II STEADY CURRENTS + MAGNETISM

D. ELECTRODYNAMICS

1. Faraday's law
2. Maxwell's correction to Ampère's law: the displacement current
3. polarization current

D. Electrodynamics

putting the time dependence into Maxwell's equations

1. Faraday's law : based on experiment

Lenz's law – induced emf acts to oppose the change producing it

$$\underbrace{\oint_C \underline{E} \cdot d\underline{l}}_{\text{electromotive force around a circuit}} = - \frac{\partial}{\partial t} \underbrace{\int_S \underline{B} \cdot d\underline{S}}_{\text{flux threading a surface } S \text{ spanning } C}$$

$\downarrow$ Stokes

$$\int_S \text{curl}\, \underline{E} \cdot d\underline{S}$$

$$\boxed{\therefore \text{curl}\, \underline{E} = - \frac{\partial \underline{B}}{\partial t}}$$

2. Maxwell's correction to Ampère's law : Displacement Current

needed to ensure conservation of charge

$$\text{div}\, \underline{J} = - \frac{\partial \rho}{\partial t}$$

Ampere's law :

$$\text{curl}\, \underline{H} = \underline{J}$$

$$\therefore \text{div curl}\, \underline{H} = \text{div}\, \underline{J} \equiv 0$$

which is incorrect if there is any time dependence but we would like

$$\operatorname{div} \operatorname{curl} \underline{H} = \operatorname{div} \underline{J} + \frac{\partial \rho}{\partial t} = 0$$

$$= \operatorname{div} \underline{J} + \frac{\partial}{\partial t}\left(\operatorname{div} \underline{D}\right)$$

(Gauss)

$$\boxed{\therefore \quad \operatorname{curl} \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}}$$

3. polarization current

we have arrived at Maxwell's equations

$$\operatorname{div} \underline{D} = \rho_f \qquad (1)$$

$$\operatorname{div} \underline{B} = 0 \qquad (2)$$

$$\operatorname{curl} \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad (3)$$

$$\operatorname{curl} \underline{H} = \underline{J}_f + \frac{\partial \underline{D}}{\partial t} \qquad (4)$$

what do they look like in terms of $\underline{B}$ and $\underline{E}$ only?

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}$$

$$\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}$$

(1) $\operatorname{div} \underline{D} = \operatorname{div}(\epsilon_0 \underline{E} + \underline{P}) = \rho_f$

$$\therefore \operatorname{div} \underline{E} = \frac{\rho_f - \operatorname{div} \underline{P}}{\epsilon_0} = \frac{\rho_f + \rho_b}{\epsilon_0}$$

(4) $\operatorname{curl}\left(\frac{1}{\mu_0}\underline{B} - \underline{M}\right) = \underline{J}_f + \frac{\partial}{\partial t}(\epsilon_0 \underline{E} + \underline{P})$

$$\therefore \operatorname{curl} \underline{B} = \mu_0 \underbrace{\operatorname{curl} \underline{M}}_{\underline{J}_b} + \mu_0 \underline{J}_f + \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t} + \mu_0 \frac{\partial \underline{P}}{\partial t}$$

$$= \mu_0 (\underline{J}_f + \underline{J}_b + \underline{J}_p) + \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

where $\underline{J}_p = \frac{\partial \underline{P}}{\partial t}$ polarisation current needed because if $\underline{P}$ changes bound charges move around

check if the continuity equation for the bound charges works:

hope: $$\operatorname{div} \underline{J}_p = -\frac{\partial}{\partial t}\rho_b$$

$$\operatorname{div}\left(\frac{\partial \underline{P}}{\partial t}\right) = -\frac{\partial \rho_b}{\partial t} \quad \checkmark \quad \text{as } \rho_b = -\operatorname{div} \underline{P}$$