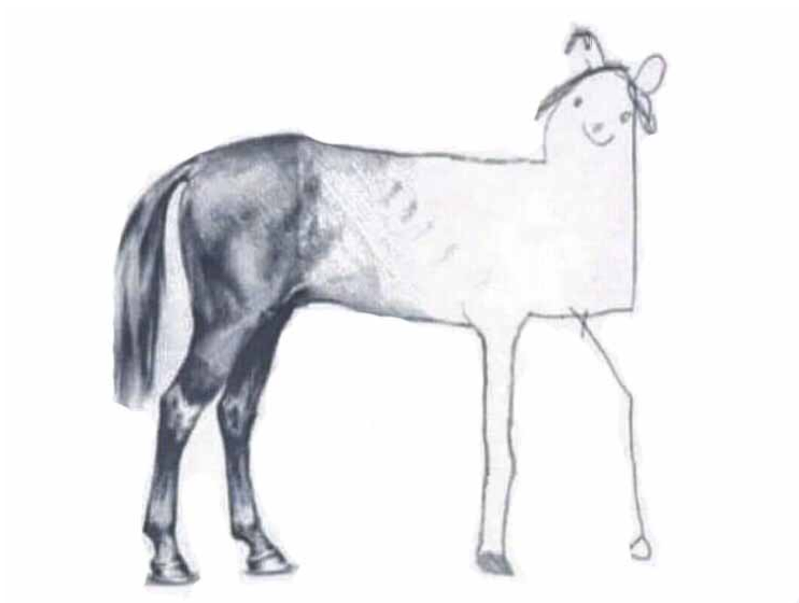
Can 4-derivative gravity make physical sense, despite having a ghost?

A) Introduction. B) Negative energy? C) Negative norm, positive energy? D) IR enhancements.



Alessandro Strumia, Pisa University, webinar at Oxford, 29/10/2020.

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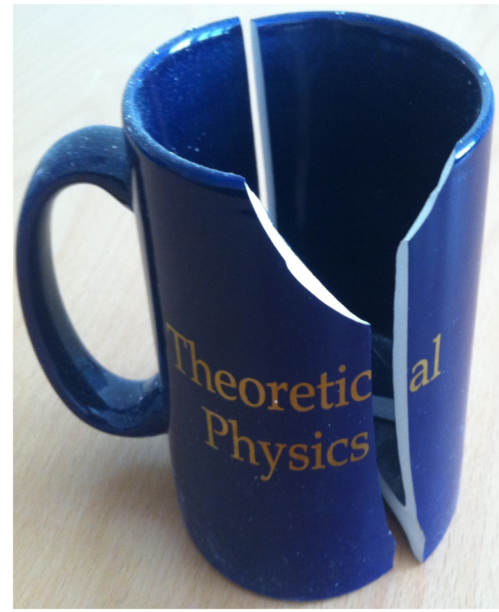
Motivation: would give a theory of quantum gravity

To get a theory of quantum gravity, write the most generic renormalizable Lagrangian with the graviton $g_{\mu\nu}$: it has dimension 0 so 4 derivatives. $R^2_{\dots}$ generated by loops, even starting from Einstein:

$$S = \int d^4x \sqrt{|\det g|}\left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{1}{2}\bar{M}_{\rm Pl}^2 R + \mathscr{L}_{\rm matter}\right]$$



Two gauge-like dimension-less constants $f_{0,2}$. Spectrum: extra states with spin 0 and 2 and masses $M_{0,2} \sim f_{0,2} M_{\rm Pl}$. Einstein gravity at lower energy. Ghost with mass M_2 key to get renormalizable quantum gravity:

$$\left(\begin{array}{c}\text{4-derivative}\\ \text{graviton propagator}\end{array}\right) = \frac{1}{k^4 - M_2^2 k^2} = \frac{1}{M_2^2}\Big[\underbrace{\frac{1}{k^2}}_{\text{graviton } g} - \underbrace{\frac{1}{k^2 - M_2^2}}_{\text{ghost } g_2}\Big] \overset{?}{=}$$

'Ghosts' are avoided like a plague by theorists and explored only by [redacted] such as Dirac, Pauli, Heisenberg, Pais, Uhlenbeck, Lee, Wick, Cutkosky, Coleman, Feynman, Hawking...

1403.4226, 1502.01334, 1512.01237, 1705.03896, 1709.04925, 1808.07883, 2007.05541. Good but not discussed: inflation, h, asymptotic freedom? Related ideas by Bender, Mannheim, Anselmi et al.

Ostrogradski classical no go

Gravity $g_{\mu\nu}(x,t) \supset$ QFT $\phi(x,t) \approx \int_p$ modes... so focus on one mode $q(t)$ with 4 time derivatives

$$\mathscr{L} = -\frac{1}{2} q \left(\frac{d^2}{dt^2} + \omega_1^2\right)\left(\frac{d^2}{dt^2} + \omega_2^2\right) q - V(q)$$

Describe canonically one 4-derivative $q(t)$ as two 2-derivative $q_{1,2}(t)$:

$$\begin{cases} q_1 = q, & p_1 = \dfrac{\delta S}{\delta \dot{q}_1} = (\omega_1^2 + \omega_2^2)\dot{q} + \dddot{q}, \\ q_2 = \dot{q}, & p_2 = \dfrac{\delta S}{\delta \dot{q}_2} = -\ddot{q} \end{cases}$$

The Hamiltonian is unbounded from below

$$H = \sum_{i=1}^{2} p_i \dot{q}_i - \mathscr{L} = p_1 q_2 - \frac{\lambda^2}{2} p_2^2 - \frac{\omega_1^2 + \omega_2^2}{2} q_2^2 + \frac{\omega_1^2 \omega_2^2}{2} q_1^2 + V(q_1).$$

(More than 2 time derivatives) $\Rightarrow$ (classical energy down to $-\infty$).

Classical free solution is ok, but interactions can give run-away evolution

B) Is negative kinetic energy hopeless?

Or maybe it's similar to negative potential energy:
meta-stability up to cosmological times?

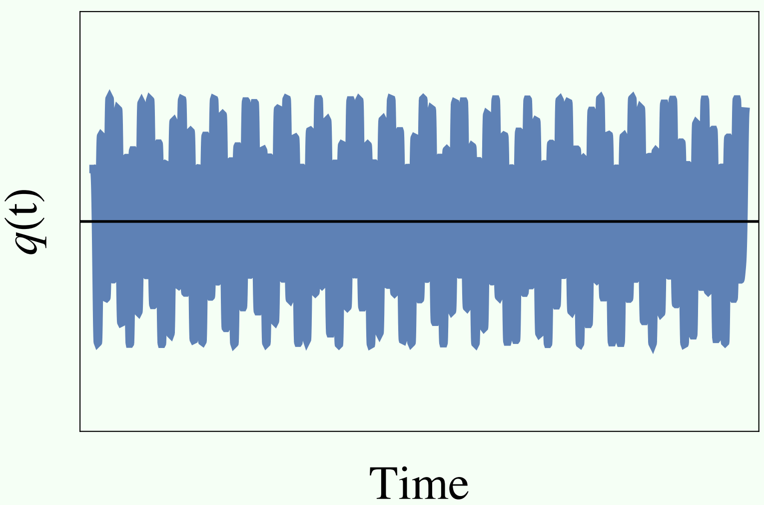
$$
\begin{array}{ccc}
\text{B1) classical mechanics} & \rightarrow & \text{B2) quantum mechanics} \\
\downarrow & \searrow & \downarrow \\
\text{B3) classical field theory} & \rightarrow & \text{B4) quantum field theory}
\end{array}
$$

B1) Classical mechanics

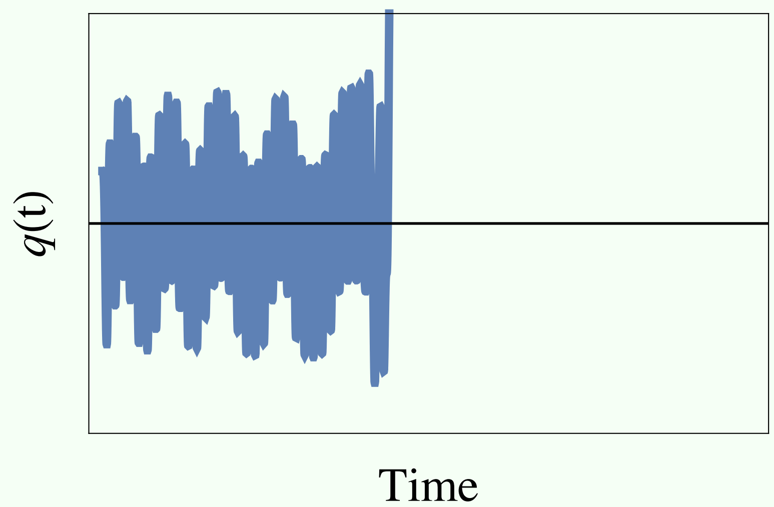
Ghost miracle?

To see: solve numerically $\ddddot{q} + (\omega_1^2 + \omega_2^2)\ddot{q} + \omega_1^2\omega_2^2 q$ = interactions e.g. λq^3:

Smaller coupling: stable



Larger coupling: runaway



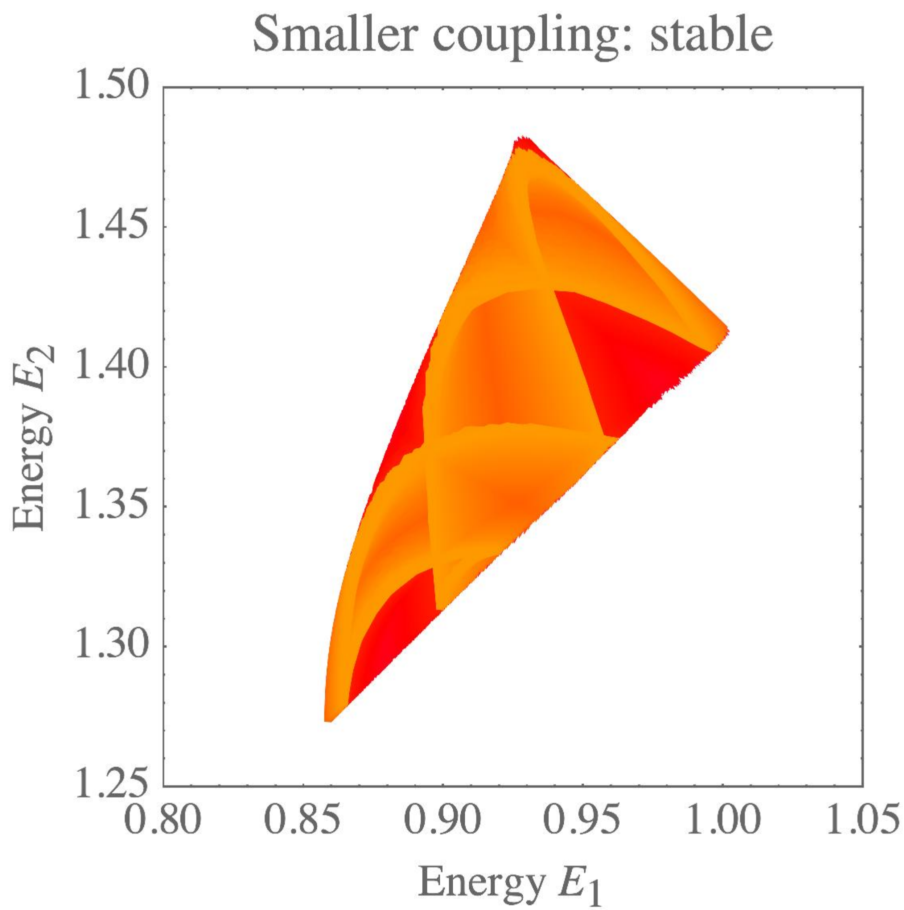
How long can be stable? And why?

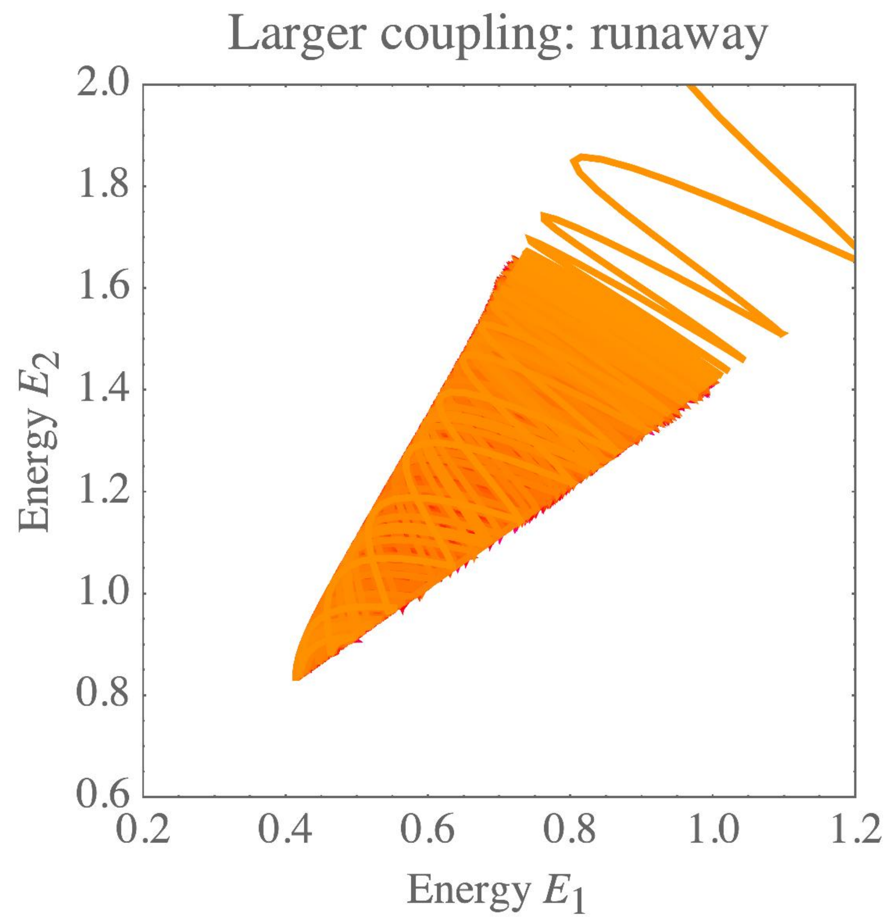
Ghost lockdown

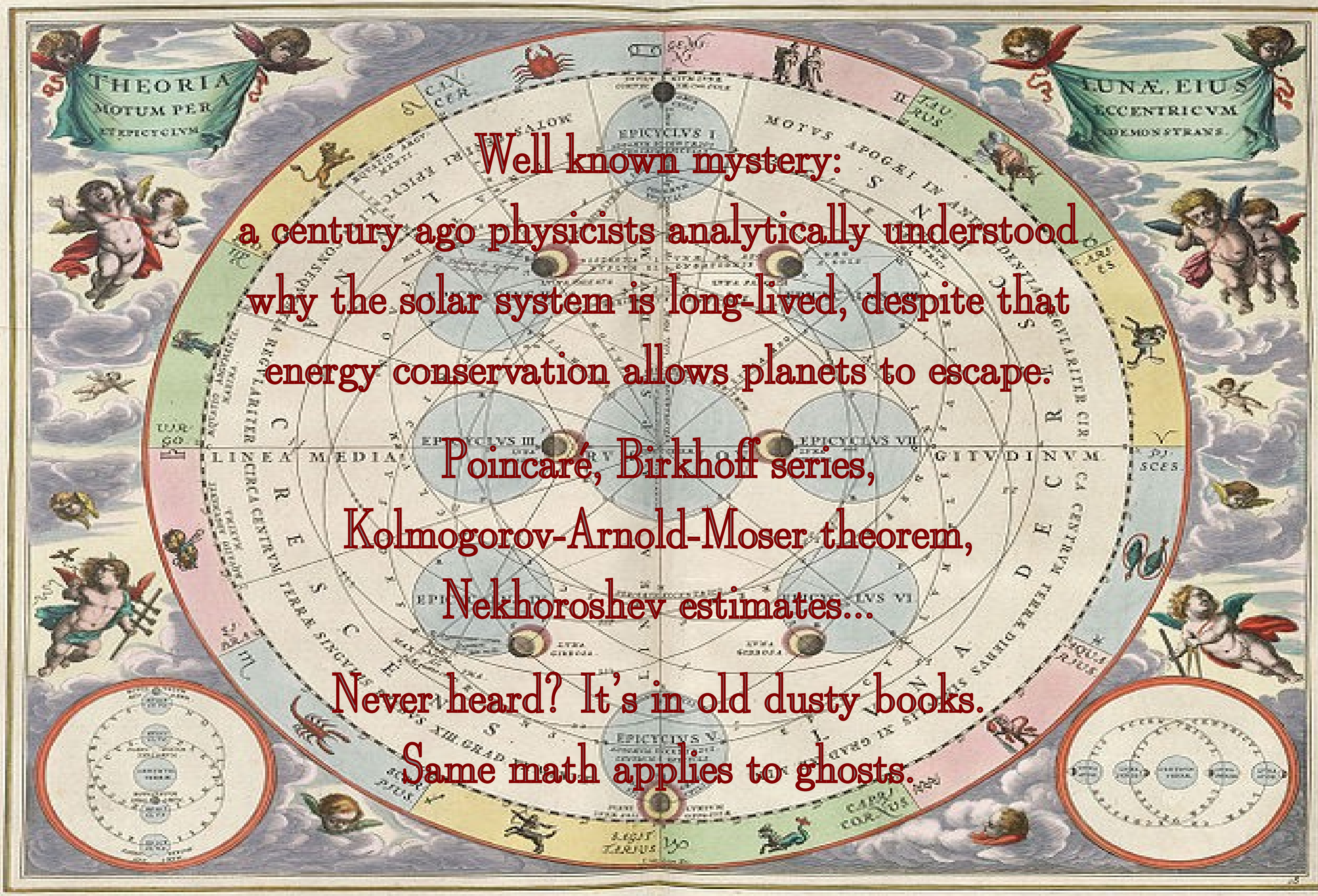
One 4-derivative $q(t)$ can be rewritten as two 2-derivative $q_{1,2}(t)$, Ostrogradski-like:

$$H = E_1 - E_2 + V \qquad E_i = \omega_i \frac{p_i^2 + q_i^2}{2} \qquad V = \frac{\lambda}{2} q_1^2 q_2^2$$

H constant, no extra constant of motion prevents run-away. Energies E_1, E_2 grow and could go everywhere, but instead remain confined to a region if the coupling λ is small







Well known mystery:

a century ago physicists analytically understood why the solar system is long-lived, despite that energy conservation allows planets to escape.

Poincaré, Birkhoff series,

Kolmogorov-Arnold-Moser theorem,

Nekhoroshev estimates...

Never heard? It's in old dusty books.

Same math applies to ghosts.

Some physical systems are ghosts

Asteroid around the Lagrange point L4 e.g. Sun/Jupiter:

$$H=\frac{\vec{p}^{\,2}}{2m}+\omega(yp_x-xp_y)-\frac{GM_S m}{|\vec{x}-\vec{x}_S|}-\frac{GM_J m}{|\vec{x}-\vec{x}_J|}$$

in the rotating frame.

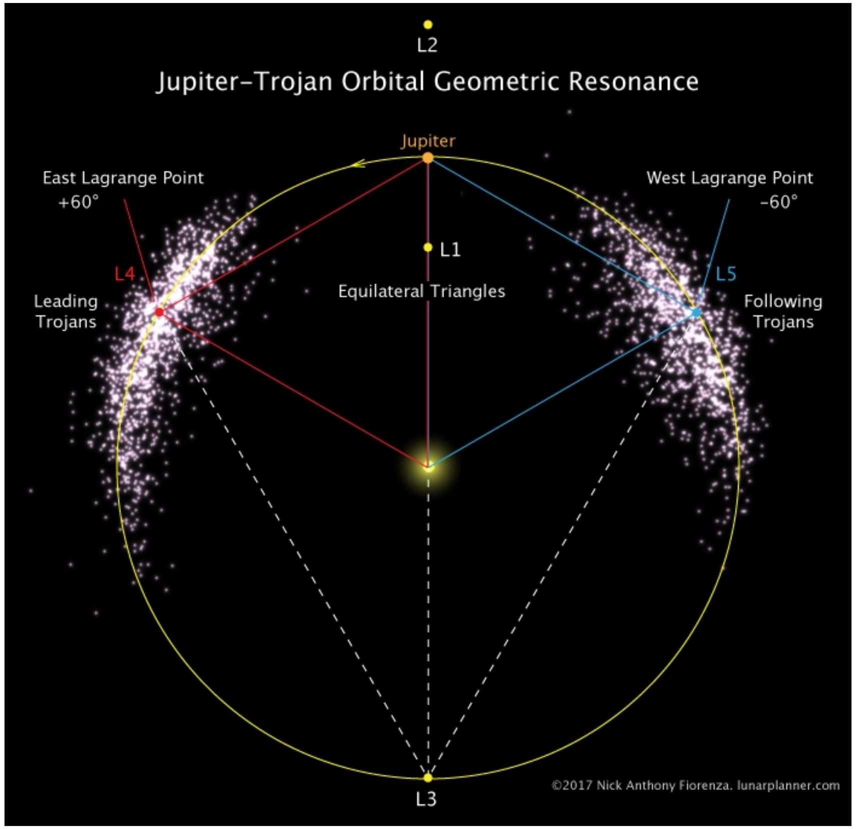
Expand as quadratic + interaction around L4

$$H_2=\frac{p_x^2+p_y^2}{2}+yp_x-xp_y+\frac{x^2}{8}-\frac{5y^2}{8}+\frac{\sqrt{27}}{4}\left(\frac{2M_J}{M_{J+S}}-1\right)xy$$

for $\omega=m=1$. Diagonalise through a canonical Sp rotation:

$$H_2=\omega_1\frac{p_1^2+q_1^2}{2}-\omega_2\frac{p_2^2+q_2^2}{2}.$$

Check, asteroids are still there!



Similarly for an electron rotating in a constant magnetic field B_z with a destabilizing potential ω_0^2

$$H=\frac{(\vec{p}-e\vec{A})^2}{2m}+e\varphi=\frac{\vec{p}^{\,2}}{2m}+\omega_B(yp_x-xp_y)+\frac{m}{2}(\omega_B^2-\omega_0^2)(x^2+y^2)\qquad \omega_B=\frac{eB_z}{2m}.$$

Birkhoff series

'Diagonalize' a classical Hamiltonian trough a canonical transformation from (q, p) to action-angle variables (J, Θ) such that

$$H(p_i, q_i) = H'(J_i)$$

makes motion trivial: $J_i = \text{cte}$ and $\Theta_i \propto t$. Harmonic oscillator:

$$q = \sqrt{\frac{2J}{m\omega}} \sin\Theta, \qquad p = \sqrt{2m\omega J} \cos\Theta.$$

Add small interactions, compute a perturbative Birkhoff series. Summarising books in 2 lines:

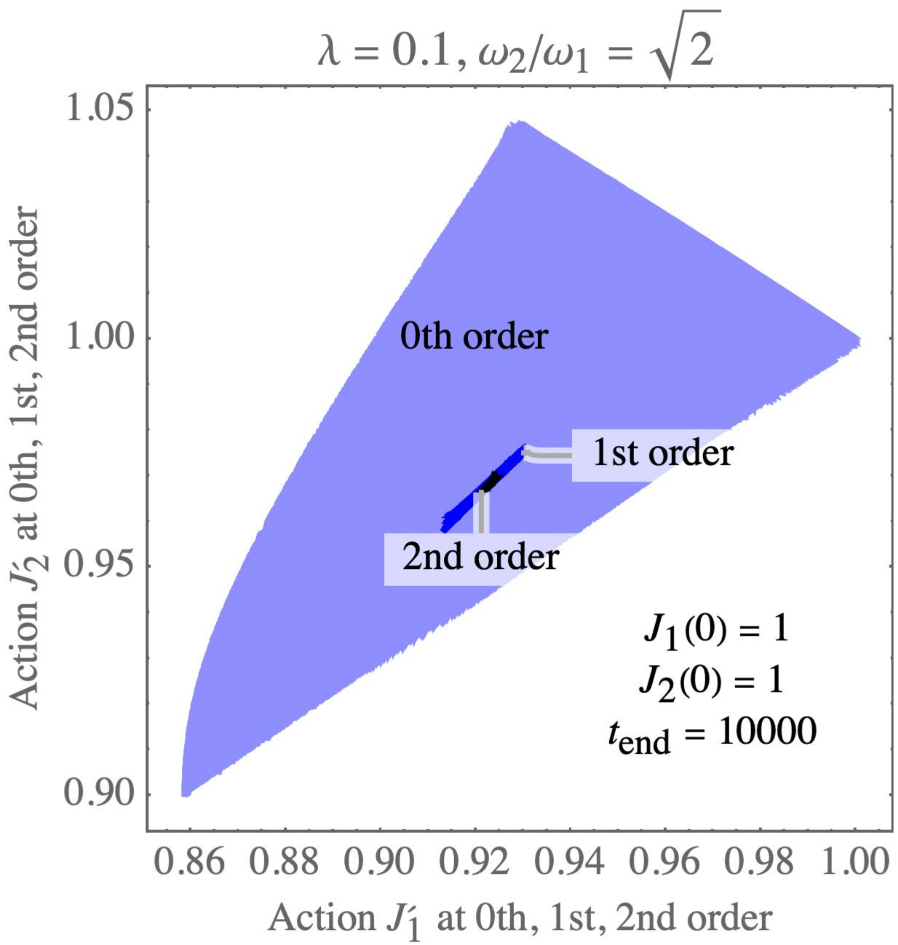
Smaller coupling $\stackrel{\sim}{\Rightarrow}$ Birkhoff series converges $\Rightarrow$ planets epycicle and stay, ghosts don't runaway.
Larger coupling $\stackrel{\sim}{\Rightarrow}$ Birkhoff series divereges $\Rightarrow$ planets motion chaotic and escape, ghosts runaway.

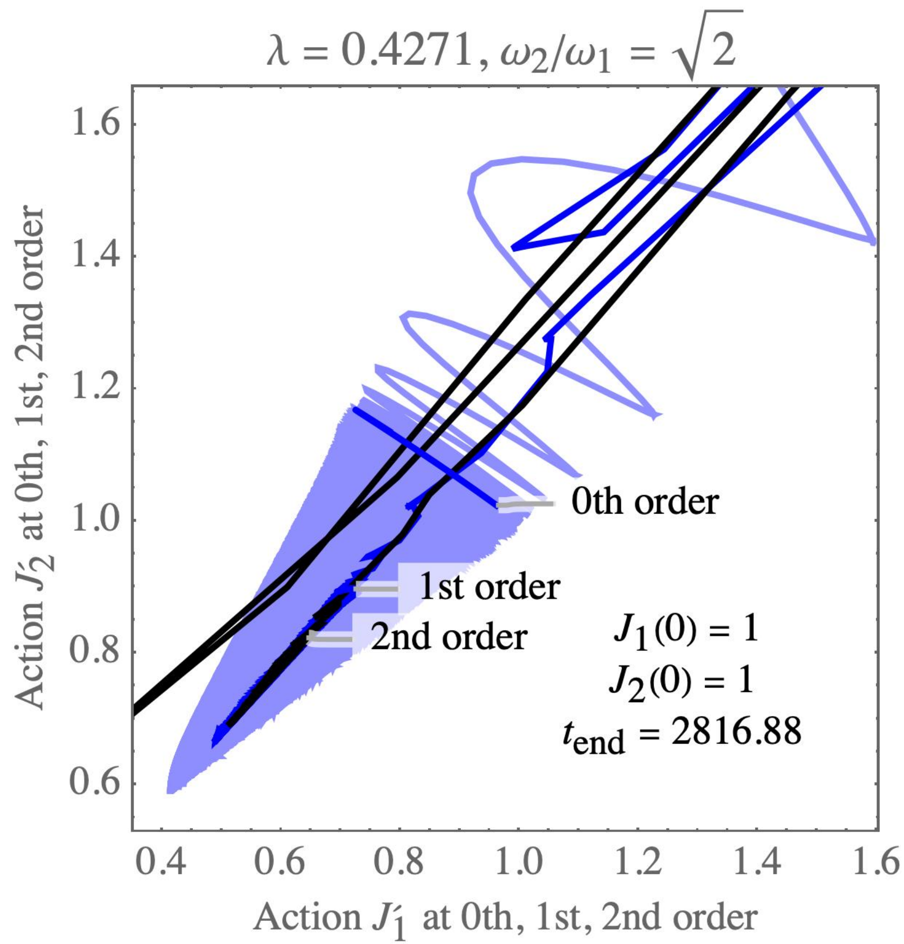
A free ghost is good. **A weakly coupled ghost remains good.**

In practice: compute more and more orders making the residual interaction smaller $\lambda \to \lambda^2 \to \lambda^3 \ldots$

$$H(p_i, q_i) = H'(J_i) + \lambda^N H_{\text{int}}(\Theta, J)$$

Example

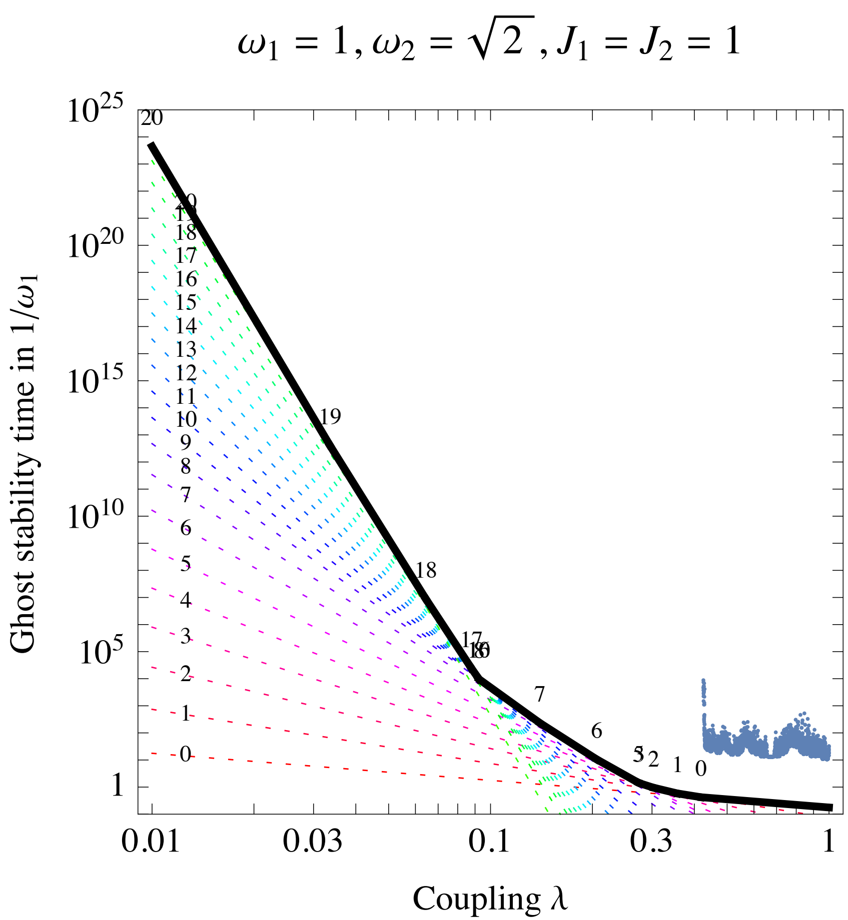


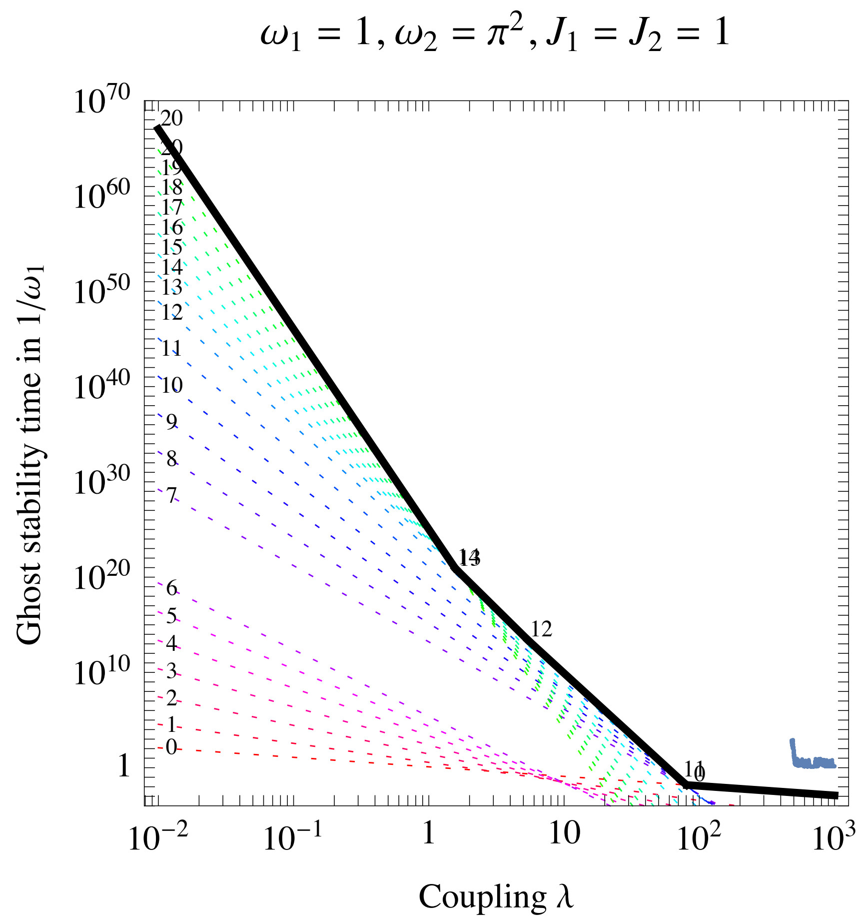


Like a hidden integral of motion

NNNNNNNNNNNNNNNNNNNNNNLO

Find order n such that small residual interaction λ^{n+1} gives maximal escape time $\tau > \max_n \tau_n$. Asymptotic series: bound strongest for finite n depending on coupling λ: non-trivial function of λ.

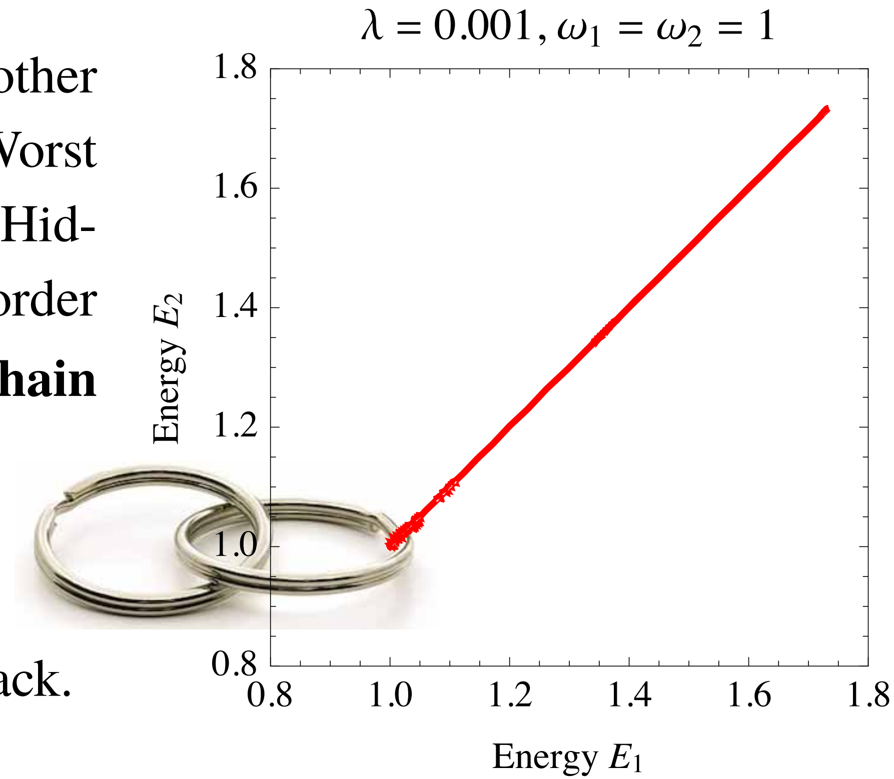




Order jumps in the figure: Birkhoff series contains $\lambda^{\cdots}/(N_1\omega_1 - N_2\omega_2)$ where $N_{1,2}$ are any integers. It can get accidentally big even at small λ when **resonances** happen: planets escape, ghosts runaway.

Resonances

To compute what happens: *resonant normal forms*, another Arnold book: diagonalise everything but the resonance. Worst resonance: leading order $N_1 = N_2 = 1$ i.e. $\omega_1 = \omega_2$. Hidden integrals of motion lost if $\Delta\omega \lesssim \lambda J/\omega_{1,2}^2$. Leading order decides. A resonant $\lambda q_1^2 q_2^2$ interaction behaves as a **loose chain**



$$H' = \omega_1 J_1' - \omega_2 J_2' + \frac{\lambda J_1' J_2'}{2\omega_1\omega_2}\left[1 + \frac{1}{2}\cos 2(\Theta_1' + \Theta_2')\right] + \cdots =$$

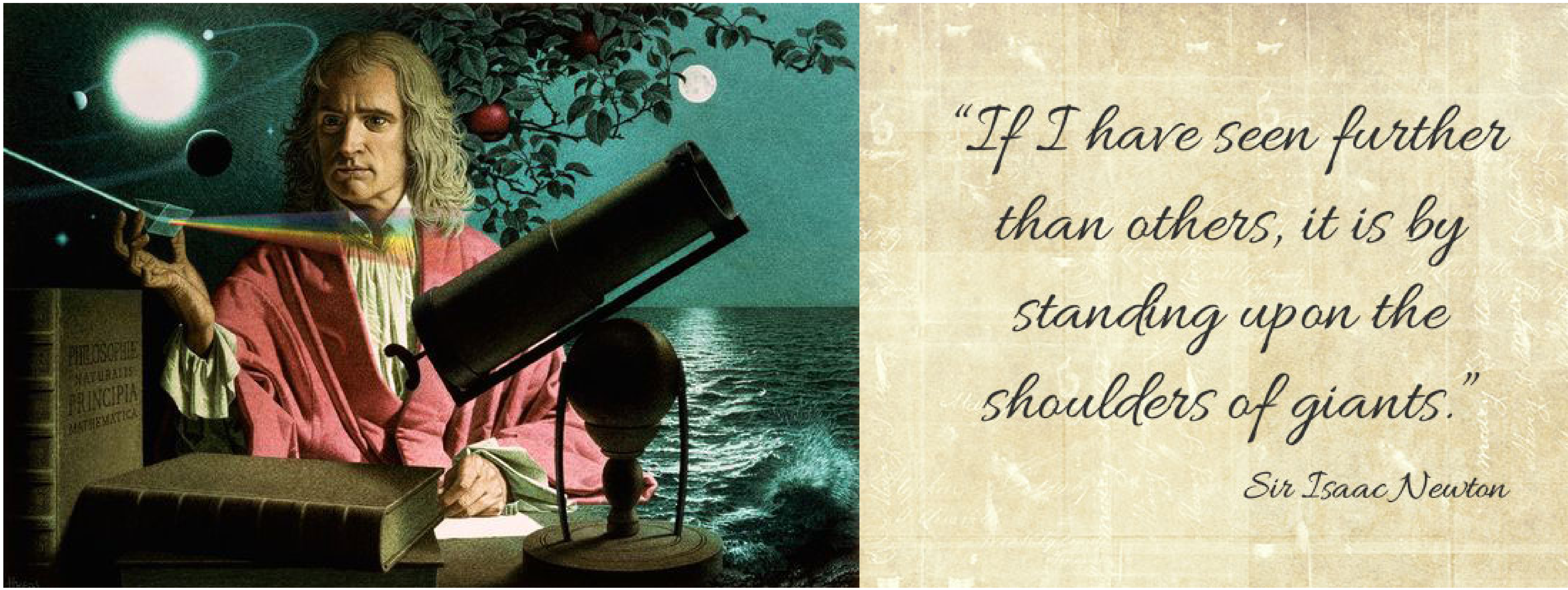
where $1/2 < 1$ means that $E_{1,2}$ vary by $\mathcal{O}(1)$ and come back.

A resonant $q_1^2 q_2$ interaction behaves as a **broken chain** i.e. run-away

$$H' = \omega_1 J_1' - \omega_2 J_2' - \frac{\epsilon}{4} J_1 \sqrt{J_2}\sin(2\Theta_1' + \Theta_2') \quad =$$

On resonance the fate depends on the model. Will be relevant in field theory.

Classical mechanics: ghosts can be meta-stable and exist



Now we get off and explore terra incognita.
Possibly without stomping on their graves.

B2) Quantum mechanics

Quantum mechanics

Adding a ghost the sign of $E - V$ no longer tells if the wave function $\psi(q_1, q_2)$ oscillates or damps

$$H = \frac{p_1^2}{2} - \frac{p_2^2}{2} + V, \qquad V = \omega_1^2 \frac{q_1^2}{2} - \omega_2^2 \frac{q_2^2}{2} + \frac{\lambda}{2} q_1^2 q_2^2, \qquad -\frac{\hbar^2}{2}\frac{\partial^2 \psi}{\partial q_1^2} + \frac{\hbar^2}{2}\frac{\partial^2 \psi}{\partial q_2^2} = (E - V)\psi.$$

Does any ψ spread into $E_1 - E_2 \approx 0$ up to large E i.e. runaway?

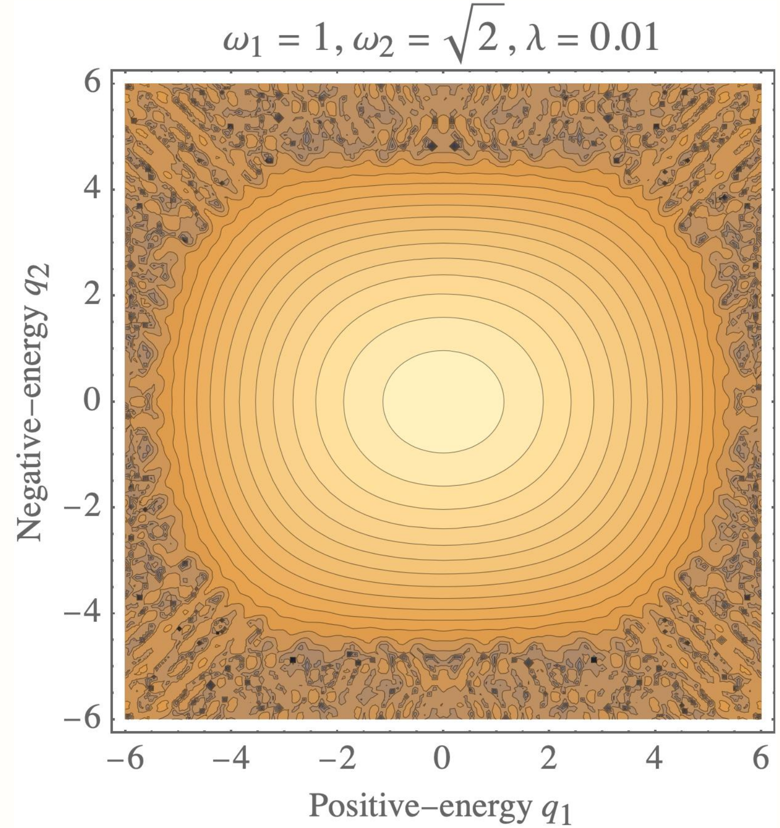
Compute numerically the bound ground state with no nodes

around $q_{1,2} \sim 0$: $\psi(q_1, q_2) \sim e^{-(\omega_1 q_1^2 + \omega_2 q_2^2)/2\hbar}$

Exponentially suppressed out-flowing probability current.

Meta-stable like a normal particle trapped in a potential barrier:

e.g. $$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + V, \qquad V = \omega_1^2 \frac{q_1^2}{2} + \omega_2^2 \frac{q_2^2}{2} - \frac{\lambda}{2} q_1^2 q_2^2.$$



K-instability (ghost run-away) is exponentially suppressed like V-instability (tunnelling).

Differences appear in the resonant case $\omega_1 \neq \omega_2$.

Quantum mechanics: getting more general

Tunnelling can be approximated semi-classically as $\psi = e^{iS/\hbar}$, such that

$$\text{Schroedinger} \overset{\hbar\to 0}{=} \text{Hamilton-Jacobi} \quad \frac{\partial S}{\partial t} = -H\left(q_i, p_i = \frac{\partial W}{\partial q_i}\right)$$

The classical action S and thereby the classical hidden integrals of motion still play a role, and keep ψ close to $q_{1,2} \sim 0$. For energy eigenstates: $S(q,t) = W(q) - Et$. Detail not yet overcomed:

WKB $\approx$ HJ approximates multi-dof $q_i(t)$ tunnelling simpler than Schroedinger:

find the classical trajectory that minimises the barrier, $W = \min\limits_{\vec{q}(t)} \int_0^{\vec{q}_{\rm release}} dq\, \sqrt{2V}$, rate $\propto e^{-2W}$.

But $(\partial W/\partial q)^2 = 2(E-V)$ has two solutions with opposite signs of W. Ground-like state at $\lambda = 0$:

- Usual HJ solution for q_1, bounce $W_1(q_1) = \lim\limits_{t_E\to\infty} S$.
- Other sign for normalizable ghost $\psi(q_2)$: classical motion backwards in time, $W_2(q_2) = \lim\limits_{t_E\to-\infty} S$

For $\lambda \neq 0$ we don't know how to use WKB to compute efficiently. And it's needed in QFT.

B3) Classical Field Theory

Classical field theory

For example two scalars $\varphi_{1,2}$, simpler than $g^{\mu\nu}$ and its ghost $g_2^{\mu\nu}$. Typical theory:

$$\mathscr{L} = \frac{(\partial_\mu\varphi_1)^2 - m_1^2\varphi_1^2}{2} - \frac{(\partial_\mu\varphi_2)^2 - m_2^2\varphi_2^2}{2} - \frac{\lambda}{2}\varphi_1^2\varphi_2^2 \,.$$

Classical field theory is sick even without ghosts. Wants to equipartition energy among infinite modes giving black body divergence cut at $\omega \lesssim T/\hbar$. Ghost is co-morbidity. Classical can be computed:

Numeric. Classical field theory can be computed numerically on smart light-cone lattice.

Analytic. Expand field $\varphi(\vec{x},t)$ as Fourier modes $q_{\vec{n}}(t)$ to use Birkhoff & co

$$\varphi(\vec{x},t) = \frac{1}{L^{d/2}}\sum_{\vec{n}=-\infty}^{\infty} q_{\vec{n}}(t)e^{i\vec{k}\cdot\vec{x}} \qquad \vec{k} = \frac{2\pi\vec{n}}{L} \qquad \omega_n^2 = m^2 + k^2$$

Resonances

Off-shell processes don't runaway. But lots of q_n with frequencies ω_n allow for lots of resonances. These on-shell processes are the usual decays, scatterings, etc.

- Resonant normal form of the complicated interaction $q_{n_1} q_{n'_1} q_{n_2} q_{n'_2}$ shows that local interactions like $\lambda\, \varphi_1^2 \varphi_2^2$ keep ghosts in chain

$$H \simeq \omega_{n_1} J_{n_1} + \omega_{n'_1} J_{n'_1} - \omega_{n_2} J_{n_2} - \omega_{n'_2} J_{n'_2} + \frac{\epsilon}{4}\Bigg[\Bigg(\frac{J_{n_1} J_{n_2}}{\omega_{n_1}\omega_{n_2}} + \frac{J_{n_1} J_{n'_2}}{\omega_{n_1}\omega_{n'_2}} + \frac{J_{n'_1} J_{n_2}}{\omega_{n'_1}\omega_{n_2}} + \frac{J_{n'_1} J_{n'_2}}{\omega_{n'_1}\omega_{n'_2}}\Bigg) + 2\sqrt{\frac{J_{n_1} J_{n'_1} J_{n'_2} J_{n'_2}}{\omega_{n_1}\omega_{n'_1}\omega_{n'_2}\omega_{n'_2}}} \cos(\Theta_{n_1} + \Theta_{n'_1} + \Theta_{n_2} + \Theta_{n'_2})\Bigg].$$

  Each resonance is benign: does not allow run-away, but violates one hidden constant of motion at $\mathcal{O}(1)$.

- One field has $N = L/a$ dof, there are $2N$ hidden constants of motion, $\sim N^2$ resonances. In the continuum limit $N^2 \gg N$: energy can flow $\varphi_1 \leftrightarrow \varphi_2$. Pictorially, **too many ghosts escape from lockdown because locked by a loose chain**.

Ambiguous situation: something good happens but not good enough?

Ghost entropy

Assume worst case scenario: mess wins, runaway possible.

Consider a system of ghost φ_2 interacting with normal φ_1 with temperatures $T_1 \neq T_2$.
Do they thermalise to same $T \sim \langle E \rangle$? No, they cannot: $T_2 < 0$ and $T_1 > 0$.
So, what happens? They maximise entropy $S = S_1 + S_2$. Since $S_2 = N_{\rm dof} \ln |T_2|$, entropy is maximal for $T_1 \to \infty$ and $T_2 \to -\infty$. Heat flows in the direction where both $|T_{1,2}|$ grow. Run-away happens.

To see how fast solve *classical* Boltzmann eq.s for generic $f(E)$ beyond the thermal limit.

(To start: $\varphi_{1,2}$ positive-energy; *quantum* Boltzmann equations are well known e.g. for $12 \leftrightarrow 1'2'$

$$\dot{\rho}_1 = -\int d\vec{k}_1 d\vec{k}_2 d\vec{k}'_1 d\vec{k}'_2 \, E_1 \, (2\pi)^{d+1} \delta(K_1 + K_2 - K'_1 - K'_2) |\mathscr{A}|^2 F \qquad \mathscr{A} = 2\hbar\lambda$$

$$F = f_1(E'_1) f_2(E'_2)[1 + f_1(E_1)][1 + f_2(E_2)] - f_1(E_1) f_2(E_2)[1 + f_1(E'_1)][1 + f_2(E'_2)]$$

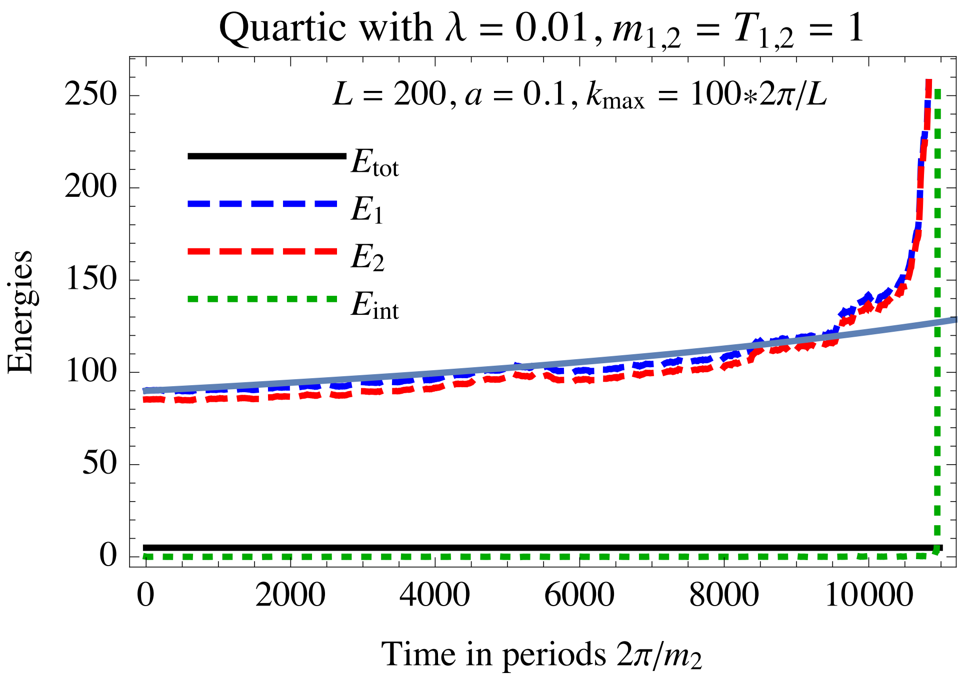
Bose-Einstein $f = 1/[e^{E/T} - 1]$ at equilibrium. Two classical limits: particle ($f \simeq e^{-E/T} \ll 1$, ignore) and wave ($f \simeq T/E \gg 1$). Thermalization rate $\dot{T}_1 \propto \lambda^2 T_1 T_2 (T_2 - T_1)$ agrees with numerics).

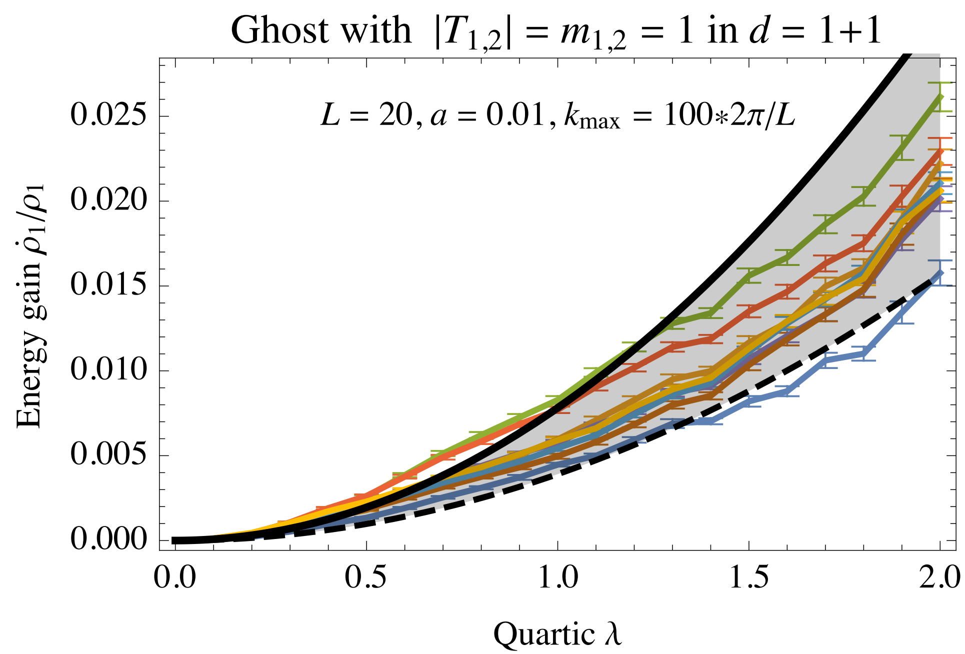
Ghost runs away in classical field theory

Next compute the ghost. Kinematics with $E < 0$ looks unusual. Trick: $\dot{\rho}_1$ remains the same using

$$\tilde{K}_\mu = -K_\mu \qquad f(E/T) = -[1 + f(\tilde{E}/T)]$$

i.e. (emission of negative energy) $\leftrightarrow$ (absorption of positive energy). No thermal equilibrium, run-away rate equals the heat flow rate $\propto \lambda^2$, not exponentially suppressed $e^{-1/\lambda}$. Analytic $\approx$ numerics:





Not a problem in 4∂ gravity: $\dot{T}/T \sim T^3/M_{\rm Pl}^2 \ll H \sim T^2/M_{\rm Pl}$. And $T \sim H_{\rm infl}$ during inflation

B4) Relativistic Quantum Field Theory

Relativistic Quantum Field Theory?

Rate for $\emptyset \leftrightarrow 11'22'$ etc from Bolztmann equation in the limit $T_1 \to 0^+$, $T_2 \to 0^-$: $F \to -1 \neq 0$ so

$$\dot{\rho}_1 = \int d\vec{k}_{\rm all}\, E_1\, (2\pi)^{d+1} \delta(K_1 + K_1' - \tilde{K}_2 - \tilde{K}_2') |\mathscr{A}|^2 = \text{coupling}^2 \cdots \int_{\sqrt{s}}^{\infty} dE\, E\, (E^2 - s)^{\frac{d}{2}-1}$$

contains a divergent dE integral over the Lorentz group, $E = (K_1 + K_1')_0$. Needed because $\emptyset$ is Lorentz invariant. Ghost production rate is infinite: large enough that K-instability is excluded?

Same problem in old computations of vacuum decay [Okun et al.]: the critical bubble can be produced with any initial speed. And $V < 0$ allows ghost bubbles with $m < 0$. Is V-instability excluded?

Later, Coleman argued that the right way of computing is not particles, but O(4)-invariant bounce. This gives finite and exponentially suppressed $\Gamma_{V\text{-tunnelling}} \propto \exp(-W)$ of action, $W \sim 1/\lambda$.

- Dvali [1107.0956] doubts that V-instability is exponentially suppressed.
- Opposite extremum: maybe K-instability too is similarly exponentially suppressed?

We don't yet know.

Coleman extended MQ to QFT using simple WKB that doesn't generalize to ghosts. Maybe only way is a brute force computation QFT $\to$ MQ as $\varphi_i(r)$ checking resonances? Next lockdown...

C) Negative norm, positive energy?

Quantization choices

Classical free solution:

$$q(t) = \frac{a_1 e^{-i\omega_1 t}}{\sqrt{2\omega_1(\omega_1^2 - \omega_2^2)}} + \frac{a_2 e^{-i\omega_2 t}}{\sqrt{2\omega_2(\omega_1^2 - \omega_2^2)}} + \text{h.c.}$$

Usual quantisation $a_1^\dagger|\tilde{0}\rangle = 0$ and $a_2|\tilde{0}\rangle = 0$ gives negative energy.

Fermions (1 derivative) too have negative classical energy, but positive-energy quantization exists...

Alternative quantization $a_{1,2}|0\rangle = 0$ gives 'negative norm' (more precisely 'indefined product'):

$$[a, a^\dagger] = -1 \qquad |E_k\rangle = \frac{(a^\dagger)^k}{\sqrt{k!}}|0\rangle \qquad \langle E_{k'}|E_k\rangle = (-1)^k \delta_{kk'}$$

and positive H eigenvalues

$$H = -\frac{p^2 + q^2}{2} = -\frac{aa^\dagger + a^\dagger a}{2} \qquad H|E_k\rangle = (k + \frac{1}{2})|E_k\rangle$$

so no run-away in transition amplitudes: $\int dt\, e^{-i(E_i - E_f)t} \to \delta(E_i - E_f)$.

Making sense of negative norm

Wrong no-go claim in the literature: negative norm gives non-normalizable wave-functions $\psi \sim e^{+x^2/2}$. Mistake: ψ computed using $\hat{q}|x\rangle = x|x\rangle$ i.e. positive norm $\langle x'|x'\rangle = \delta(x - x')$. Must use:

$$\text{Pauli-Dirac negative-norm coordinate representation} \qquad \hat{q}|x\rangle = i x|x\rangle, \qquad \hat{p}|x\rangle = +\frac{d}{dx}|x\rangle.$$

Then $\hat{q}$ and $\hat{p}$ are self-adjoint with respect to the indefinite norm $\langle x'|x\rangle = \delta(x' + x)$:

$$\langle x'|\hat{q}^\dagger|x\rangle \equiv \langle x|\hat{q}|x'\rangle^* = [ix'\delta(x + x')]^* = ix\delta(x + x') = \langle x'|q|x\rangle.$$

In this way, anti-symmetric $\psi(x)$ (odd levels of harmonic oscillator) have negative norm

$$\langle \psi'|\psi\rangle = \int dx\, \psi'^*(x)\psi(-x).$$

Ground state: solve $\langle x|a|0\rangle = 0$ with $\hat{a} = (\hat{q} + i\hat{p})/\sqrt{2}$, get $\psi_0 \propto e^{-x^2/2}$. Normalizable wave functions. Adding interactions, real $\hat{H}(\hat{q}, \hat{p})$ is self-adjoint. Time evolution $e^{-i\hat{H}t}$ conserves the negative norm.

Born rule? Different attempts to get probabilities seem to converge to a simple idea. In some 'good' theories unusual $\hat{H}$ gives usual diagonalization: eigenvalues $E_\pm$ are real, eigenstates $\psi_\pm$ evolve in time picking usual phases $e^{iE_\pm t}$. The constant negative norm is $|\psi_+|^2 - |\psi_-|^2$, the positive norm $|\psi_+|^2 + |\psi_-|^2$ is constant too: 'good' theory with negative norm describes non-local theory with positive norm. 'Good' $\sim$ means weak coupling $|H_{ij}| \lesssim |H_{ii} - H_{jj}|$. Are relativistic QFT 'good'? I don't know. Pessimistically not, particle decay is mixing between ∞ degenerate states.

D) Infra-red divergences

What is a ghost?

An important part of the physics is trivial: $2 \to 4$ derivatives improve UV, worsen IR.

Compute tree-level scatterings, to avoid higher-order pinching subtleties.

Compute observable cross sections among asymptotic matter states. Like Lee-Wick and experimentalists: reconstruct g_2 from its decay products. What is a ghost? A ghost is what it does.

Result: IR-enhanced cross sections do **not** follow naive dimensional analysis:

$$\text{Expect NDA } \sigma \sim \frac{\text{dimensionless couplings } f_{0,2}}{s}. \qquad \text{Get} \qquad \sigma \sim \sigma_{\text{Einstein}} \sim \frac{(s/M_{\text{Pl}}^2)^n}{M_{\text{Pl}}^2}.$$

Bad news: ghosts don't make miracles, like cancelling $\sigma_g - \sigma_{g_2}$.
Good news: ghosts don't make miracles, like cancelling $\sigma_g - \sigma_{g_2}$.

Cross section mediated by one gravi-ghost

Consider $2 \to 3$ or more such that one ghost has free $s_g \equiv k^2$. Exchange of massless ghost is IR-divergent: $P(k^2) = 1/k^4$ as $k^2 \to 0$ even if $k_\mu \neq 0$, not soft. So massive is IR enhanced by $1/M_2^2$. Decompose as Scattering $\times$ Propagator $\times$ Decay.

Phase space: $d\Phi = d\Phi_{\rm scattering} ds_g d\Phi_{\rm decay}/2\pi$.

Factorise around gravi-ghost peak:

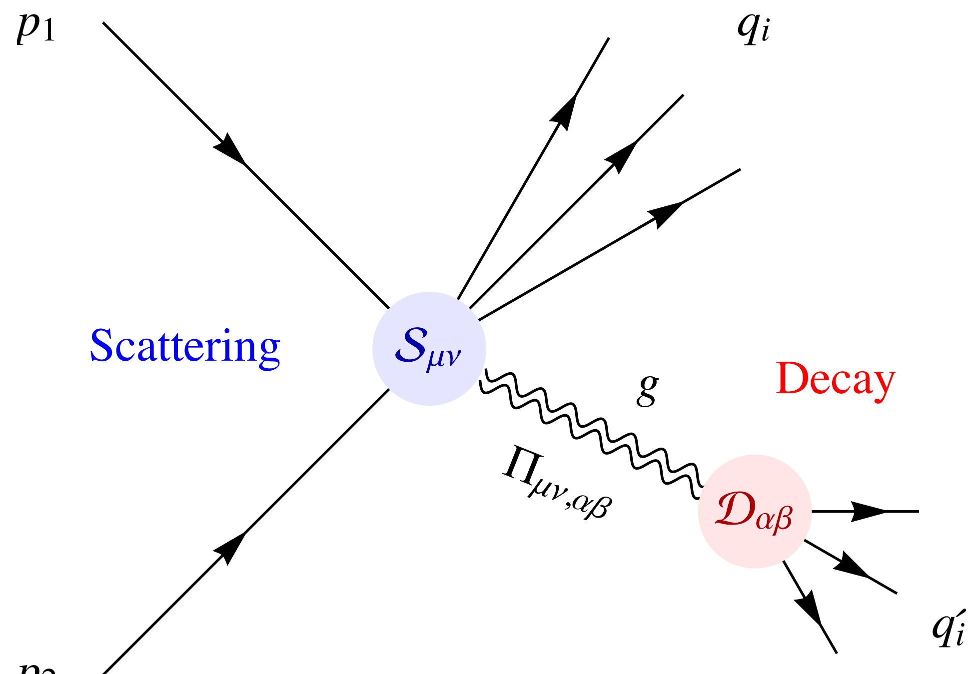
$$|P|^2 \simeq \frac{\pi}{M_2^5|\Gamma_2|}\delta(s_g - M_2^2).$$

Ghost decay width agrees with its Im(propagator)

$$d\Gamma = -\frac{f_2^2}{M_2^3}|D|^2 d\Phi_{\rm decay}.$$

Ghost production cross section:

$$d\sigma = -\frac{1}{2I}\frac{f_2^2}{M_2^2}|S|^2 d\Phi_{\rm scattering}.$$



Example

$$\sigma(e\bar{e} \to \gamma g_2) \overset{s \gg M_2^2}{\simeq} -\frac{e^2 f_2^2}{48\pi M_2^2} = -\frac{e^2}{24\pi \bar{M}_{\rm Pl}^2}$$

like $-\sigma(e\bar{e} \to \gamma g)$ in Einstein.

Ghostrahlung

Cross section mediated by n gravi-ghosts: same IR enhancement for each ghost

$$\sigma \sim \frac{\overbrace{(\text{couplings})^p}^{\substack{\text{naive}\\ \text{dimensional}\\ \text{analysis}}}}{4\pi s} \times \overbrace{\begin{cases} \left(\frac{f_2^2}{(4\pi)^2}\int_0^s \frac{s\,ds_g}{s_g^2}\right)^n & M_2 = 0, \text{ IR divergent} \\ \left(\frac{s}{M_{\rm Pl}^2}\right)^n & M_2 \neq 0, \text{ IR enhancement} \end{cases}}^{\text{IR enhancement}}$$

E.g. production of two ghosts: $\sigma(SS^* \to g_2 g_2) \stackrel{s\gg s_g}{\simeq} \frac{f_2^4 s}{960\pi s_g^2} \sim \frac{s}{240\pi \bar{M}_{\rm Pl}^4}$. **Big** but **IR**.

IR understood in QED, QCD, gravity: $m_{\gamma,G,g} = 0$ give long-range forces, invalidating LSZ. Compute what is observed, IR divergences canceled by virtual effects in the soft $k \to 0$ and collinear regions.

Newton potential $V \propto 1/r$ becomes $V \propto r$ for $M_2 = 0$: no free particles. IR enhancements in agravity involve a new Minkowskian region, soft $k^2 \sim M_2^2$ but non-soft k_μ. Agravity cross sections grow as big as in Einstein at $s \sim M_{\rm Pl}^2$. Einstein: Planckian gravitons are **strongly coupled**, make black holes?

Agravity: gravitons are **weakly coupled** and fly away carrying energy. Resummation of IR-enhanced ghost radiation presumably downgrades $s \gg M_{\rm Pl}^2$ down to Planckian: perturbative classicalization?

Conclusions: can negative kinetic energy be ok?

Newton stopped at 2 derivatives: $F = ma$. More make quantum gravity renormalizable, give IR enhancements and ghosts. Two possible quantizations:

1) Positive energy, negative norm.
   - Wave-functions normalizable, norm conserved by interactions.
   - Probabilistic interpretation? Need to compute.
2) Negative energy, positive norm
   - **Classical mechanics**. Weakly-coupled ghosts could runaway but instead undergo lockdown, can be meta-stabile up to cosmologically large time. Seen in physical systems like Trojan asteroids. Resonances allow for partial or total energy flow.
   - **Classical field theory**. Too many ghosts escape lockdown: infinite benign resonances give energy flow. No thermal state, heat flows from negative $T_{\rm ghost}$ to positive $T_{\rm normal}$ because entropy maximal for $|T| \to \infty$. Rate $\propto$ coupling^2, small in agravity where coupling $\sim E/M_{\rm Pl}$.
   - **Quantum mechanics**: K-instability exponentially suppressed like V-instability if $\omega_1 \neq \omega_2$. But we don't know how to simply compute *à la* WKB.
   - **Quantum field theory**: ghost runaway rate $\propto \lambda^2\times$ (divergent integral over Lorentz group). Might signal analogous of Coleman O(4) bubbles?