# II STEADY CURRENTS AND MAGNETISM

## B. MAGNETIC MULTIPOLES

1. magnetic vector potential $\underline{A}$
2. summary of formulae for $\underline{B}$, $\underline{A}$, $\underline{J}$
3. multipole expansion of $\underline{A}$ and the magnetic dipole

# B. MAGNETIC MULTIPOLES

## 1. Magnetic vector potential $\underline{A}$

$\text{div}\,\underline{B} = 0$

$\therefore$ can write $\underline{B} = \text{curl}\,\underline{A}$

$\underline{A}$ is not uniquely defined — can add any function with zero curl to it

one convenient choice is to ensure $\text{div}\,\underline{A} = 0$

How do I calculate $\underline{A}$ from $\underline{J}$?

$$\text{curl}\,\underline{B} = \mu_0 \underline{J} = \text{curl}\,\text{curl}\,\underline{A} = \text{grad}\,\text{div}\,\underline{A} - \nabla^2 \underline{A}$$

Ampère's law

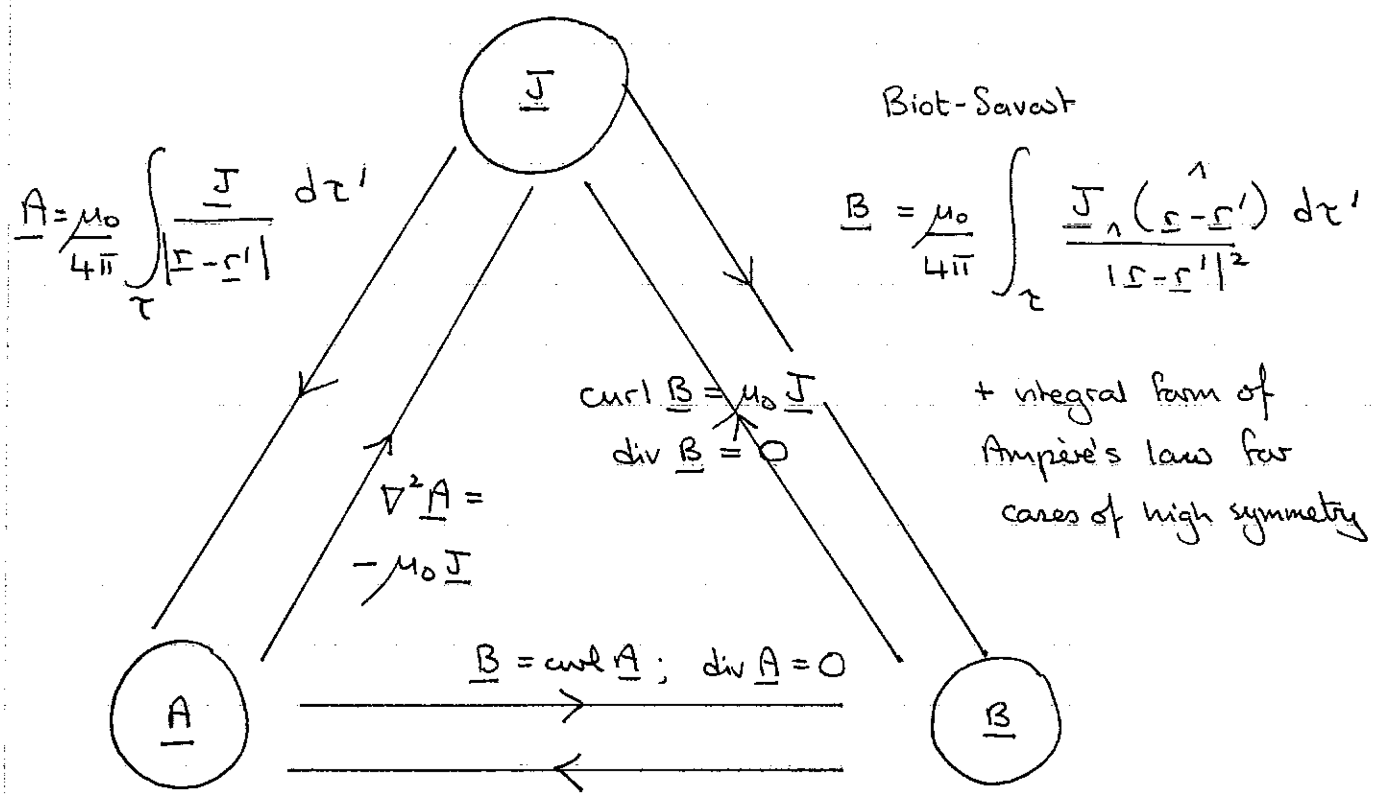
zero by construction

$$\therefore \nabla^2 \underline{A} = -\mu_0 \underline{J} \qquad \text{compare} \qquad \nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

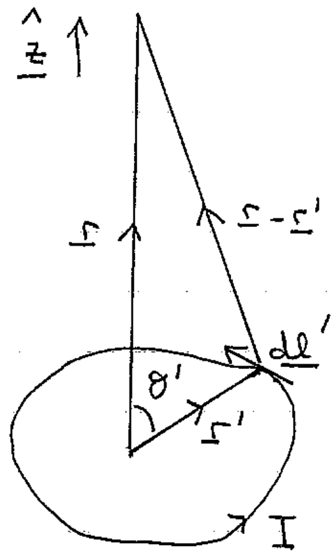
$$\Downarrow$$

$$\underline{A} = \frac{\mu_0}{4\pi}\int_\tau \frac{\underline{J}}{|\underline{r}-\underline{r}'|}\,d\tau' \Leftarrow \qquad V = \frac{1}{4\pi\varepsilon_0}\int_\tau \frac{\rho}{|\underline{r}-\underline{r}'|}\,d\tau'$$

## 2. Summary of links between $\underline{A}$, $\underline{J}$, $\underline{B}$



## 3. Multipole expansion of $\underline{A}$




recall

$$(\underline{r}-\underline{r}')^{-1} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta')$$

vector potential of a current loop

$$\underline{A}(\underline{r}) = \frac{\mu_0 I}{4\pi}\oint \frac{\underline{d\ell}'}{|\underline{r}-\underline{r}'|}$$

$$= \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\theta')\,\underline{d\ell}'$$

monopole term $\quad n=0$

$$\underline{A}_0(\underline{r}) = \frac{\mu_0 I}{4\pi r}\oint \underline{d\ell}' = \underline{0} \qquad \text{as expected}$$

dipole term $\quad n=1$ $\qquad$ (z-axis along $\underline{r}$)

$$\underline{A}_1(\underline{r}) = \frac{\mu_0 I}{4\pi r^2}\oint r'\cos\theta'\,\underline{d\ell}' = \frac{\mu_0 I}{4\pi r^2}\oint \underline{r}'\cdot\hat{\underline{r}}\;\underline{d\ell}'$$

V8 : with $\underline{a} = \hat{\underline{r}}$

$$= \frac{\mu_0 I}{4\pi r^2}\int_S \underline{dS}'\wedge\hat{\underline{r}} \equiv \frac{\mu_0}{4\pi r^2}\,\underline{m}\wedge\hat{\underline{r}}$$

where $\underline{m} = I\int_S \underline{dS}' = I\,\underline{S}$

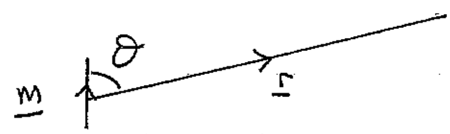
magnetic dipole moment $\qquad$ current $\qquad$ vector area of loop

## to find the dipolar field $\underline{B}_1(\underline{r})$

(N.B. new problem, new co-ordinate system)

$\underline{m}$ along $\hat{\underline{z}}$ ; spherical polars

$$\underline{A}_1(\underline{r}) = \frac{\mu_0}{4\pi r^2}\, \underline{m} \wedge \hat{\underline{r}}$$




$$= \frac{\mu_0 m \sin\theta}{4\pi r^2}\, \hat{\underline{\varphi}}$$

$$\underline{B}_1(\underline{r}) = \text{curl}\ \underline{A}_1(\underline{r})$$

$$= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}(r\sin\theta\, A_\phi)\, \hat{\underline{r}} - \frac{1}{r\sin\theta}\frac{\partial}{\partial r}(r\sin\theta\, A_\phi)\, \hat{\underline{\theta}}$$

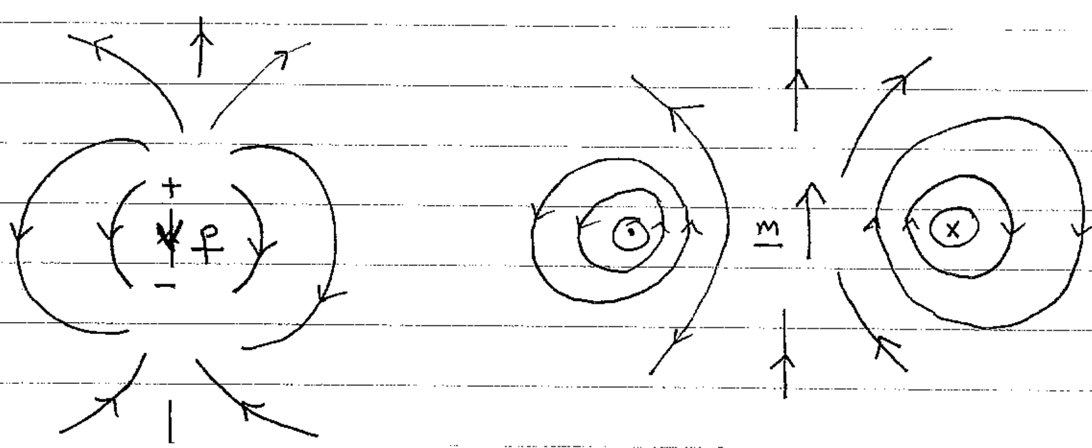
curl in sph. polars

(if $A_r, A_\theta = 0$)

$$= \frac{\mu_0 m}{4\pi}\left( \frac{2\cos\theta}{r^3}\, \hat{\underline{r}} + \frac{\sin\theta}{r^3}\, \hat{\underline{\theta}} \right)$$

same form as electric dipole

(N.B. for a real pair of charges | current loop fields only same at sufficiently large r )



# STEADY CURRENTS AND MAGNETISM

## C. MAGNETIZABLE MATERIALS

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2. bound currents
3. Ampère's law in magnetizable materials and $\underline{H}$
4. linear materials and the relative permeability $\mu$
5. boundary conditions for $\underline{B}$, $\underline{H}$ in a linear medium
6. magnetic scalar potential $\phi$
7. ferromagnets