Field Theory in Condensed Matter   Hilary Term 2012   Prof. J. Cardy

Problem Set 2

Although everyone in the class is welcome to try these problems, only the papers of students in Theoretical Physics will be marked. They should be put in the pigeon hole of Adam Nahum in Theoretical Physics by **Friday March 9 at 4 pm.** They will be reviewed at a Problems Class the following week on **Wednesday March 14, 2-4 pm,** in the **Fisher Room**, which everyone in the class is welcome to attend. Any questions before the due date should be directed to me at `j.cardy1@physics.ox.ac.uk`
Parts of questions marked † may be found slightly harder. However anyone seriously interested in learning QFT should try to tackle them.

1. In the $O(N)$ model, consider the correlation function $\langle\sum_{a=1}^{N}\phi_a(y)^2\,\phi_b(y_1)\phi_b(y_2)\rangle$, and work out which subset of diagrams contribute to this in the large N limit. By summing these diagrams explicitly, work out how the Fourier transform of this correlator depends on the momentum q conjugate to y, when the renormalised mass $m=0$. Hence find the anomalous dimension of the operator $\sum_{a=1}^{N}\phi_a^2$, and show that your result is consistent with the scaling relation relating this to the value for ν we found in class.

2. In the lectures, in computing how the renormalised mass m vanishes as $m_0^2\to m_{0c}^2$, we omitted the borderline cases $d=4$ and $d=2$. Fill in the gaps.

3. In the $O(N)$ model considered in the lectures, show that, at least to 2-loop order, all loop corrections to the 2-point function vanish if we set $N=-2$. †Can you prove it to all orders? Are all the correlation functions trivial?

4. A Brownian particle in a neutral heat bath carries an electric charge q and moves in a uniform magnetic field $\mathbf{B}$. For simplicity you may assume that the particle is confined to two dimensions, and that $\mathbf{B}$ is perpendicular to the motion. Write down the equations of motion including dissipation and noise, and show that the Einstein relation

still holds. Calculate the response and correlation functions (note these are now matrices with components labelled by x and y) and check the fluctuation-dissipation relation.

5. In Problem Set 1 we considered scalar ϕ^3 field theory. Consider the model A dynamics of this theory. Draw the 1-loop corrections to the inverse response function $\Gamma^{(1,1)}(\omega, k)$, write them down as Feynman integrals, and show that when $\omega = 0$ they reduce to the known 1-loop corrections in the static theory. †Compute the 1-loop renormalisation of the diffusion constant in $6 - \epsilon$ dimensions.