

CALCULUS ANSWERS

A1(a) (i) $(\cos x + 3x^2 \sin x)e^{x^3}$ (ii) $(x^3 \cos x + 3x^2 \sin x)e^{x^3 \sin x}$ (iii) $\frac{-\tanh(1/x)}{x^2}$

(b) (i) $-\frac{1}{\sqrt{1-x^2}}$ (ii) $\frac{1}{(1+2x)}$

(c) (i) $x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x \right)$ (ii) $\frac{2}{x \ln 10}$

(d) (i) $\frac{2x - (y^2/x)e^{y \ln x}}{(y \ln x + 1)e^{y \ln x} - 2y}$

(e) $\frac{dy}{dx} = \coth \theta$; $\frac{d^2y}{dx^2} = -\operatorname{cosech}^3 \theta$

A6 $\frac{d^8y}{dx^8} = x^2 \sin x - 16x \cos x - 56 \sin x$

B1(a) (i) $\frac{-1}{(1+2ax+x^2)^{1/2}} + C$ (ii) $e - 1$ (iii) $2/3$ (iv) 4

(b) (i) $\sin^{-1}\left(\frac{x-1}{2}\right) + C$ (ii) $\pi/4$

(c) (i) $\ln\left(\frac{x}{\sqrt{1+x^2}}\right) + C$

(d) (i) $\sin x - x \cos x + C$ (ii) $x \ln x - x + C$

(e) 1

B3 (a) $\sinh 1$ (b) $\pi/2$ (c) $S.A. = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$

B4 (a) $32/3$ (b) $64/3$ (c) (i) 1 (ii) 1

C2 (a)(i) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ (ii) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$ (iii) $x - \frac{x^3}{3} + \dots$

(b) 0.515038072 ; error in last digit

C4 0.9461

C6(a) (i) 1 (ii) 1 (iii) 0

(c) (i) 0 (ii) 0 (iii) -1

(d) (i) maximum (ii) ∞

D2(a) (i) $\frac{x}{(x^2+y^2)^{1/2}}$ (ii) $-\frac{y}{x^2+y^2}$ (iii) $y^x \ln y$

D3 9%

D4 (a) $na \cos^{n-1} at \sin^{n-1} at (\cos^2 at - \sin^2 at)$ (b) $x + 2x \ln x - \frac{1}{x(\ln x)^2}$

$$\text{D5 } \frac{\partial w}{\partial r} = -2(x^2 + y^2)^{\frac{1}{2}} e^{-x^2 - y^2}, \quad \frac{\partial w}{\partial \theta} = 0$$

$$\text{D7 (a) } \left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial z}{\partial u}\right)_v 2x + \left(\frac{\partial z}{\partial v}\right)_u 2y$$

$$\text{(b) } \left(\frac{\partial z}{\partial u}\right)_v = \frac{1}{2(x^2 - y^2)} \{x \left(\frac{\partial z}{\partial x}\right)_y - y \left(\frac{\partial z}{\partial y}\right)_x\}$$

$$\text{(c) } \left(\frac{\partial z}{\partial u}\right)_v - \left(\frac{\partial z}{\partial v}\right)_u = \frac{1}{2(x-y)} \{ \left(\frac{\partial z}{\partial x}\right)_y - \left(\frac{\partial z}{\partial y}\right)_x \}$$

$$\text{D8 } f(x, y) = e^6 + 3e^6(x - 2) + 2e^6(y - 3) + \frac{9e^6}{2}(x - 2)^2 + 6e^6(x - 2)(y - 3) + 2e^6(y - 3)^2$$

$$\text{D9 (i) } x = y = 0 \text{ (minimum)}$$

$$\text{(ii) } x = y = 0 \text{ (maximum); } x = y = 1/3 \text{ (saddle)}$$

$$\text{(iii) } x = y = \pi/3 \text{ (maximum)}$$

$$\text{D10 (i) exact } f = xy$$

$$\text{(ii) not exact}$$

$$\text{(iii) exact } f = \frac{1}{2}(x^2 + y^2 + z^2)$$

$$\text{(b) } 0$$

$$\text{E5 (i) } -\pi \text{ (ii) oscillates (iii) } \sqrt{2}/4 \text{ (iv) } 0 \text{ (v) } 1$$

$$\text{E6 } y(x) = -3x + 4x^3$$