Dynamics of stellar discs

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Story so far

- Fluctuations in Φ cause stars to diffuse through action space
- Flux is

$$\begin{aligned}\mathbf{F}(\mathbf{J}) &= \mathbf{F}_1(\mathbf{J}) + \mathbf{F}_2(\mathbf{J}) \\ &= -\mathbf{D}_1(\mathbf{J}) f_0 - \mathbf{D}_2(\mathbf{J}) \cdot \frac{\partial f_0}{\partial \mathbf{J}}\end{aligned}$$

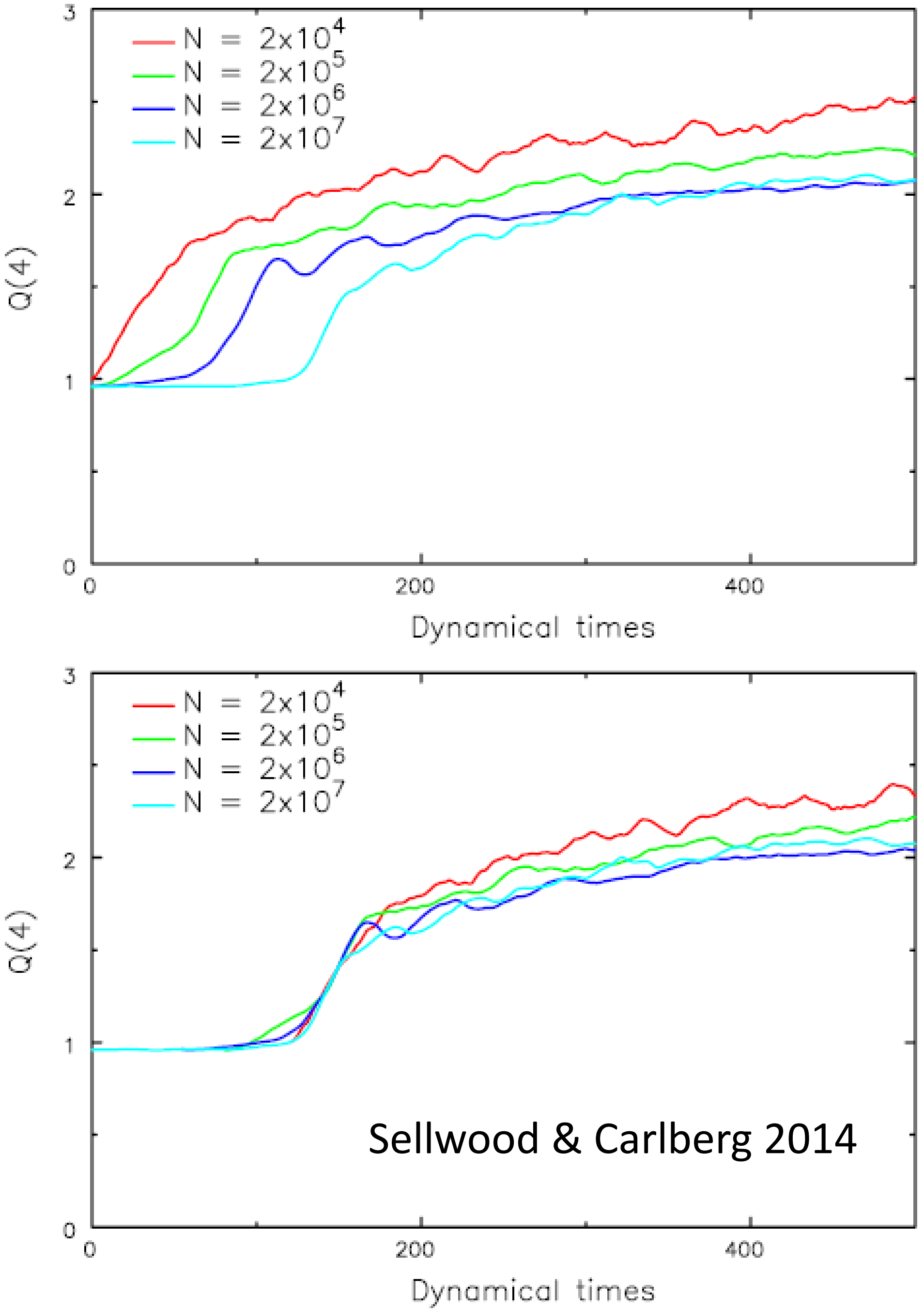
$$\mathbf{D}_1(\mathbf{J}) = -\tfrac{1}{2}(2\pi)^4 m \sum_{\mathbf{n}\mathbf{n}'} \int \mathrm{d}^3\mathbf{J}' \left|E_{\mathbf{n}\mathbf{n}'}(\mathbf{J}, \mathbf{J}', -\mathrm{i}\mathbf{n} \cdot \mathbf{\Omega}_0)\right|^2 \mathbf{n}' \cdot \frac{\partial f_0}{\partial \mathbf{J}'} \delta(\mathbf{n}' \cdot \mathbf{\Omega}_0' - \mathbf{n} \cdot \mathbf{\Omega}_0)\, \mathbf{n}$$

$$\mathbf{D}_2(\mathbf{J}) = \tfrac{1}{2}(2\pi)^4 m \sum_{\mathbf{n}\mathbf{n}'} \int \mathrm{d}^3\mathbf{J}' \left|E_{\mathbf{n}\mathbf{n}'}(\mathbf{J}, \mathbf{J}', -\mathrm{i}\mathbf{n} \cdot \mathbf{\Omega}_0)\right|^2 f_0(\mathbf{J}') \delta(\mathbf{n}' \cdot \mathbf{\Omega}_0' - \mathbf{n} \cdot \mathbf{\Omega}_0)\, \mathbf{n} \otimes \mathbf{n}$$

- So star at J is disturbed by stars at J’ that resonate with it
- D_1 drives stars back towards low J and low E while D_2 causes them to diffuse to high J and high E
- Einstein first recognised the need for the “dynamical friction term” D_1 in his model of Brownian motion

N-body discs

- “Stable” discs always develop O(1) spiral structure/a bar
- More particles -> it takes longer
- But duration of non-linear phase is independent of N

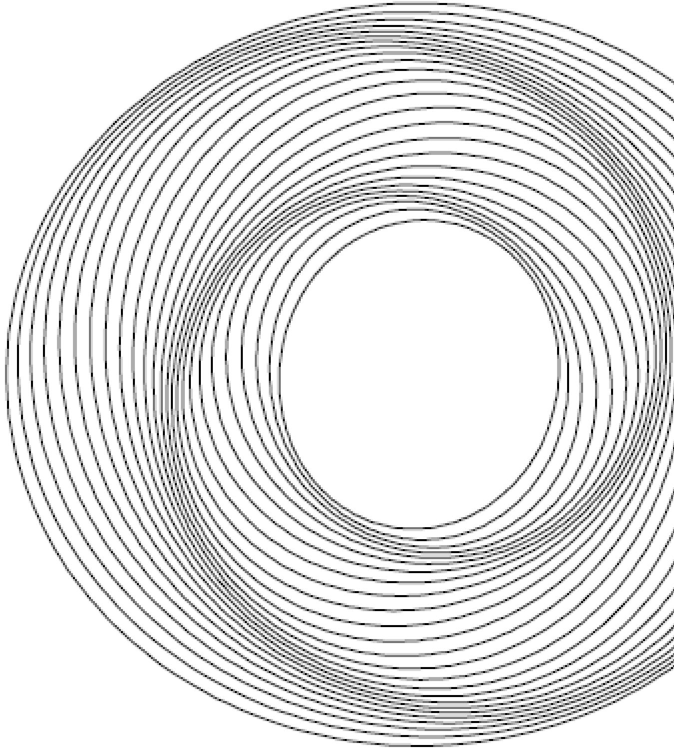


Wave mechanics of discs

- Self gravity couples orbits so they can exchange angular momentum
- Disc becomes elastic medium that supports waves
- Study of these waves easiest when tightly wound
- Then

$$\Sigma = H(R)e^{i[m\phi + kR]} \quad \Rightarrow \quad \Phi = -\frac{2\pi G}{|k|}H(R)e^{i[m\phi + kR]}$$

- Obtain local relations



(b)

Binney & Tremaine, Galactic Dynamics

Dispersion relation WKB waves

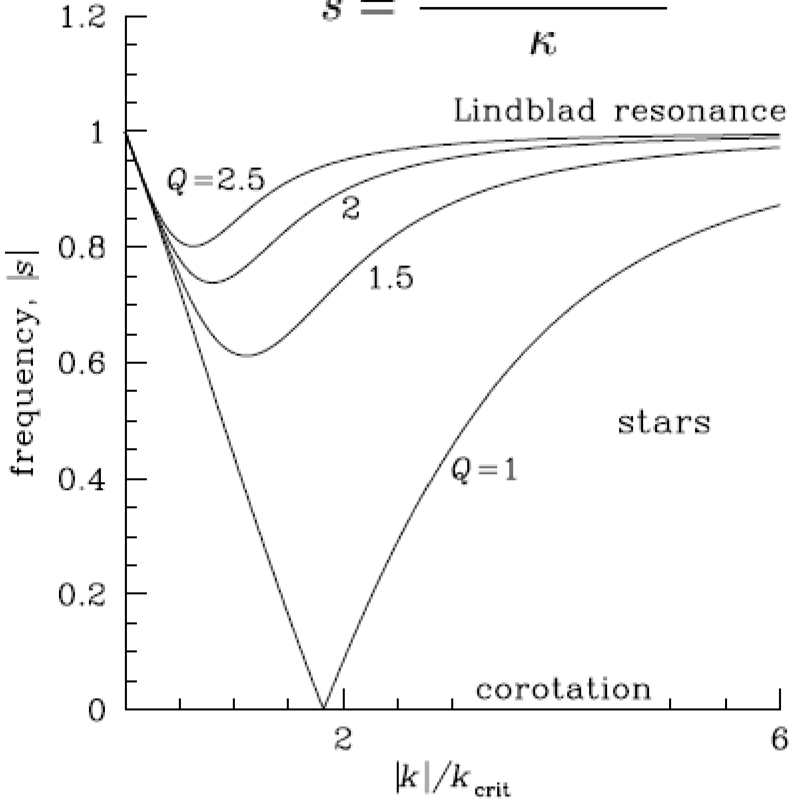
$$Q = \frac{\sigma_R \kappa}{3.36 G\Sigma}$$

- Dispersion reln same for leading (k<0) or trailing waves
- Disc stable to axisymmetric disturbance for $k<k_{crit}$ even at Q=0

$$X = \frac{k_{\rm crit} R}{m} = \frac{\kappa^2 R}{2\pi G \Sigma m}$$

- Waves excluded from a region around corotation
- Excluded region grows with Q
- Long and short branches
  - but long branch of doubtful utility

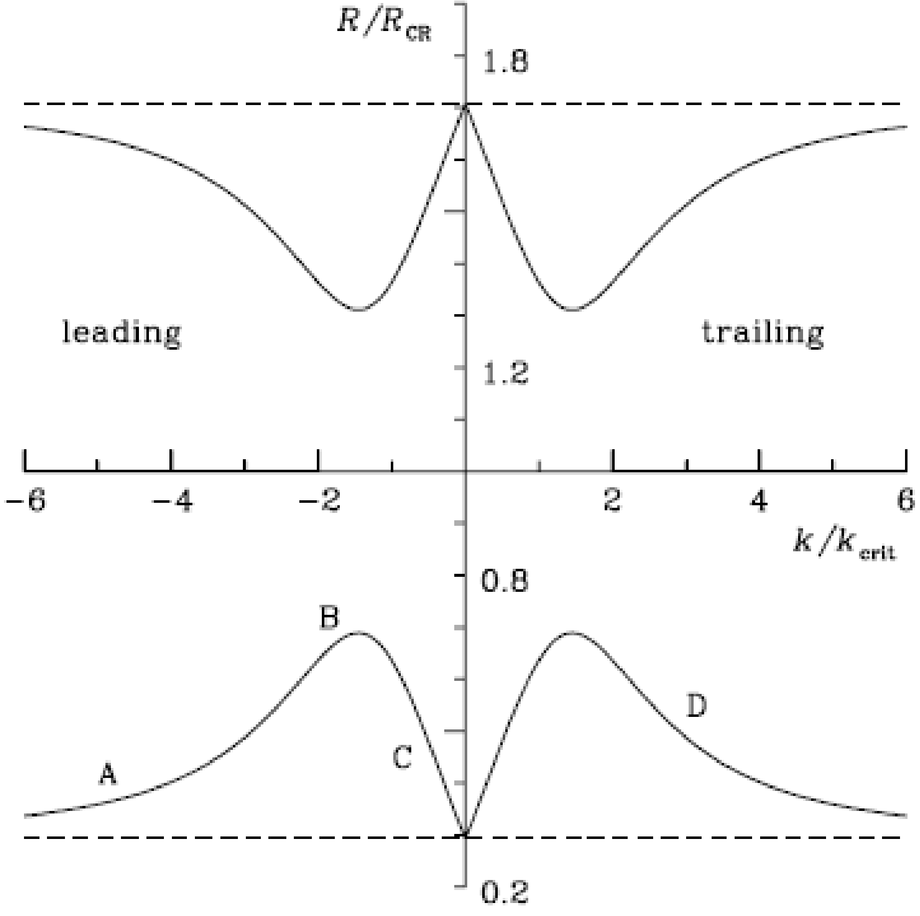
$$s = \frac{m(\Omega_p - \Omega)}{\kappa}$$



Binney & Tremaine, Galactic Dynamics

Group velocity

- Tightly wound leading wave packet moves towards CR
- |k| decreases as it goes (unwinds)
- At edge of forbidden region mathematically V_{group} reverses as it transfers to long branch
- Actually WKB approx. breaks down and it's "swing amplified" to more powerful short trailing wave
- Short trailing wave moves away from CR towards Lindblad resonance
    - |k| -> infty as resonance approached
- Near LR wave resonantly absorbed (Landau damped)



Binney & Tremaine, Galactic Dynamics

Swing amplifier (Julian & Toomre 1966)

- Near CR waves cannot be handled by WKB because not tightly wound
- Studied with "shearing sheet"

$$\mathcal{L} = \tfrac{1}{2}[\dot{x}^2 + (R_\mathrm{g} + x)^2 (\dot{y}/R_\mathrm{g} + \Omega_\mathrm{g})^2] - \Phi(R_\mathrm{g} + x)$$

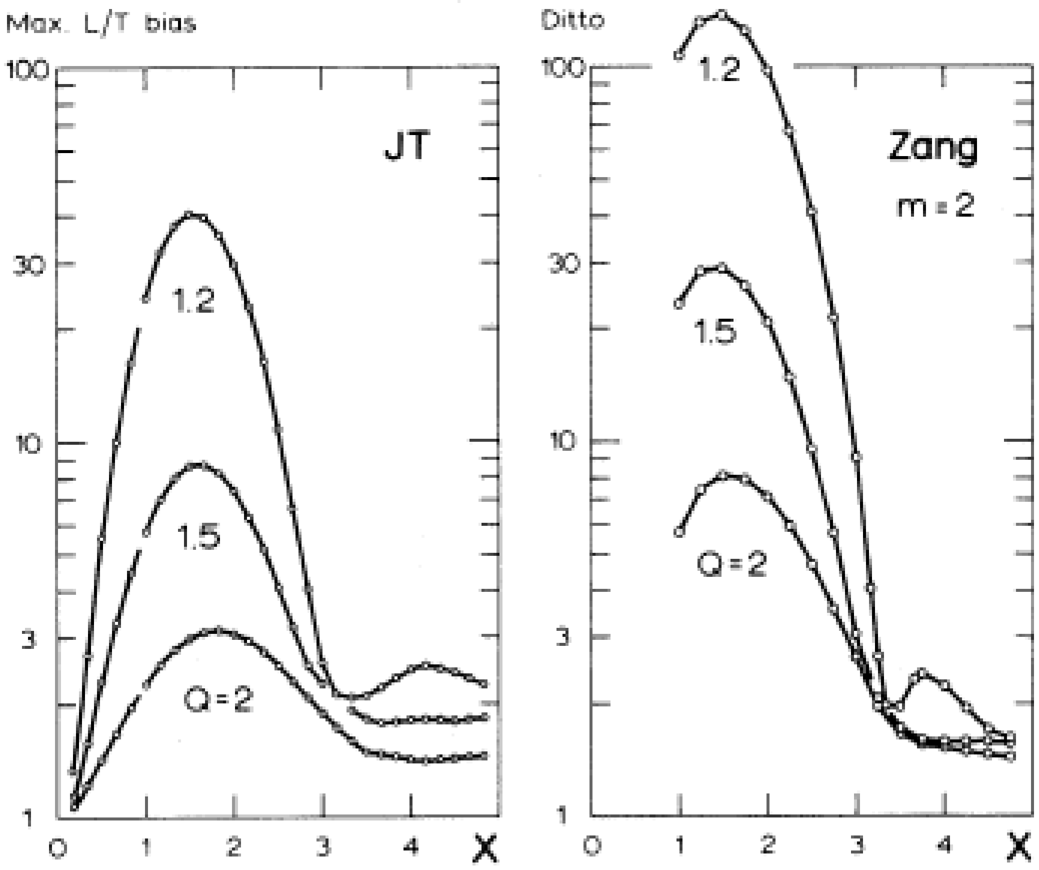
$$H = \tfrac{1}{2}\left(p_x^2 + \frac{p_y^2}{(1 + x/R_\mathrm{g})^2}\right) - \Omega_\mathrm{g} R_\mathrm{g} p_y + \Phi$$

- P_y=const controls mean value of x, which oscillates harmonically
- JT impose $\Sigma(x,y) = \exp(\mathrm{i}[k_x x + k_y y]) \quad \Rightarrow \quad \Phi(x,y) = \dfrac{\Sigma(x,y)}{|k|}$

JT66 continued

- As shear swings wave from leading to trailing, gravity strongly amplifies it
- Then it runs to a LR and there Landau damps

$$X = \frac{k_{\rm crit}R}{m} = \frac{\kappa^2 R}{2\pi G\Sigma m}$$



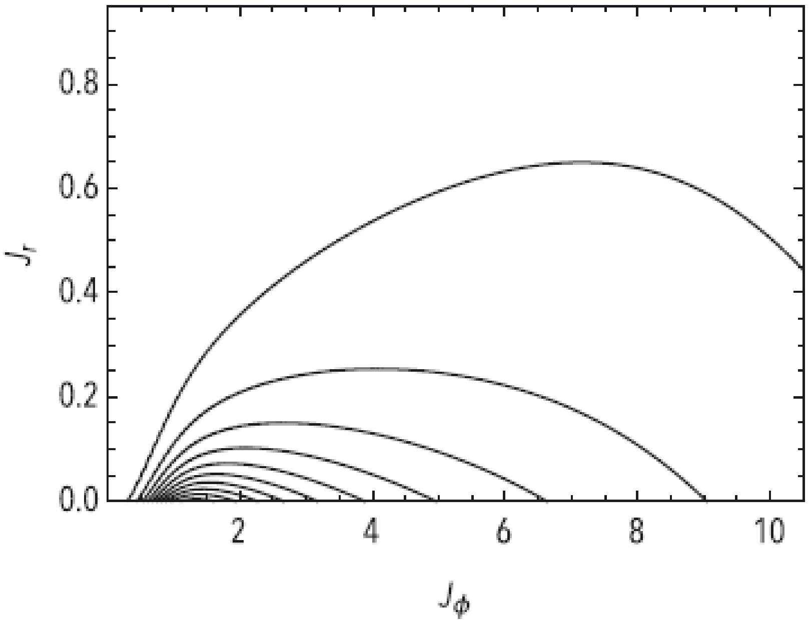
Balecu-Lenard for discs

Fouvry et al 2015 A&A 584, A129

- Need to compute $E_{nn}(J,J',p)$ using basis functions that can represent both tightly wound & loosely wound disturbances
- Considered only strictly planar discs – currently technically too difficult to treat the 3d case

Model disc

- Assume unperturbed Φ = v_0^2 ln(R)
  - generates constant circular speed v_0
- A DF $f_0 = C\,J_\phi^{\,q} \exp(-E/\sigma^2)$ with $q = v_0^2/\sigma^2-1$ exactly generates the required surface density
- But taper this DF at small R (gravity of the bulge) and at large R (gravity of the dark halo)
- Also in the middle use only ξ times the full DF because the dark halo contributes significant gravity at all R



$$T_{\rm inner}(J_\phi) = \frac{J_\phi^{\nu_{\rm i}}}{(R_{\rm i}V_0)^{\nu_{\rm i}} + J_\phi^{\nu_{\rm i}}},$$

$$T_{\rm outer}(J_\phi) = \left[1 + \left[\frac{J_\phi}{R_0 V_0}\right]^{\mu_{\rm t}}\right]^{-1}$$

Implementation

- Choose a system of orthogonal potential-density pairs

$$\Phi^{\alpha}(r,\phi) = \mathrm{e}^{\mathrm{i}l\phi}\,\Phi_n^l(r) \qquad \rho^{\alpha}(r,\phi) = \mathrm{e}^{\mathrm{i}l\phi}\,\rho_n^l(r).$$

- Φ^l_n a polynomial and ρ^l_n a polynomial times half power of $1 - r^2/r_0^2$
- Compute their form in AA coordinates

$$\hat{\Phi}^{(\alpha)}(\mathbf{n},\mathbf{J}) = \delta_{\alpha_2,n_2}\frac{1}{\pi}\int_{r_\mathrm{p}}^{r_\mathrm{a}} \mathrm{d}r\,\Phi_n^l(r)\cos[n_1\theta_1 + n_2(\theta_2 - \phi)].$$

- Compute

$$\epsilon_{\alpha\alpha'}(p) \equiv \delta_{\alpha\alpha'} + \frac{(2\pi)^3}{\mathcal{E}}\mathrm{i}\int \mathrm{d}^3\mathbf{J}\,\sum_{\mathbf{n}} \frac{\mathbf{n}\cdot\frac{\partial f_0}{\partial \mathbf{J}}}{p + \mathrm{i}\mathbf{n}\cdot\mathbf{\Omega}_0}[\hat{\Phi}^{(\alpha)}(\mathbf{n},\mathbf{J})]^*\hat{\Phi}^{(\alpha')}(\mathbf{n},\mathbf{J})$$

- Hence compute $E_{nn'}$

Compute D_1 and D_2

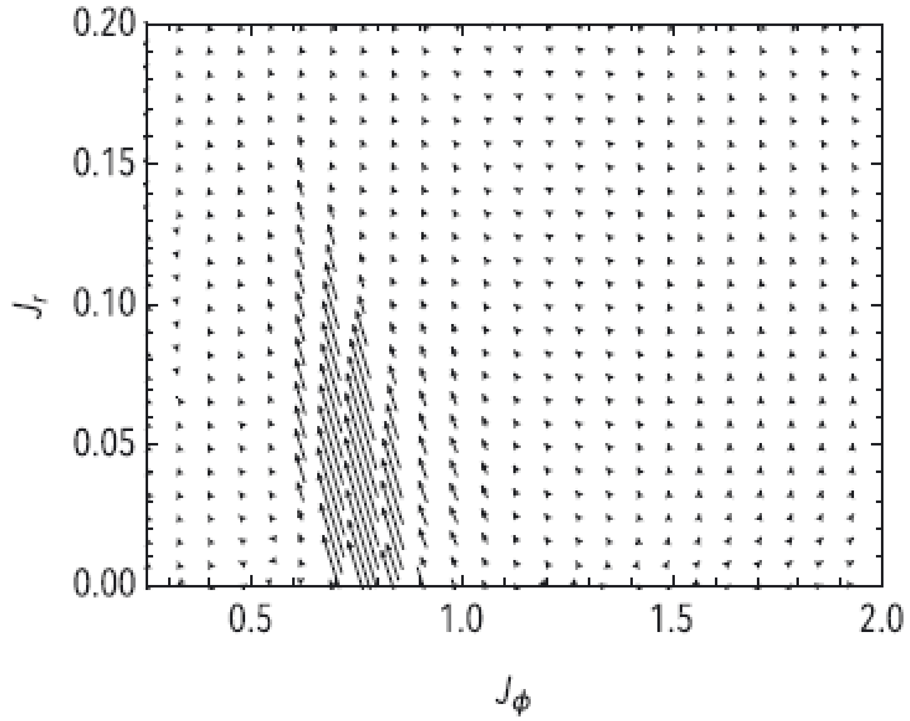
- Costly because for each (n,J) have to find resonant (n’,J’) – they lie along a line in J space on which n’O’ is constant
- Number of vectors n’ for which resonance is possible increases rapidly with |n|

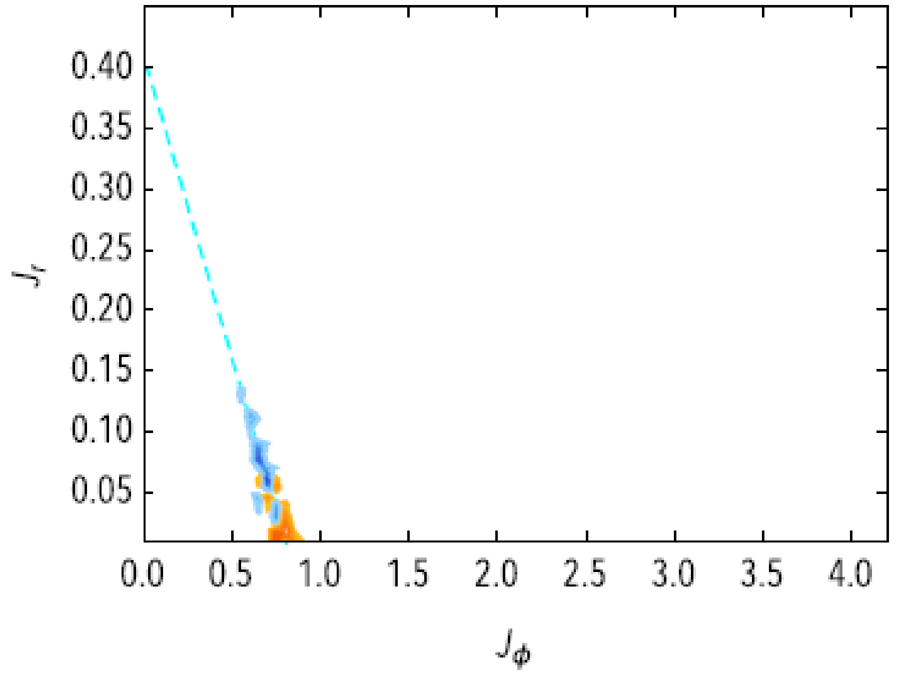
$$\mathbf{D}_1(\mathbf{J}) = -\tfrac{1}{2}(2\pi)^4 m \sum_{\mathbf{n}\mathbf{n}'} \int \mathrm{d}^3\mathbf{J}' \left| E_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',-\mathrm{i}\mathbf{n}\cdot\mathbf{\Omega}_0)\right|^2 \mathbf{n}'\cdot\frac{\partial f_0}{\partial \mathbf{J}'}\delta(\mathbf{n}'\cdot\mathbf{\Omega}_0' - \mathbf{n}\cdot\mathbf{\Omega}_0)\,\mathbf{n}$$

Results

- Obtain F and div F strongly concentrated along sloping line

$$2\Omega_\phi - \Omega_r = \text{constant} \equiv 2\omega_\mathrm{p}$$





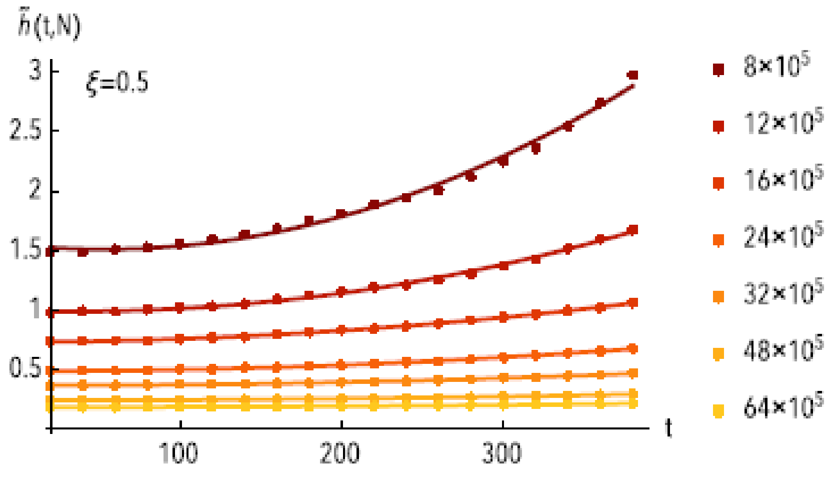
$$\tilde{h}_0(N) = \int \mathrm{d}\boldsymbol{J} \, \left\langle [F - \langle F_0 \rangle]^2 \right\rangle ,$$

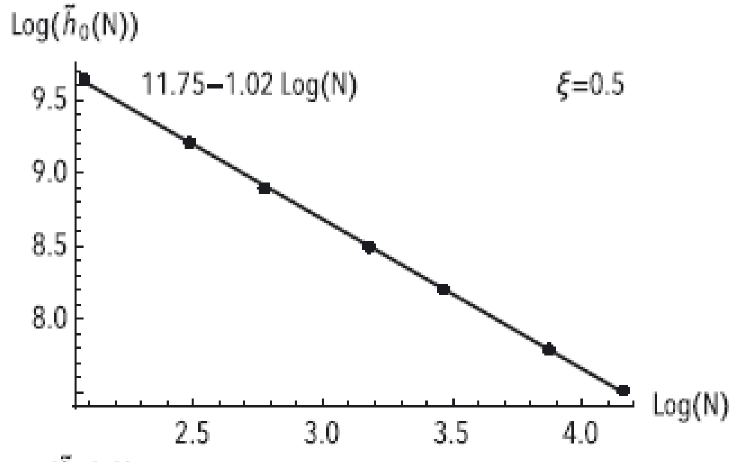
$$\tilde{h}_1(N) = 2 \int \mathrm{d}\boldsymbol{J} \, \langle [F - \langle F_0 \rangle] \, F' \rangle$$

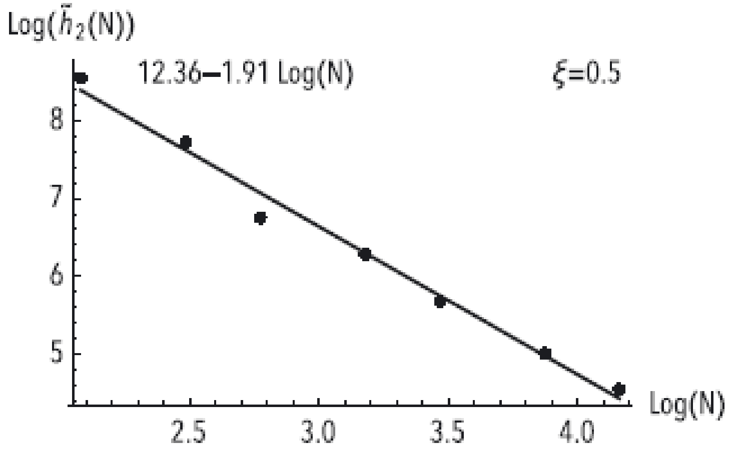
$$\tilde{h}_2(N) = 2 \int \mathrm{d}\boldsymbol{J} \, \left\langle [F']^2 + [F - \langle F_0 \rangle] \, F'' \right\rangle$$

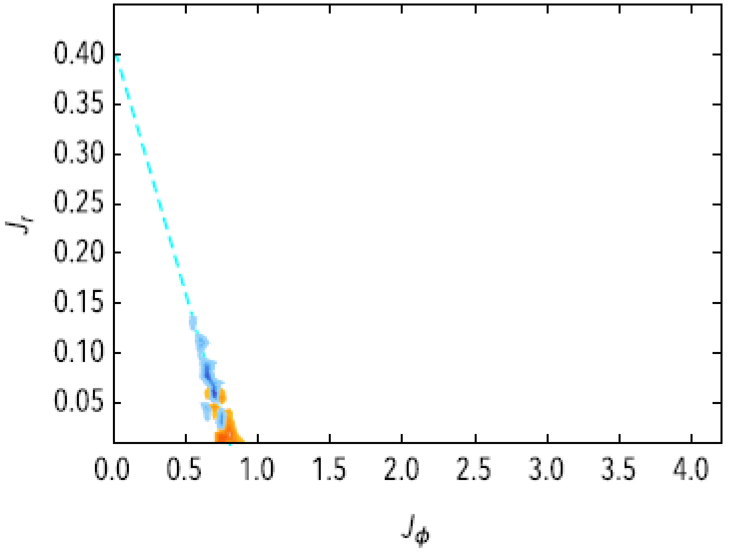
Comparison with N bodies

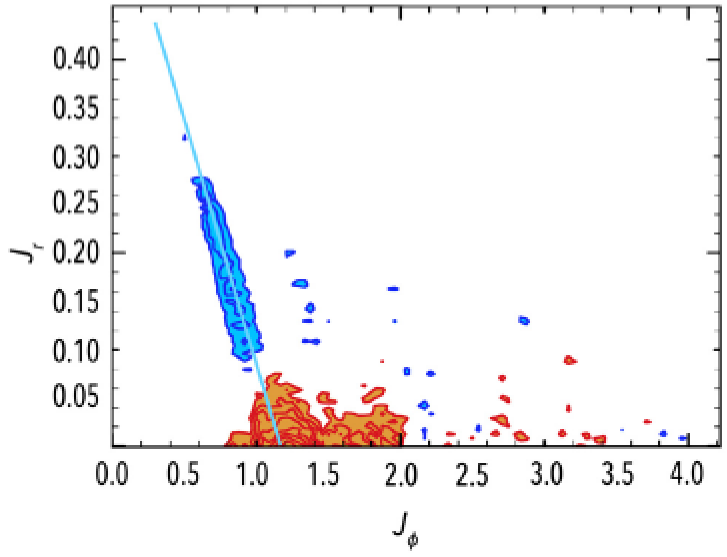
- Compute $h(t) = \int d^2J\, [f(J,t) - f(J,0)]^2$
- Fit to quadratic in t: $h(t) = h_0 + h_1 t + h_2 t^2/2$
- Explore dependence on N and ξ
- $h(\xi{=}0.6)/h(\xi{=}0.5)$ = 29(NB) or 42(BL)
- N-body noise >1000 times as loud as Spitzer-Chandrasekhar predicted because particles dressed



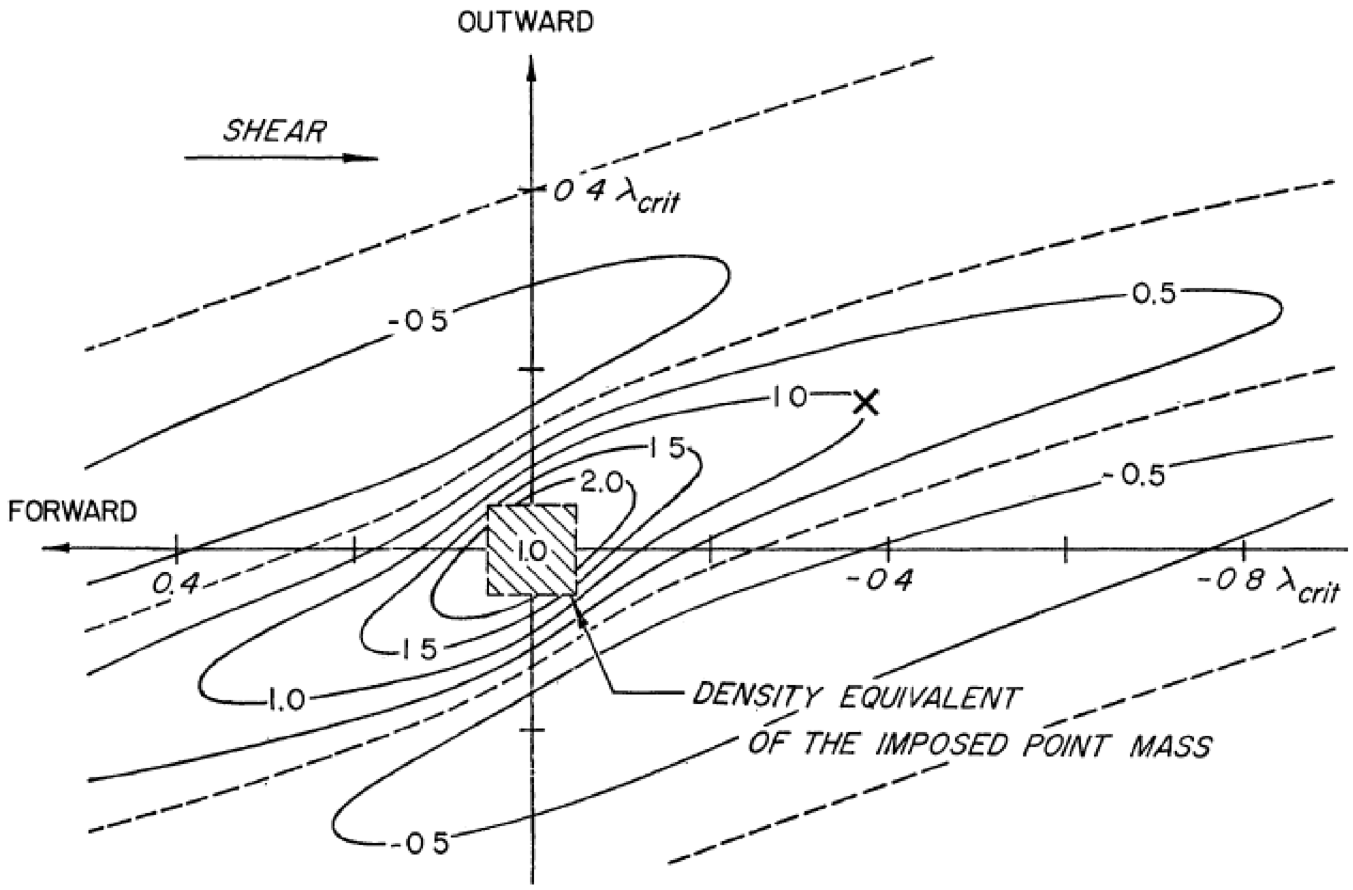








“Debye sphere” of a mass in a disc
Julian & Toomre 1966 ApJ 146 810

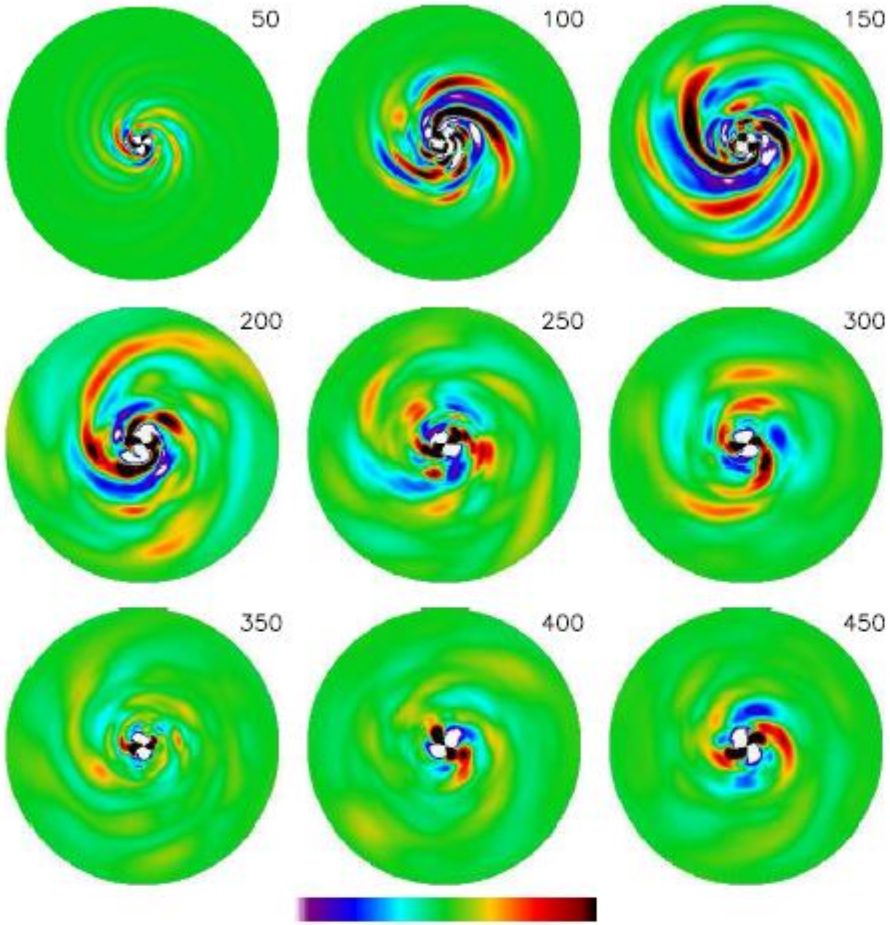


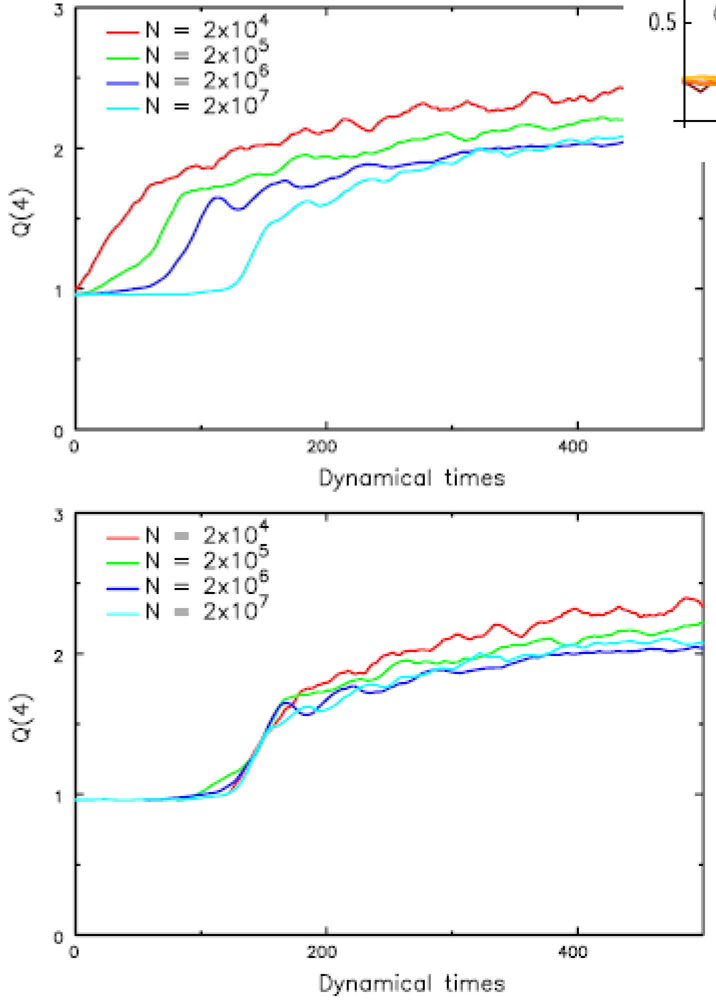
Q = 1.4 v_c = const

Consequences of resonant heating

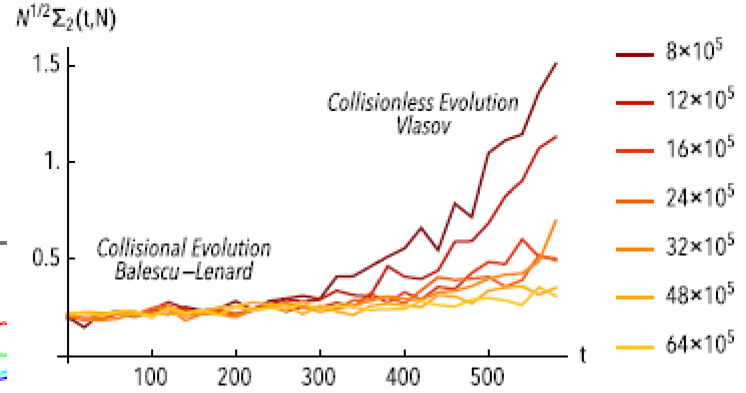
(Sellwood & Carlberg 2014 ApJ 785, 137)

- Initial conditions generate leading wave, amplified and absorbed at its LR (what Fouvry et al compute)
- Later noise generates an amplified trailing wave that approaches its LR, which lies inside ILR of first wave
- The feature in the DF generated by resonant absorption of the first wave is too narrow for the WKBJ approx to hold
- So feature reflects back to CR some of the second wave
- There the reflected portion re-amplified
- Eventually all wave E absorbed at LR
- So the feature generated in DF at LR of 2nd wave stronger than the feature at ILR of first wave
- Second feature is an even more effectively silvered mirror!
- Soon the disc is an effective laser in which favoured modes grow exponentially
- The Poisson noise has made the disc unstable at a *collisionless* level

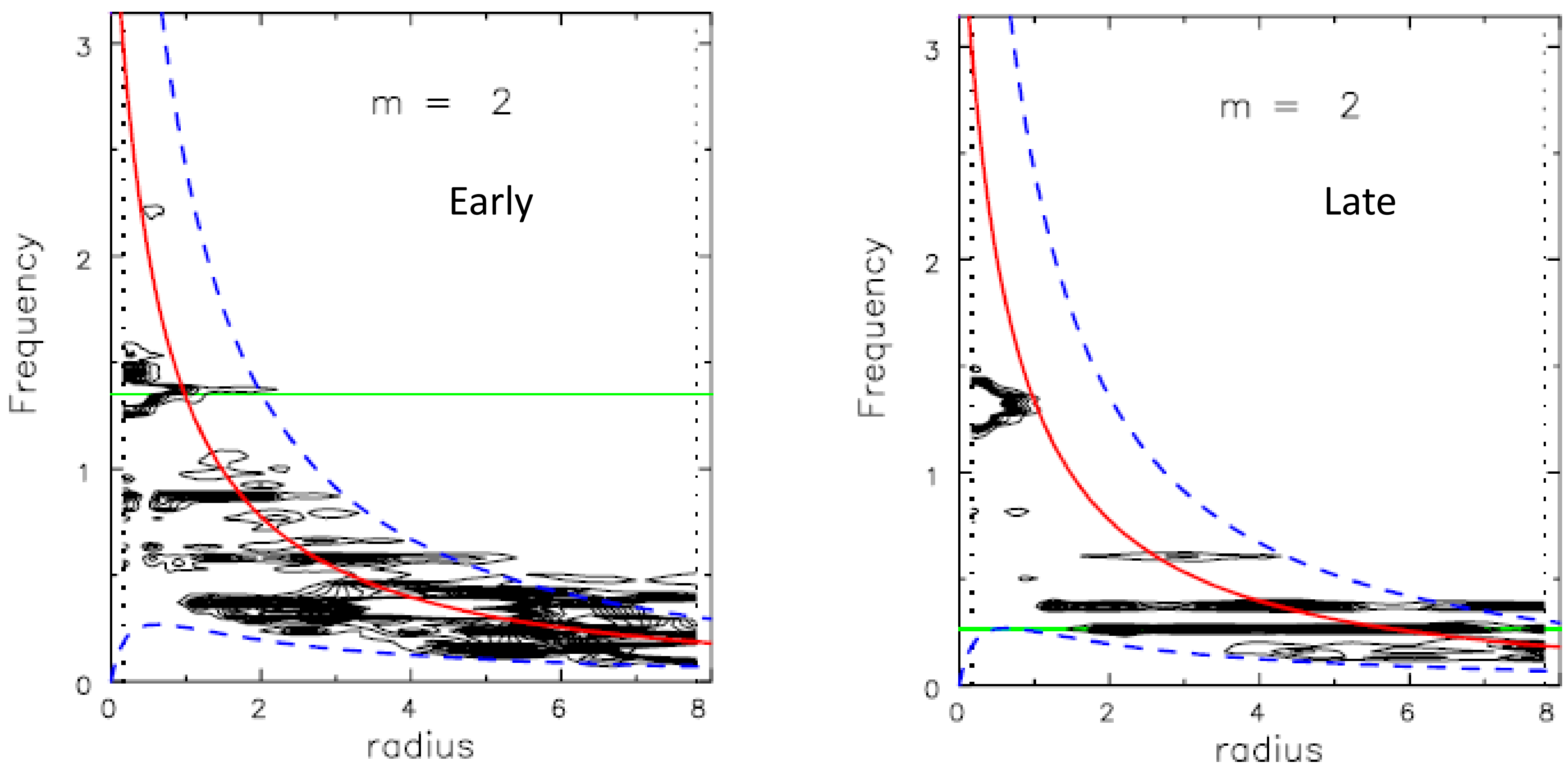




Sellwood & Carlberg 2014



Fouvry et al 2015



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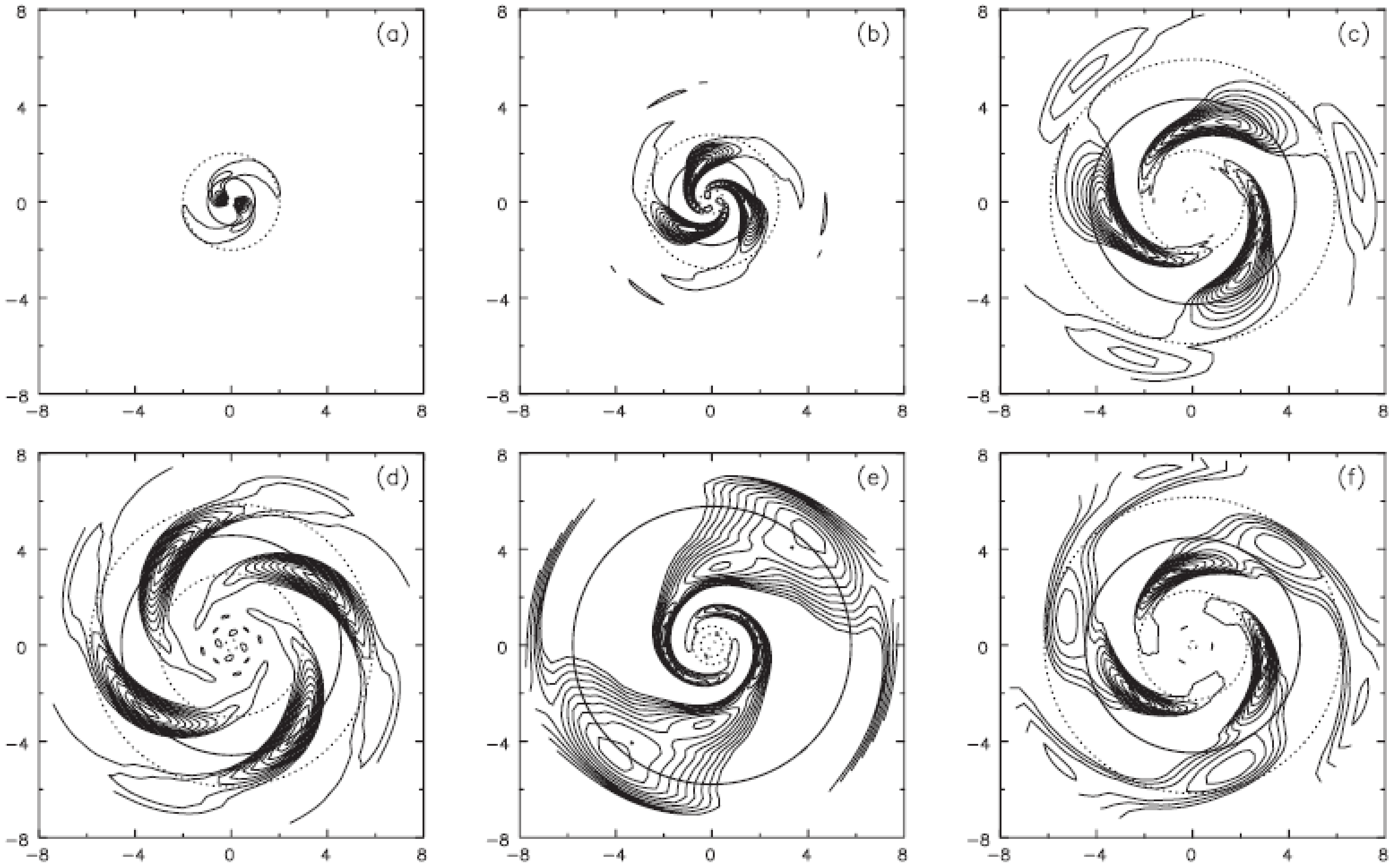

Sellwood & Carlberg 2014

Summary

- Evolution of mean-field model unambiguously driven by randomy excited global oscillations
- Poisson noise is important even with 10^8 particles
  - Because in a cool disc noise is strongly swing amplified
- WKB approx. is useful near LRs but seriously misleading near CR
  - Real excitations are concentrated in WKB-forbidden region around CR
- Landau damping dumps E of excitations very locally
  - Leads to WKB breakdown even near LRs so waves partially reflected
- Noise manoeuvres disc into state that's unstable at collisionless level
- The BL eqn is hard to implement
  - its value is as guide to physics of relaxation