II STEADY CURRENTS AND MAGNETISM

C. MAGNETIZABLE MATERIALS

1. Magnetization: definition and physical origins

When a magnetizable material is placed in a magnetic field $\underline{B}$ it acquires a magnetic dipole moment. This is measured by the magnetization $\underline{M}$ defined as the magnetic dipole moment per unit volume.

Why does the field induce a magnetic dipole moment?

(i) all materials

the field changes the shape of the electron orbits by a small amount to give an extra dipole moment $\underline{M} \propto -\underline{B}$

this is diamagnetism ; typically a very small effect

(ii) materials with unpaired electrons

some atoms have an intrinsic magnetic dipole moment due to the angular momentum and spin of the unpaired electron. in a field a small excess point along $\underline{B}$

this is paramagnetism
$\underline{M} \propto \underline{B}$ unless field is very large (Curie's law)

(i), (ii) linear materials i.e. $\underline{M} \propto \underline{B}$

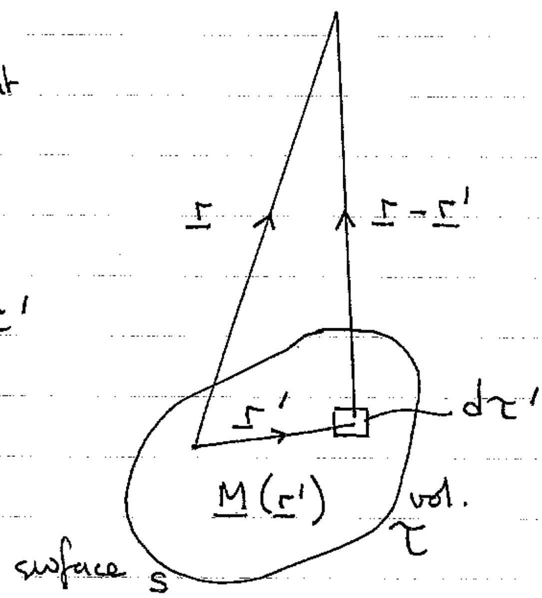
(iii) ferromagnets

non-linear – see section C7

2. bound currents

> The vector potential (∴ the $\underline{B}$-field) of an object with magnetization $\underline{M}$ is the same as the vector potential produced by
>
> a volume current density $\underline{J_b} = \text{curl}\ \underline{M}$
>
> plus a surface current density $\underline{K_b} = \underline{M} \wedge \hat{\underline{n}}$

vector potential due to a dipole moment per unit volume $\underline{M}$



$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int_\tau \frac{\underline{M}(\underline{r}') \wedge \widehat{(\underline{r}-\underline{r}')}}{|\underline{r}-\underline{r}'|^2}\, d\tau'$$

$$\Big\downarrow \text{ vector algebra}$$

$$\frac{\mu_0}{4\pi} \int_\tau \frac{\text{curl}'\ \underline{M}(\underline{r}')}{|\underline{r}-\underline{r}'|}\, d\tau' + \frac{\mu_0}{4\pi} \int_S \frac{\underline{M}(\underline{r}') \wedge \hat{\underline{n}}}{|\underline{r}-\underline{r}'|}\, dS'$$

vector potential of a volume current density $\underline{J_b} \equiv \text{curl}\ \underline{M}$

vector potential of a surface current density $\underline{K_b} \equiv \underline{M} \wedge \hat{\underline{n}}$

3. Ampère's law in magnetised materials and $\underline{H}$

$$\text{curl}\,\underline{B} = \mu_0(\underline{J}_f + \underline{J}_b) = \mu_0(\underline{J}_f + \text{curl}\,\underline{M})$$

$$\text{curl}\left(\frac{\underline{B}}{\mu_0} - \underline{M}\right) = \underline{J}_f$$

define $$\boxed{\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}} \qquad (1)$$

$$\text{curl}\,\underline{H} = \underline{J}_f \qquad (2)$$

4. Linear materials and the relative permeability μ

linear means $\underline{M} \propto \underline{B}$ (equivalently $\underline{M} \propto \underline{H}$, $\underline{B} \propto \underline{H}$)

write $\underline{M} = \chi_m \underline{H}$

magnetic susceptibility

choice of $\underline{H}$ not $\underline{B}$ a matter of definition

from (1) $$\underbrace{\underline{H}(1+\chi_m)}_{\text{define as relative permeability } \mu} = \frac{\underline{B}}{\mu_0}$$

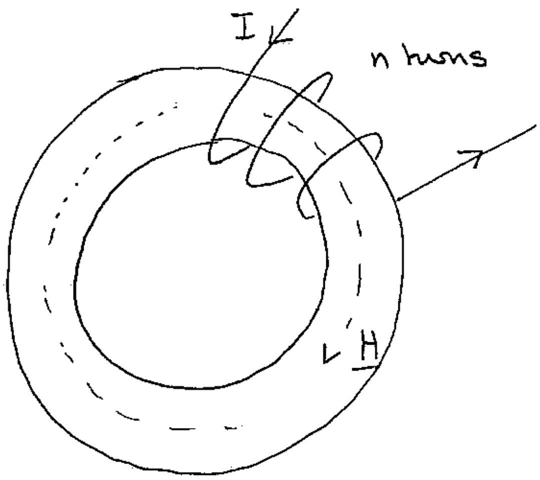
$$\boxed{\underline{B} = \mu\mu_0 \underline{H}} \qquad (3)$$

using (2) and (3)

$$\text{curl}\,\underline{B} = \mu\mu_0 \underline{J}_f$$

N.B.1: $\underline{H}$ is sometimes called the 'magnetic field' and $\underline{B}$ the 'magnetic flux density'.

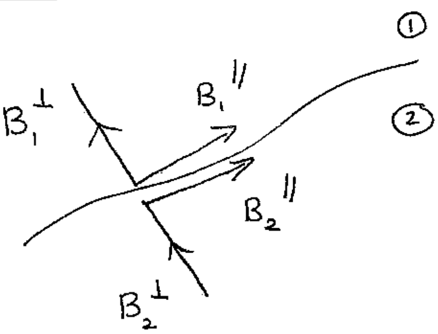
N.B.2: $\underline{H}$ is directly accessible experimentally:



$$\int \underline{H} \cdot d\underline{\ell} = nI$$

5. <u>Most convenient boundary conditions on $\underline{B}$, $\underline{H}$</u>

general magnetostatic boundary conditions are (Sec. A6)



$$\underline{B_1^{\perp} = B_2^{\perp}} \qquad \text{fine}$$

$$B_1^{\parallel} - B_2^{\parallel} = \mu_0 (I_f^s + I_b^s) \qquad \text{inconvenient}$$

but

$$H_1^{\parallel} - H_2^{\parallel} = I_f^s$$ and, if there are no free surface currents,

$$\underline{H_1^{\parallel} = H_2^{\parallel}}$$

for a boundary between two magnetizable materials with no free currents

$$\left.\begin{array}{ll} B^{\perp} & (\text{'B normal'}) \\ H^{\parallel} & (\text{'H tangential'}) \end{array}\right\} \text{continuous}$$

6. <u>Magnetic scalar potential</u>

and no time dependence

<u>IF</u> $J_f = 0$, $\text{curl}\, \underline{H} = 0$

then, by analogy with V, can define a magnetic scalar potential ϕ by

$$\underline{H} = -\text{grad}\,\phi$$
$$\therefore\ \underline{B} = -\mu\mu_0\,\text{grad}\,\phi$$

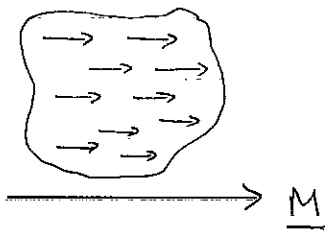
$\text{div}\,\underline{H} = 0$ $\qquad\therefore\quad \nabla^2\phi = 0$

and can use the usual Laplace equation formalism.

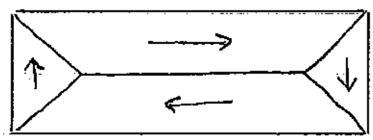
7. Ferromagnets

(i) microscopic picture

Atomic dipoles want to align because of the quantum mechanical exchange interaction : short range and strong locally a small ferromagnetic particle looks like



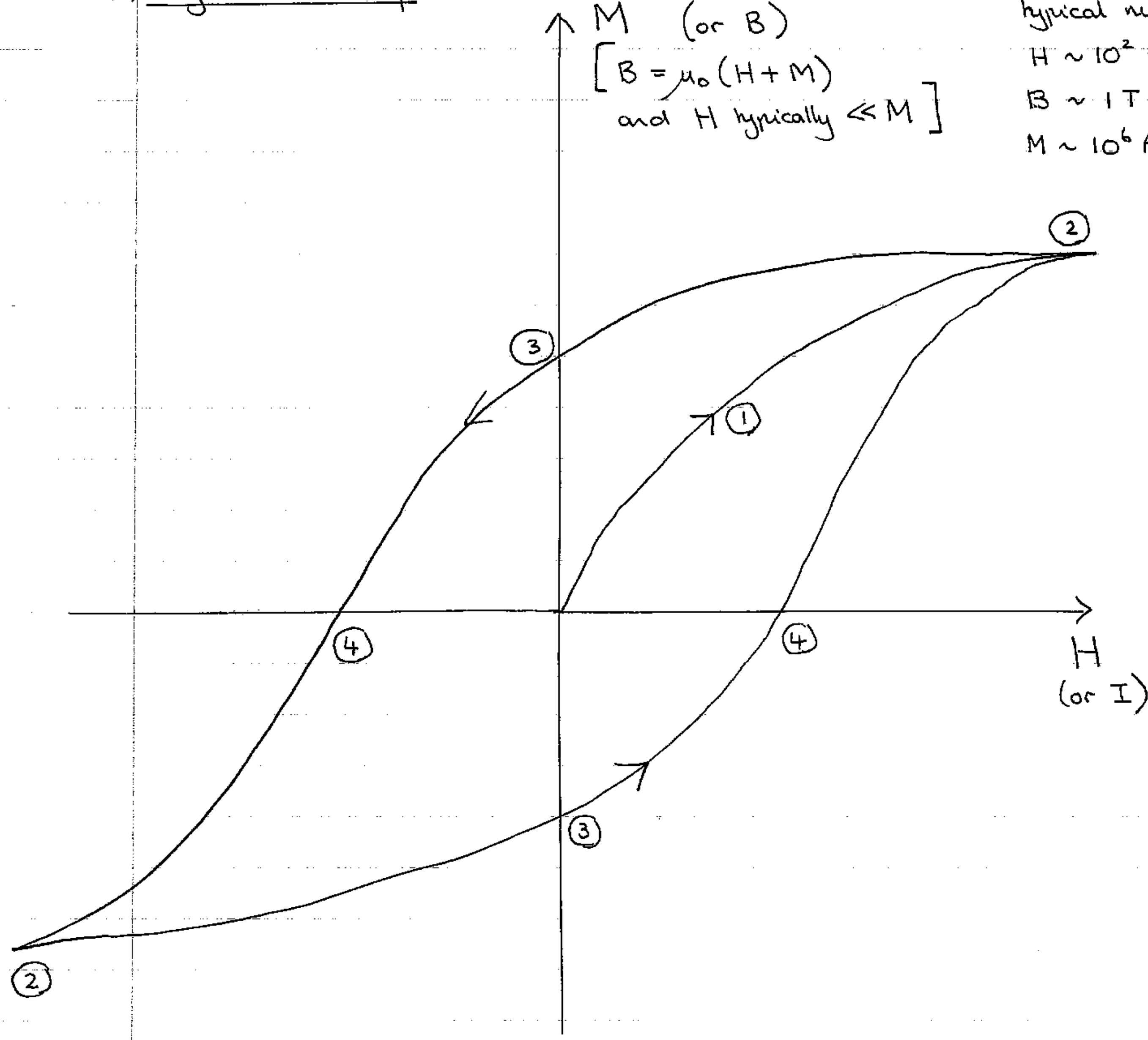
but there is also the dipolar interaction : weak, but long range. to minimise their dipolar energy the atomic dipoles form domains with $\underline{M}$ in different directions



for the whole sample $\underline{M} = 0$

In an applied field the domains along the field grow at the expense of those not $\parallel$ to the field.

(ii) Hysteresis loop



$[B = \mu_0(H+M)$
and H typically $\ll M]$

typical numbers
$H \sim 10^2\ \mathrm{A\,m^{-1}}$
$B \sim 1\ \mathrm{T}$
$M \sim 10^6\ \mathrm{A\,m^{-1}}$

(1) H increases from zero; domains tend to align along field

(2) all domains aligned SATURATION

(3) $M \neq 0$ even when H returned to zero REMANENCE
useful for magnetic memories

(4) field in opposite direction needed to reduce M to zero
COERCIVE FORCE

NB (i) called hysteresis loop because M depends not only on H but also on the history of the sample - memory effects

(ii) example of a non-linear constitutive relation $\quad B$ not $\propto H$

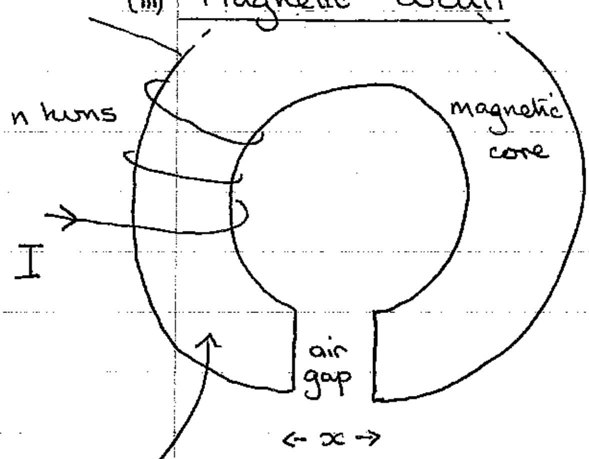
how B depends on H

($B = \mu_0\mu H$ still written but μ depends on H)

(iii) HARD material; large remanence, large coercive force; hard to move domain walls; useful for permanent magnets

(iv) SOFT material; small remanence, small coercive force; easy to move domain walls; useful for electromagnets, transformers, motors.

(iii) Magnetic circuit



field lines loop around the core to minimise energy - they prefer to be in the magnetic material

directions of $\underline{B}, \underline{H}, \underline{M}$? - see problem set 2

simple design of an electromagnet

length of path in core $\quad l$
length of gap $\quad x$
fields in core $\quad H_c \; ; \; B_c$
fields in gap $\quad H_g \; ; \; B_g$

"4 equations"

1. Ampère's law
$$H_c l + H_g x = nI$$
2. $B^{\perp}$ continuous
$$B_c = B_g$$
3. constitutive relation in gap
$$B_g = \mu_0 H_g$$
4. constitutive relation in material
$$B = f(H)$$
e.g. $B = \mu\mu_0 H$ if linear material
hysteresis loop if ferromagnet