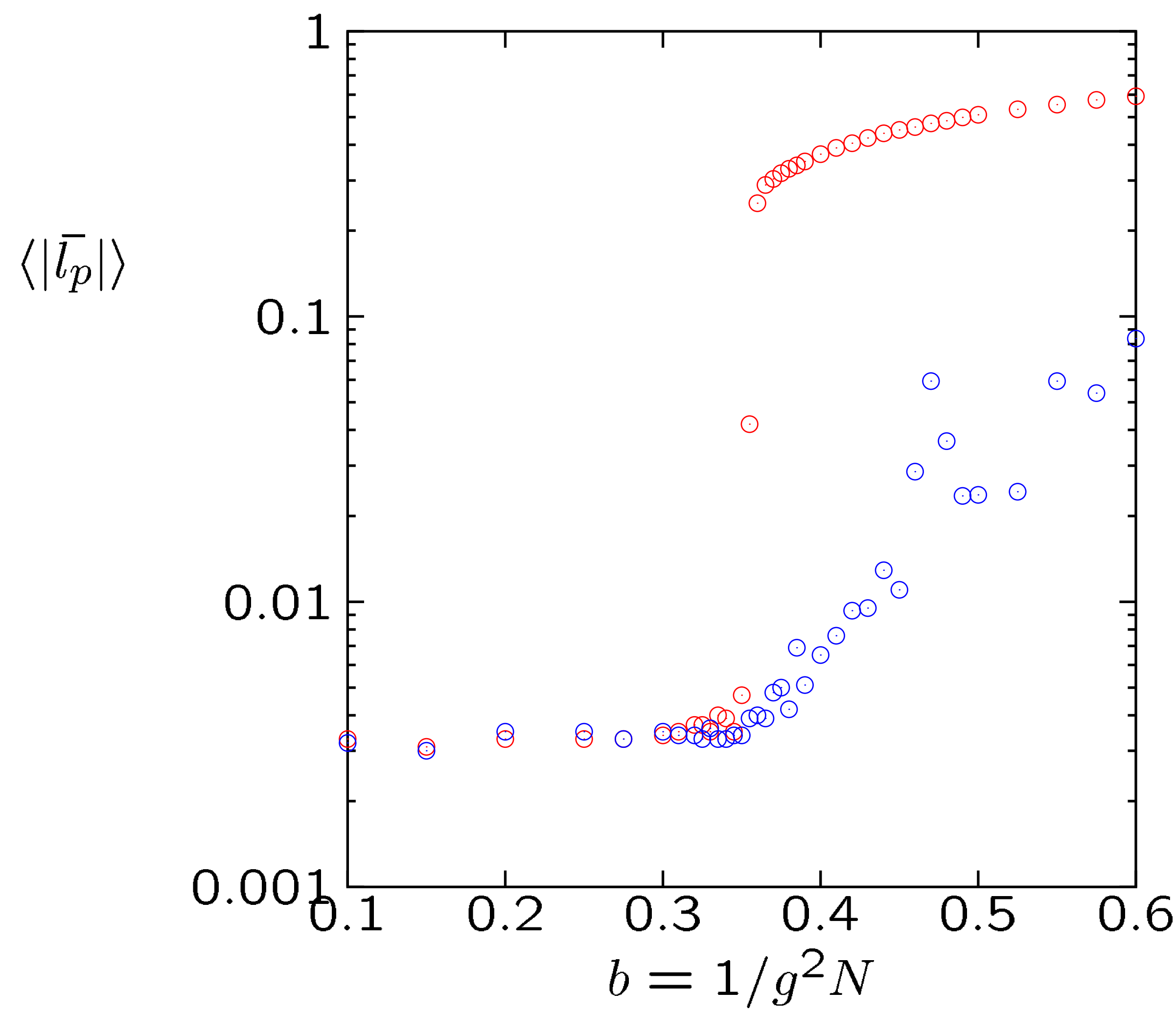
$\rightarrow$

usually modify the Polyakov loop operator:

$$\langle l_p \rangle \quad \rightarrow \quad \langle |\bar{l}_p| \rangle$$

(usual) Polyakov loop order parameter

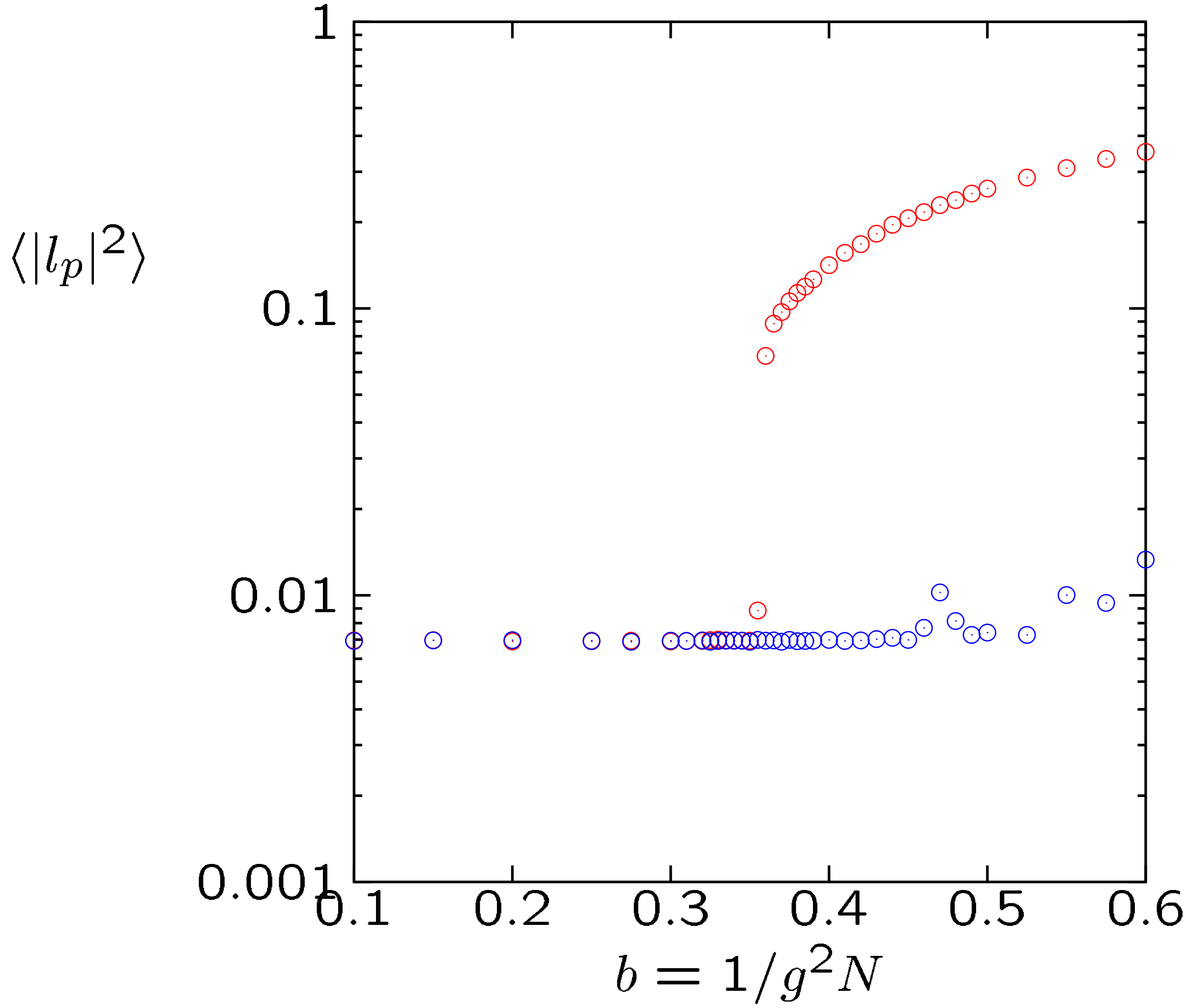
SU(12)



time-like Polyakov loop, ∘

space-like Polyakov loop, ∘

adjoint Polyakov loop



time-like adjoint Polyakov loop, ○

space-like adjoint Polyakov loop, ○

adjoint Polyakov loop

$\langle |l_p|^2 \rangle = 1/N^2$
$=$ value random matrix (Haar measure)
in both strong and weak coupling confined phases

now

our normal Polyakov loop is a normalised trace in the fundamental representation: $l_p = \frac{1}{N}\mathrm{Tr}_f l$

and so the loop in the adjoint representation is:
$\mathrm{Tr}_a l = N^2 |l_p|^2 - 1$

$\rightarrow$

$\langle \mathrm{Tr}_a l \rangle = 0$ in the confining phase
$\langle \mathrm{Tr}_a l \rangle = O(1)$ in the deconfined phase

$\leftrightarrow$

confined phase : adjoint string does not (easily) break
deconfined phase : adjoint string easily breaks

good order parameter even if we use an adjoint action

N counting of free energies (heuristic)

$$Z = e^{-\frac{F}{T}} = \sum_n e^{-\frac{E_n}{T}} \quad (2)$$

$$= \sum_{c=singlet} e^{-\frac{E_c}{T}} + \sum_{g=gluons} e^{-\frac{E_g}{T}} \quad (3)$$

$$= e^{-\frac{F_c}{T}} + e^{-\frac{F_g}{T}} \quad (4)$$

and at $T = T_c$ we have

$$F_c = F_g$$

but

$F_g \sim N^2$ colour singlet entropy $\sim N^0$

so reason that $T_c \not\to 0$ as $N \to \infty$ is that

$$E_c = hadron\, masses + E_{vac}$$

and

$$E_{vac} \sim -N^2 \sim gluon\, condensate$$

so

$$F_g = -E_{vac} \text{ at } T = T_c$$

T-dependence of the string tension?

extra free energy of 2 sources a distance r apart

$$\Delta F(r) = V_{eff}(r,T) \overset{r\to\infty}{\simeq} \sigma_{eff}(T) r$$

but

$$\exp\{-\Delta F(r)/T\} \overset{r\to\infty}{\simeq} \exp\{-m_l(l) r\}$$

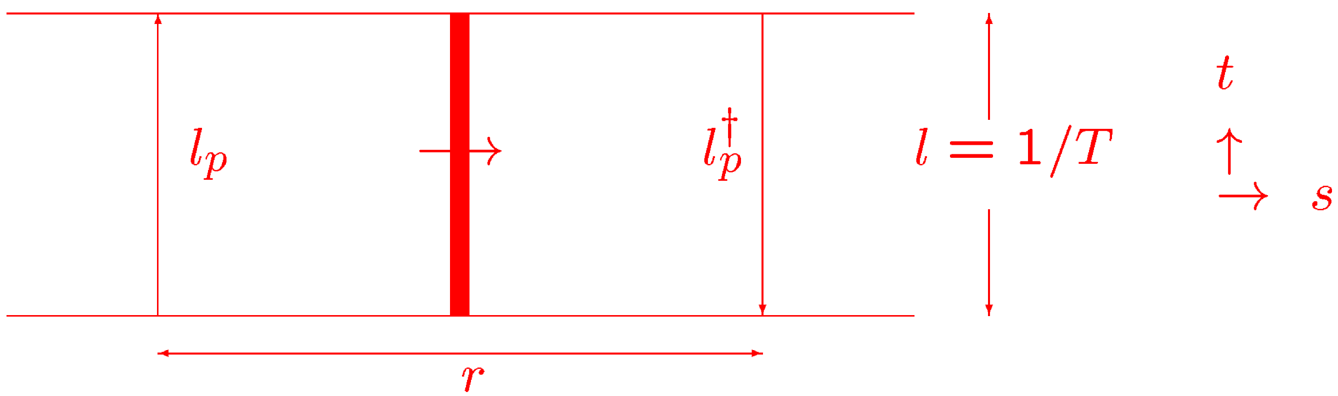
where $m_l(l = 1/aT)$ is the lightest mass coupling to the Polyakov loop that represents the world-line of the source, i.e. it is the mass of the lightest flux loop that winds around the time-torus, so

$$\sigma_{eff}(T) = m_l(l = 1/T).T$$

calculate from space-like correlations of time-like Polyakov loops

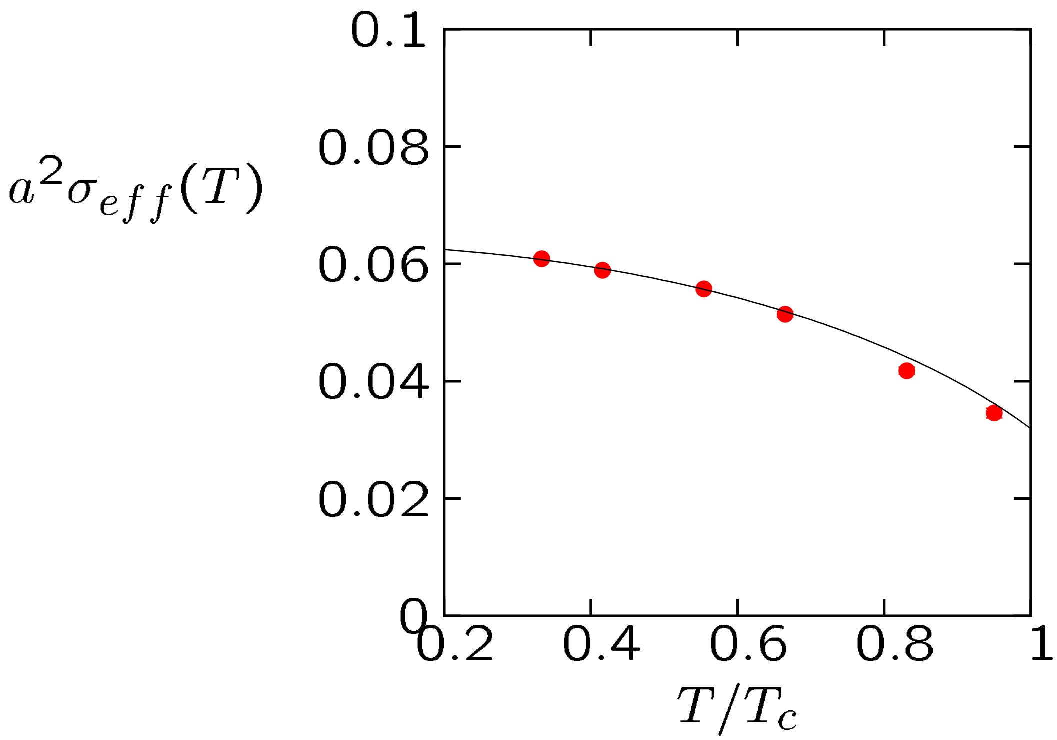
$$\langle l_p^\dagger(r) l_p(0) \rangle \stackrel{t\to\infty}{\propto} \exp\{-m_p(l = 1/T)r\}$$

in pictures



D=3+1 SU(6)

H.Meyer, M.Teper: hep-lat/0411039



Nambu-Goto fit shown:

$$a^2\sigma_{eff}(l = 1/aT) = \frac{am_l(l)}{l} = a^2\sigma\left(1 - \frac{2\pi}{3}\frac{1}{\sigma l^2}\right)^{\frac{1}{2}}$$

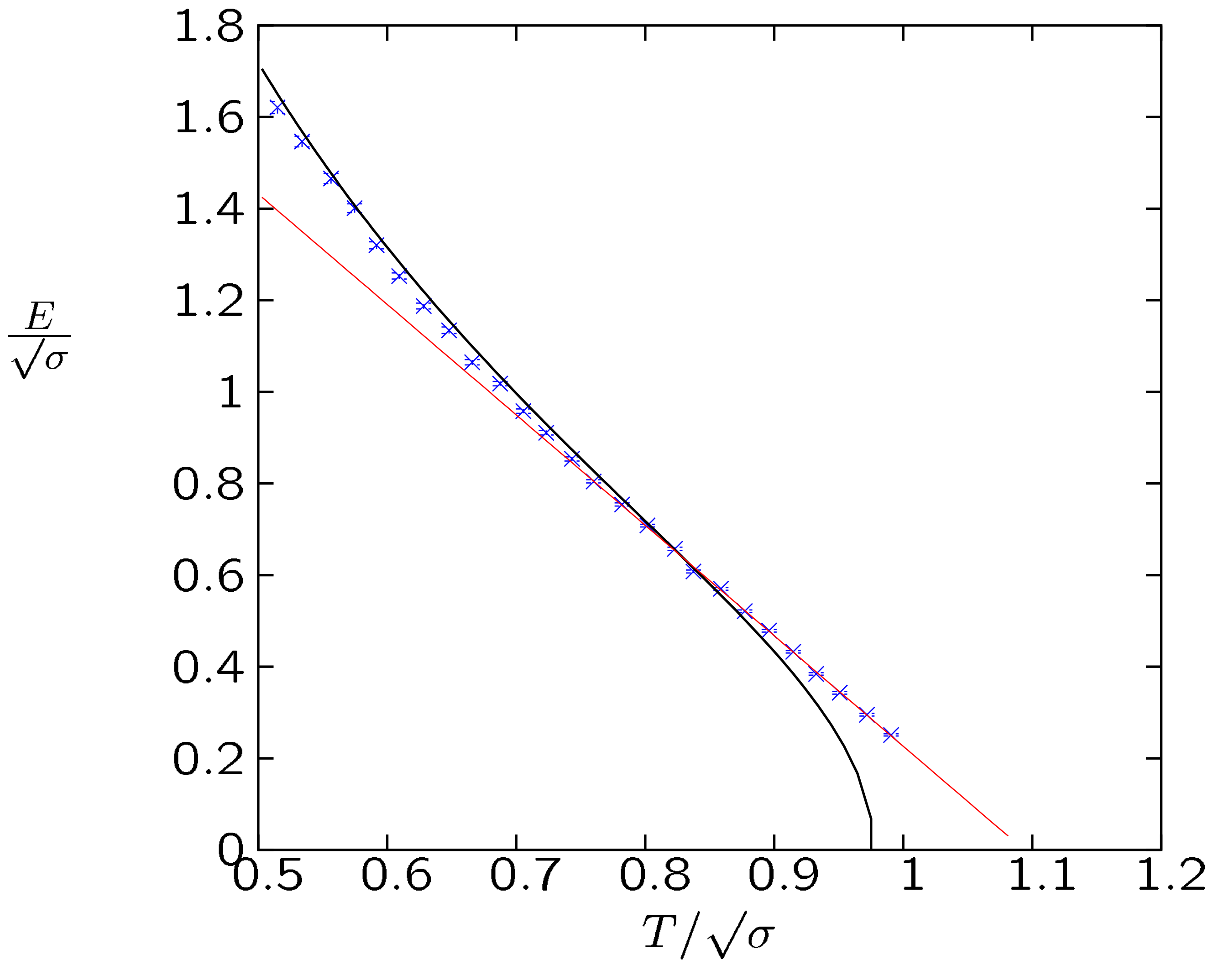
$\xrightarrow{T\to 0}$

$$\sigma_{eff}(T) = \sigma - \frac{\pi}{3T^2} + O(\frac{1}{T^4})$$

what of 2nd order transitions?

NG $\rightarrow$ critical behaviour?

e.g. SU(2), D=2+1:



- critical: $\propto (T - T_c)^{\nu=1}$
- NG: $\frac{\sqrt{\sigma}}{T}\left(1 - \frac{\pi}{3}\frac{T^2}{\sigma}\right)^{1/2}$

strings naturally deconfine

suppose we are at some finite T; what is the probability $P(l)$ of finding a string (long flux tube) of length $l \gg 1/\sqrt{\sigma}$ in the thermal 'vacuum'?

roughly $\quad P(l) \propto n(l) e^{-E(l)/T}$

where $n(l)$ is the number of strings of length l and

$$n(l) \propto e^{cl}$$

neglecting powers of l and the string thickness

$$P(l) \propto e^{cl} e^{-\frac{\sigma l}{T}} \propto e^{-\frac{\sigma_{eff}(T) l}{T}}$$

with

$$\sigma_{eff}(T) = \sigma - cT \to 0 \quad : \quad T \to T_c \equiv \sigma/c$$

So a theory with string-like confining flux tubes naturally deconfines at some T_c in the sense that the thermal vacuum contains a condensate of such flux tubes

$\Rightarrow$

at $T = T_c$ it costs no energy to increase the distance between (already distant) sources of fundamental flux – the effective string tension vanishes

example: Nambu-Goto

$$\sigma_{eff}(T) = \sigma \left(1 - \frac{T^2}{\sigma} \frac{\pi(D-2)}{3}\right)^{\frac{1}{2}}$$

this looks like some kind of Hagedorn transition ... can we see a corresponding non-analyticity in the partition function:

$$Z = \sum_{states} \exp -\frac{E}{T} \qquad ?$$

This is a sum over glueball states, and we know that the lightest glueball m_G satisfies

$$\frac{m_G}{T_c} \simeq \frac{3\sqrt{\sigma}}{0.6\sqrt{\sigma}} \simeq 5$$

so it surely looks unlikely that there is any non-analyticity in Z at T_c if the first term contributes only

$$e^{-m_G/T_c} \sim e^{-5} \sim 0.007$$

however:

- there is a class of highly excited, massive glueballs composed of closed loops of fundamental flux of length $l \gg 1/\sqrt{\sigma}$
- the Boltzmann weight of such a state will be $\exp -M_g(l)/T$ where $M_g \simeq \sigma l$, and the number of such states will again be $\propto \exp +cl$ where this classical counting of states should be reliable in this large quantum number limit ('classical-quantum correspondence')

$\Rightarrow$

This contribution to Z will diverge precisely at the same $T = T_c$ where strings condense!

It is not the lightest masses that are relevant to the non-analyticities of Z – it is the density of states at asymptotically high energies

Of course this simple picture of an apparently 2nd order ($\sigma_{eff}(T) \to 0$ as $T \to T_c^-$) deconfining transition ignores the interaction between overlapping strings or glueballs – possible 'excluded volume ' effects – which may be important and may change the picture qualitatively

$\Rightarrow$

argument for a Hagedorn transition is best at $N = \infty$ where colour singlet objects – glueballs, closed flux tubes – do not interact

But

we will find that the phase transition is first order, not second order, for larger N

Why?

there is a whole ensemble of states, those composed of a finite number of gluons per unit volume, that the string theory knows nothing about – and while they are unimportant at small T, where they are suppressed by the Boltzmann factor, the entropy is $O(N^2)$ and so at some T the free energy, $F = E - TS$, of the 'gluon plasma' will become lower than that of the colour singlet ensemble of states whose entropy is only $O(N^0)$.

and

at this point there will be a jump between these two distinct ensembles that will be characteristically a first order phase transition

nonetheless lurking somewhere beyond this 1st order transition there may be a stringy Hagedorn transition that one might be able to locate using the strong metastability of the first order deconfining transition ...

‘Hagedorn’ string condensation – heuristics

probability flux tube of length l
$\propto$ number paths length l $\times$ Boltzmann suppression
$\propto \quad \exp\{+cl\} \exp\{-\sigma l/T\} \quad \propto \quad \exp\{-(\sigma - cT)l/T\}$

$\rightarrow$

$\sigma_{eff}(T) \rightarrow 0$ as $T \rightarrow T_H = \sigma/c$

$\rightarrow$

a second-order deconfining transition
BUT neglects interactions (e.g. excluded volume effects) so argument best for $N = \infty$

although Nambu-Goto values match well on to SU(2) in D=3+1 and SU(3) in $D = 2 + 1$:
$T_c/\surd\sigma = 0.7091(36)$ vs 0.691 in SU(2) D=3+1
$T_c/\surd\sigma = 1.007(4)$ vs 0.977 in SU(3) D=2+1

Can we hope to see it through the metastable window provided by the strongly first order deconfining transition?

Searching for the Hagedorn transition : SU(12)

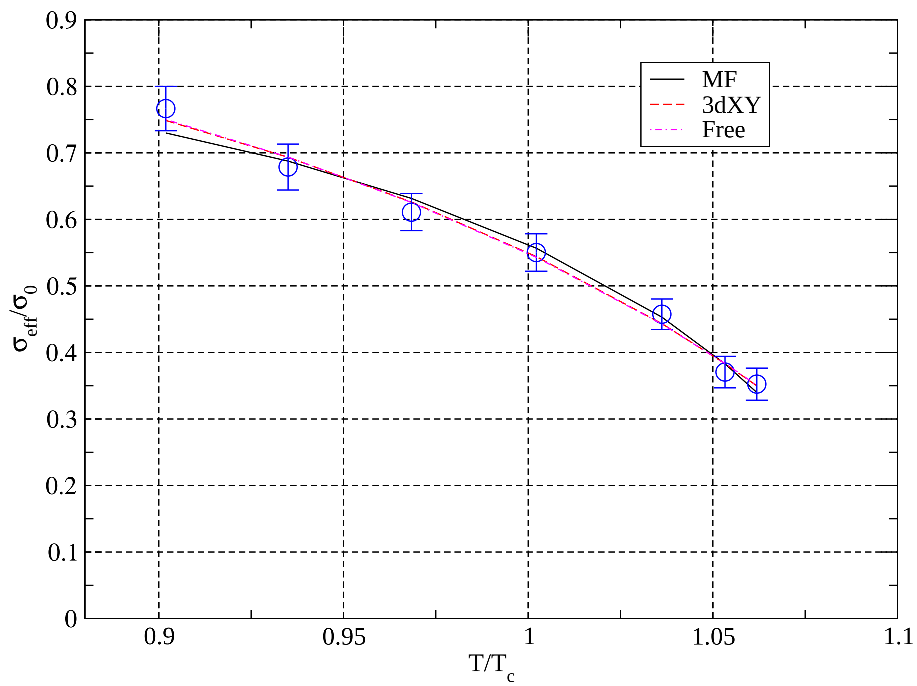
B. Bringoltz, M.Teper: hep-lat/0508021

use strong metastability of the 1st order deconfining transition to stay in the confining phase for

$$T > T_c$$

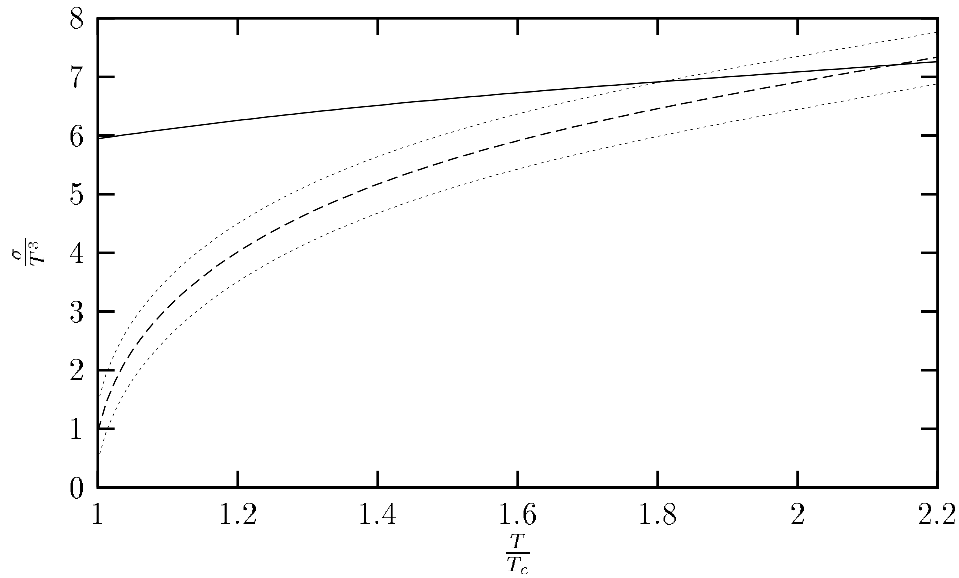
and try to extrapolate to

$$\lim_{T \to T_H} \sigma_{eff}(T) = 0$$



The 't Hooft string tension ...

F. Bursa, M. Teper: hep-lat/0505025



SU(4) 't Hooft string tension in units of T (with 2-loop perturbative result using $g^2(T) \simeq g^2_{MFI}(a)$).

$\Rightarrow$

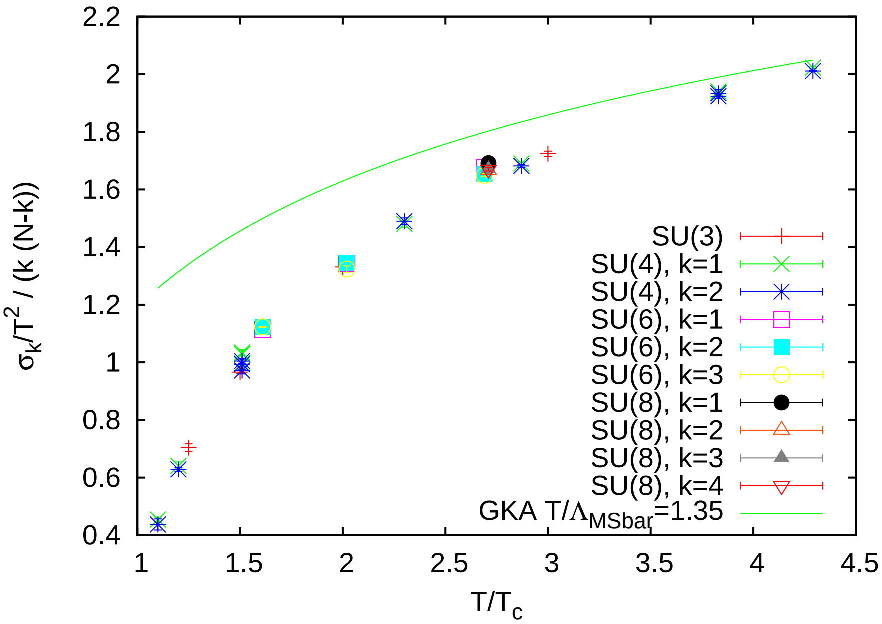
hint of approximate duality

$T \leftrightarrow \frac{1}{T}$

between the confining and 't Hooft string tensions

also ...

de Forcrand, Lucini, Noth: hep-lat/0510081



$\Rightarrow$

tension very small at $T = T_c$

small deconf-deconf wall tensions at $T = T_c$

$\Rightarrow$

$\sigma_{cd} \stackrel{T=T_c}{=} 2\sigma_{k=N/2}$

$\Leftrightarrow$

small conf-deconf wall tension at $T = T_c$

Strongly Coupled Gluon Plasma - at large N?

B. Bringoltz, M.Teper: hep-lat/0506034

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp(-\beta S_W).$$

now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W \rangle = N_p \langle u_p \rangle$

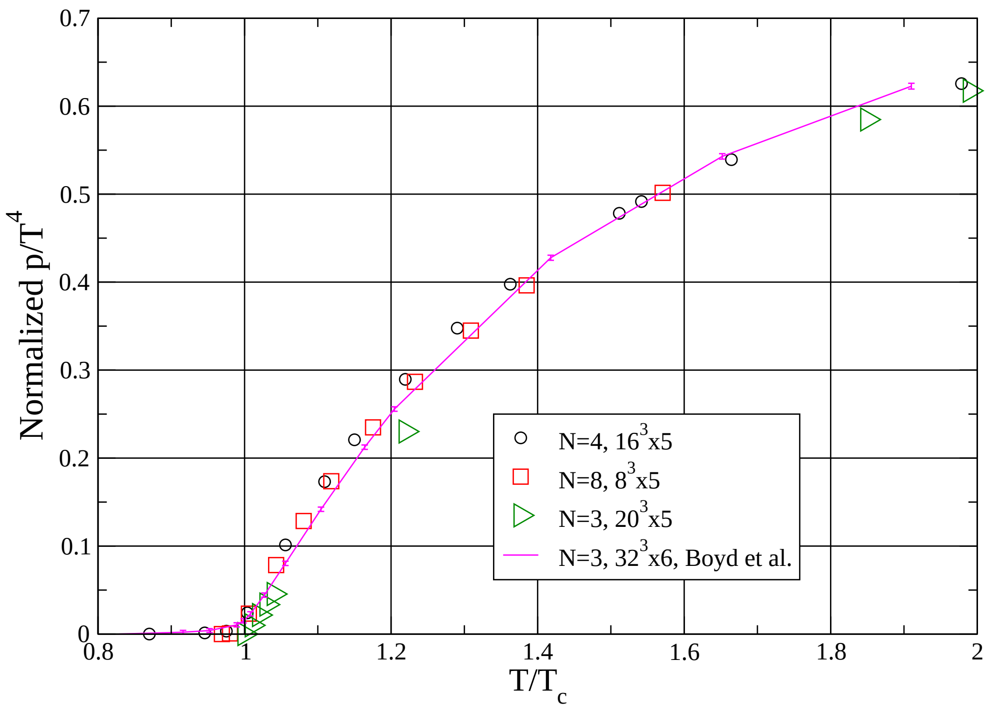
so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta' (\langle u_p \rangle_T - \langle u_p \rangle_0)$.

i.e. $\frac{p(T)}{T^4} = 6L_t^4 \int_{\beta_0}^{\beta} d\beta' (\langle u_p \rangle_T - \langle u_p \rangle_0)$.

similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial \beta}{\partial \log(a(\beta))}$.

Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

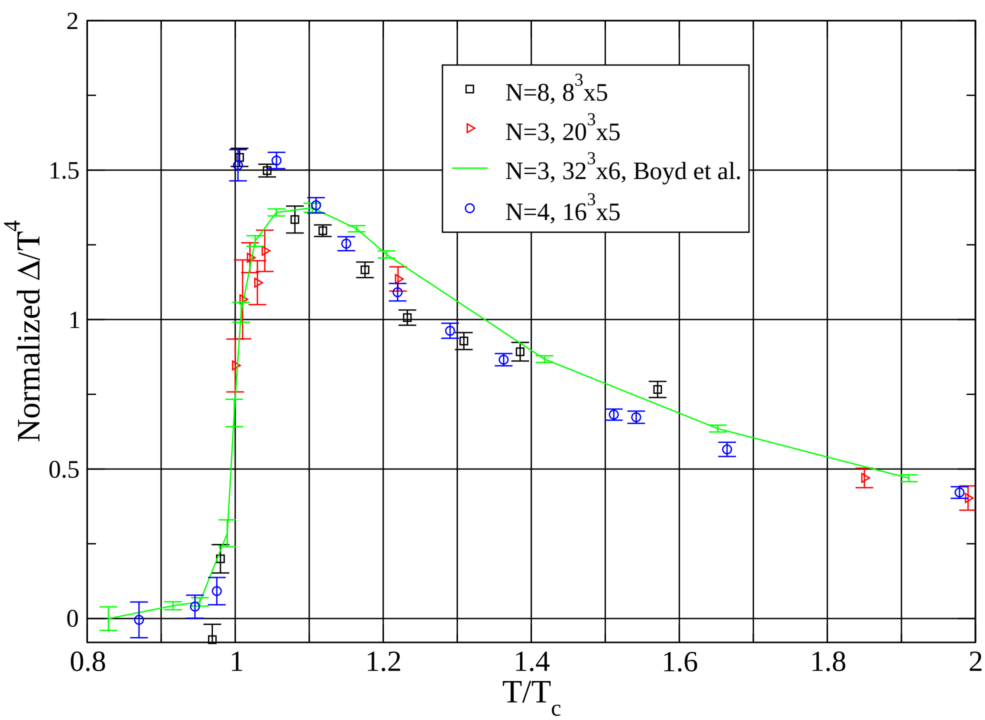
$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)

$$\Delta \equiv \epsilon - 3p$$

B. Bringoltz, M.Teper: hep-lat/0506034

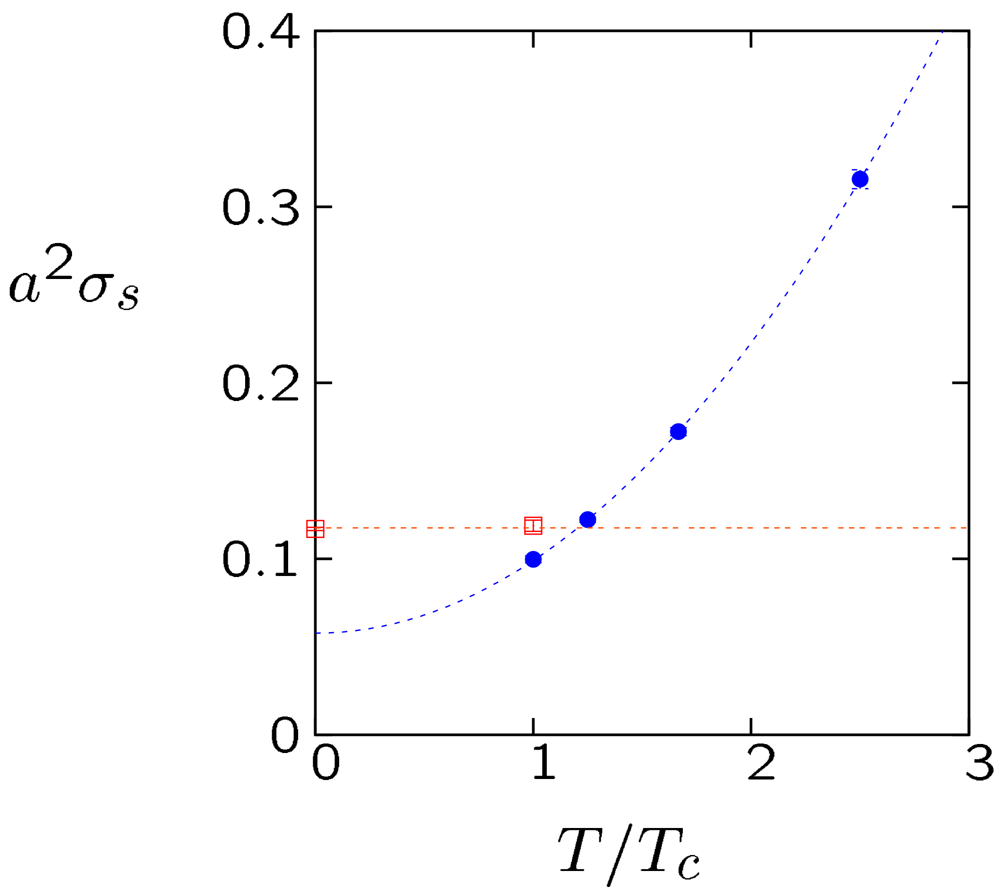
$\Delta = 0$ in Stefan-Boltzman gas



$N = 8$ spatial string tension

Lucini,Teper,Wenger: hep-lat/0502003

$a = 1/5T_c$:

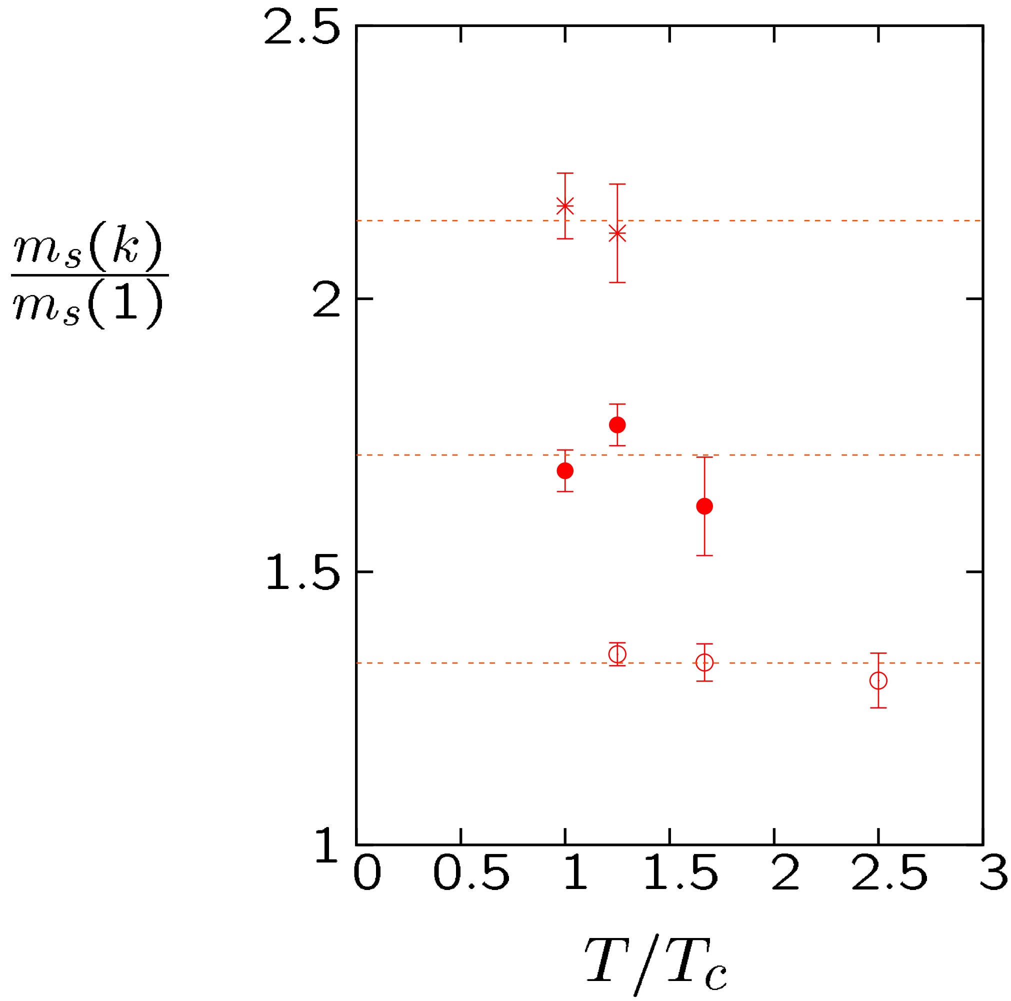


quadratic and constant fits shown

spatial k-string tensions

Lucini,Teper,Wenger: hep-lat/0502003

$a = 1/5T_c$:



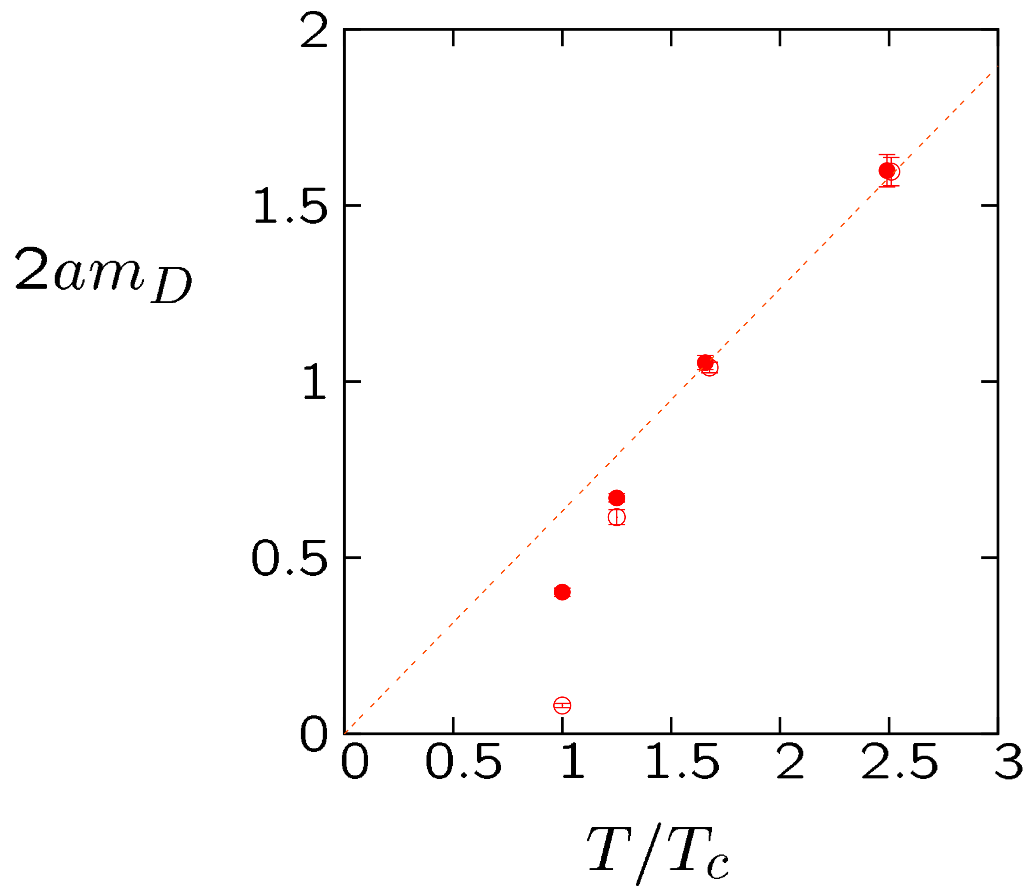
$m_s(k=2)/m_s(k=1)$ in SU(4),$\circ$, and SU(8),$\bullet$, and $m_s(k=3)/m_s(k=1)$ in SU(8),$\star$. Casimir scaling shown.

Debye electric screening mass

Lucini,Teper,Wenger: hep-lat/0502003

from the lightest mass coupling strongly to the vacuum-subtracted time-like Polyakov loop

$a = 1/5T_c$:



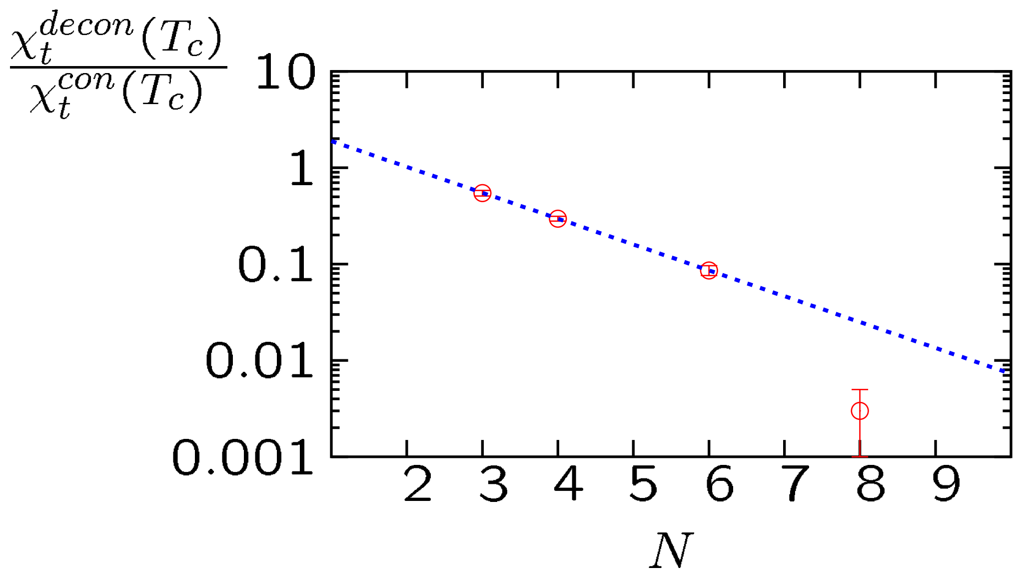
$N = 8$, $\bullet$, $N = 3$, $\circ$.

no topological fluctuations in deconfined phase ...

Lucini,Teper,Wenger: hep-lat/0401028

(Del Debbio,Panagopoulos,Vicari: hep-lat/0407068)

$\chi_t \equiv \langle Q^2 \rangle / V$ in confining/deconfining phases at $T = T_c$



$\Rightarrow$

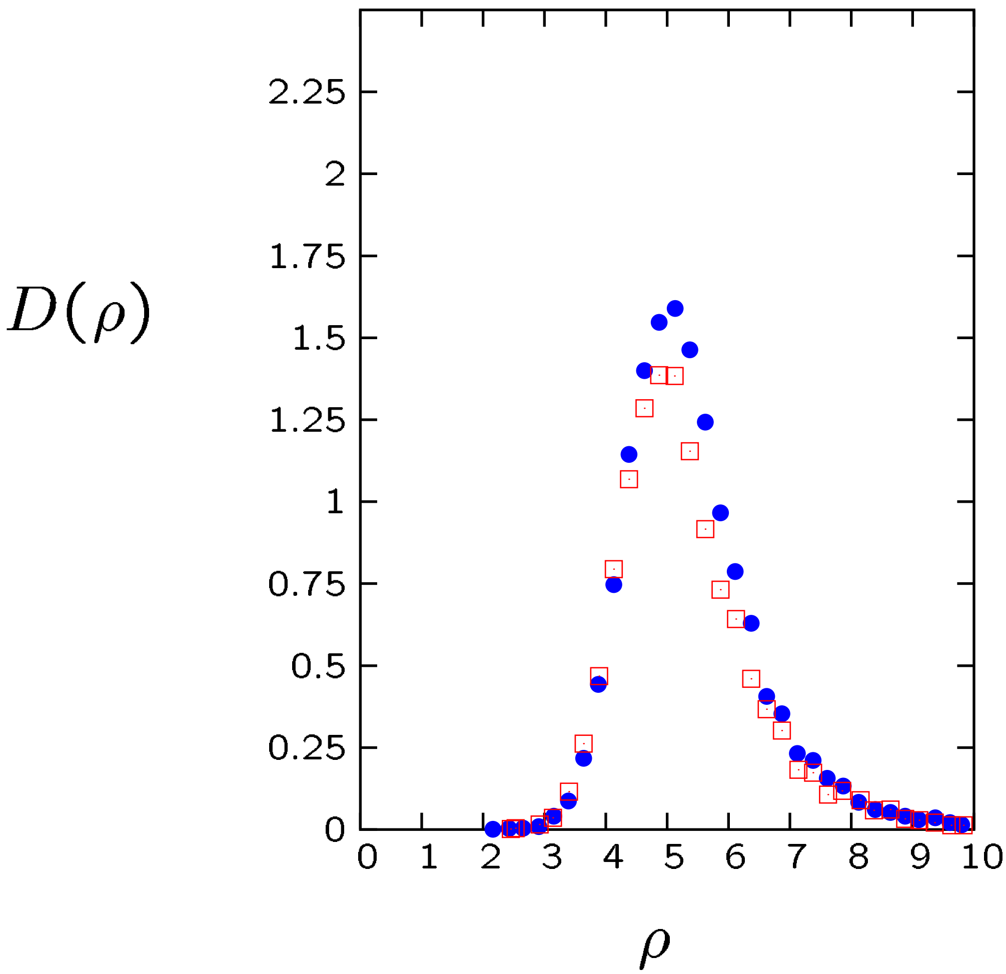
deconfined topological fluctuations

vanish with N exponentially fast

instanton density at $T = T_c$ in confined phase

Lucini,Teper,Wenger: hep-lat/0401028

SU(8) at $a = 1/5T_c$ for $T = 0$, $\bullet$,and $T = T_c$, $\square$,in confined phase.

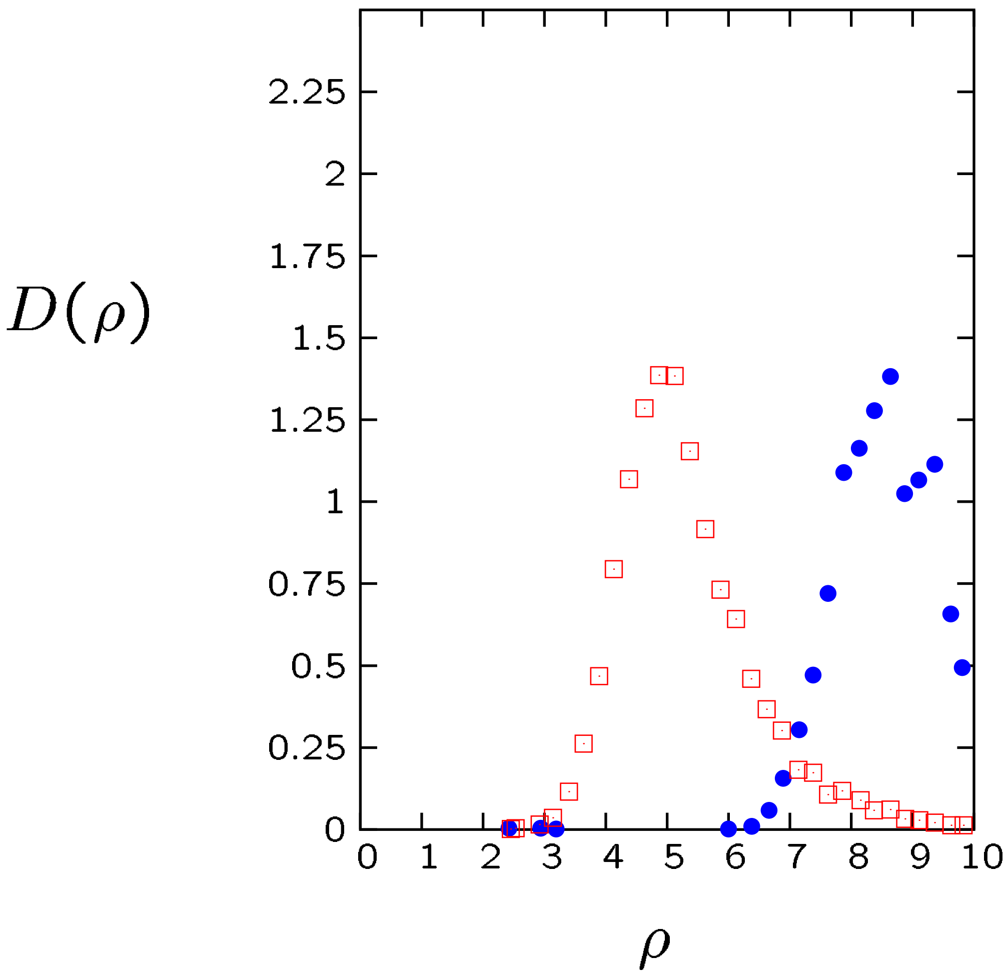


$\Rightarrow$
$D(\rho)$ is independent of T in confined phase as $N \to \infty$

instanton density at $T = T_c$ in deconfined phase

Lucini,Teper,Wenger: hep-lat/0401028

SU(8) at $a = 1/5T_c$ for deconfined • ,and confined, □ ,phases.



$\Rightarrow$

$D(\rho) \equiv 0$ in deconfined phase as $N \to \infty$

not so surprising ...

Lucini,Teper,Wenger: hep-lat/0401028

- for small instantons, $\rho \ll 1/\sqrt{\sigma}$,

$$D(\rho) \sim \exp\{-8\pi^2/g^2(\rho)\} \sim \exp\{-c(\rho)N\}$$

with $c(\rho) = 8\pi^2/g^2(\rho)N$

- for T high enough for perturbation theory in $g^2(T)$ to be good, Gross,Pisarski,Yaffe RMP 53(19981)43

$$D(\rho, T) \sim D(\rho, T = 0) \exp(-\frac{2N}{3}\{\pi\rho T\}^2 - \gamma(\rho T))$$

and instantons with $\rho T > 1$ again vanish exponentially in N

So

the lattice results suggest that these two regions of exponential suppression overlap for any T in the deconfined phase, so that all instantons vanish $\propto \exp\{-cN\}$

dimensional cascade at $N = \infty$ ...

F. Bursa, M. Teper: hep-lat/0511081

• take a space-time volume $l_0 l_1 l_2 l_3$ with $l_0 \ll l_1 \ll l_2 \ll l_3$ and rescale all lengths by some common factor ξ

$\longrightarrow$

decrease ξ so as to increase $T = 1/al_0 \Rightarrow$ a deconfining phase transition at $T = T_c$ such that $\langle l_p(\mu = 0)\rangle \not= 0$ for$T > T_c$

$\longrightarrow$

increase $T = 1/al_0$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a $l_1 l_2 l_3$ gauge-adjoint Higgs theory

$\longrightarrow$

increase $T = 1/al_1 \Rightarrow$ a deconfining phase transition at $T = T_c^{D=3}$ such that $\langle l_p(\mu = 1)\rangle \not= 0$ for$T > T_c^{D=3}$

$\longrightarrow$

increase $T = 1/al_1$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a $l_2 l_3$ gauge-adjoint Higgs theory

$\longrightarrow$

increase $T = 1/al_2 \Rightarrow$ a deconfining phase transition at $T = T_c^{D=2}$ such that $\langle l_p(\mu = 2)\rangle \not= 0$ for$T > T_c^{D=2}$

$\longrightarrow$

increase $T = 1/al_2$ further $\Rightarrow$ dimensional reduction to a dimensionally reduced a l_3 gauge-adjoint Higgs theory

.....

• sequence of $N = \infty$ finite volume transitions on an l^4 space-time volume, as l is decreased

R. Narayanan, H. Neuberger: hep-lat/0704.2591,0509014,0303023

• Gregory-Laflamme transitions in AdS/CFT M. Hanada, T. Nishioka : arXiv:0706.0188

In Summary:

• $SU(\infty)$ has a 'strong' first order deconfining transition in both $D = 4$ and $D = 3$.
It is second order for $N < 3$ in $D = 4$ and for $N < 4$ in $D = 3$

• the 'strong coupling gluon plasma' (e.g. pressure anomaly) above T_c is a large-N phenomenon
$\Rightarrow$
certain explanations are excluded (e.g. some bound states surviving close to T_c)
AdS/CFT has hope of being relevant

• some quantities, such as the spatial string tension σ_s, show only the dimensional dependence
$\sigma_s \propto T^2$
with no $g^2(T)$ dependence visible, while for some others, e.g. the Debye mass, this only becomes so at larger N
$\Rightarrow$
a 'conformal window' above and near T_c?
but
some quantities, such as the 't Hooft string tension, show a very strong additional T dependence ...

some further observations:

- the transition is first order for $N \geq 3$ in $D = 3 + 1$, and for $N \geq 4$ in $D = 2 + 1$

- at this point the $O(N^2)$ gluon plasma free energy is entirely balanced by the $O(N^2)$ confining vacuum energy (gluon condensate)

- we have gone beyond T_c, about halfway to 'T_H', in the confined metastable phase

- the 't Hooft tension decreases rapidly as $T \to T_c^+$ suggesting some approximate duality with the (Wilson) string tension, with a dual Hagedorn transition just below T_c

- topological fluctuations vanish as $\exp\{-cN\}$ at all T in the deconfined phase (calorons?)

- a single phase transition at larger N where the whole Z_N group is broken

- a cascade of finite volume transitions at $N \to \infty$ that are essentially deconfining transitions on ever more dimensionally reduced gauge + adjoint scalar theories

- adjoint Polyakov loops make an elegant order parameter at larger N

(bosonic) strings in D=4?

can long confining flux tubes in $D = 4$ SU(N) gauge theories be described by some string theory?

but

there are no consistent string theories in D=4 : we need D=26 for bosonic, and D=10 for SUSY strings

yes, but

while the properties of confining flux tubes are not that well known, because the physics is strongly coupled, there exist weakly coupled examples, such as Nielsen-Olesen vortices in the Abelian-Higgs model, that provide explicit examples of string like objects in D=4

so

effective string theories, for long strings, should certainly exist in $D = 4$

indeed

the typical inconsistency in quantising a free bosonic string of length R in D=4 is a breakdown of Lorentz covariance: e.g. generators of rotations are anomalous

J. Arvis, Phys. Lett. 127B(1983)106

$$[L^i, L^j] = -L^{ij} + F(R)$$

but one sees that

$$F(R) \propto 1/R^2 \stackrel{R\to\infty}{\longrightarrow} 0$$

so that the inconsistencies disappear in any D for long enough strings

P. Olesen, Phys. Lett. 160B(1985)144

$\star$same for $D = 3\star$

analysing effective string theories

• field theory approach (non-covariant 'gauge fixing' of the string theory)
M. Luscher, K. Symanzik, P. Weisz : Nucl. Phys. B173 (1980) 365; M. Luscher : Nucl. Phys. B180 (1981) 317;
M. Luscher, P. Weisz : JHEP 0407 (2004) 014

• covariant effective string approach
J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681;
J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

In both approaches the starting point is to consider a long (open or closed) string of length r and to consider those corrections allowed by symmetry in powers of $1/r$ – the corrections being to the spectrum of the free Nambu-Goto string theory

Nambu-Goto free string theory

$$\int \mathcal{D}X e^{-\frac{i}{\sigma}\times \text{Area}}$$

a string breaks spontaneously the transverse translation invariance

$\rightarrow$

D-2 Goldstone bosons – massless transverse oscillations of frequencies quantised by the string length

$\rightarrow$

these massless modes determine the effective action of long strings

spectrum of a string of length l winding once around a spatial torus with zero transverse momentum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

for states with total momentum $2\pi q/l$ along the string and with left and right oscillators summing to N_L and N_R

$$N_L = \sum_{k>0} n_L(k)\, k, \qquad N_R = \sum_{k'>0} n_R(k')\, k'$$
$$N_L - N_R = q, \qquad \prod_k a_k^{+n_k} |0\rangle$$

J. Arvis, Phys. Lett. 127B(1983)106

Nambu-Goto ground state $N_L = N_R = 0$:

$E^2(l) = (\sigma\, l)^2 - \frac{\pi(D-2)\sigma}{6}$

$\Rightarrow$ tachyon of mass $\mu^2 = -\frac{\pi(D-2)\sigma}{6}$

$\leftrightarrow$ from zero-point energies of oscillators - the Casimir energy of a (periodic) string

$\leftrightarrow$ 'Hagedorn' deconfinement

$\Rightarrow$ string theory does not exist for $l \leq l_c = \sqrt{\frac{\pi(D-2)}{6\sigma}}$

$\Rightarrow$ spectrum exists and well-behaved for $l > \sqrt{\frac{\pi(D-2)}{6\sigma}}$

field-theoretic approach

• string of length r with fixed ends and world sheet co-ords

$$z = (z_0, z_1), \qquad 0 \leq z_1 \leq r$$

and displacement vector $h(z)$ with effective action (D=2)

$$S = \sigma r T + \mu T + S_0 + S_1 + \cdots\,, \qquad S_0 = \frac{1}{2}\int d^D z\,(\partial_a h \partial_a h)$$

one finds

$$S_1 = \tfrac{1}{4} b \int d^{D-1} z \left\{ (\partial_1 h \partial_1 h)_{z_1=0} + (\partial_1 h \partial_1 h)_{z_1=r} \right\}, \quad b = [l]^1$$

$$S_2 = \tfrac{1}{4} c_2 \int d^D z\,(\partial_a h \partial_a h)\,(\partial_b h \partial_b h)\,, \qquad c_2 = [l]^2$$

$$S_3 = \tfrac{1}{4} c_3 \int d^D z\,(\partial_a h \partial_b h)\,(\partial_a h \partial_b h)\,, \qquad c_3 = [l]^2$$

since the couplings of S_i are increasing powers of length, they will contribute with increasing powers of $1/r$

imposing symmetries and open-closed duality

$\Rightarrow \qquad b = 0, \quad c_3 = -2c_2$

$\Rightarrow$

• $D = 2 + 1$

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{1}{6}\right) - \frac{\pi^2}{2\sigma l^3}\left(n - \frac{1}{6}\right)^2 + O(l^{-4})$$

which is exactly the Nambu-Goto spectrum to this order in $1/l$, and the lowest few degeneracies in the spectrum are also maintained

• $D = 3 + 1$

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{1}{3}\right) - c_2\frac{\pi^2}{l^3}\left(n - \frac{1}{3}\right)^2 + O(l^{-4})$$

which is the Nambu-Goto spectrum up to $O(1/l)$ – the 'Luscher correction' – but not necesarily beyond that

Note: the corrections to E_n that are higher order in $1/l$ directly reflect the coefficients of corresponding operators S_i in the action – as do the splittings of the free string theory degeneracies in the spectrum.

$\Rightarrow$

accurate calculation of the energy spectrum as a function of l provides a *direct* way to determine the effective string action

covariant effective string theory approach

J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681

imagine integrating out all massive modes leaving an integration over the massless $D-2$ transverse oscillators with action
$S_o = \frac{1}{\sigma}\int d\tau^+ d\tau^- \partial_+ X^\mu \partial_- X_\mu$
and some determinants from the massive modes. These should be made out of the induced metric
$h_{ab} = \partial_a X^\mu \partial_b X_\mu$
and suppose the determinant is as for Polyakov but replacing the intrinsic metric e^ϕ with the induced metric h_{+-}. Then the determinant (in conformal gauge) is e^{iS_L} where
$S_o = \frac{26-D}{48\pi}\int d\tau^+ d\tau^- \partial_+\phi\partial_-\phi$
becomes
$S_L = \frac{26-D}{48\pi}\int d\tau^+ d\tau^- \frac{\partial_+^2 X\cdot\partial_- X \partial_+ X\cdot\partial_-^2 X}{(\partial_+ X\cdot\partial_- X)^2}$

Instead of pursuing this, encode the determinants in an effective string action, with conformal invariance as the sole restriction.
Since we are quantising around a long string, we do not mind non-polynomial terms since the denominator will always be large. One finds:
$S = \int d\tau^+ d\tau^- \frac{1}{\sigma}\partial_+ X^\mu \partial_- X_\mu + \frac{\beta}{4\pi}\frac{\partial_+^2 X\cdot\partial_- X \partial_+ X\cdot\partial_-^2 X}{(\partial_+ X\cdot\partial_- X)^2} + O(1/l^3)$

coordinate invariance not anomalous
$\Rightarrow \qquad \beta = \frac{D-26}{12}$
which translates into the usual expression for the Luscher string correction

we can now continue to one higher order and we find
J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right) - \frac{\pi^2}{2\sigma l^3}\left(n - \frac{D-2}{6}\right)^2 + O(l^{-4})$$

i.e. identical to Nambu-Goto to this order in $1/l$ for both $D = 2+1$ *and* $D = 3+1$

Note:
the effective action is only valid for very long strings – $l\surd\sigma \gg 1$ – as is obvious from the denominators in the effective action.
$\Rightarrow$ it tells us nothing about light glueballs since these are composed of small closed loops
it tells us nothing about k-strings or other multiple strings, since the interaction between these (at the origin of their binding) will in general involve the exchange of small closed loops

$\Rightarrow$
what we learn about confining flux tubes with $l\surd\sigma \not\gg 1$ will tell us whether what we have is just an effective string theory for very long flux tubes or possibly an effective string theory on all scales ...

Numerical calculations :

Euclidean $D = 3, 4$

- potential between static sources

$$V(r) = -\frac{c_f \alpha_s(r)}{r} \qquad D = 4 \quad r \ll \frac{1}{\sqrt{\sigma}}$$

$$V(r) = \frac{c_f g^2}{2\pi} \ln g^2 r \qquad D = 3 \quad r \ll \frac{1}{\sqrt{\sigma}}$$

$$V(r) = \sigma r - \frac{\pi(D-2)}{24r} + O\left(\frac{1}{r^3}\right) \qquad r \gg \frac{1}{\sqrt{\sigma}}$$

and corresponding excitations.

M.Luscher, P.Weisz : hep-lat/0207003

H.Meyer , hep-lat/0607015

N. Hari Dass, P.Majumdar, hep-lat/0608024 , hep-lat/0702019

M. Caselle, M. Pepe, A. Rago hep-lat/040600

Michele Caselle, Martin Hasenbusch, Marco Panero hep-lat/0501027, hep-lat/0403004

- flux tubes wound around a spatial torus

$$E(l) = \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \qquad l \geq l_c = \frac{1}{T_c}$$

and corresponding excitations.

A.Athenodorou, B.Bringoltz, M.Teper arXiv:0709.0693

J.Kuti: hep-lat/0511023

- Wilson loops vs Nambu-Goto

$$E(l) = \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \qquad l \geq l_c = \frac{1}{T_c}$$

and corresponding excitations.

M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch, P. Provero hep-lat/9609041

I will focus on the spectrum of strings that are closed around a spatial torus of length l :

- the winding states are flux 'tubes' for all l down to the phase transition at $l = l_c = 1/T_c$ at which one loses confinement

- this phase transition is first order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$

- thus it is possible that we may have a simple string description of the closed string spectrum for all possible lengths (at large N)

- such a simple string description is most likely at $N \to \infty$ where complications such as mixing, e.g string $\to$ string + glueball, will go away

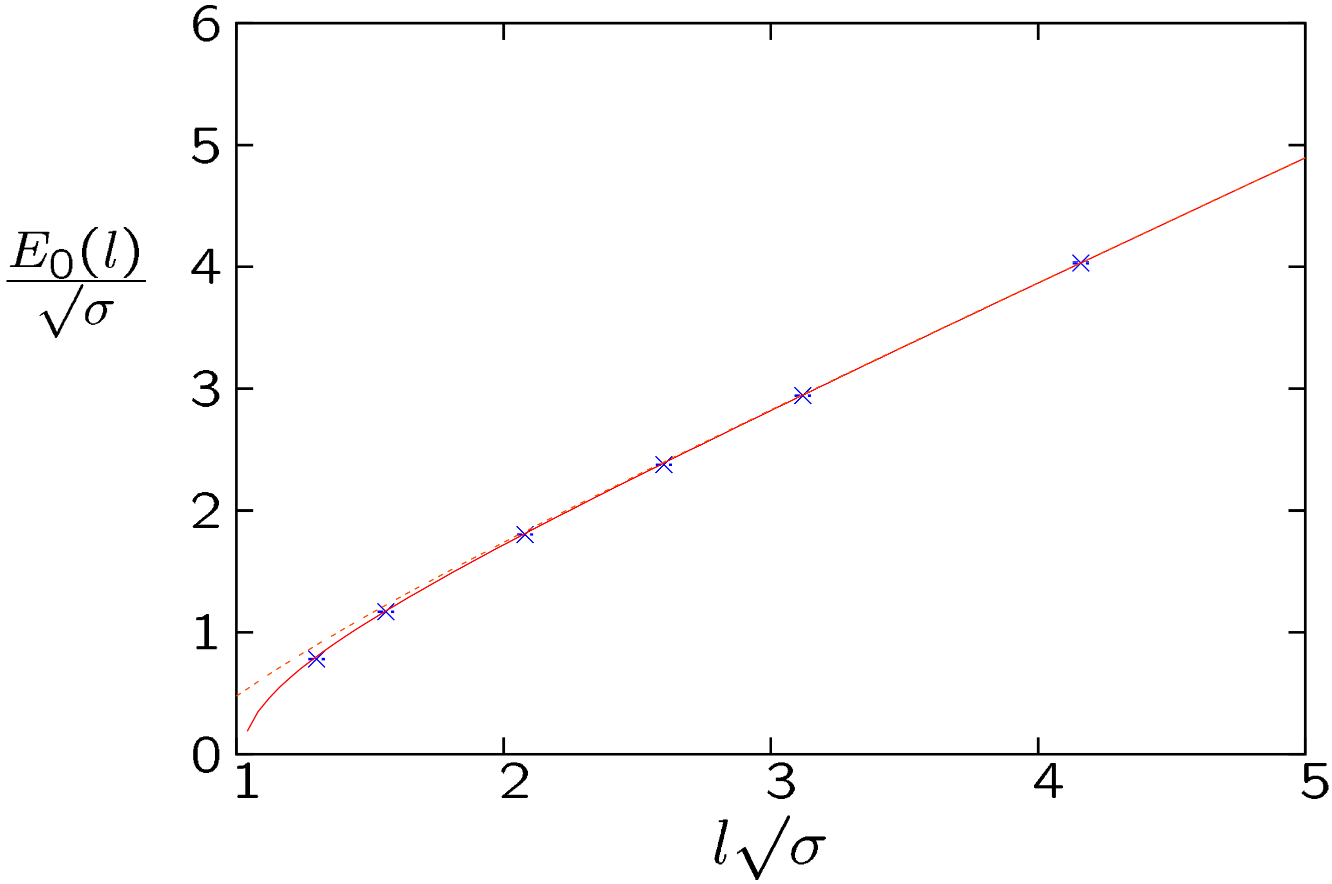
- by contrast, for the potential $V(r)$ between static sources there is a cross-over in r between flux tubes and a Coulomb potential, over some ill-determined distance, and so it is not straightforward to investigate the properties of shorter strings – although there may be a string theory description that includes the Coulomb potential, that is a much more challenging goal

and mostly D=3 ... from:
A.Athenodorou, B.Bringoltz, M.Teper arXiv:0709.0693
B.Bringoltz, M.Teper hep-th/0611286
A.Athenodorou, B.Bringoltz, M.Teper in progress

The spectrum of flux tubes that are closed around a spatial torus of lnegth l : SU(N) $D = 2 + 1$

- linear confinement?
- how good are our energy calculations?
- bosonic string universality class?
- what happens as $l \to l_c$?
- $E_n(l)$: expansion in $1/l$ or covariant Nambu-Goto?

D=2+1 ; SU(5)



Luscher:...

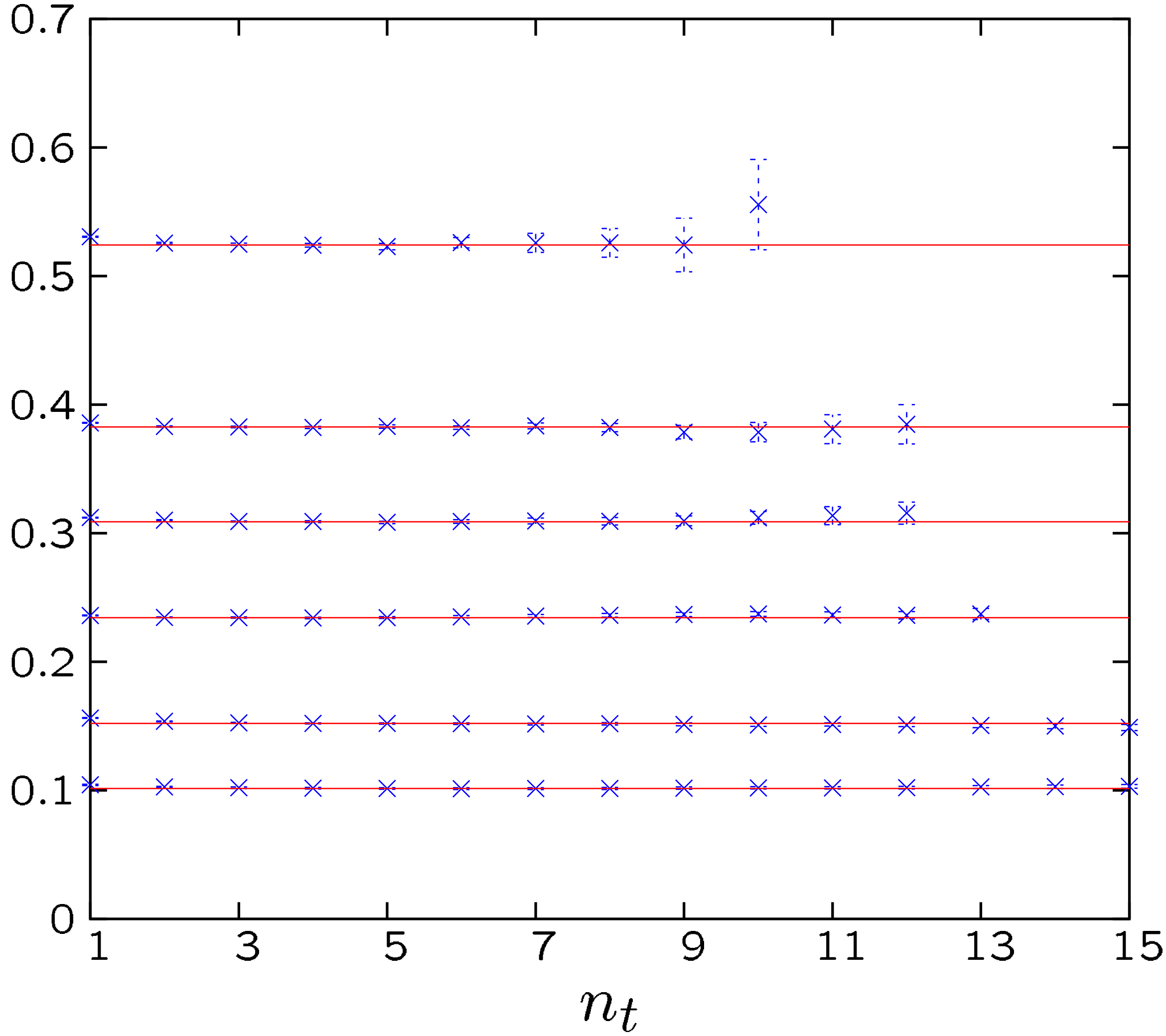
$$E_0(l) = \sigma l - \frac{\pi}{6l}$$

Nambu-Goto:-

$$E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

how good are the energies?

$aE_{eff}(t) = -\ln C(t)/C(t-1)$



lattice sizes and MC ‘statistics’

transverse and longitudinal sizes need to be large enough in units of the loop mass i.e. increase as $l \downarrow$

l	lattice	MC sweeps
10	$10 \times 40 \times 120$	$0.5M$
12	$12 \times 32 \times 80$	$1.0M$
16	$16 \times 32 \times 56$	$1.5M$
20	$20 \times 32 \times 40$	$2.0M$
24	$24 \times 24 \times 32$	$2.0M$
32	$32 \times 32 \times 32$	$1.5M$

effective string theory
– universality class?

central charge appears in the string ‘Casimir’ energy

$$E_0(l) = \sigma l - \frac{c\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right)$$

where

$$c = 1,\ 1.5,\ 0$$

for bosonic, Neveu-Schwartz, Ramond strings respectively

to determine the central charge numerically, calculate the ground state energy $E_0(l)$ for a sequence of increasing lengths, and fit an effective central charge, $c_{eff}(l)$, to neighbouring values of l, i.e.

$$c_{eff}(l,l') = \frac{6}{\pi(D-2)} \frac{\frac{E(l)}{l} - \frac{E(l')}{l'}}{\frac{1}{l'^2} - \frac{1}{l^2}}$$

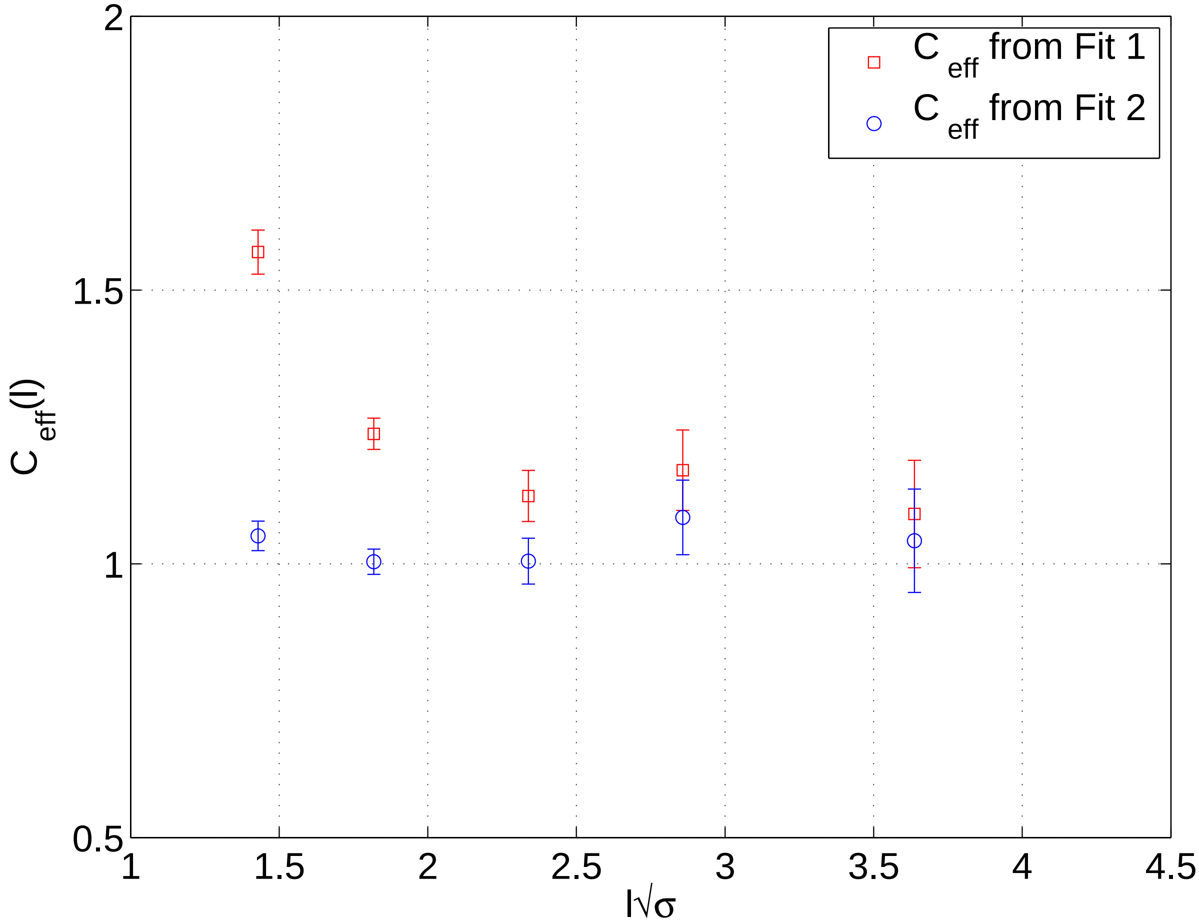
alternatively do the same for Nambu-Goto, solving

$$E_0(l) = \sigma l \left(1 - c_{eff}(l,l')\frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}}$$

$$E_0(l') = \sigma l' \left(1 - c_{eff}(l,l')\frac{\pi(D-2)}{3\sigma l'^2}\right)^{\frac{1}{2}}$$

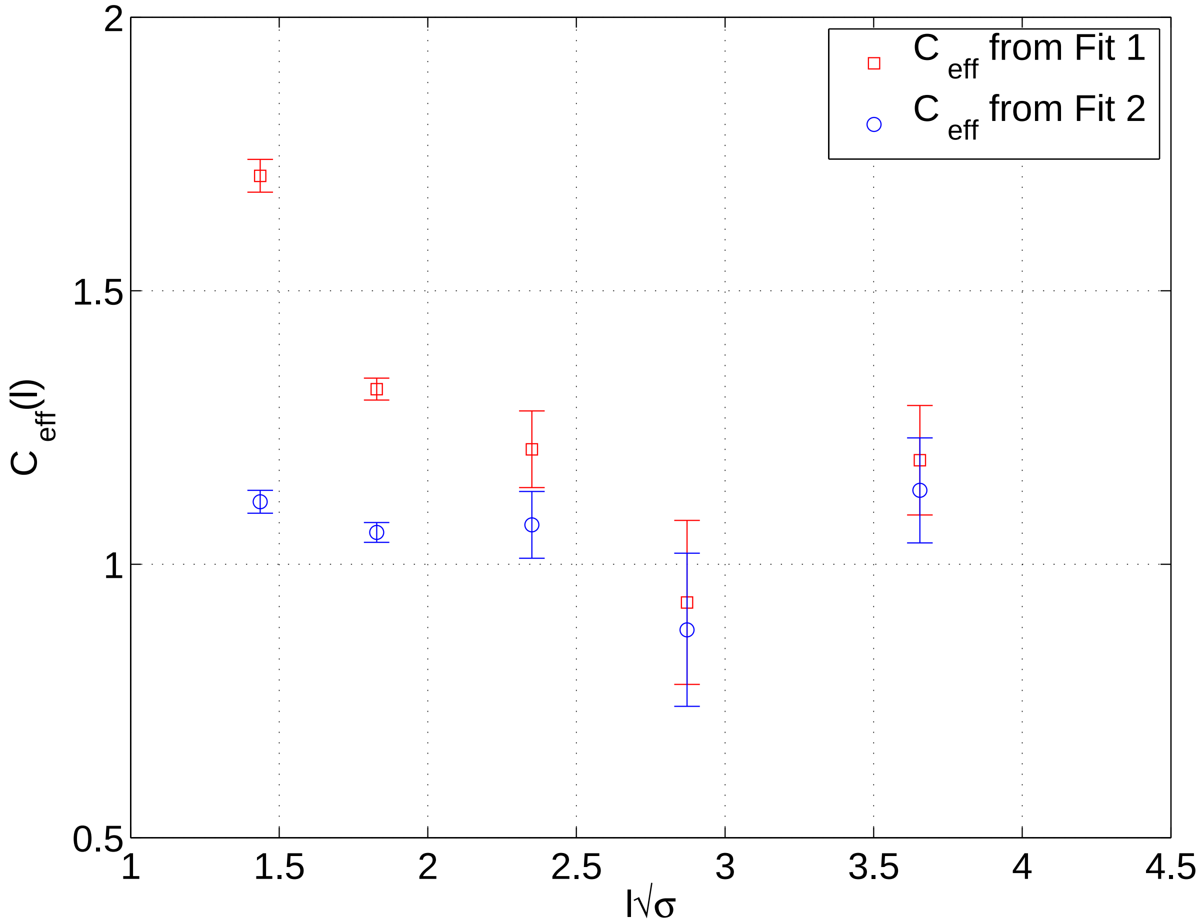
we refer to these as Fit 1 and Fit 2 respectively

SU(5) : $l_c\sqrt{\sigma} \simeq 1.07$



○ : c_{eff} from Luscher
○ : c_{eff} from Nambu-Goto

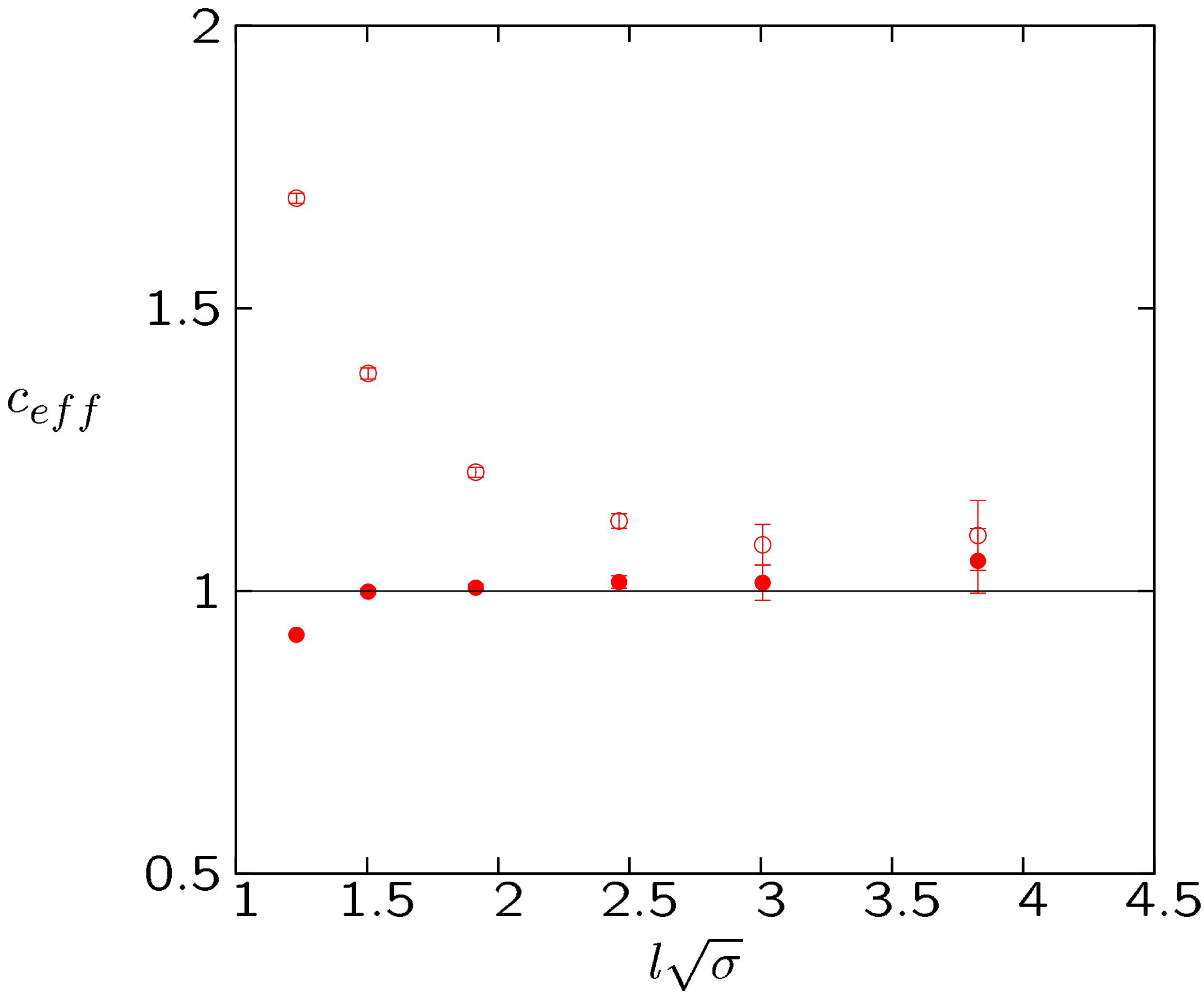
SU(3) : $l_c\sqrt{\sigma} \simeq 1.04$



○ : c_{eff} from Luscher

○ : c_{eff} from Nambu-Goto

SU(2) : $l_c\sqrt{\sigma} \simeq 0.94$



○ : c_{eff} from Luscher
● : c_{eff} from Nambu-Goto

$$l \to l_c$$

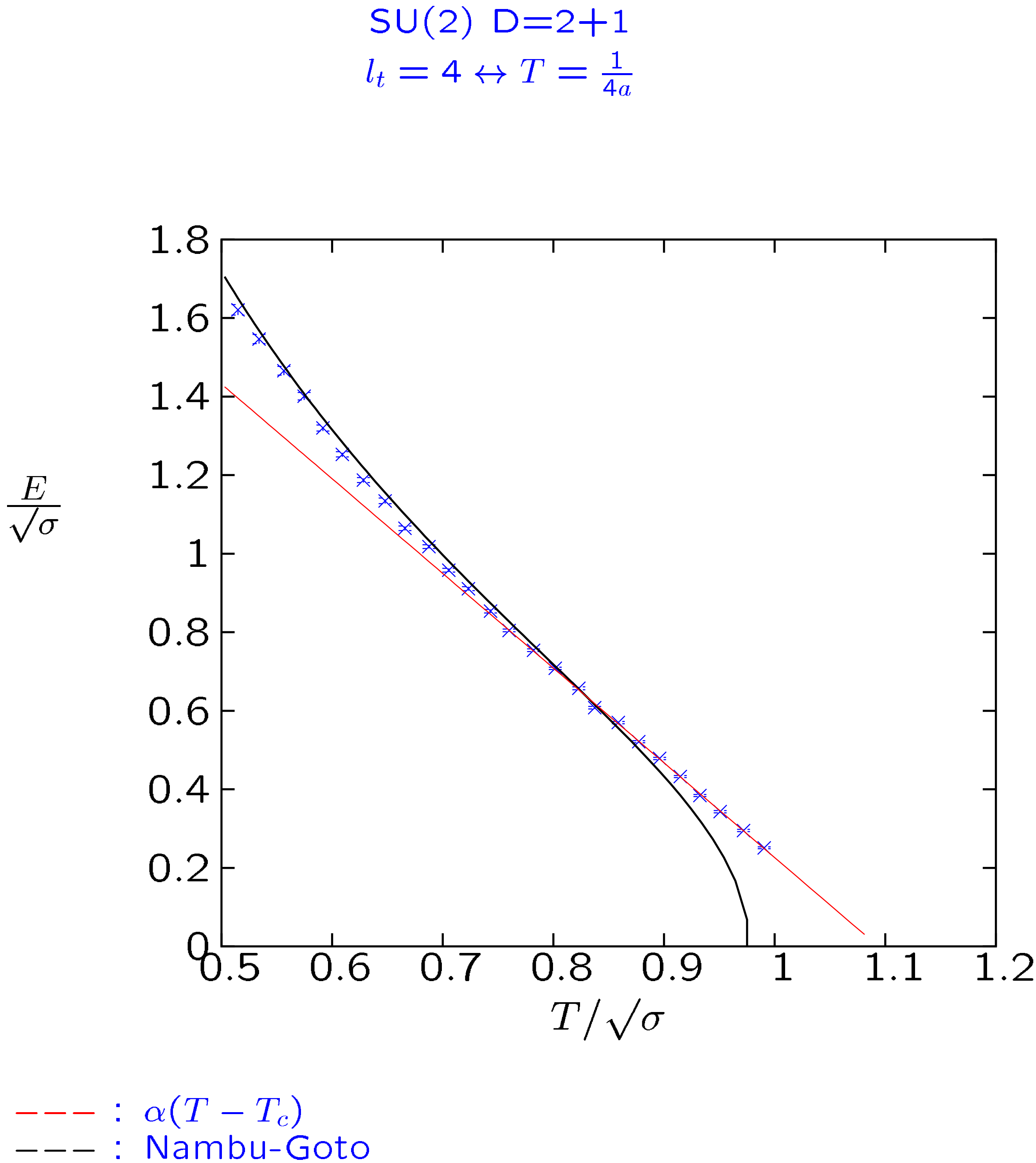
for small N the phase transition to the deconfined phase with no winding flux loops, becomes 2nd order, and then

$$E_0(l) \stackrel{l \to l_c^+}{\propto} (l - l_c)^\gamma$$

where γ is determined by the critical exponents, which will be the same as that of the Z_N spin model in $D-1$ dimensions Svetitsky-Yaffe
e.g. $\gamma = 1.0$
for SU(2) in D=2+1 and Ising in $D = 2$

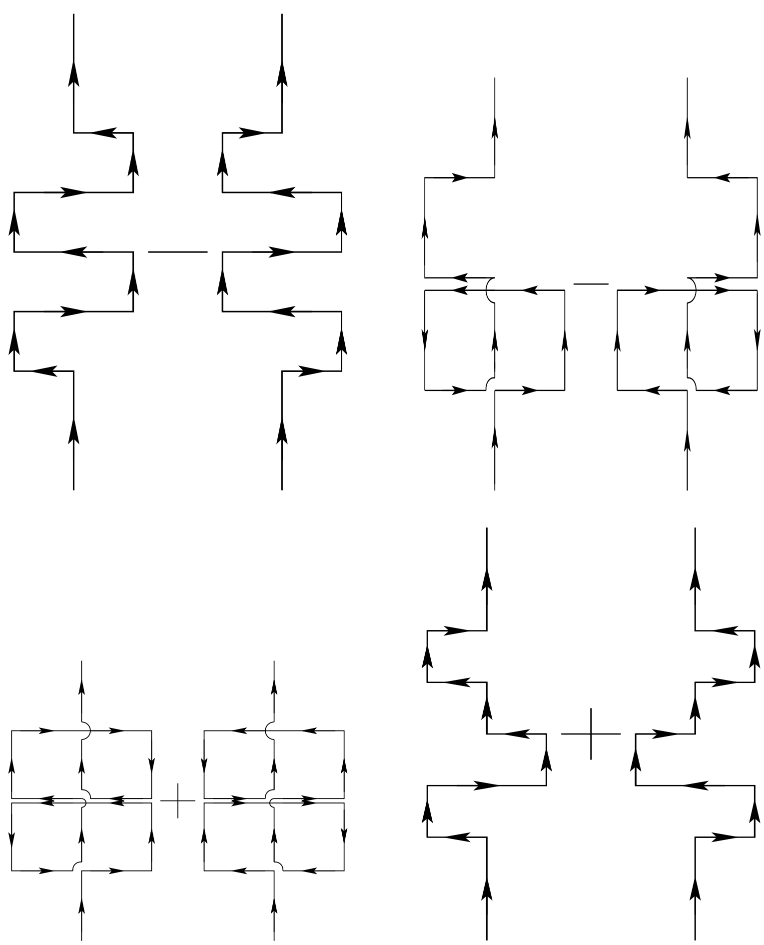
in contrast for Nambu-Goto we have
$E_0(l) \stackrel{l \to l_c^+}{\propto} (l - l_c)^{\frac{1}{2}}$
so at some point the Nambu-Goto fit $must$ break down for small N

E.g.


SU(2) D=2+1
$l_t = 4 \leftrightarrow T = \frac{1}{4a}$
$\frac{E}{\sqrt{\sigma}}$
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
$T/\sqrt{\sigma}$
: $\alpha(T - T_c)$
: Nambu-Goto

Excited States

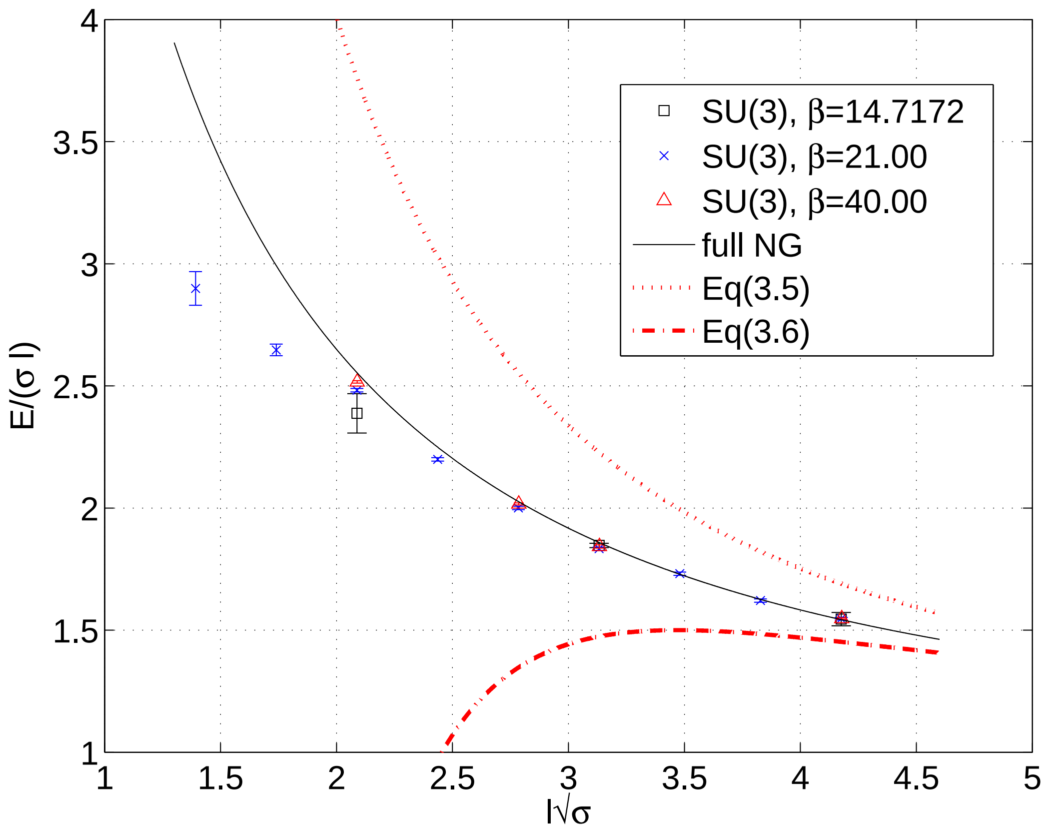
to have good overlaps onto excited string states, we need to include many mor operators in our variational basis – in particular operators that 'look' excited and ones that have an intrinsic handedness so that we can construct $P = -$ as well as $P = +$, e.g.



typically we have 100-200 operators in our basis ...

first excited state : $N = 3$

no parameter: σ from ground state

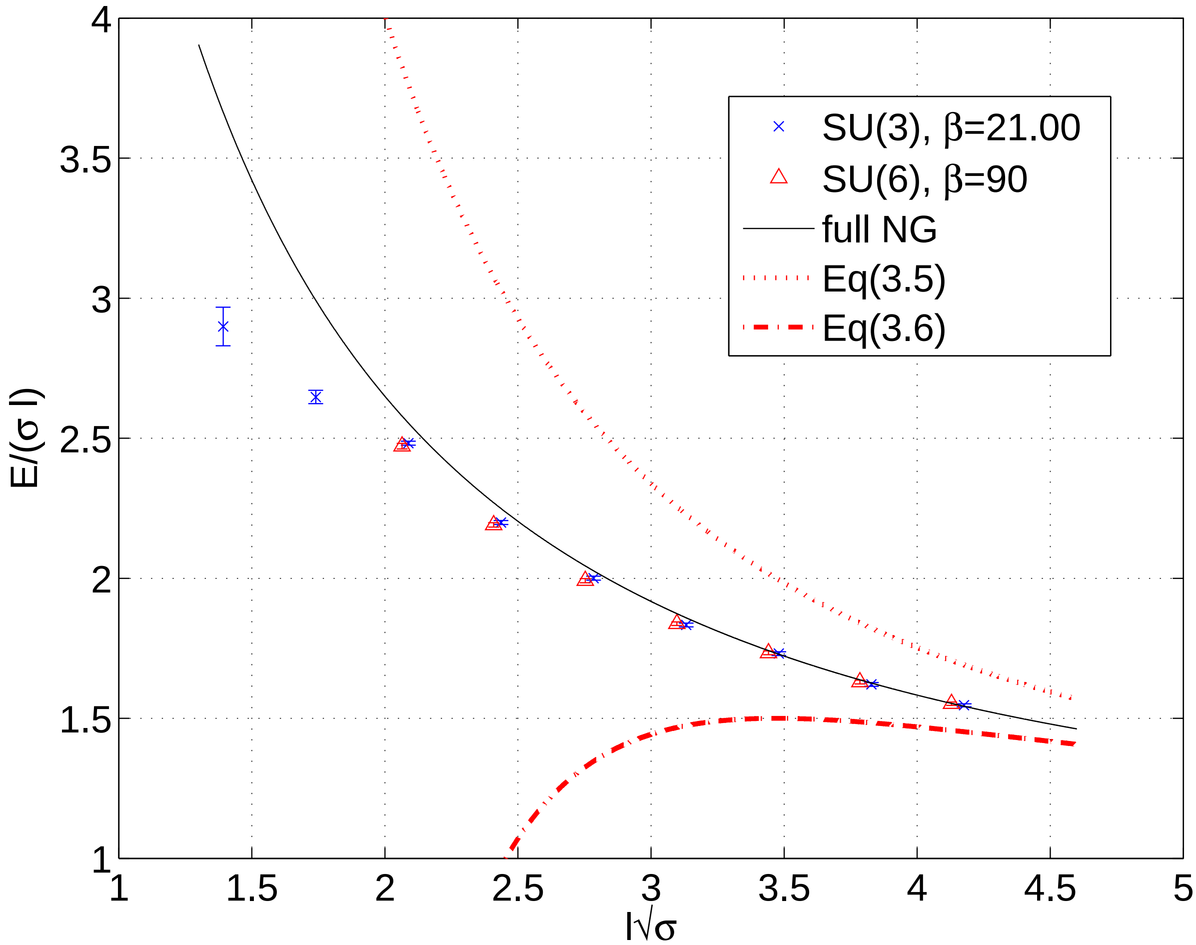


— Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

... Luscher 1980: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right)$

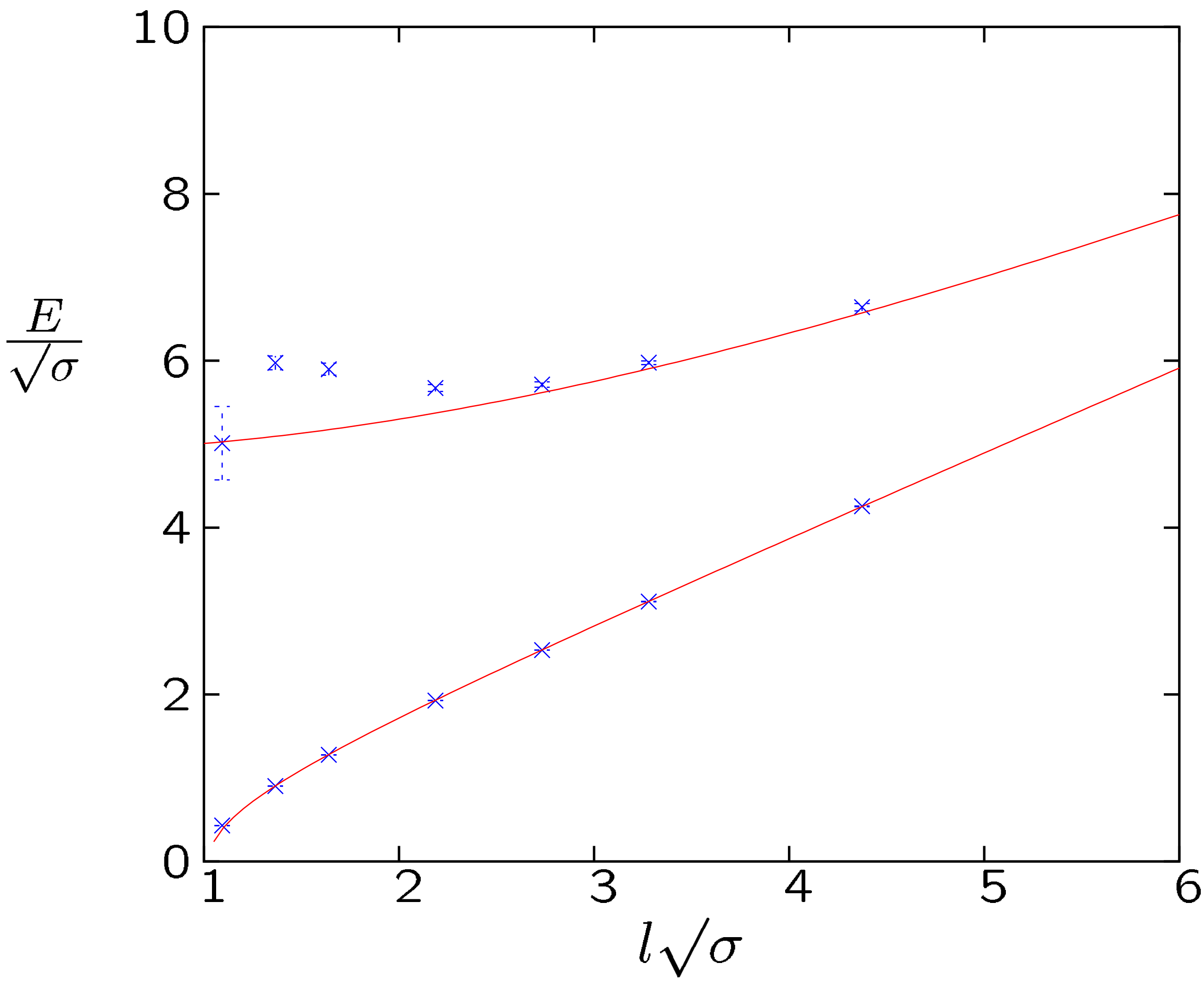
- - Luscher 2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma\, l^3}\left(n - \frac{1}{24}\right)^2$

first excited state : N-dependence?



SU(2) : closed string spectrum

$a\sqrt{\sigma} = 0.2733$; $l_c\sqrt{\sigma} \simeq 1.0$



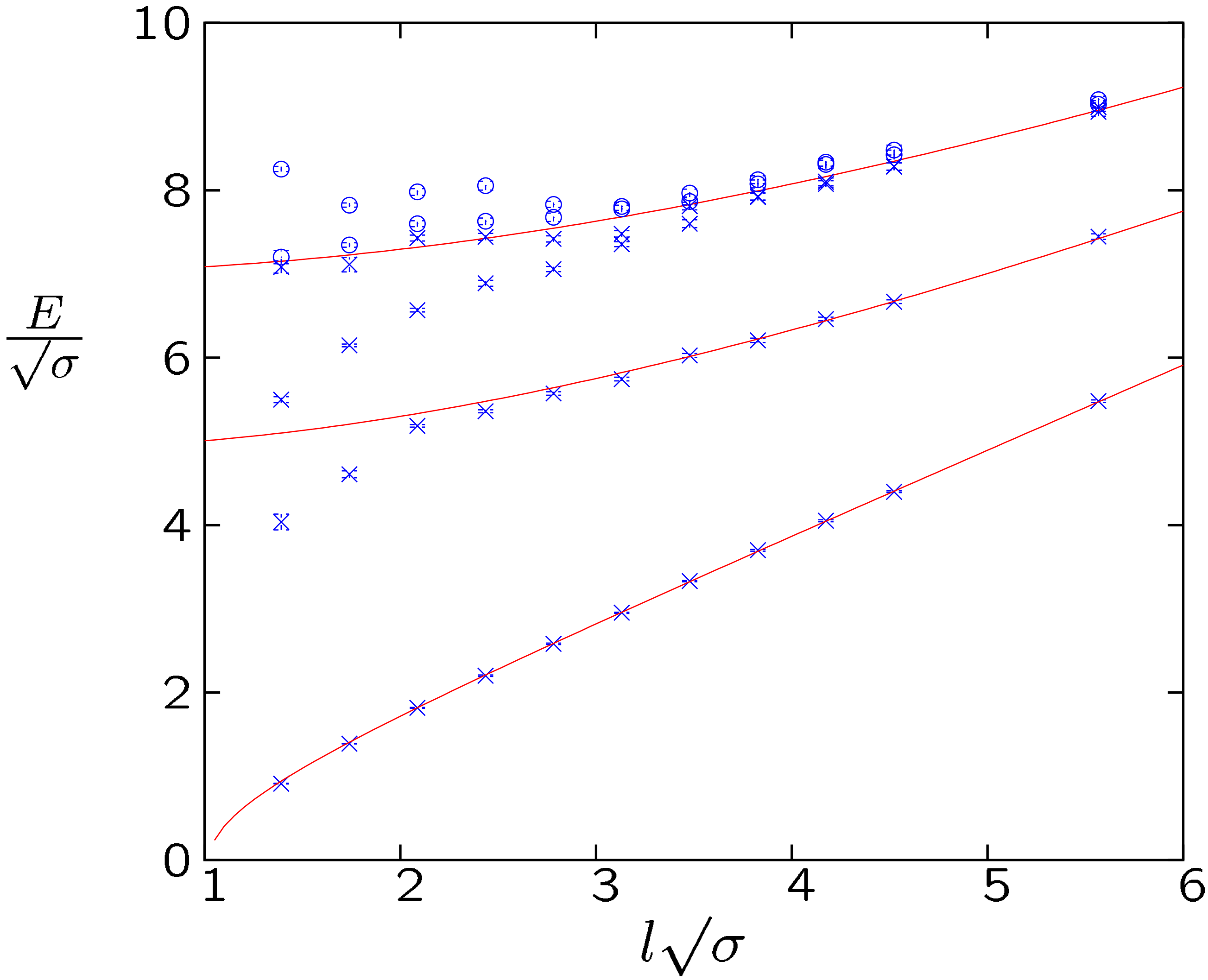
- : Nambu-Goto (σ from ground state)

$\times$: +ve parity

$\circ$: -ve parity

SU(3) : closed string spectrum

$a\sqrt{\sigma} = 0.17395(7)$; $l_c\sqrt{\sigma} \simeq 1.0$



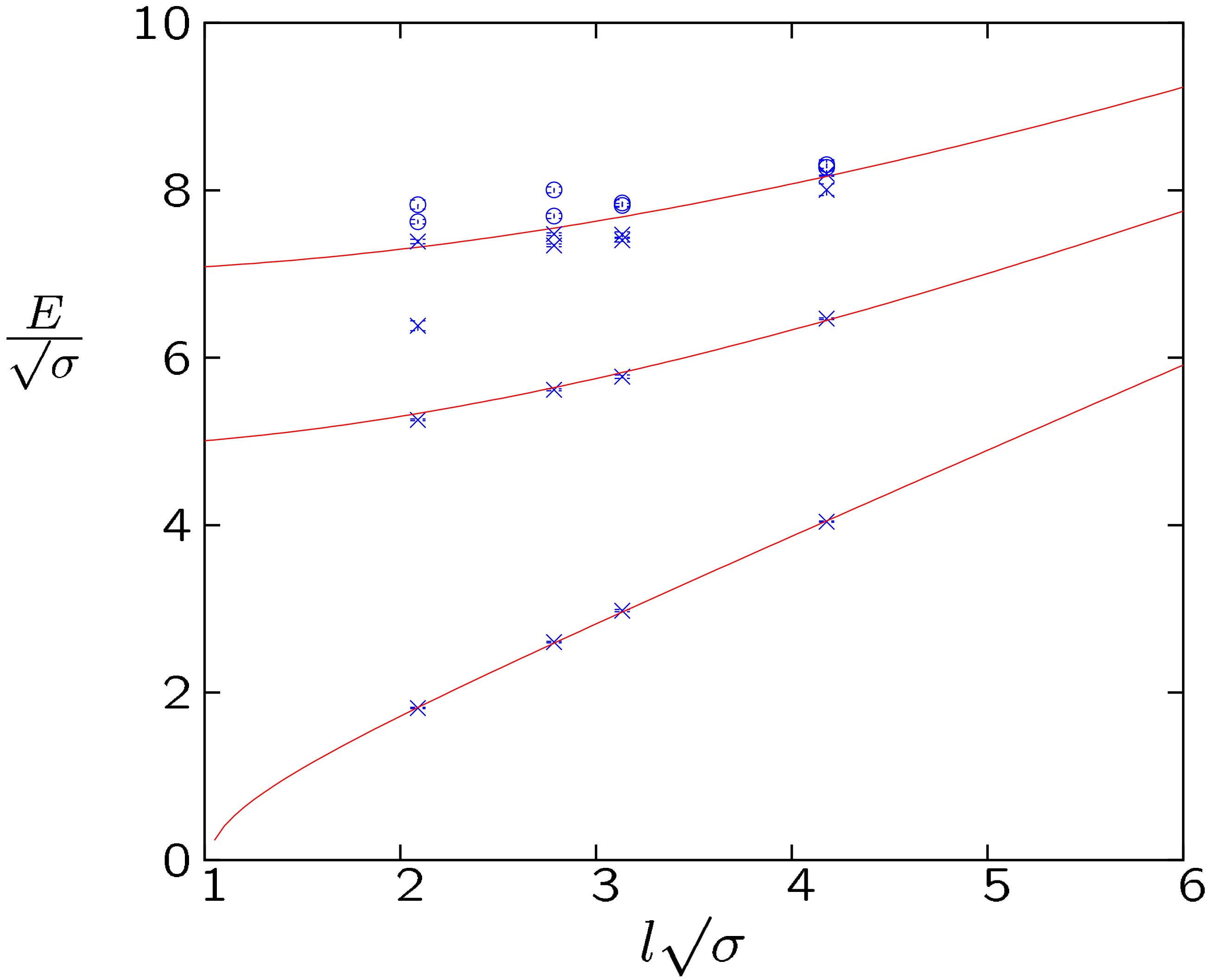
- : Nambu-Goto (σ from ground state)

× : +ve parity

○ : -ve parity

SU(3) : smaller a

$a\sqrt{\sigma} = 0.08705(7)$; $l_c\sqrt{\sigma} \simeq 1.0$



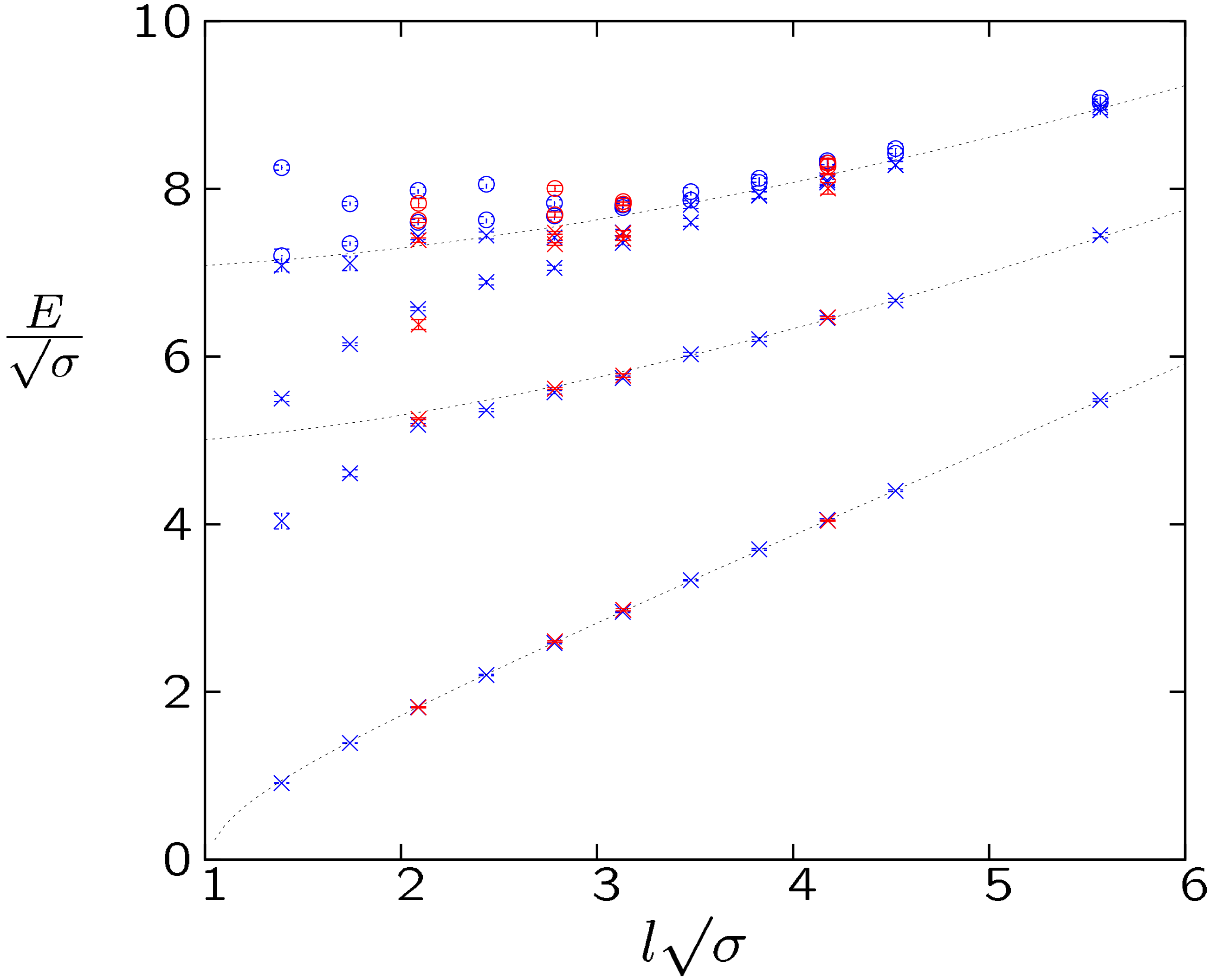
- : Nambu-Goto (σ from ground state)

$\times$: +ve parity

$\circ$: -ve parity

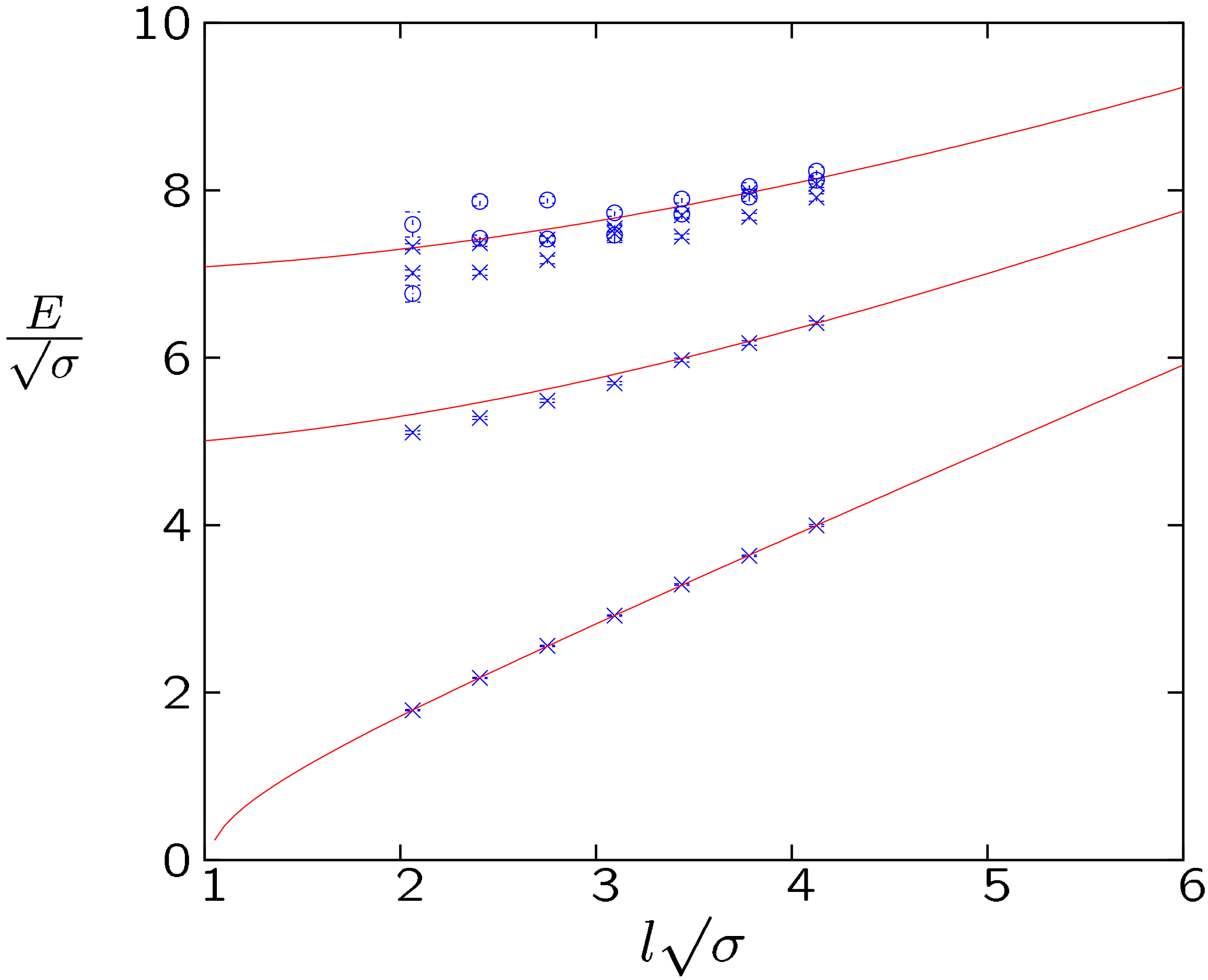
SU(3) : continuum limit?

$a\sqrt{\sigma} \simeq 0.174$ vs $a\sqrt{\sigma} \simeq 0.087$



SU(6) : closed string spectrum

$a\sqrt{\sigma} = 0.17193(7)$; $l_c\sqrt{\sigma} \simeq 1.1$

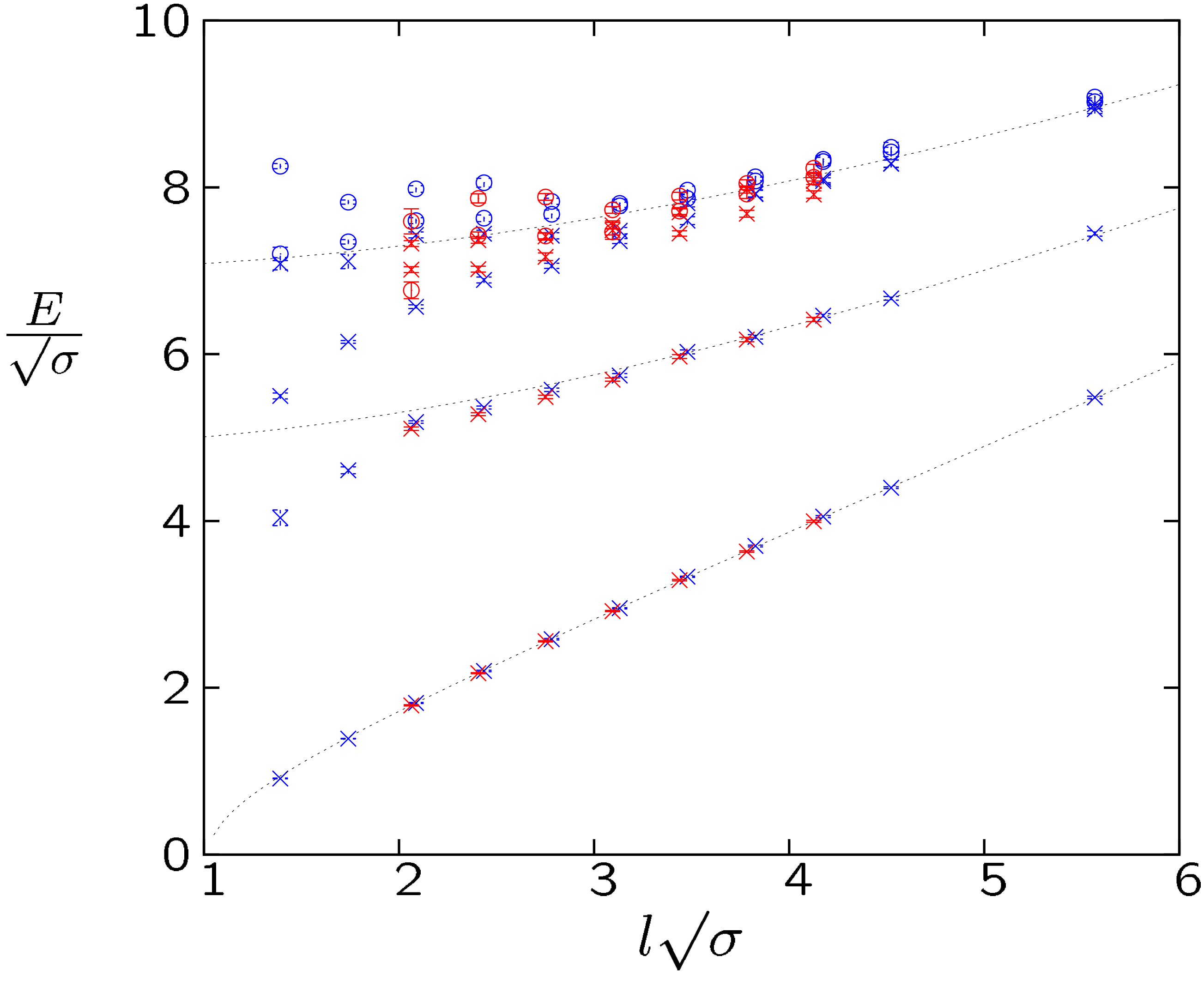


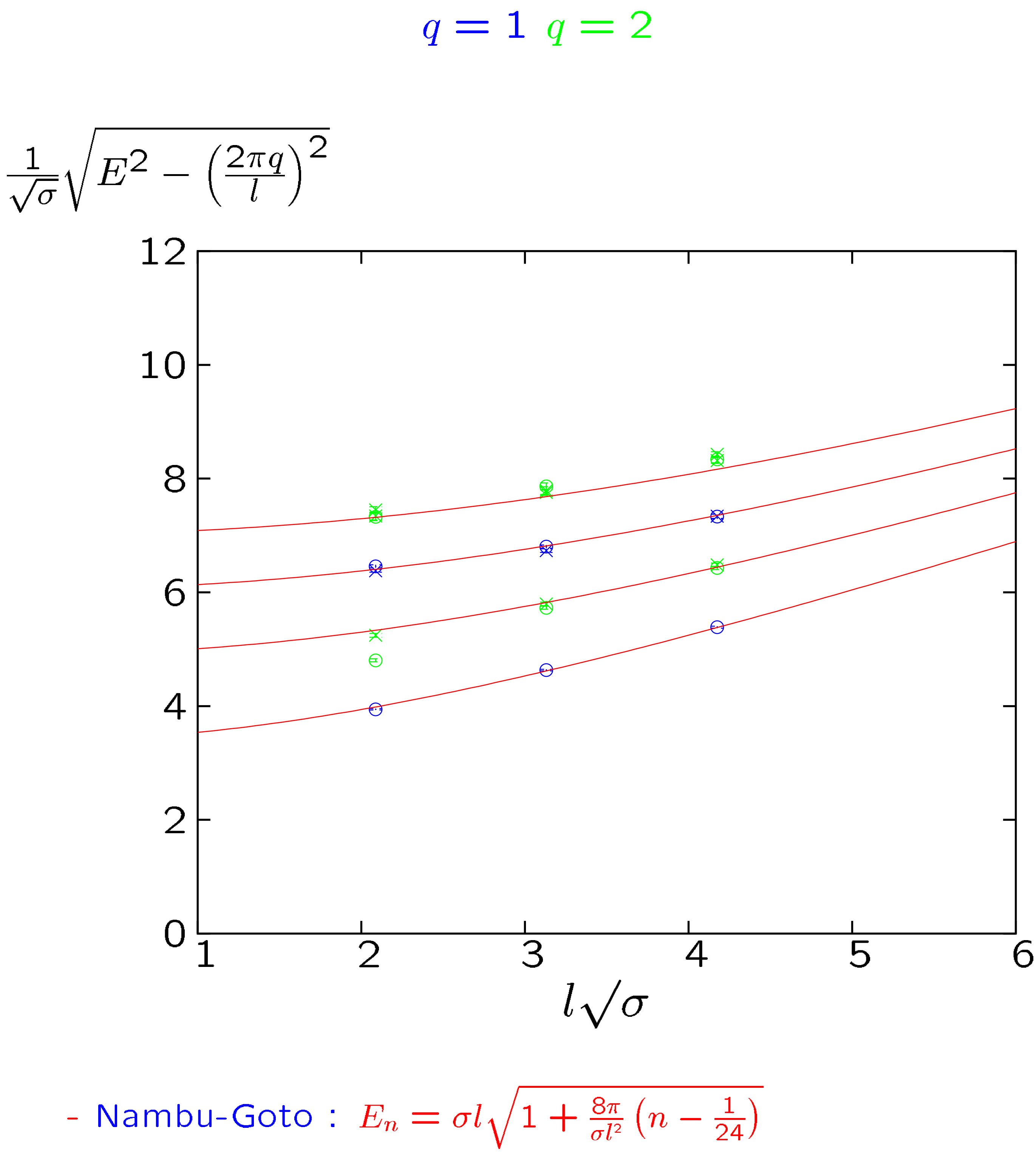
- : Nambu-Goto (σ from ground state)

$\times$: +ve parity

$\circ$: -ve parity

SU(3) vs SU(6) : same a





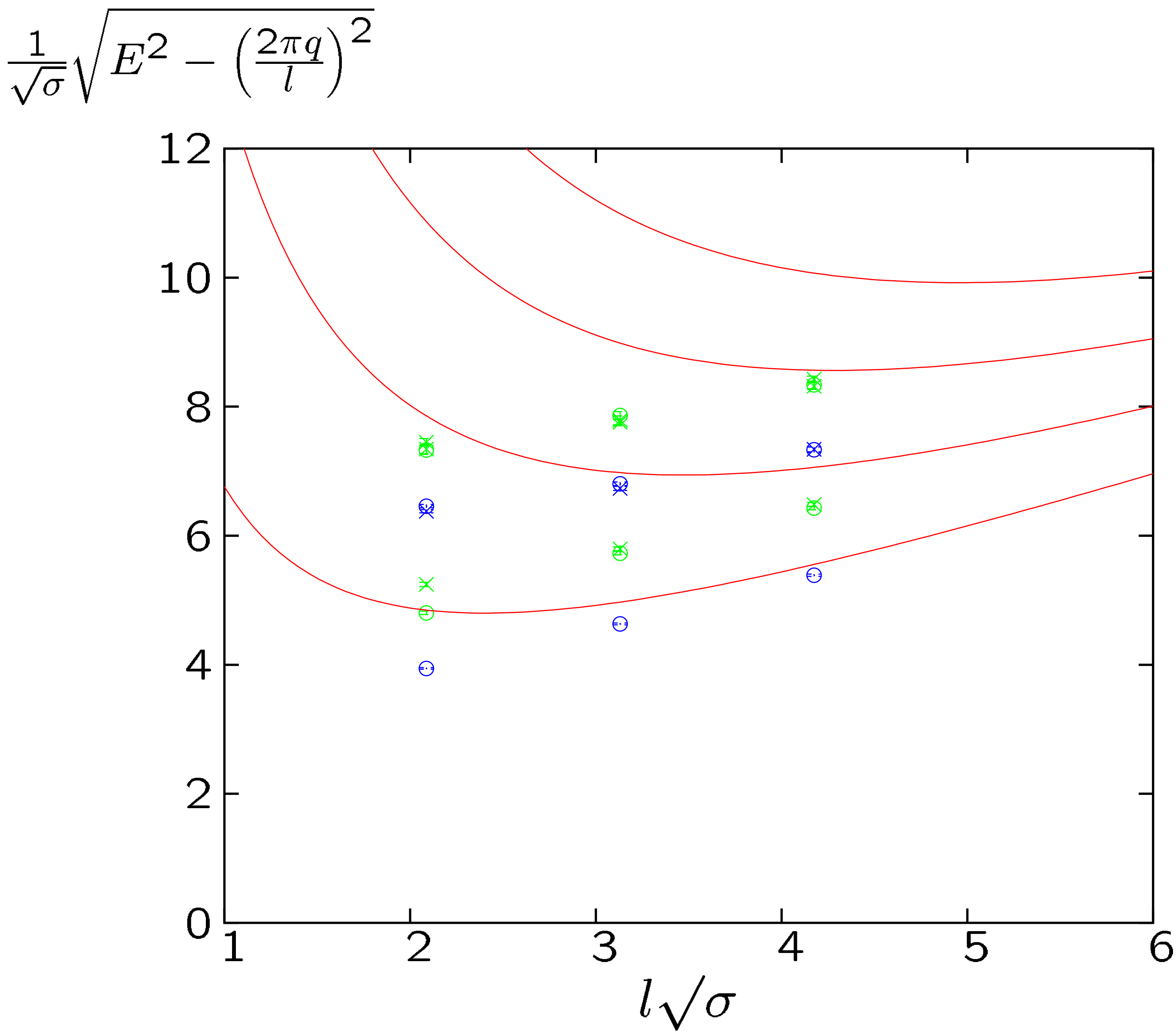
$q = 1$ $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$
- Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

content of NG states:

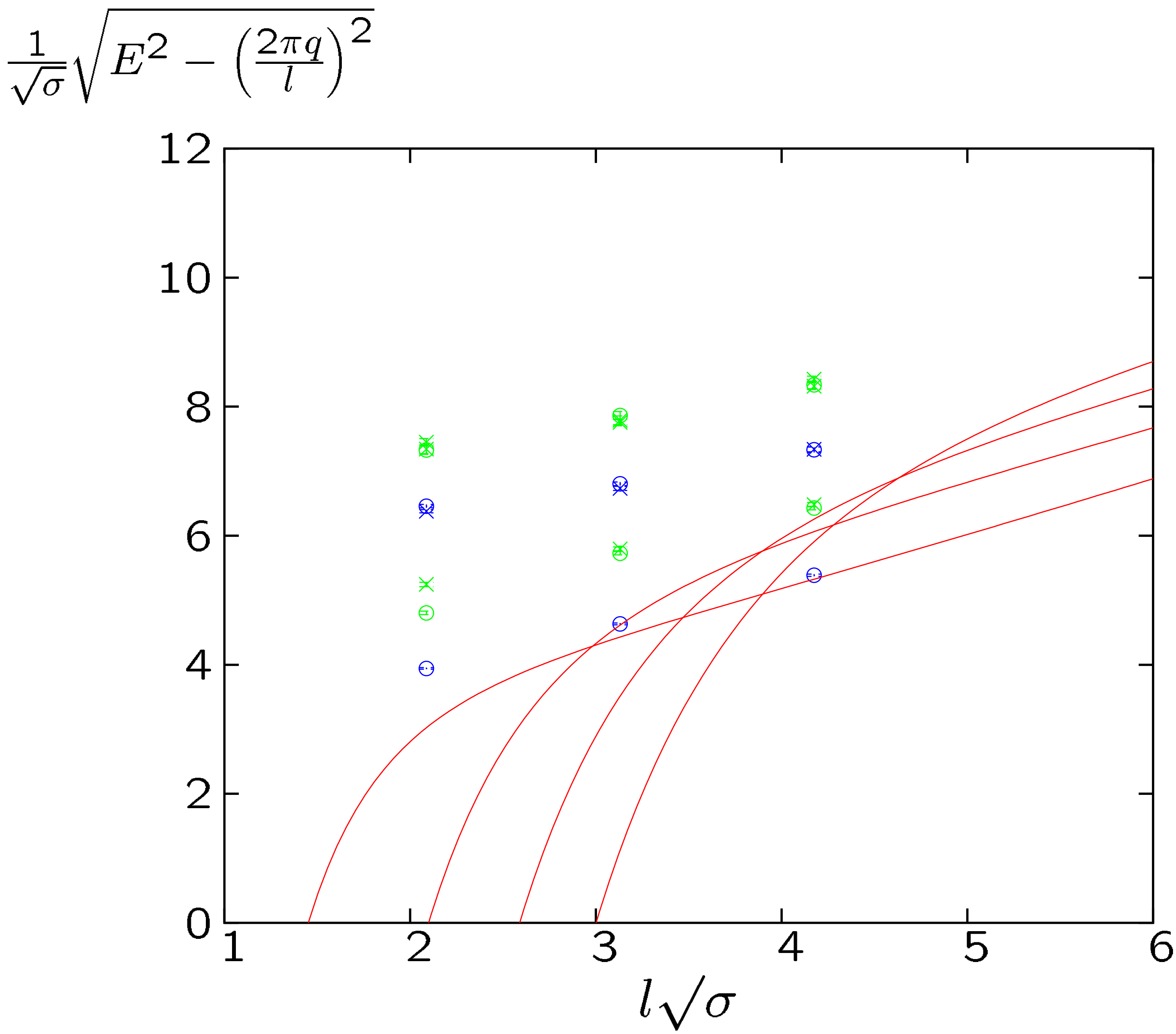
$a^R(k=1)\|0\rangle$	P=−, q=1
$a^R(k=2)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^L(k=1)\|0\rangle$	P=+, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=1
$a^R(k=3)a^L(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=2

the individual sets of degeneracies tell us something specific about the interactions amongst the corresponding phonons – usually that they are very weak ... although there certainly should be more to be said than that

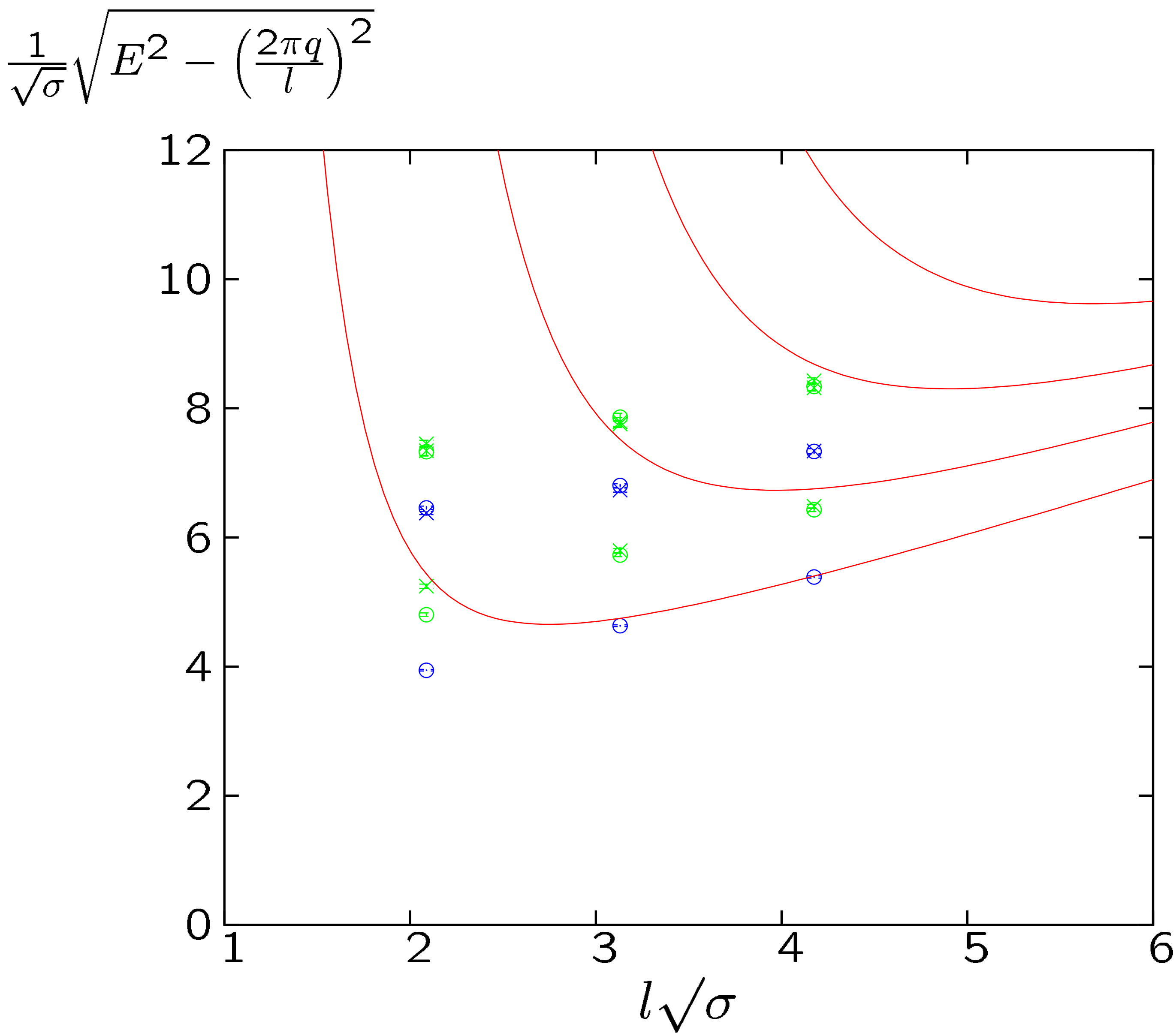
[also $q=-1, q=-2$ degenerate within errors]



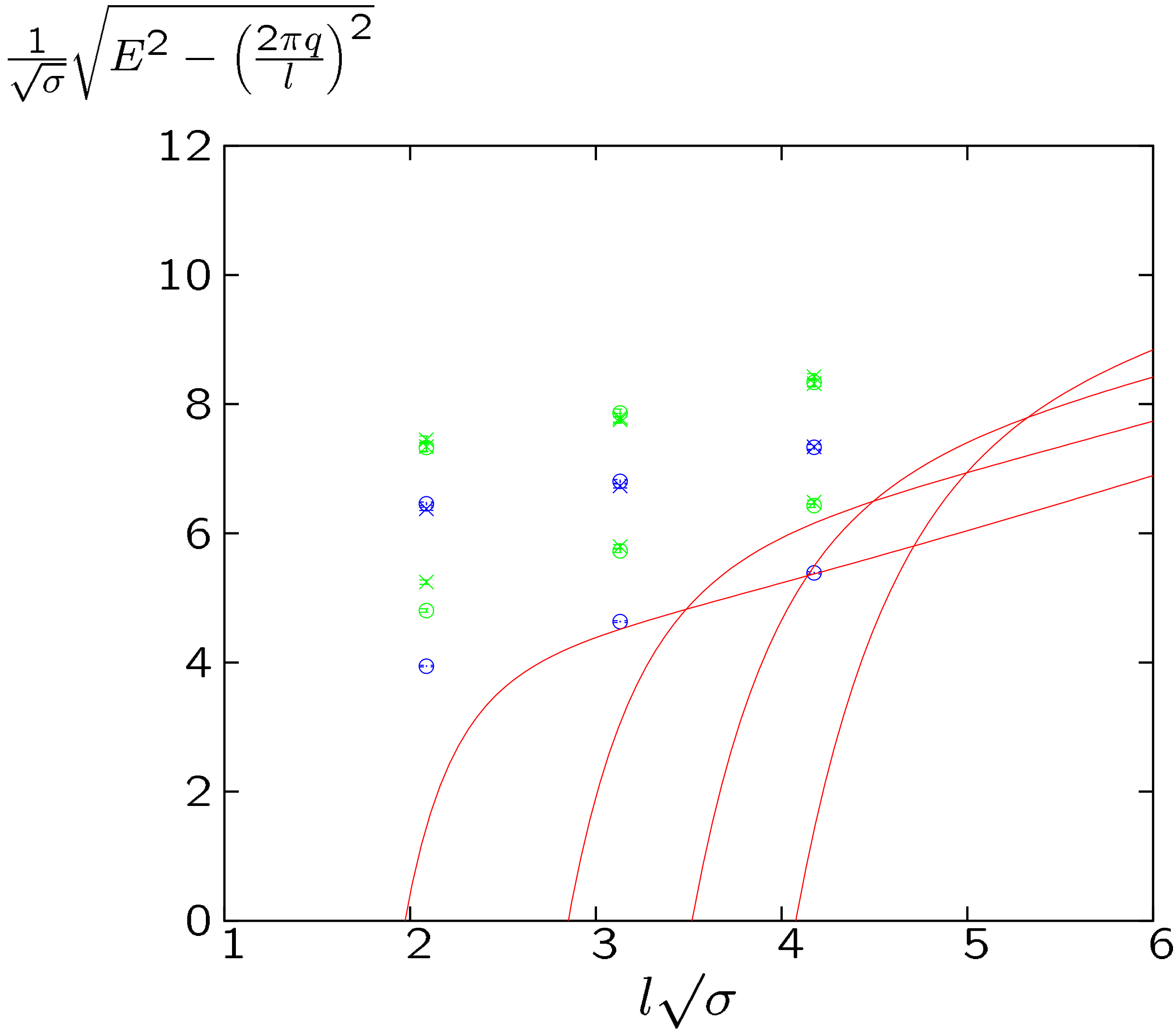
- Luscher,Symanzik,Weisz 1980: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right)$



- Luscher-Weisz 2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma\, l^3}\left(n - \frac{1}{24}\right)^2$



- one more order of NG ...



$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$

- … and another …

Why ?

the covariant Nambu-Goto expression e.g. for $q = 0$,

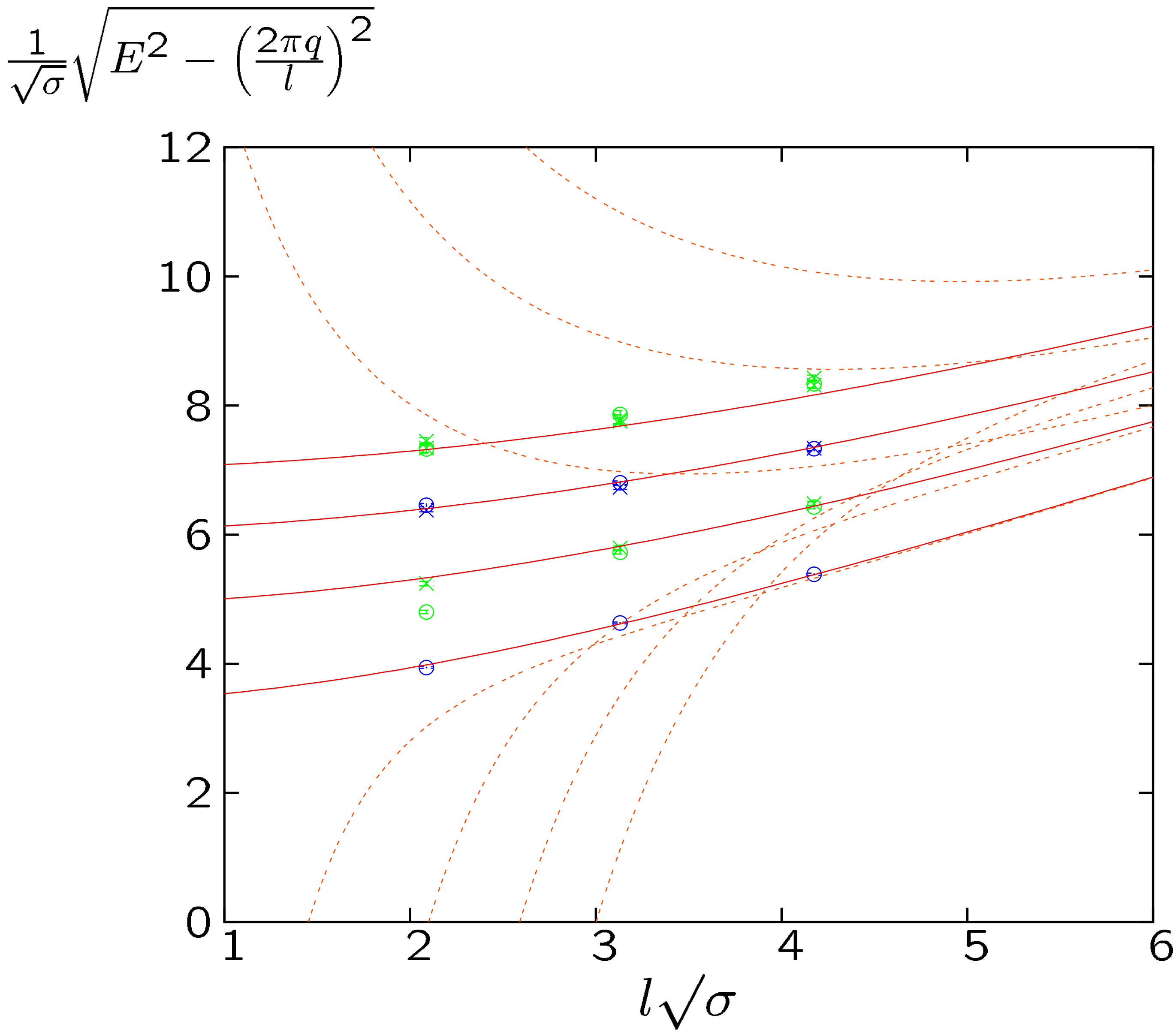
$$E(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}}$$

can only be expanded as a power series in $1/l\sqrt{\sigma}$ when

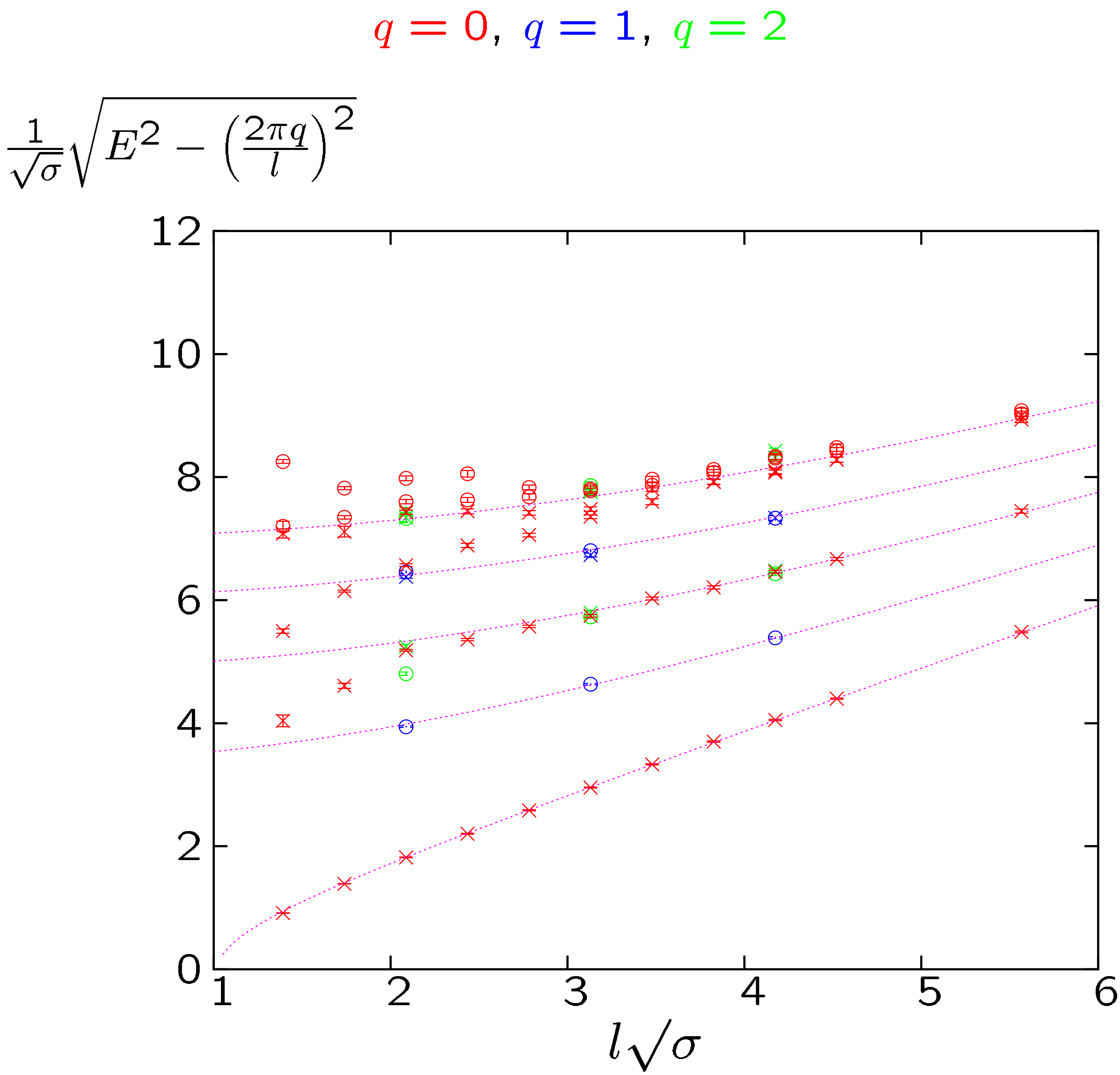
$$x \equiv \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right) \leq 1$$

whereas in practice we have a very good fit by Nambu-Goto even down to

$$x \sim 12 \qquad : \qquad l\sqrt{\sigma} \sim 2\ ,\ n = 2$$



$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$



$q = 0$, $q = 1$, $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$

content of NG states:

$\vert 0\rangle$	P=+, q=0
$a^R(k=1)\vert 0\rangle$	P=−, q=1
$a^R(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=0
$a^R(k=2)\vert 0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)\vert 0\rangle$	P=+, q=2
$a^R(k=2)a^L(k=1)\vert 0\rangle$	P=+, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=1
$a^R(k=2)a^L(k=2)\vert 0\rangle$	P=+, q=0
$a^R(k=1)a^R(k=1)a^L(k=2)\vert 0\rangle$	P=−, q=0
$a^R(k=2)a^L(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=0
$a^R(k=1)a^R(k=1)a^L(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=0
$a^R(k=3)a^L(k=1)\vert 0\rangle$	P=+, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=2

observed near-degeneracies for $l \geq 2/\sqrt{\sigma} \sim 1\text{fm} \sim$ width flux tube!

• in D=2+1 SU(N) gauge theories, confining flux tubes belong to the universality class of a simple bosonic string theory

• more than that, the Nambu-Goto covariant free string spectrum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

is very accurate down to values of $l\sqrt{\sigma}$ where an effective string theory expansion, $x = l\sqrt{\sigma}$,

$$\frac{E_n}{\sqrt{\sigma}} = x\left(1 + \frac{c}{x^2}\right)^{\frac{1}{2}} = x + \frac{c}{2x} - \frac{c}{8x^3} + \cdots$$

makes no sense (is far past its range of convergence)

• So, since in the range of $l\sqrt{\sigma}$ where such a power expansion is relevant, any difference with Nambu-Goto is totally negligible, it is clear that there is a challenge here to incorporate string corrections to Nambu-Goto in some 're-summed' or 'non-perturbative' way ...

What about $D = 3+1$?

earlier work has provided good evidence that for SU(2) and SU(3) the universality class of the effective string theory describing long flux tubes is bosonic e.g. for the ground state

$$E_0(l) = \sigma l - \frac{\pi}{3l} + O(l^{-2})$$

and there is some evidence that excited states tend towards the corresponding behaviour:

$$E_n(l) = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{12}\right) + O(l^{-2})$$

but only when they are very long (and hard to calculate!):

$$l\sqrt{\sigma} \gg 1$$

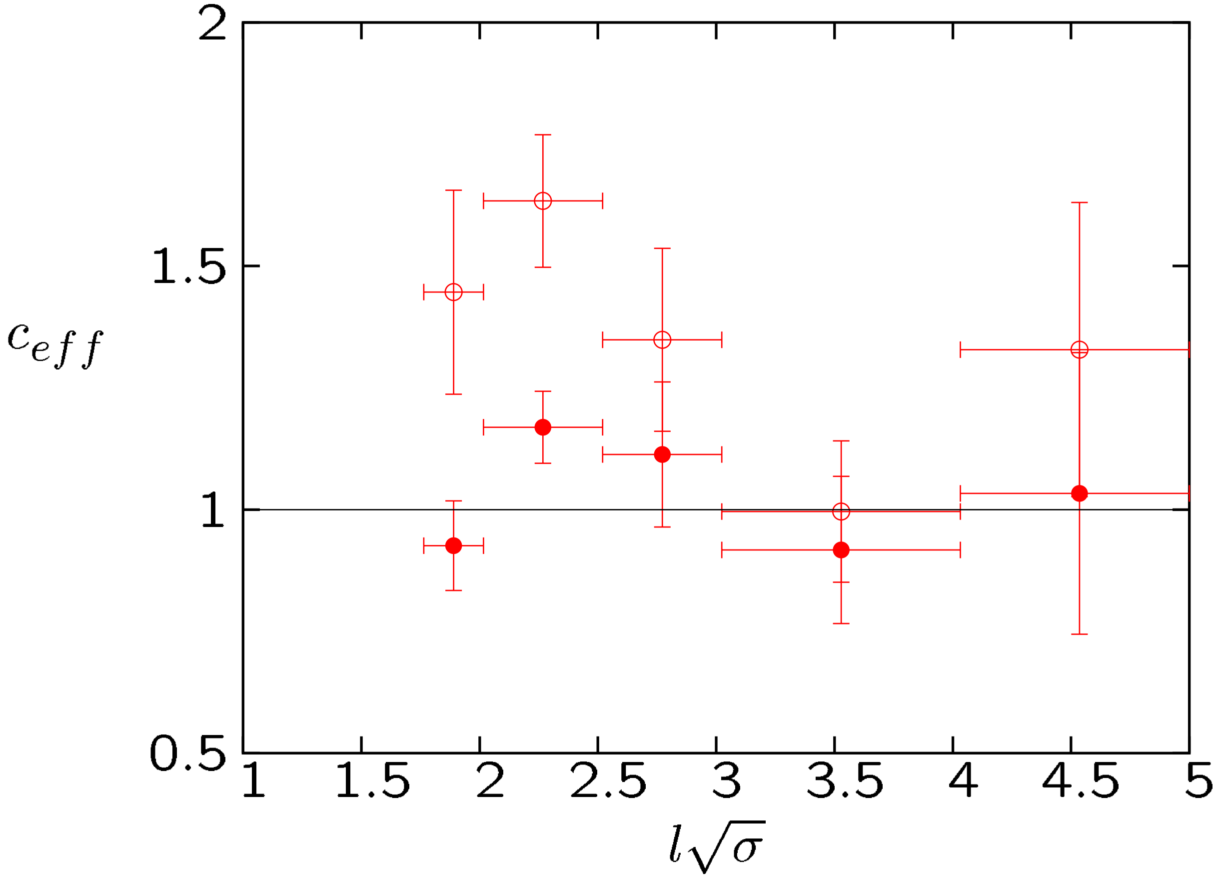
here I show some preliminary indications that what we found for $D = 2+1$ SU(N) gauge theories i.e. an amazingly precocious onset of Nambu-Goto behaviour:

$$E_n(l) = \sigma l (1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{12}\right)^{\frac{1}{2}}$$

is also the case in $D = 3+1$ – but only an indication so far ...

D=3+1 ; SU(6) ; $l_c \simeq 1.6$

H. Meyer, M. Teper: hep-lat/0411039



Luscher: ○

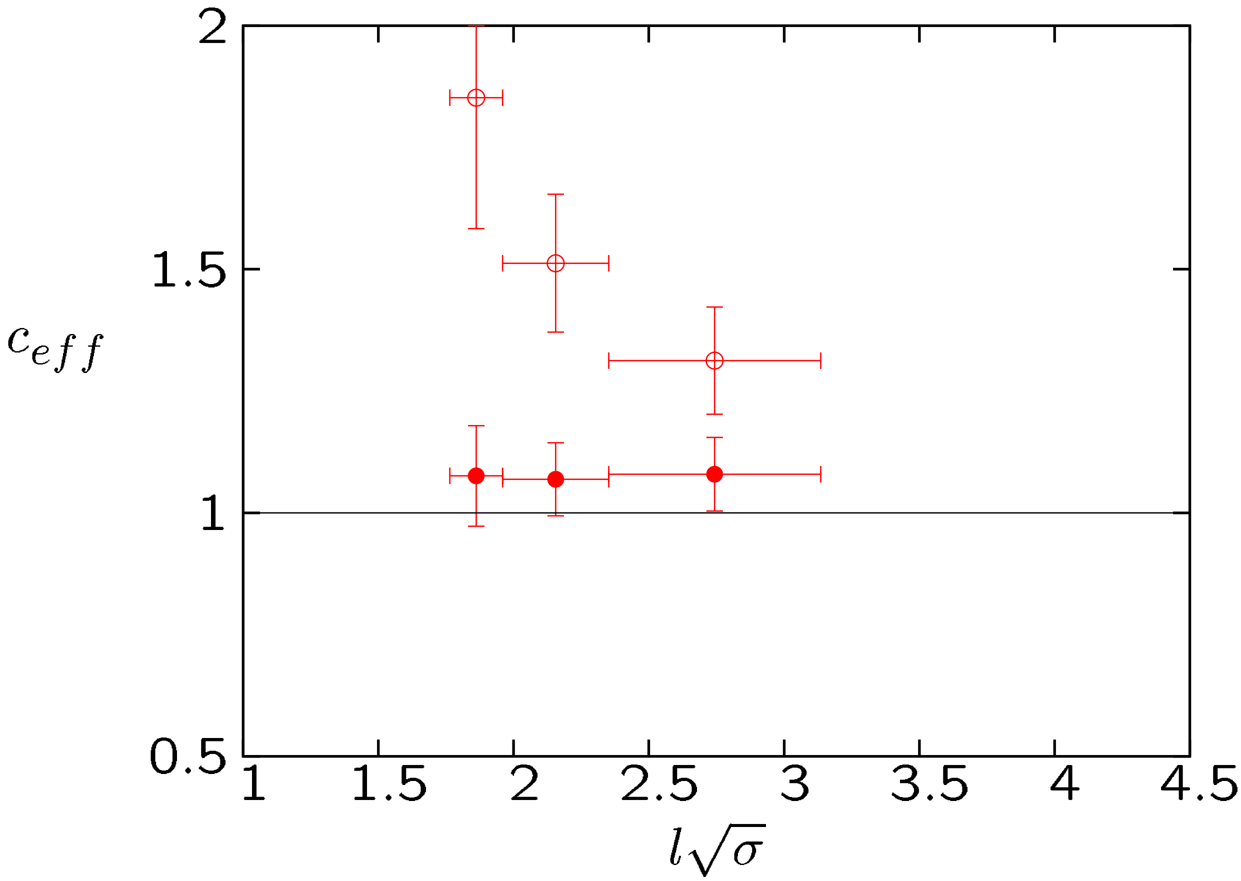
$$E_0(l) = \sigma l - c_{eff}\frac{\pi}{3l}$$

Nambu-Goto: ●

$$E_0(l) = \sigma l \left(1 - c_{eff}\frac{2\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

D=3+1 ; SU(3) ; $l_c \simeq 1.6$

B.Bringoltz, A.Athenedorou, M.Teper: in progress



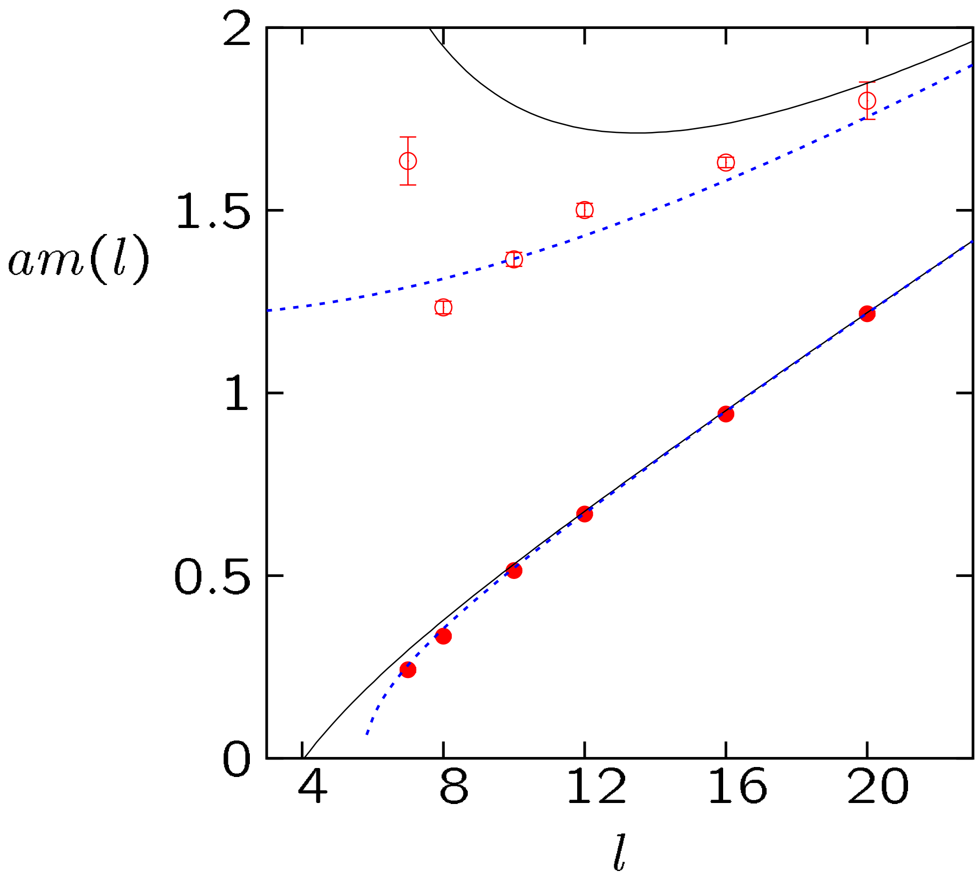
Luscher:○

$$E_0(l) = \sigma l - c_{eff}\frac{\pi}{3l}$$

Nambu-Goto:●

$$E_0(l) = \sigma l \left(1 - c_{eff}\frac{2\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

H. Meyer, M. Teper: hep-lat/0411039



linear + bosonic string correction (—):

$$E_n(l) = \sigma l + \frac{4\pi}{l}(n - \frac{1}{12})$$

Nambu-Goto string action ($\cdots$):

$$E_n(l) = \sigma l[1 + \frac{8\pi}{\sigma l^2}(n - \frac{1}{12})]^{\frac{1}{2}}$$

flux tubes multiply wound around the torus

- confining flux tubes wound once around a spatial torus have a spectrum that is amazingly close to that of the free bosonic Nambu-Goto string model

- strings that wind more than once may overlap and so non-NG interaction effects might show up here

- and a dependence on the specific SU(N) group might also show up here, e.g.
in SU(2) a doubly wound loop could overlap with the vacuum, as $f \otimes f$ contains a singlet, which cannot happen for larger N.

- stringy interaction effects are not only outside Nambu-Goto but also lie beyond the scope of the Polchinski-Strominger approach since that does not deal properly with the small closed loops whose exchange is presumably part of the interaction.

- a particularly interesting subset of such loops (and products of such loops) are k-strings

k-strings

Consider a source that is the product of k fundamental sources.

The flux tube between such a static source and its conjugate is a k-string.

It may be energetically favourable to screen such a source with gluons from the vacuum into another representation

However, since the gluons are adjoint and transform trivially under the centre, the source, after any screening, will transfrom the same way under $z \in Z_N$ i.e.
$\phi_k \to z^k \phi_k$

So screening leaves sources in the same k-class subject to the constraint $z^N = 1$
Typically a source will be screened so that it acts as a source for the lightest string of given k.

Thus k is a good quantum number. Similarly k-strings can wind around a torus,

Question:

Is the lightest k-string composed of k fundamental strings, $\sigma_k = k\sigma_{k=1}$ or do we have bound states $\sigma_k < k\sigma_{k=1}$?

k-strings in $D = 4$

Lucini, Teper, Wenger: hep-lat/0404008

also Pisa group

Casimir scaling:

$$\frac{\sigma_k}{\sigma} = \frac{k(N-k)}{N-1}$$

'MQCD':

$$\frac{\sigma_k}{\sigma} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}$$

σ_k/σ			
(N,k)	Casimir scaling	this paper	'MQCD'
(4,2)	1.333	1.370(20)	1.414
(4,2)	1.333	1.358(33)	1.414
(6,2)	1.600	1.675(31)	1.732
(6,3)	1.800	1.886(61)	2.000
(8,2)	1.714	1.779(51)	1.848
(8,3)	2.143	2.38(10)	2.414
(8,4)	2.286	2.69(17)	2.613

k-strings in $D = 3$

Bringoltz, Teper : arXiv:0708.3447

k	N	σ_k/σ	Casimir
2	4	1.3553(23)	1.3333..
2	5	1.5275(26	1.5
2	6	1.6242(35)	1.6
2	8	1.7524(51)	1.7142..
3	6	1.8590(63)	1.8
3	8	2.174(19	2.143..
4	8	2.373(12)	2.286..

higher k and/or N less reliable
systematic error due to excited states not estimated here: but will typically slightly reduce the ratios

Casimir scaling good at $\sim 2\%$ level
– better than $D = 4$
– and main systematic error will improve comparison

$D = 3$ ratios ten times more accurate than $D = 4$: so will focus on the former below

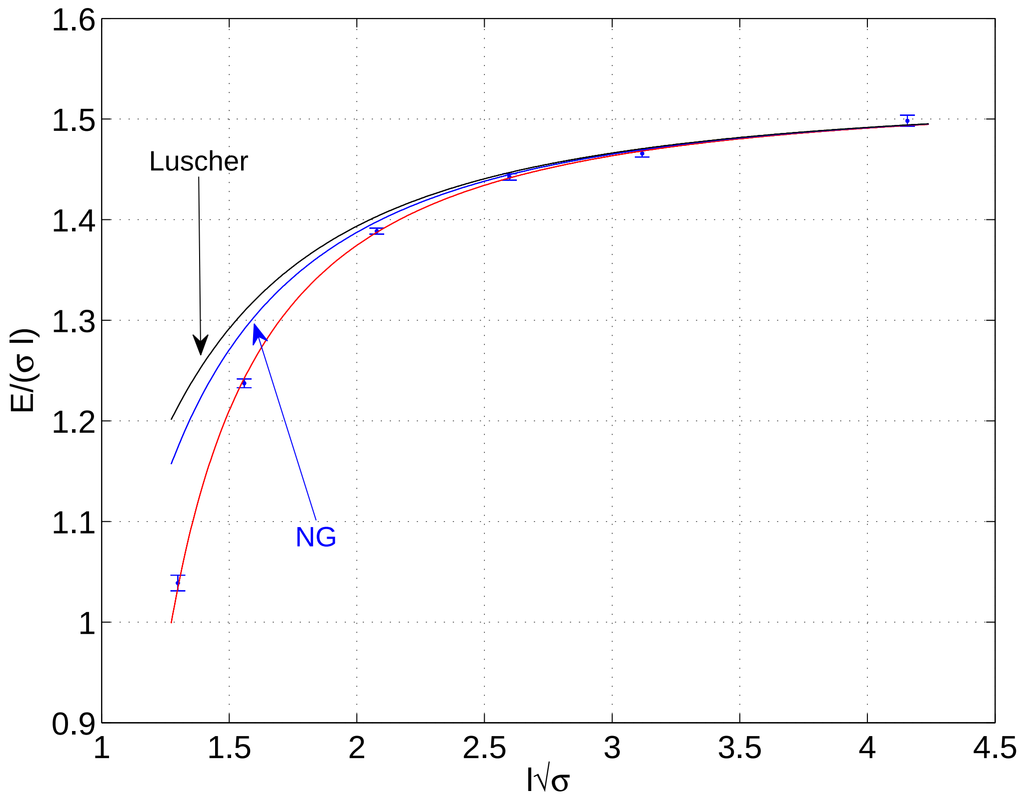
string corrections?

fit $k = 2$ ground state to Nambu-Goto:

$E_k(l) = \sigma_k l \left(1 - \frac{\pi(D-2)}{3\sigma_k l^2}\right)^{\frac{1}{2}}$

and just Luscher

$E_k(l) = \sigma_k l - \frac{\pi(D-2)}{6l}$



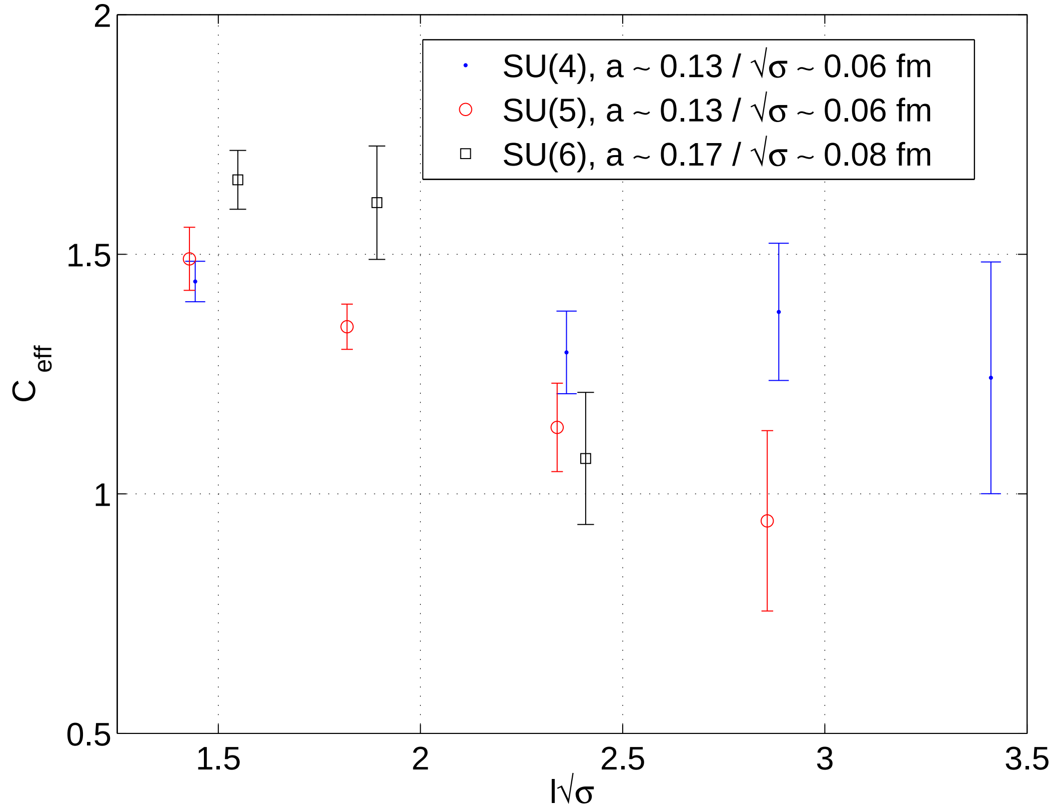
$\Rightarrow$

Much larger deviations at smaller l than for $k = 1$ flux tube

string corrections?

fit ground state to central charge of Nambu-Goto:

$$E_k(l) = \sigma_k l \left(1 - C_{eff}\frac{\pi(D-2)}{3\sigma_k l^2}\right)^{\frac{1}{2}}$$



$\Rightarrow$

using NG for $l\sqrt{\sigma} \sim 3$ looks OK

Corrections: $O(1/N)$ or $O(1/N^2)$?

e.g.

Casimir scaling gives an $O(1/N)$ correction:

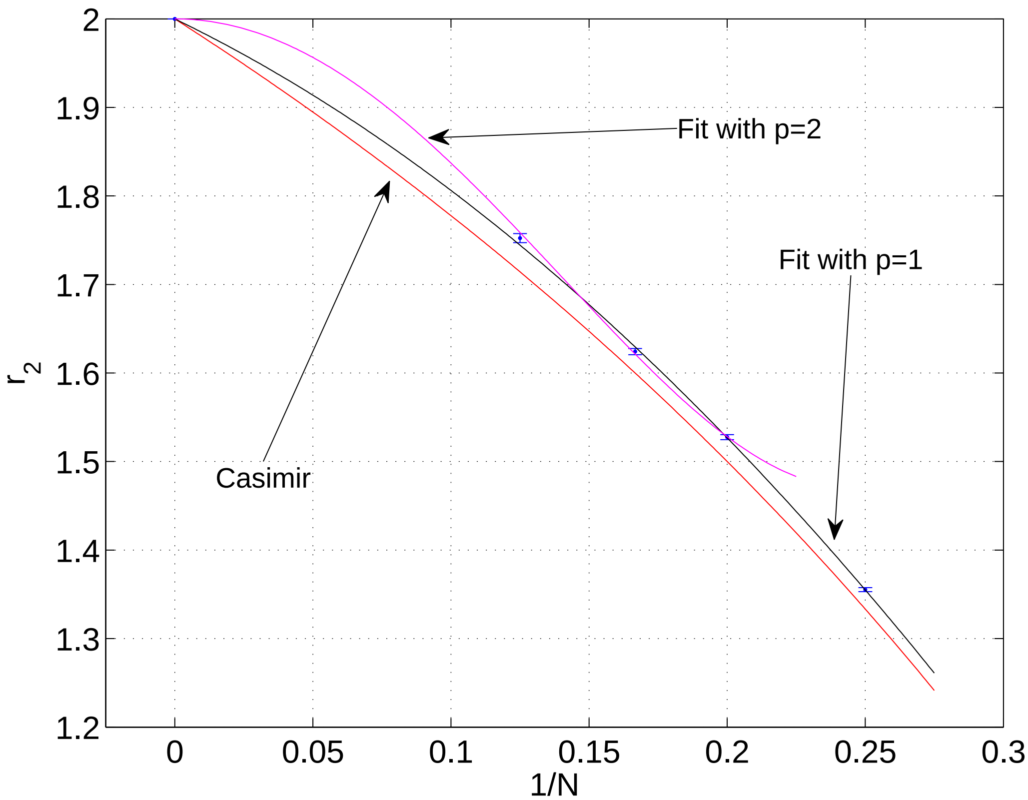
$\frac{\sigma_k}{\sigma} = \frac{k(N-k)}{N-1} = k - \frac{k(k-1)}{N-1}$

while the (MQCD) sine formula gives a more conventional $O(1/N^2)$ correction.

since an $O(1/N)$ correction can feed into an $O(1/N)$ correction to glueball masses – think of excited glueballs composed of closed k-strings – this is an interesting issue ...

$$\frac{\sigma_{k=2}}{\sigma}$$

fit $\frac{\sigma_{k=2}}{\sigma} = 2 - \frac{a}{N^p} - \frac{b}{N^{2p}}$

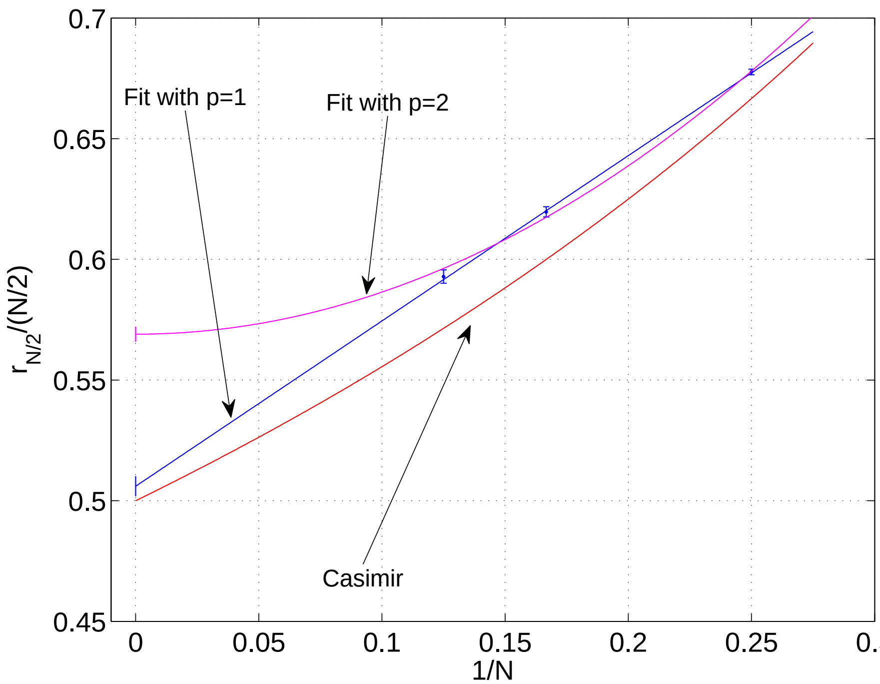


$\Rightarrow$

$p = 1$ fit OK ; $p = 2$ fit poor

$$\frac{\sigma_{k=N/2}}{\sigma}$$

fit $\frac{\sigma_{k=\frac{N}{2}}}{\sigma} = a + \frac{b}{N^p}$



$\Rightarrow$

$p = 1$ fit good, and $a = 0.506(4)$ consistent with Casimir scaling
$p = 2$ fit poor

$\Rightarrow$ evidence is for $O(1/N)$ corrections

Excited states in a given k sector

• does the spectrum know about the group or just about the center?
e.g.
$k = 2a : \{\mathrm{Tr}l\}^2 - \mathrm{Tr}l^2$
$k = 2s : \{\mathrm{Tr}l\}^2 + \mathrm{Tr}l^2$
with a ground state – bound state or resonance – in each representation?

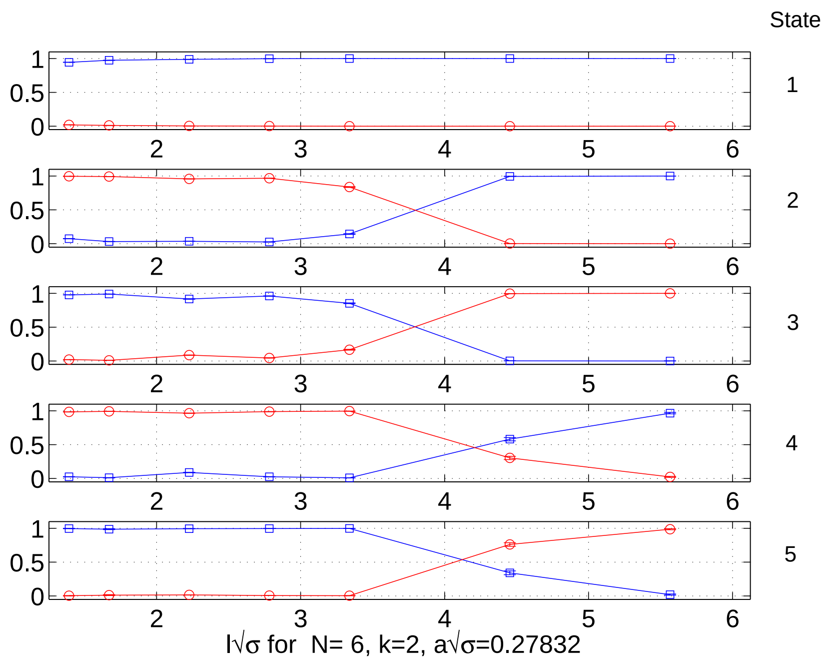
• is there a Nambu-Goto tower of excited states built on each of these ground states?

Excited states in the $k = 2$ sector

- Calulate the spectrum using the full basis of $k = 2$ operators as a function of l

- Out of the original operators form separate orthonormal bases of $k = 2a$ and $k = 2s$ operators (warning: not exact here!)

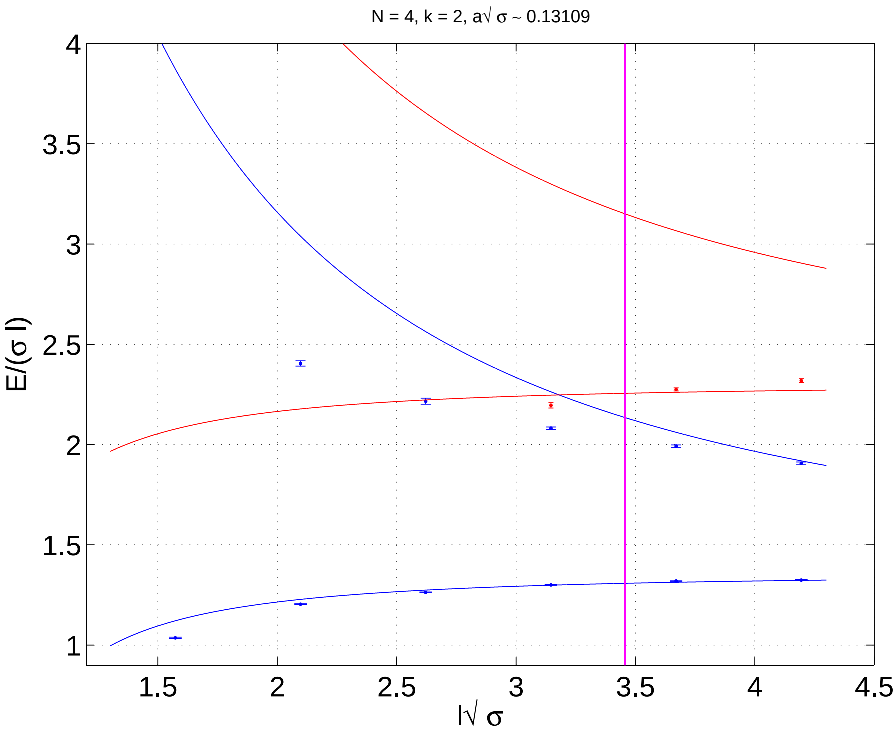
- Calculate the projection of each eigenvector onto the $k = 2a$ and $k = 2s$ bases separately:



⇒

the states organise themselves into represnetations of SU(N), leading to accidental crossing degeneracies at certain l

with some evidence for a Nambu-Goto tower ...



so once we look at multiply wound strings and k-strings, we find :

k-strings are strongly bound

for each k we have a complex spectrum of excited states

k-strings know about SU(N) e.g. the low-lying spectrum of $k = 2$ strings appears to fall spontaneously into symmetric and antisymmetric representations

there is some evidence that on each 'ground state', in each representation, we have an approximate Nambu-Goto tower of excited states

String spectrum in D=2+1 SU(N) gauge theories

- non-perturbative linear confinement
- scale set by $g^2N \sim [m]$
- dimensionless coupling for physics on scale l is lg^2N

$\longrightarrow$
UV freedom and IF slavery, just like $D = 3 + 1$

QUESTION:

consider the spectrum of confining flux tubes around a spatial torus of length l:

there are no sources and (for $N \geq 3$) there is a first order phase transition at $l = l_c = \frac{1}{T_c}$

so this sector of string states should be describable by some effective string theory $\forall l \geq l_c$ and this provides a starting point for finding the effective string theory that describes gauge theories (Polyakov, 't Hoost, ..)

this effective string theory should be particularly simple at $N = \infty$ where the are no decays or mixings or screening or binding between strings and glueballs

SU(N) string tension: Karabali-Nair prediction

Barak Bringoltz, MT: hep-th/0611286

Karabali and Nair analytic Hamiltonian formalism: e.g.

hep-th/0309061, arXiv:0705.2898, 0705.0394

(also: Freidel, Leigh, Minic hep-th/0604184)

$\Rightarrow$

$$\frac{\sqrt{\sigma}}{g^2 N} = \sqrt{\frac{1-\frac{1}{N^2}}{8\pi}}$$

within $\sim 3\%$ of 'old' lattice calculations for all N

$\Rightarrow$

need calculations where *all* systematic (=hard) as well as statistical (= easy) errors are controlled to $\ll 1\%$.

Note:

the $D = 2 + 1$ (Hamiltonian) expectation value of a Wilson loop taken with respect to the KKN ground state wave-functional, involves 2 dimensional integrations that to 'leading order' turn out to be precisely like the $D = 1 + 1$ Euclidean expectation value of a Wilson loop

$\Rightarrow$

for sources in a representation $\mathcal{R}$ the corresponding string tension satisfies Casimir scaling:

$$\frac{\sigma_{\mathcal{R}}}{\sigma_f} = \frac{C_{\mathcal{R}}}{C_f}$$

where C is the quadratic Casimir

BUT

just as in $D = 1 + 1$ there is no screening e.g. of adjoint

$\Rightarrow$

KKN, to this approximation, *cannot* be exact, except possibly at $N = \infty$ where screening vanishes in $D = 3$

moreover

just as in $D = 1 + 1$ there are no $O(1/r)$ corrections to Wilson loops

$\Rightarrow$

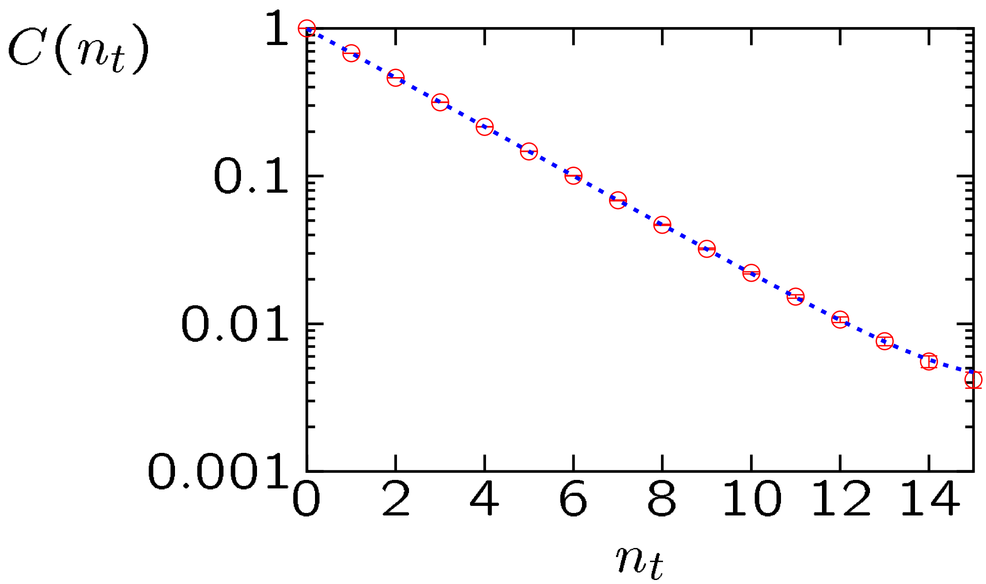
even at $N = \infty$ KKN cannot be exact for finite Wilson loops, although it might be for the string tensions – this is what we will try to establish

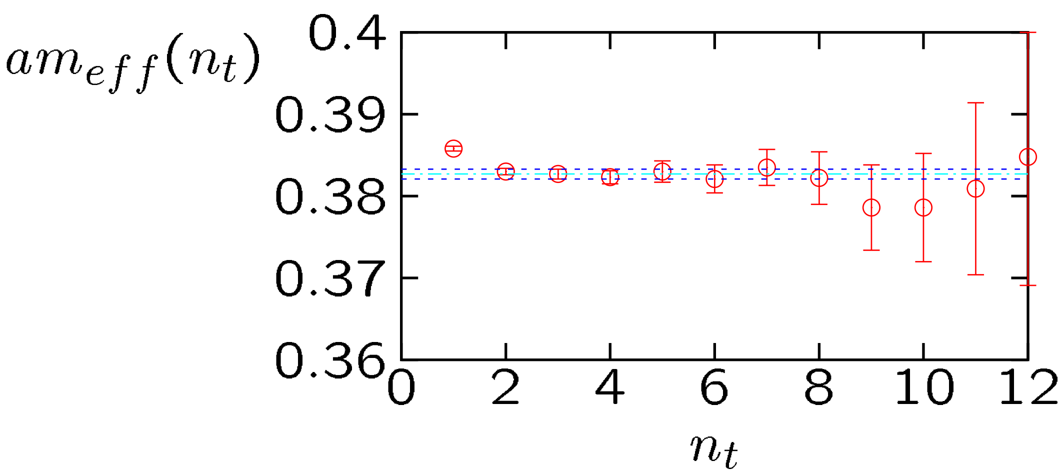
systematic errors

Barak Bringoltz, MT: hep-th/0611286

- corrections in powers of $1/l$ to $E(l) = \sigma l - c\frac{\pi}{6l}$
- explicit bound on finite volume corrections
- excited string state contributions to ground state string correlators
- correlated fits
- $O(a^4)$ corrections to usual $O(a^2)$ continuum extrapolations
- $O(1/N^4)$ corrections to usual $O(1/N^2)$ large-N extrapolations

excited state correction e.g. SU(5) $24^2 32$

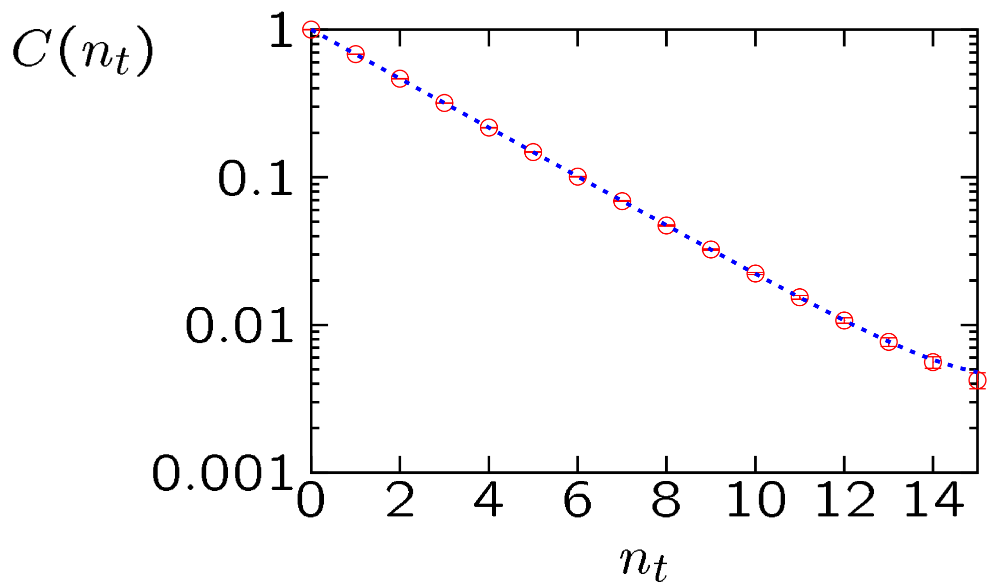


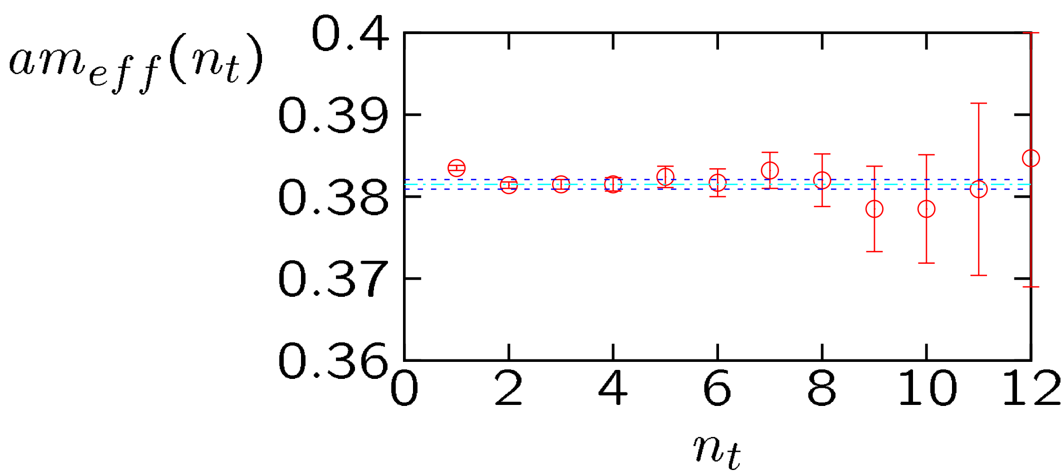


$\Rightarrow \qquad am = 0.3827 \pm 0.0006$

subtract a maximal (positivity) contribution of the first excited state from $C(t)$ i.e.

$C(t) \rightarrow C(t) - 0.015\,\exp\{-0.74t\}$





$\Rightarrow \qquad am = 0.3827(6) \longrightarrow 0.3815(6)$

i.e. 2σ systematic error

continuum limit :

since :

$$\beta \to \frac{2N}{ag_3^2} \quad ; \quad \beta \to \infty$$

where g_3^2 is the dimensionful coupling that sets the mass scale of the continuum $D = 3$ gauge theory, it is usual and convenient to define

$$\beta \equiv \frac{2N}{ag^2}$$

So we can extrapolate to the continuum limit:

$$\frac{\beta}{2N} a\sqrt{\sigma(a)} \to \frac{\sqrt{\sigma}}{g_3^2} \quad ; \quad \beta \to \infty$$

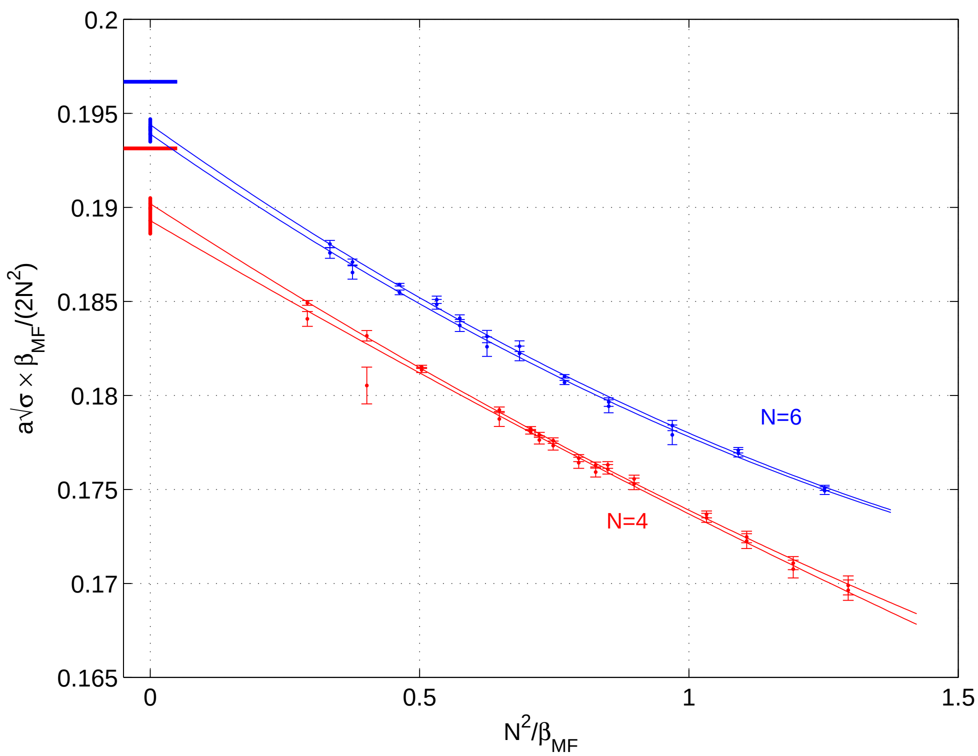
for example by using :

$$\frac{\beta}{2N} a\sqrt{\sigma(a)} = \frac{\sqrt{\sigma}}{g_3^2} + \frac{c_1}{\beta} + \frac{c_2}{\beta^2}$$

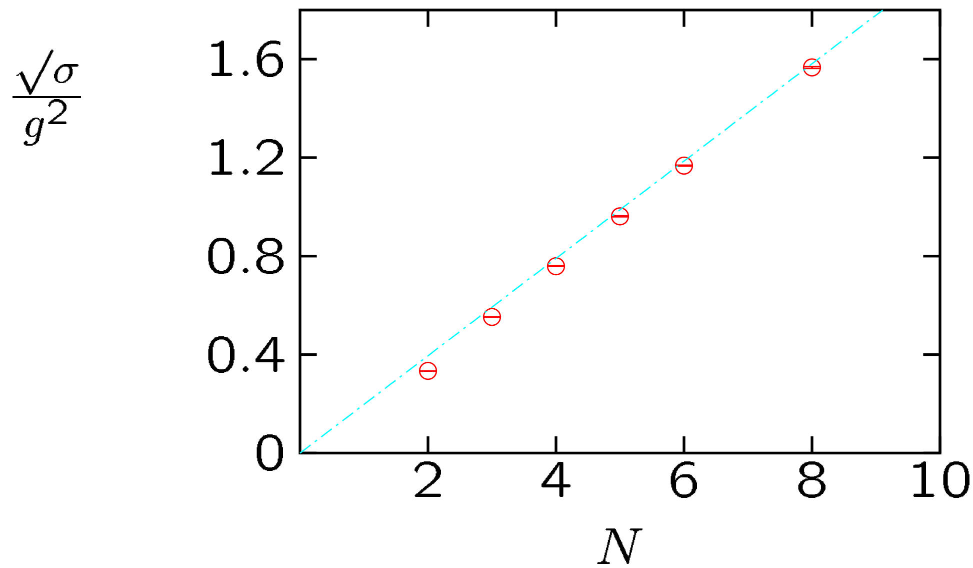
we shall frequently use g^2 in place of g_3^2, hopefully without confusion ...

some continuum limits :

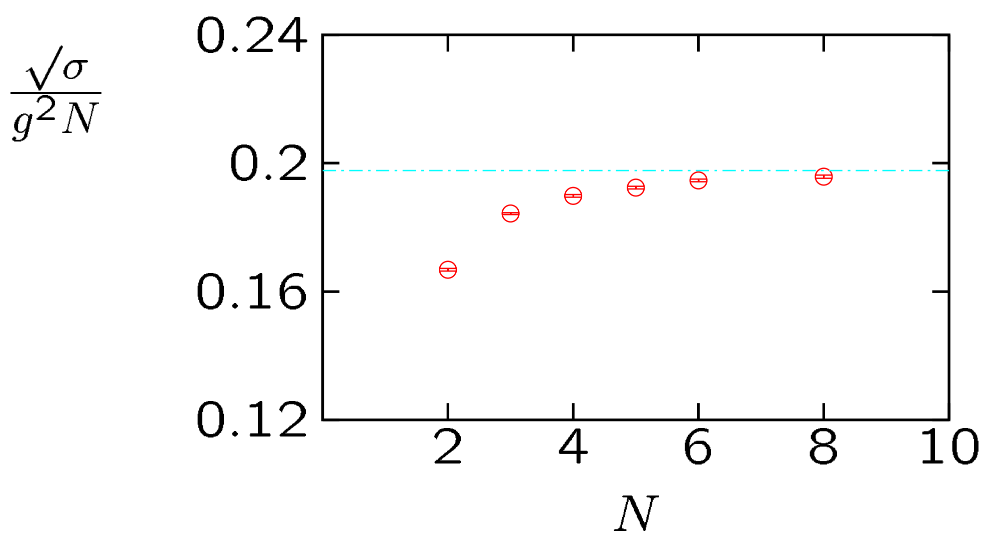
$\frac{\sqrt{\sigma(a)}}{g^2 N}$ vs. $ag^2 N$



smooth limit keeping $g^2 N$ fixed?



$\downarrow \quad : \quad g^2 \to g^2 N$

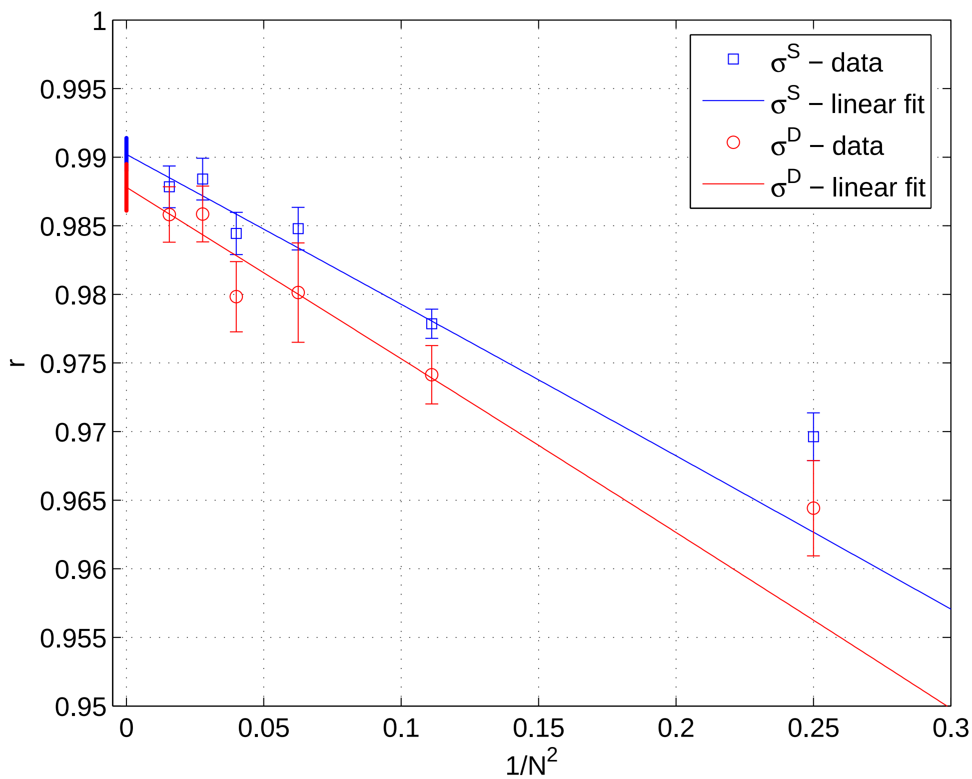


$$\lim_{a \to 0} \frac{\sqrt{\sigma}}{g^2 N}$$

N	KKN	lattice
2	0.1728	0.1668(4)
3	0.1881	0.1843(3)
4	0.1931	0.1898(4)
5	0.1954	0.1924(4)
6	0.1967	0.1946(4)
8	0.1979	0.1958(5)
∞	0.1995	0.1977(4)

$$N \to \infty :$$

$$r \equiv \frac{\left(\sqrt{\sigma}/g^2N\right)_{\mathrm{KKN}}}{\left(\sqrt{\sigma}/g^2N\right)_{\mathrm{Lattice}}}$$



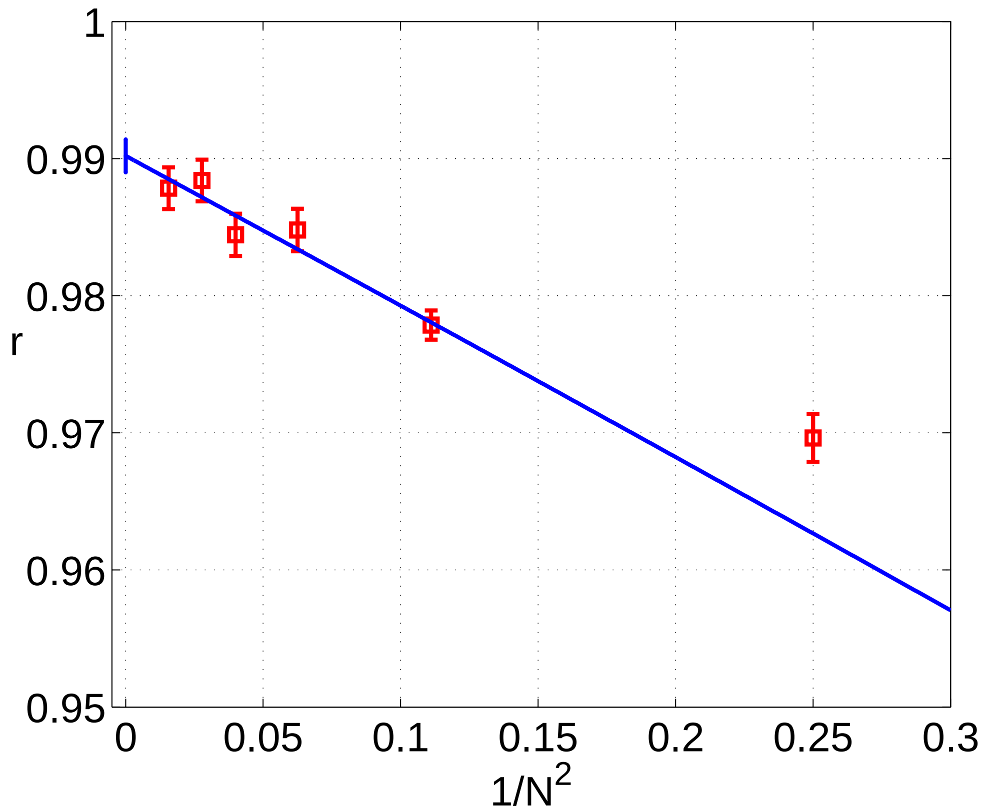
$\Rightarrow$

$$\lim_{N\to\infty} \frac{\sqrt{\sigma}}{\sqrt{\sigma_{\mathrm{KKN}}}} = 0.9901 \pm 0.0010 - 0.0025$$

convincing $\sim$ 6 sd discrepancy

$N \to \infty$:

$$r \equiv \frac{\left(\sqrt{\sigma}/g^2N\right)_{\text{KKN}}}{\left(\sqrt{\sigma}/g^2N\right)_{\text{Lattice}}}$$



$\Rightarrow$

$$\lim_{N\to\infty} \frac{\sqrt{\sigma}}{\sqrt{\sigma_{\text{KKN}}}} = 0.9901 \pm 0.0010 - 0.0025$$

convincing $\sim$ 6 sd discrepancy

the topological susceptibility

the topological susceptibility, χ, is given by the fluctuations of the topological charge, Q, per unit space-time volume V:

$$\chi \equiv \frac{\langle Q^2 \rangle}{V}$$

the Witten-Veneziano realisation of 't Hooft's resolution of the $U_A(1)$ problem (that the η' is anomalously heavy for a Goldstone boson), leads to the large-N relation:

$$\chi = \frac{f_\pi^2 m_{\eta'}^2}{4N_f} + O(m_q, 1/N)$$

which putting in experimental numbers comes to

$$\chi \simeq (180\text{MeV})^4$$

and lattice calculations of the SU(3) topological suceptibility obtain values close to this.
However all this assumes that the SU(3) value of χ differs from its SU(∞) value by modest $O(1/N^2)$ corrections and that the same is true of the combination $f_\pi^2 m_{\eta'}^2$.
For progress towards establishing the latter see:

R. Narayanan, H. Neuberger hep-lat/0501031;

G. Bali, F. Bursa arXiv:0708.3427

Here I show some results demonstrating the latter.

lattice technique

replace the continuum topological charge density by a corresponding lattice operator:

$$\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \longrightarrow \epsilon_{\mu\nu\rho\sigma} U_{\mu\nu} U_{\rho\sigma}$$

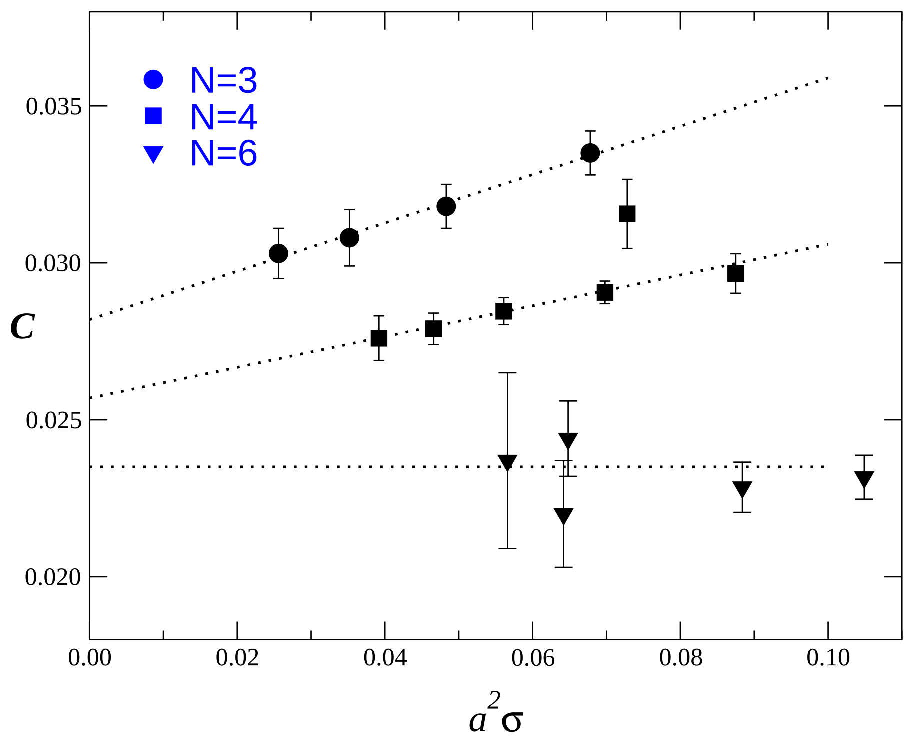
where $U_{\mu\nu}(n)$ is the plaquette matrix in the μ, ν plane and at the lattice site $x = an$ When the fields are smooth:

$$Q_L(x) = \frac{1}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} ReTr\{U_{\mu\nu}(x) U_{\rho\sigma}(x)\} \stackrel{a \to 0}{\longrightarrow} a^4 Q(x) + O(a^6)$$

where $Q(x)$ is the continuum topological charge density. To obtain a smooth lattice field from a rough Monte Carlo generated lattice field we apply a standard iterative 'cooling' (= smoothing) procedure.

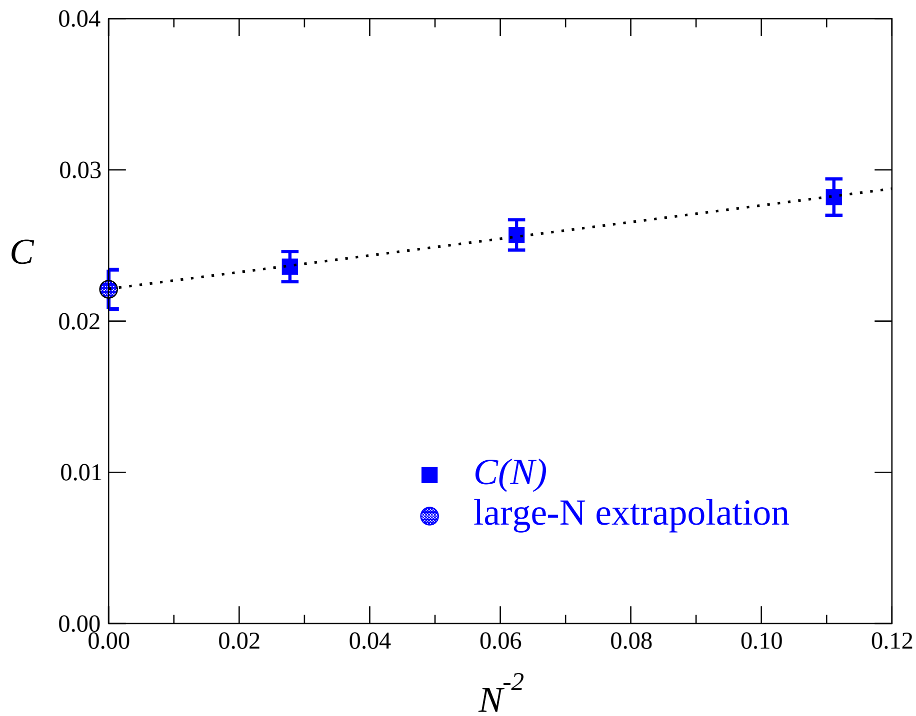
to the continuum limit ...

L. Del Debbio, H. Panagopoulos, E. Vicari hep-th/0204125 ...



N-dependence

L. Del Debbio, H. Panagopoulos, E. Vicari hep-th/0204125
(see also: B. Lucini, M. Teper hep-lat/0307017)



$$\chi^{\frac{1}{4}}_{N=3} \simeq 180(2)\mathrm{MeV} \quad \longrightarrow \quad \chi^{\frac{1}{4}}_{N=\infty} \simeq 170(3)\mathrm{MeV}$$

a question ...

$\langle Q^2 \rangle$ measures the *fluctuations* of Q around $\langle Q \rangle = 0$: why does it not vanish as $N \to \infty$ i.e.

$$\langle Q^2 \rangle \overset{N\to\infty}{=} \langle Q \rangle^2 = 0?$$

It does not vanish because at a generic value of $\psi = \theta/N$

$$\langle Q \rangle = O(N) \leftrightarrow \langle Q^2 \rangle = O(N^2)$$

so the finite limit that we observe of $\lim_{N\to\infty} \langle Q^2 \rangle / V$ is in fact the $O(1/N^2)$ correction.

B. Lucini, M. Teper, U. Wenger hep-lat/0401028 let us suppose for the moment that we have added a θ-term to the action, i.e. $\delta S = i\theta Q$, and that we are working with a generic nonzero value of θ so that $\langle Q \rangle \neq 0$. We have

$$\langle Q \rangle = -i \frac{d}{d\theta} \ln Z(\theta) = i \frac{d}{d\theta} \epsilon(\theta) V$$

where we have used $Z = \exp -\epsilon V$ where ϵ is the vacuum energy per unit volume. Now we expect from the usual large-N counting arguments that $\epsilon \propto N^2$ and that a smooth large-N limit is reached if one keeps θ/N fixed i.e.

$$\epsilon(\theta) = N^2 h(\theta/N)$$

Plugging this in and using the notation $\psi \equiv \theta/N$, we immediately see that

$$\langle Q \rangle = i \frac{d}{d\theta} \epsilon(\theta) V = NVi \frac{d}{d\psi} h(\psi) \propto N$$

size distribution of instantons

look at peaks in smoothened lattice topological charge density and use the classical formula

$$Q_{peak} = \frac{6}{\pi^2 \rho^4}$$

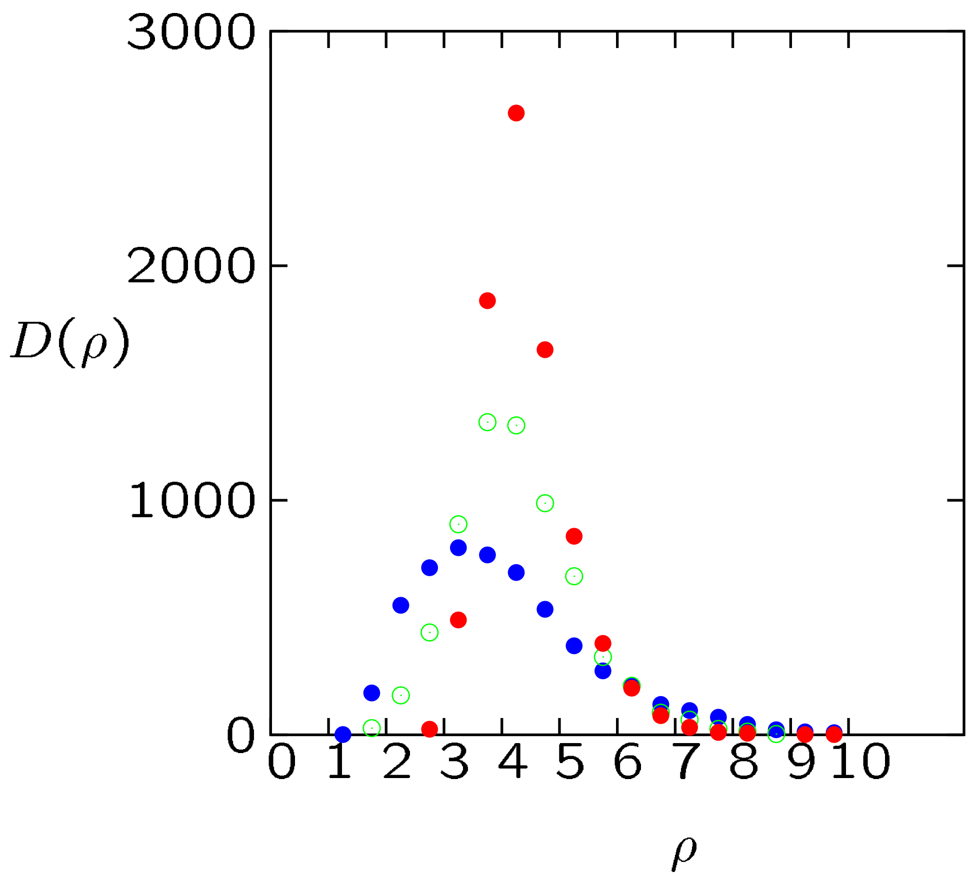
to determine the instanton size ρ

Warning : for small ρ the peaks are huge and the method is reliable; but for large ρ the ‘peaks’ become very wide and low bumps that eventually might be anything ...

Instanton size density

B. Lucini, M. Teper hep-lat/0103027

$$D(\rho) \stackrel{N\to\infty}{\longrightarrow} \delta(\rho - \rho_c) \ ?$$



SU(3) • ; SU(6) ∘ ; SU(12) •

$$\rho_c \simeq \frac{1}{T_c}$$

• vanishing of small instantons?

for small instantons, $\rho \ll \Lambda_{QCD}$, the coupling becomes small, $g^2(\rho) \ll 1$, and the density is dominated by the action factor;

$$D(\rho)d\rho \overset{\rho\to 0}{\propto} e^{-\frac{8\pi^2}{g^2(\rho)}} = e^{-\frac{8\pi^2}{\lambda(\rho)}N} \quad : \quad \lambda \equiv g^2 N$$

which translates to:

$$D(\rho) \overset{\rho\to 0}{\propto} \rho^{\frac{11N}{3}-5}$$

which is more-or-less reproduced below ...

• vanishing of larger instantons?

as we increase ρ we see from the more complete semi-calssical expression

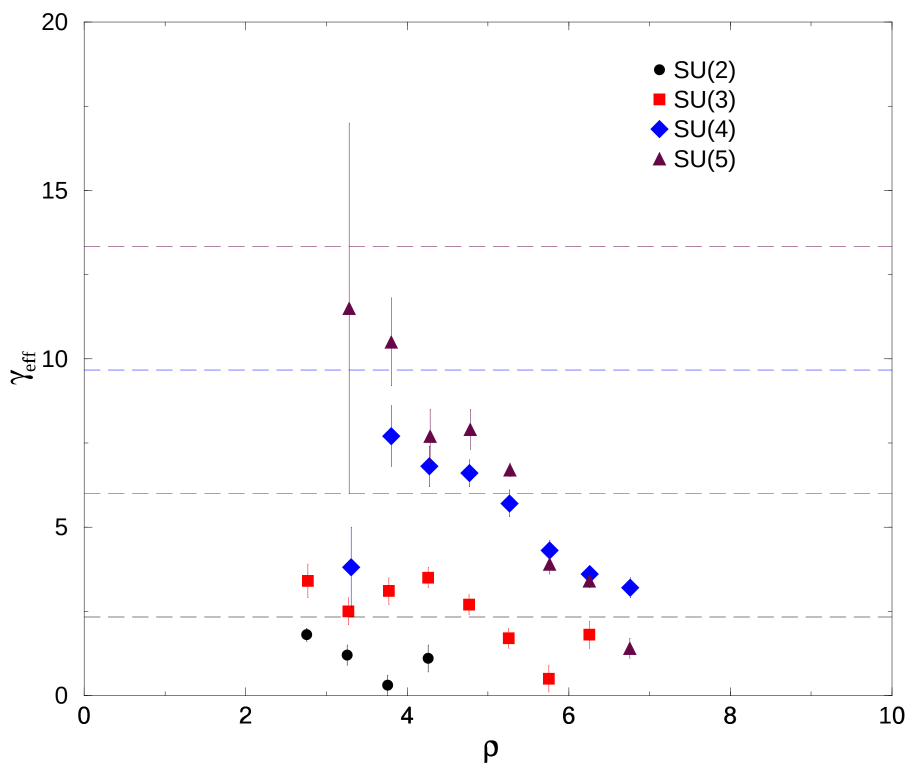
$$D(\rho)d\rho \propto \frac{d\rho}{\rho}\frac{1}{\rho^4}\left\{\frac{b^2}{\lambda^2(\rho)}e^{-\frac{8\pi^2}{\lambda(\rho)}}\right\}^N$$

that the suppression with N becomes weaker, and depnds on details, such as the effective action for overlapping instantons etc.
In any case, for large instantons we lose our ability to calculate ...

however we expect very large instantons to be suppressed because the theory has a mass gap, which means we cannot have arbitrarily large coherent chromoelectric and magnetic fields that would exist in the core of an arbitrarily large instantons ...

vanishing of small instantons

fit $D(\rho) \propto \rho^{\gamma_{eff}(\rho)}$



vanishing of small instantons

$\Rightarrow$

sequence of lattice fields gets stuck in a given topological sector:

MC is ‘local’ so the fields in the sequence change almost continuously – one link matrix at a time

$\rightarrow$

to change Q we need to shrink an (anti) instanton down to the size of a hypercube, $\rho \sim a$, and then eject it through the hypercube, leaving an innocuous gauge singularity behind.

$\rightarrow$

this necessarily means passing through sizes where $g^2(\rho)$ is small, and the probability of this is suppressed exponentially in N

Solution? use a subvolume $v \ll V$ which is small enough not to care that the total charge in V is constant, and which is large enough, $v \gg \Lambda_{QCD}^{-4}$, that it encompasses relevant length scales.

... this still leaves many interesting questions about how topology is encoded in large-N fields ...

Interlaced θ-vacua in SU(N) gauge theories

consider the gauge action with a θ term

$$S[g^2,\theta] = \frac{1}{4g^2}\int d^4x \mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{16\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr} F_{\mu\nu}F_{\rho\sigma}$$

Since

$$\frac{1}{16\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr} F_{\mu\nu}F_{\rho\sigma} = Q = \text{integer}$$

we know that $S[\theta]$ and hence the vacuum energy $E(\theta)$ are periodic in θ

$$E(\theta) = E(\theta + 2\pi) \qquad \forall N$$

On the other hand, we expect that for a smooth $N \to \infty$ limit, we need to factor N from S so that the couplings to keep fixed are $1/g^2N$, θ/N, ... i.e.

$$E(\theta) = N^2 h(\theta/N)$$

How do we reconcile these two apparently irreconciliable demands? E.Witten hep-th/9807109 suggestion: E(θ) is a multi-branched function

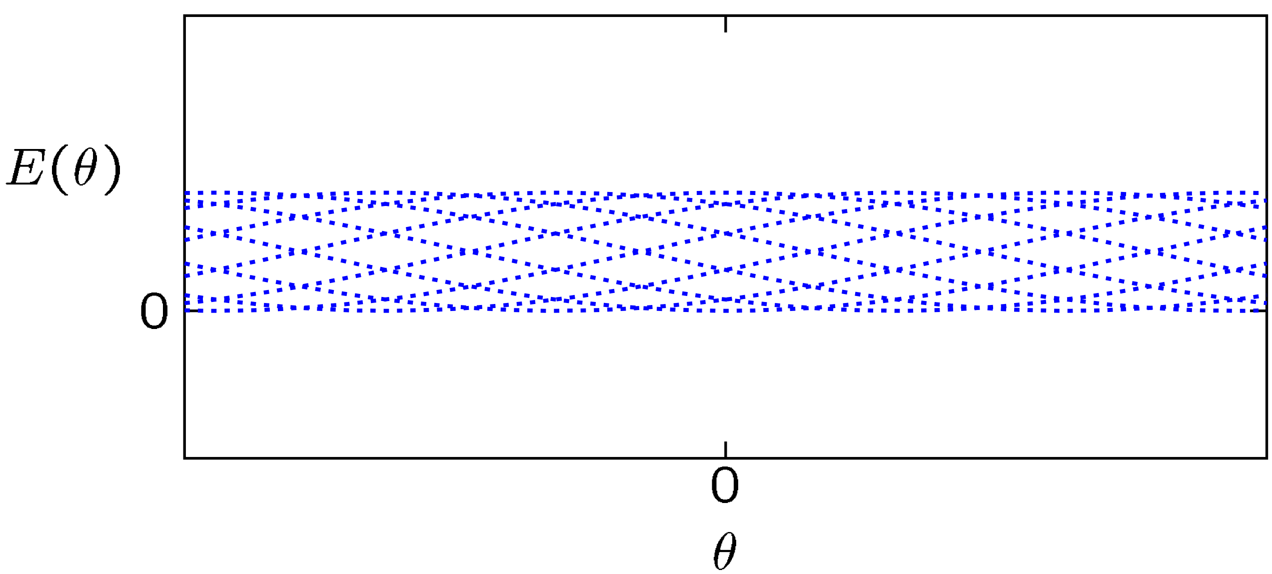
E.Witten hep-th/9807109

$$E_k(\theta) = N^2 h\left(\frac{\theta + 2\pi k}{N}\right) \quad ; \quad E(\theta) = \min_k E_k(\theta)$$

so that: $E(\theta) = E(\theta + 2\pi)$

while each $E_k(\theta)$ is periodic in $2\pi N$

e.g. N=10:



domain wall tension between different 'k-vacua' is $O(N)$

so as $N \to \infty$ these will all become stable ...

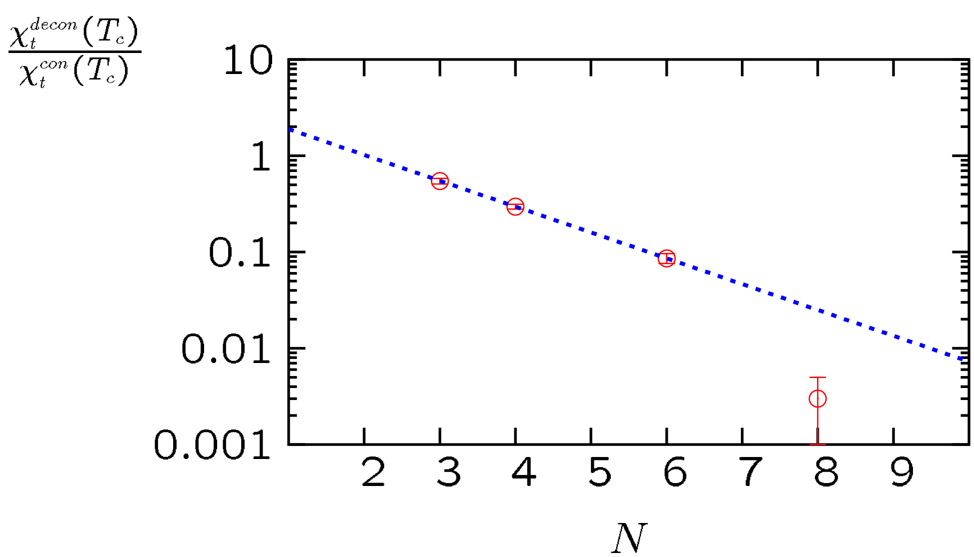
Witten: AdS/CFT ; Shifman: $\mathcal{N} = \infty$ SUSY

So:

$T \leq T_c$
we have $N/2$ vacua at any given θ

there is very nice lattice evidence for this scenario:
L.Del Debbio, H. Panagopoulos, E. Vicari, hep-th/0204125, arXiv:0706.1479

$T > T_c$
topology disappears exponentially in N:



so no interlaced vacua – just naive 2π periodicity in θ
with exponentially small $E(\theta)$ variation

$\longrightarrow$

So as $N \to \infty$ there are $N/2$ stable vacua at $\theta = 0$

For large N the lowest vacua are close to their minima $\to$ we can use a quadratic approximation for $E_k(\theta)$, $\to$

$$E_k(\theta = 0) = \frac{1}{2}\chi_t(2\pi k)^2$$

where χ_t is the topological susceptibility

Note:

$$E_{k=1} = 2\pi^2\chi_t \sim (360MeV)^4 \sim \text{gluon condensate}$$

Unfortunately, we have searched for such quasistable (in MC time) states but have not (yet) found any sign of them