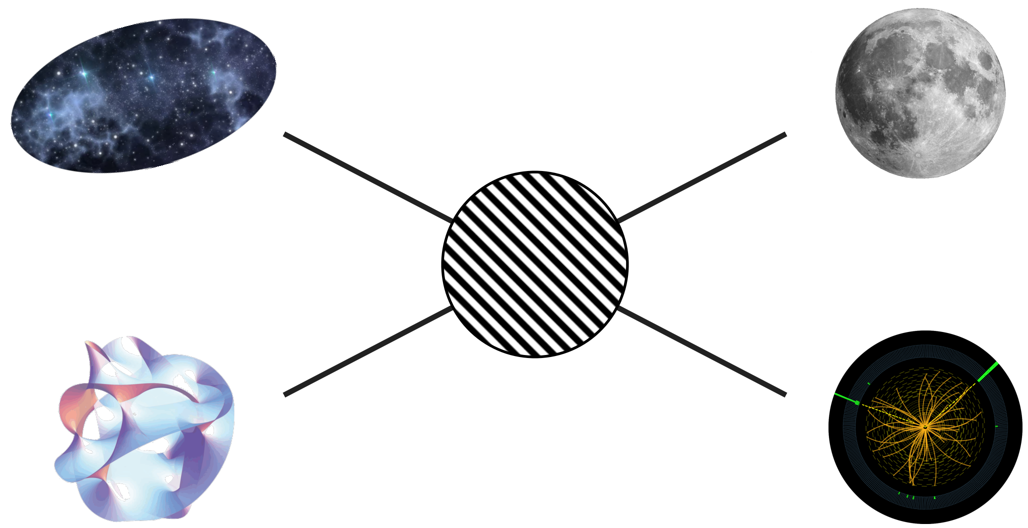
Exploring Fundamental Physics Effectively



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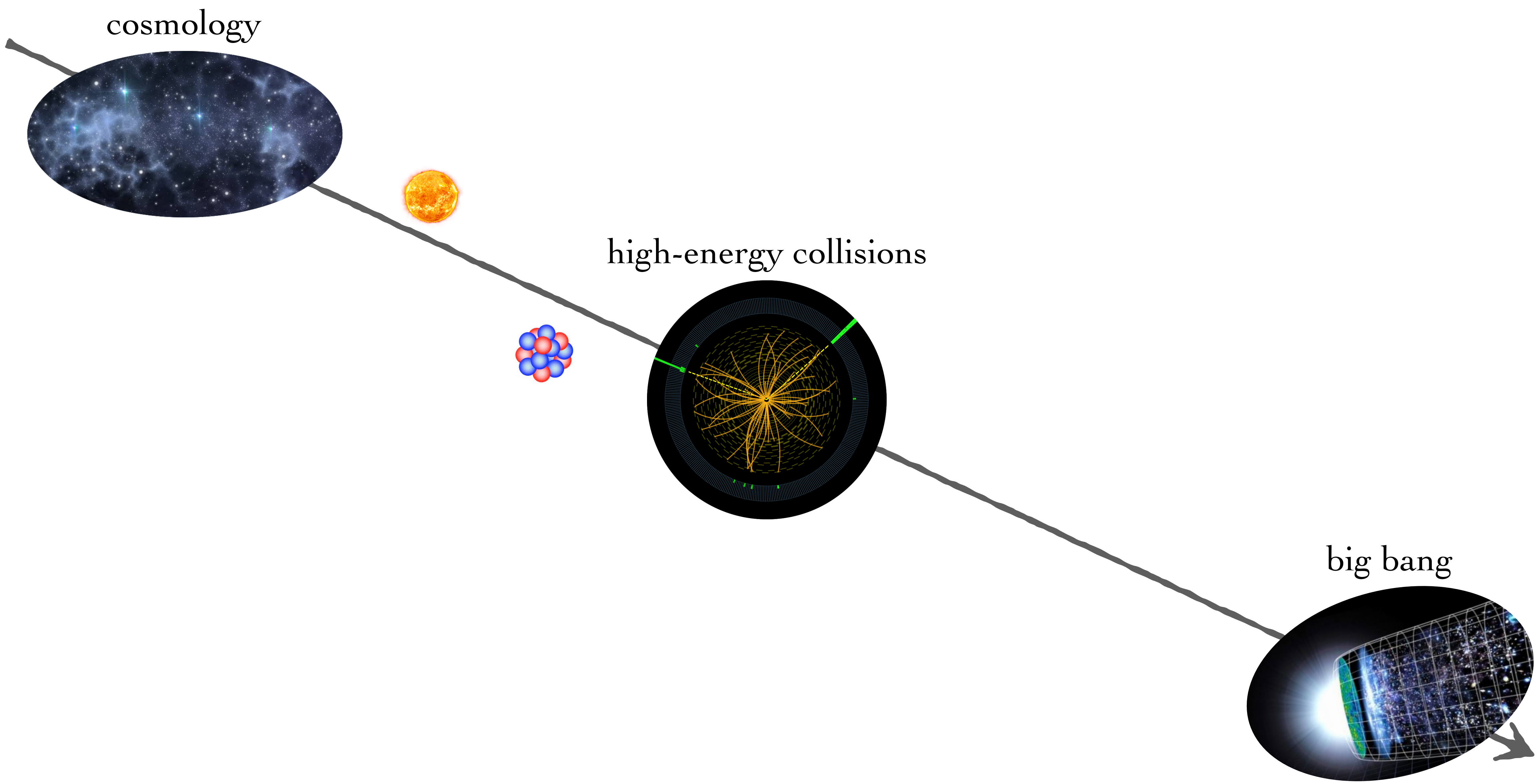
with B.Bellazzini, M.Lewandowski, F.Riva and F.Sgarlata

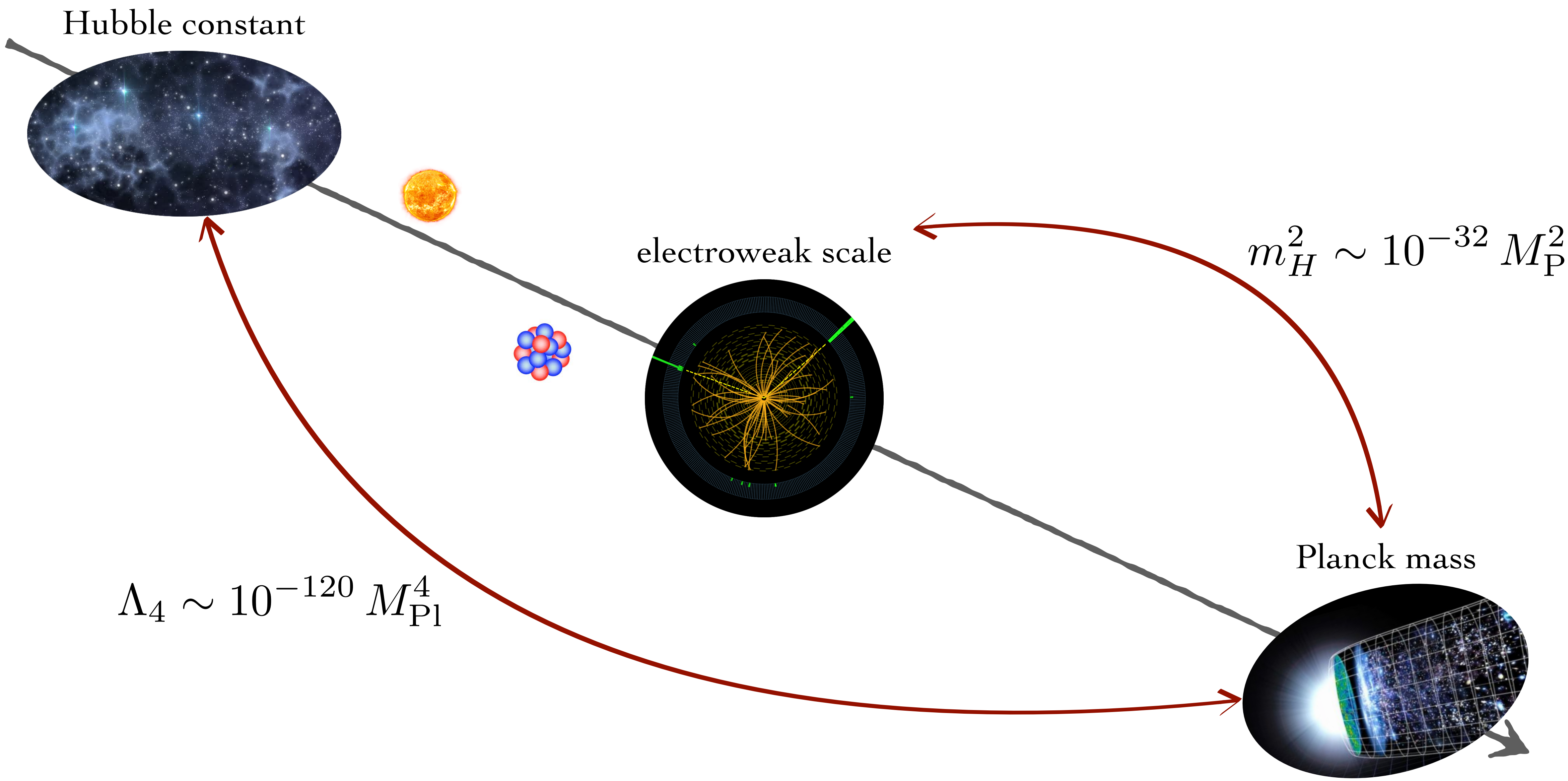
Effective Field Theory is the single most powerful organising principle in physics.



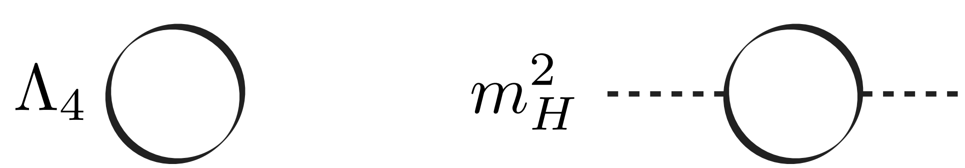
EFT lies at the heart of our understanding of nature.

Intrinsic of fundamental physics is to understand connection between energy scales.

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} M_{\rm Pl}^2 R + m_H^2 |H|^2 + \Lambda_4 + \cdots \right]$$



Quantum Mechanics and Relativity turn these questions into problems.



Outline

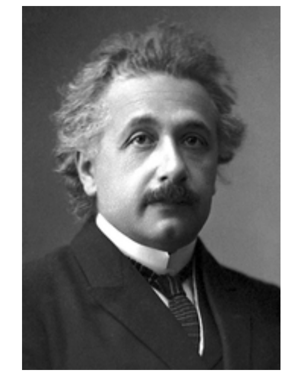
- Fundamentals
  - QFT, EFT
- S-matrix Constraints
  - dispersion relations
- Applications
  - composite Higgs
  - fermion compositeness
  - massive gravity
  - weak gravity conjecture
- Conclusions

Fundamentals

Quantum Mechanics

$\hbar$

Relativity



c

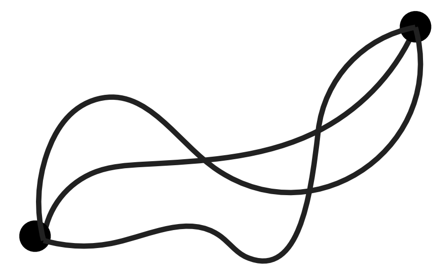
Born
Dirac
Pauli
Feynman
…

Quantum Field Theory

Cornerstones of QFT

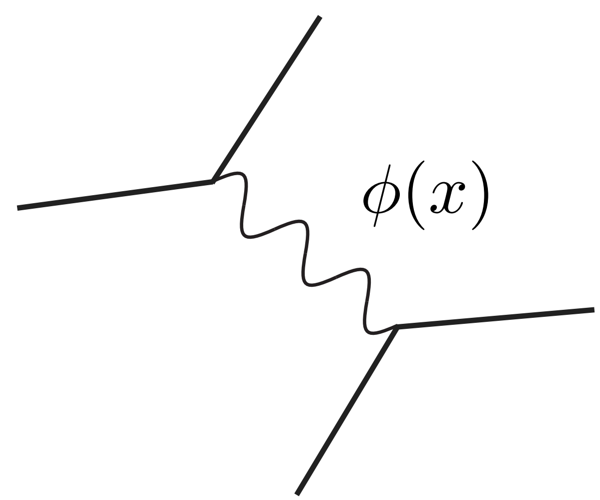
- Unitarity:

$$\sum \text{prob.} = 1$$

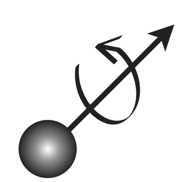


$$\sum_{\text{paths}} e^{i\mathcal{S}/\hbar}$$

- (manifest) Locality:



- Poincaré invariance:



$\frac{n}{2}\hbar$

		little group
massive:	spin	$SU(2)$
massless:	helicity	$ISO(2)$

- Causality:

$$[\phi(x_A), \phi(x_B)] = 0 \qquad (x_A - x_B)^2 > 0$$

(spacelike)

- Gauge (redundancy) invariance:

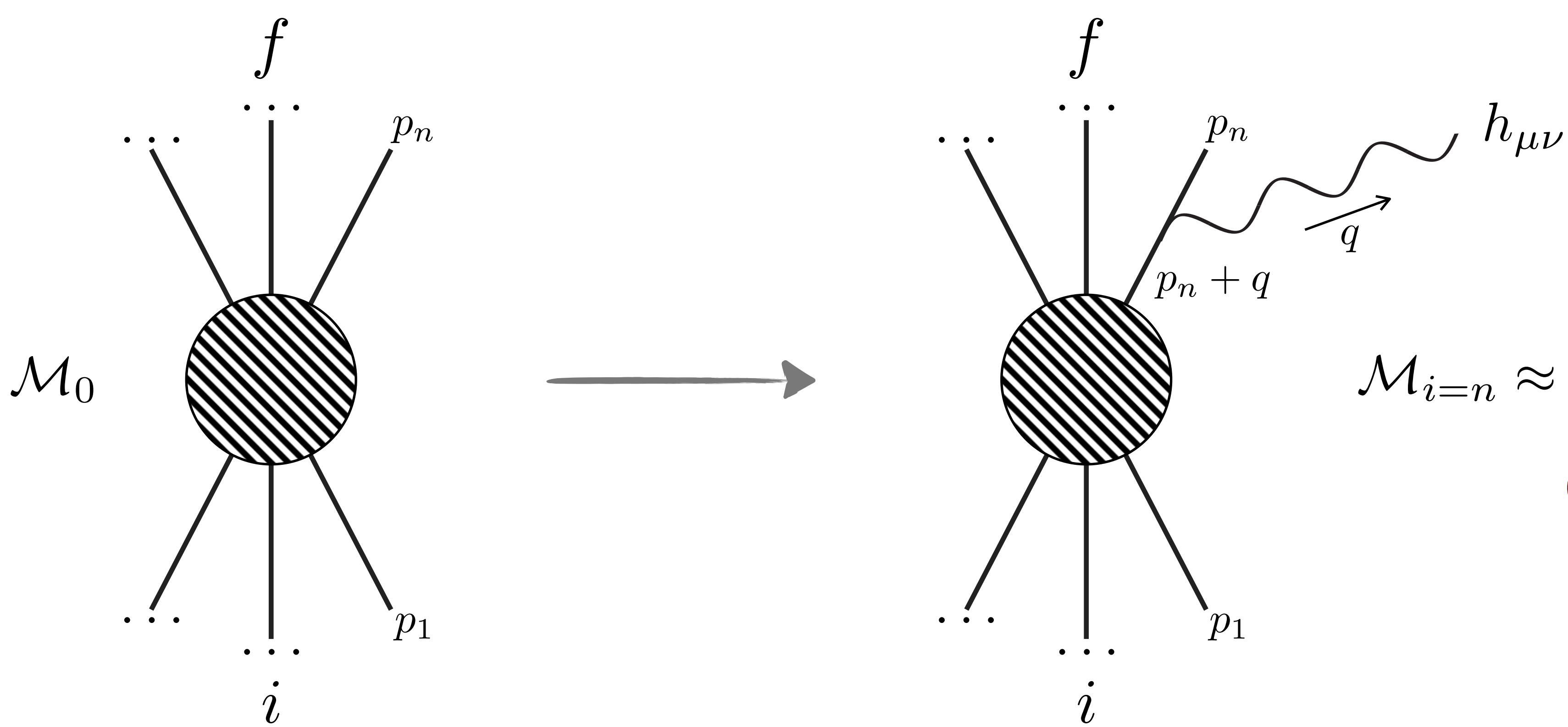
  - h = ± 1 (2 d.o.f.), $A_\mu \to A_\mu + \partial_\mu \lambda$
  - h = ± 2 (2 d.o.f.), $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \partial_\mu \xi^\alpha \partial_\nu \xi_\alpha$

These principles alone make QFT a very powerful and constrained framework.

Effective Field Theories

Physics at long distances ~ low energies.

Weinberg soft theorem (e.g. gravity h = ± 2)



$$\mathcal{M}_{i=n} \approx -\kappa_n \frac{p_n^\mu \, p_n^\nu \, \epsilon_{\mu\nu}}{p_n \cdot q} \mathcal{M}_0$$

coupling at q = 0

$$\mathcal{M} \approx \mathcal{M}_0 \left[\sum_{\text{initial}} \kappa_i \frac{p_i^\mu \, p_i^\nu}{p_i \cdot q} - \sum_{\text{final}} \kappa_j \frac{p_j^\mu \, p_j^\nu}{p_j \cdot q} \right] \epsilon_{\mu\nu}$$

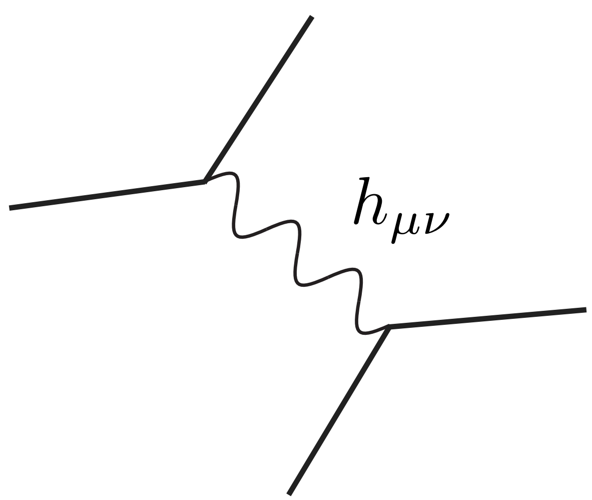
gauge invariance

$$\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + q_\mu \lambda_\nu + q_\nu \lambda_\mu$$

$$\mathcal{M}(\epsilon') = \mathcal{M}(\epsilon) \longrightarrow \sum_{\text{initial}} \kappa_i p_i^\nu = \sum_{\text{final}} \kappa_j p_j^\nu$$

equivalence principle

$$\kappa_{i,j} = \kappa = \frac{1}{M_{\text{Pl}}} \;\forall\, i, j$$



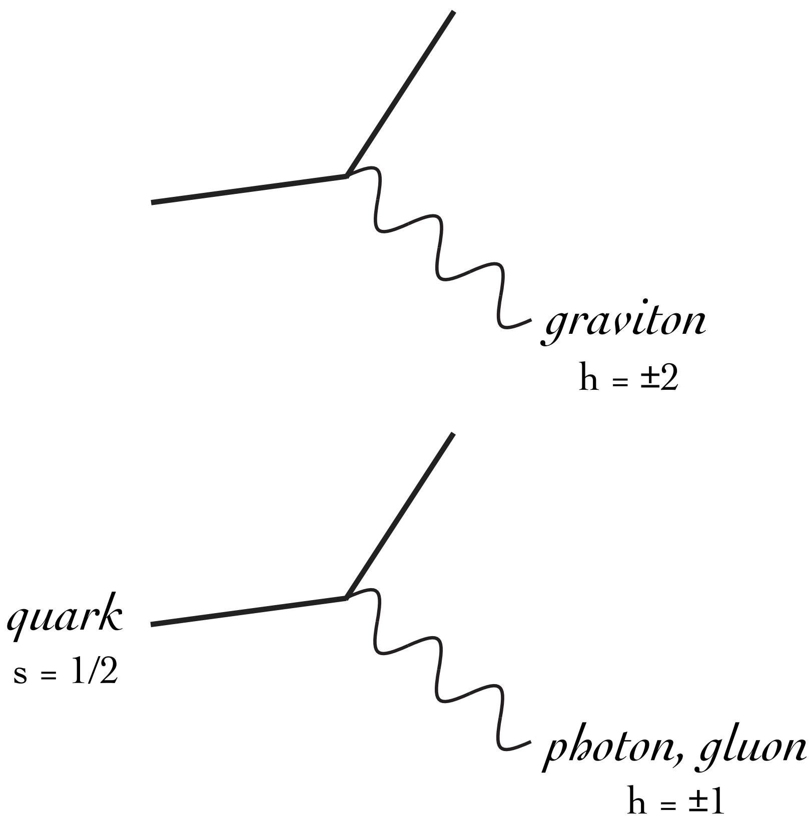
energy-momentum tensor

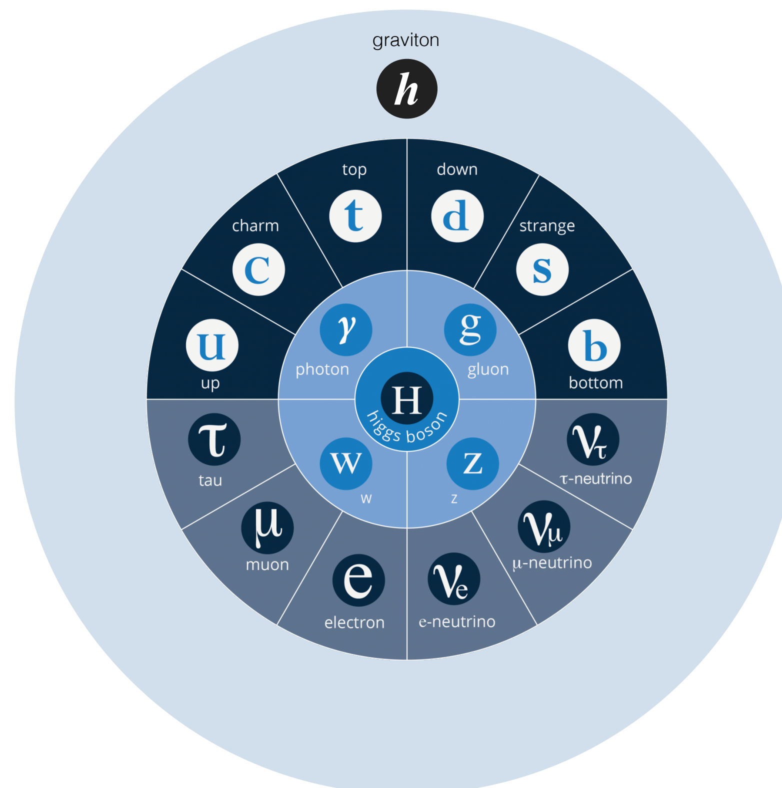
$$\mathcal{L}_{\text{int}} = h_{\mu\nu} T^{\mu\nu} \,, \quad \partial_\mu T^{\mu\nu} = 0$$

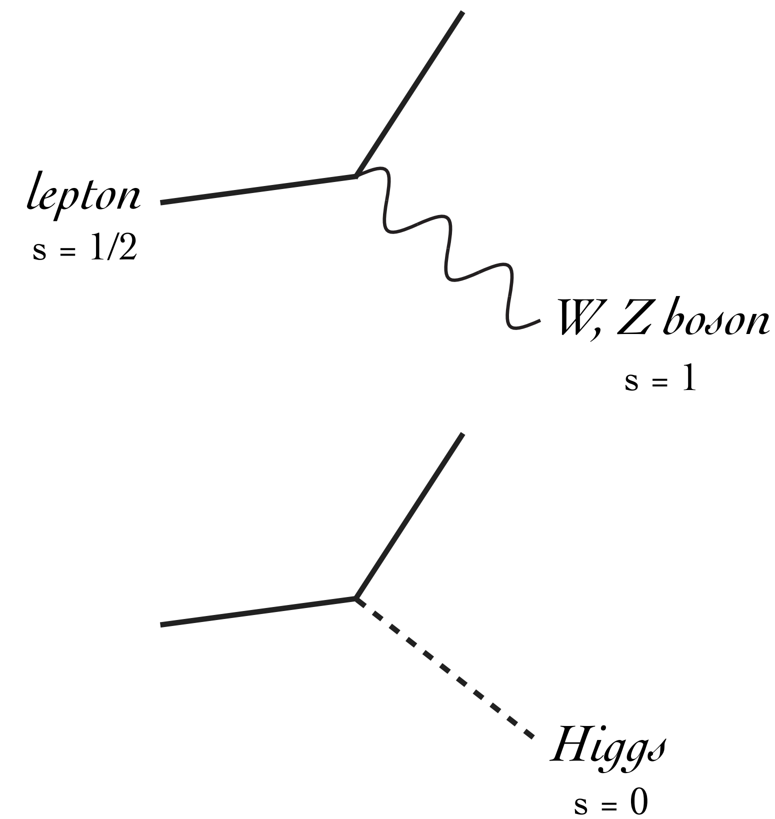
GR is the *unique* Lorentz invariant low-energy theory for interacting massless spin-2 particles.

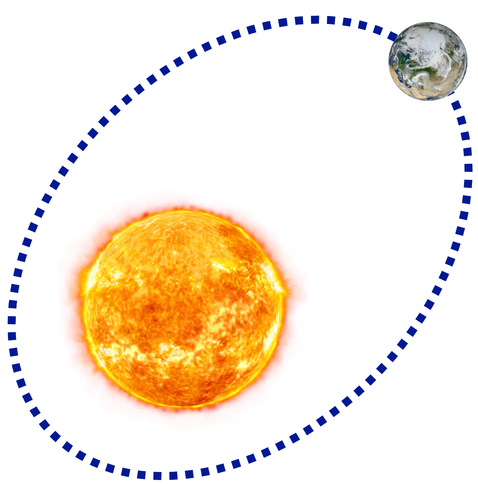
- h = ± 1: *charge conservation* $Q_{\text{initial}} = Q_{\text{final}}$
- |h| ≥ 3: *no long-range force* $\kappa_{q=0} = 0$

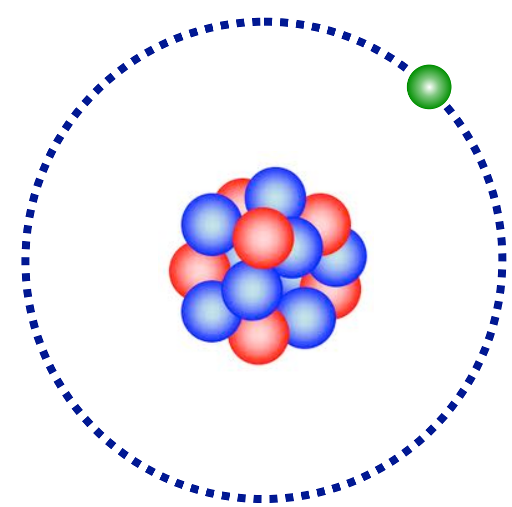
Leading 2-derivative action is fundamentally fixed (massless particles up to spin 2).



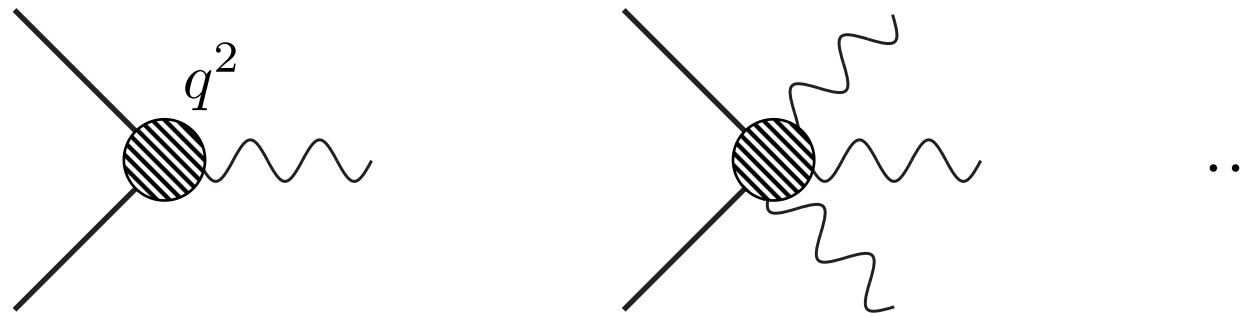








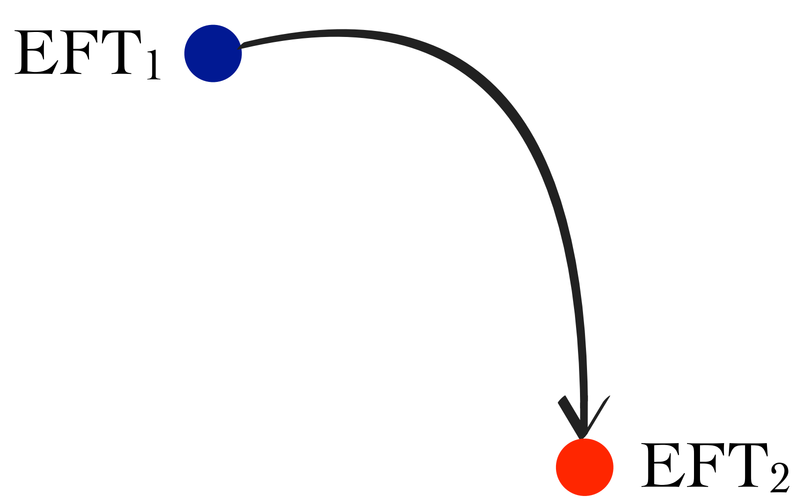
This is precisely the world we see.*



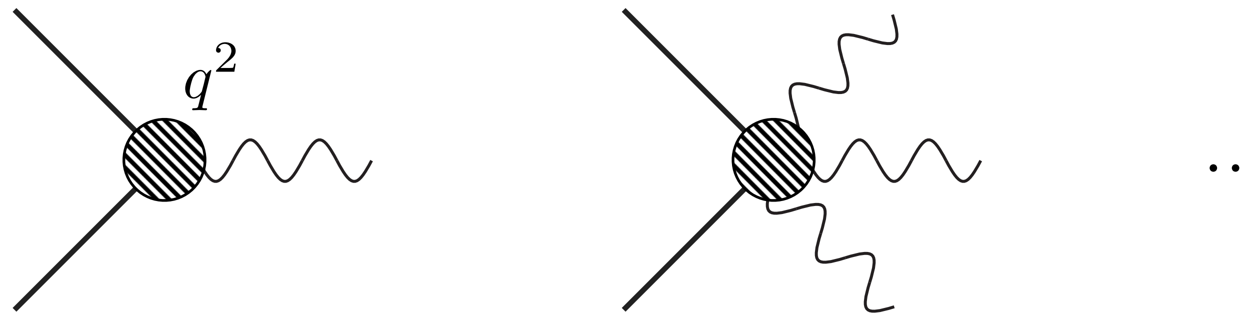
$$\mathcal{M} \sim \left(\frac{E}{\Lambda}\right)^n \ll 1$$

Other fundamental interactions have been neglected, since irrelevant at low energies, $E \ll \Lambda$.

any QFT:



EFT = expansion in fields/derivatives consistent with *symmetries* of the system.

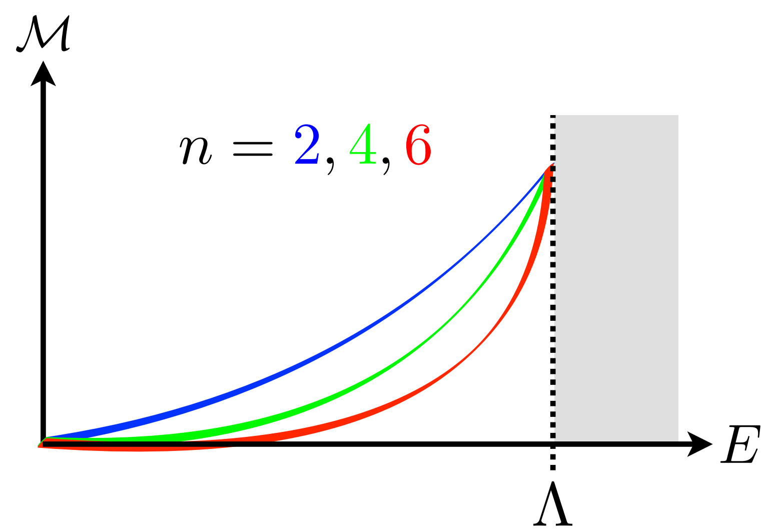


$$\mathcal{M} \sim \left(\frac{E}{\Lambda}\right)^{n}$$

softness

cutoff

These interactions are crucial to unravel what lies beyond the Standard Model.



From the fundamental properties of QFT we can also learn about them; not everything goes.

Naive Dimensional Analysis and Symmetries

Dimensions of masses, (strong coupling) scales, and couplings.

$e^{i\frac{p\cdot x}{\hbar}}$

$$T \sim \frac{L}{c} \qquad E \sim \frac{\hbar}{T}$$

$$\rightarrow \quad \mathcal{S} = \int d^4x \mathcal{L} \sim \hbar \sim EL \quad \rightarrow$$

$$m \sim \Lambda \sim \partial^{-1} \sim \frac{1}{L}$$

$$\phi \sim f \sim \sqrt{\frac{E}{L}}$$

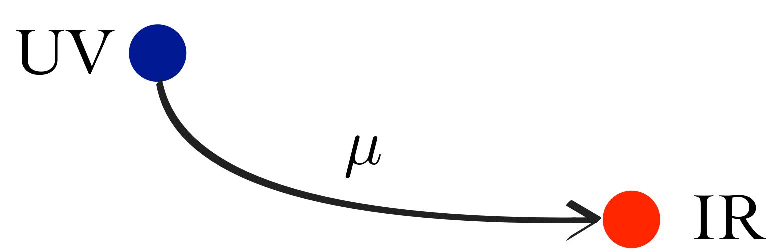
$$g_* \sim \frac{1}{\sqrt{EL}}$$

$$\downarrow$$

$$\Lambda \sim g_* f$$

$$\mathcal{M}(2 \to 2) \sim g_*^2$$

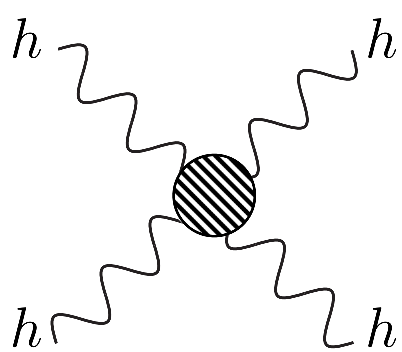
Symmetries are preserved by the renormalization group flow.*



- General Relativity:

$$\mathcal{S}_{\rm EH} = \tfrac{1}{2} M_{\rm Pl}^2 \int d^4x \sqrt{-g}\, R \;=\; O(\partial^2 h^n)$$

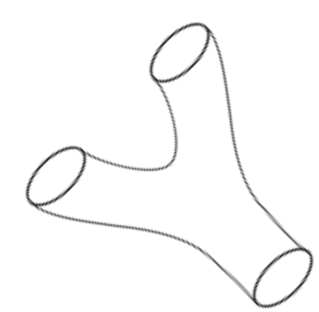
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



$$\mathcal{M} \sim \left(\frac{E}{M_{\rm Pl}}\right)^2 \quad \xrightarrow{\text{NDA}} \quad \Lambda \sim g_s M_{\rm Pl}$$

Gravity needs UV completion.

Theoretical prediction of e.g. *string theory.*



- Standard Model pre-LHC:

non-Abelian massive spin-1 theory
(3 = 2 + 1 d.o.f.)

$$\mathcal{L}_{s=1} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} v^2 A^a_\mu A^{a\mu} + \cdots$$

Stueckelberg trick

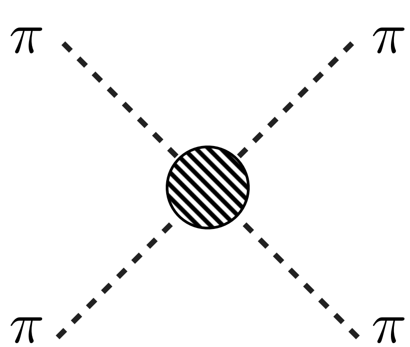
$$A_\mu \to \hat{A}_\mu \equiv A_\mu + \partial_\mu \pi$$

manifest gauge invariance

$$A_\mu \to A_\mu - \partial_\mu \theta \quad \text{(2 d.o.f.)}$$

$$\pi \to \pi + \theta \quad \text{(1 d.o.f.)}$$

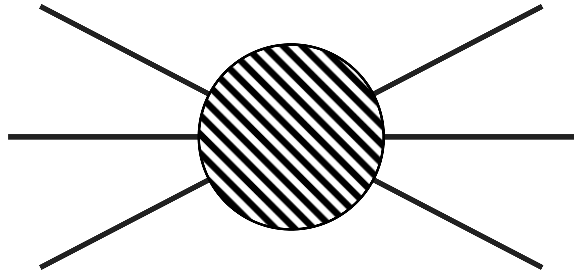
$$\mathcal{L}_\pi = \frac{1}{2}(\partial \pi^a)^2 + \frac{1}{v^2}(\pi^a \partial \pi^a)^2 + \cdots$$



$$\mathcal{M} \sim \left(\frac{E}{v}\right)^2 \xrightarrow{\text{NDA}} \Lambda \sim g_h v$$

Theoretical prediction of the *Higgs boson.*

S-matrix Constraints



S-matrix Theory

$$S = \mathbb{I} + i(2\pi)^4 \delta(p) \mathcal{M}$$

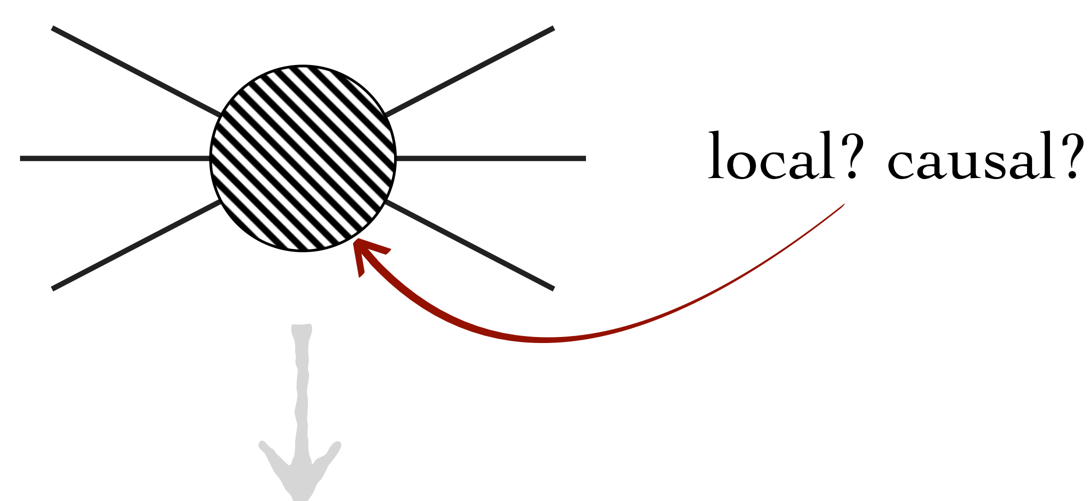
- Unitarity:

$$S^\dagger S = \mathbb{I}$$ → optical theorem

- Poincaré invariance:

  particle = (m, s)

  Little group covariant amplitudes.

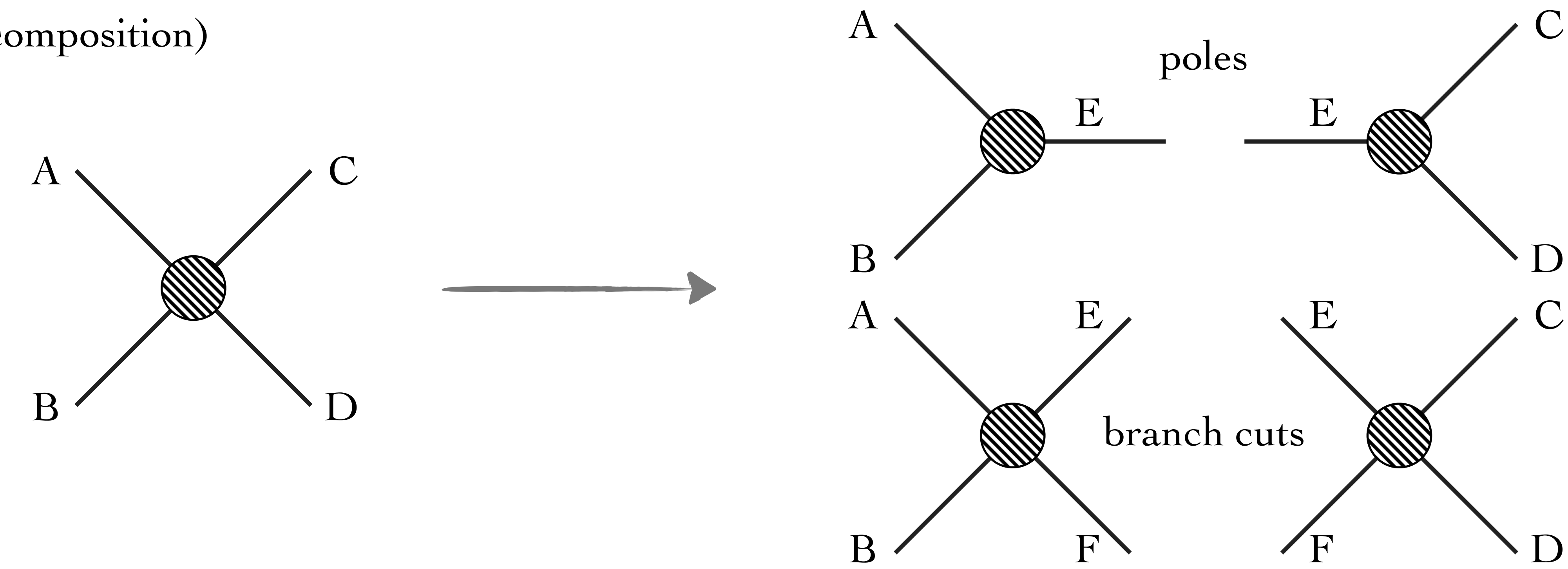
- 

Properties of scattering amplitudes in the complex plane of kinematical variables.

2 to 2 scattering Mandelstam variables: *s, t, u*

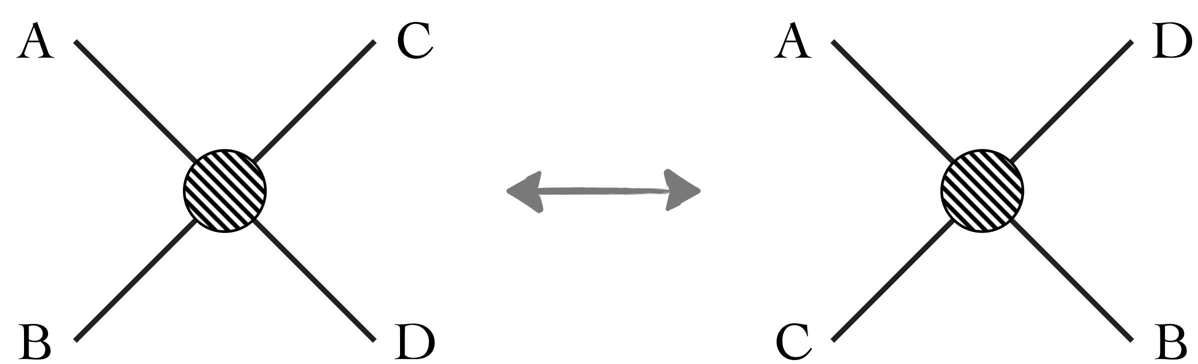
- Analyticity:
  (~ cluster decomposition)

$\mathcal{M}(s,t,u)$



- Crossing symmetry:

$\mathcal{M}(s,t,u)$



$\mathcal{M}(t,u,s)$

- Polynomial boundedness:
  (Froissart-Martin bound)

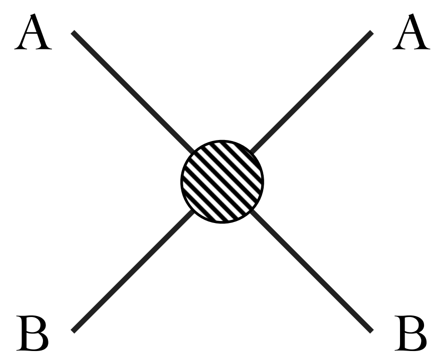
forward limit: t = 0

$$|\hat{\mathcal{M}}(s)| < c \cdot s \log^2 s$$

Satisfied in massive* QFT (and perturbative string theories).

(Gell-Mann, Goldberger, Thirring ’54)

IR-UV Connection



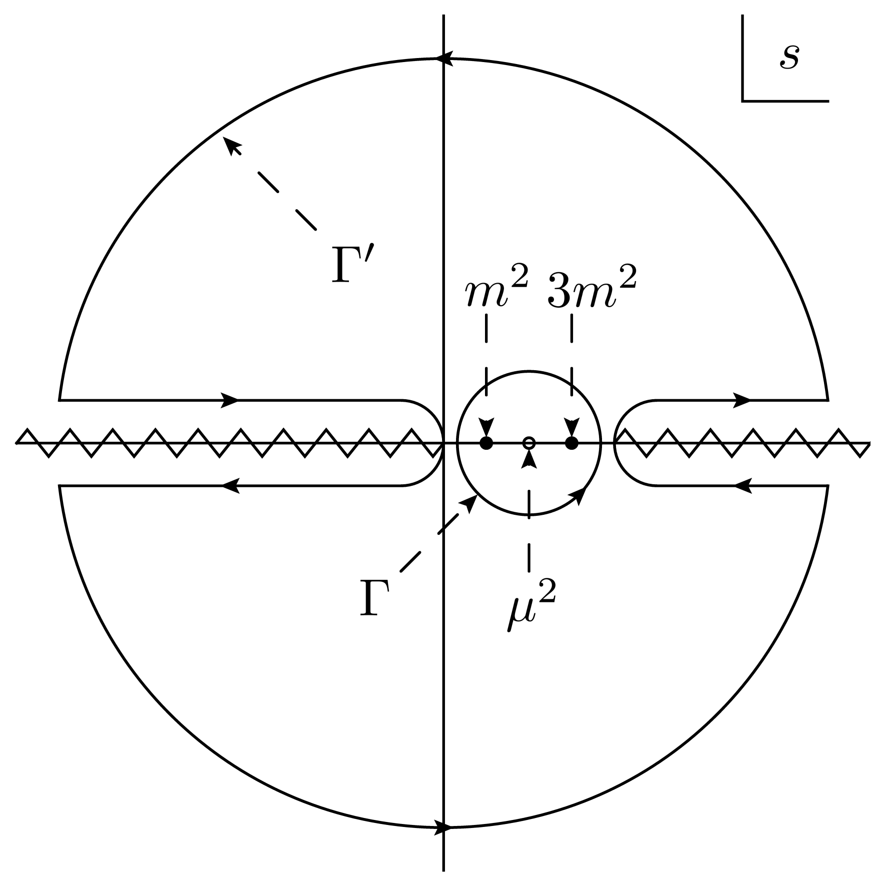
$t \to 0$

$$\frac{1}{2\pi i}\oint_{\Gamma} \frac{\hat{\mathcal{M}}_{AB}(s)}{(s-\mu^2)^3}$$

forward elastic 2 to 2 scattering (s = 0)

Γ

Γ'



$\sum$ IR residues

$$\int_{4m^2}^{\infty} ds\, [\text{branch cuts}] + \text{boundary}$$

IR tree

Froissart bound

crossing symmetry

optical theorem

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$

Positivity constraint from a dispersion relation.

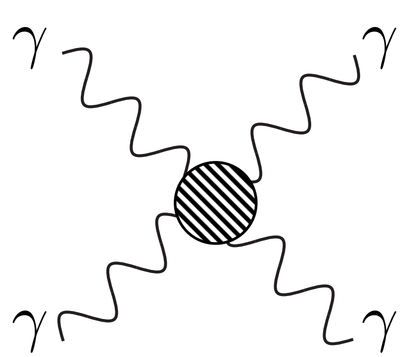
$$\times \left[1 + O\left(\frac{m^2}{\Lambda^2}, \frac{\mu^2}{\Lambda^2}\right)\right]$$

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

UV theories with *canonical* S-matrices give rise to EFTs satisfying positivity constraints.

- Theory of interacting photons (Euler-Heisenberg):

$$\mathcal{L}_{U(1)} = -\frac{1}{4}F^2_{\mu\nu} + \frac{c}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{\tilde{c}}{\Lambda^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

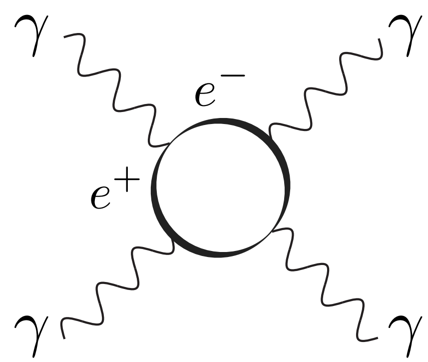


$$\hat{\mathcal{M}}_{\uparrow\uparrow,\uparrow\downarrow} = |\#|(c,\tilde{c})\frac{s^2}{\Lambda^4}$$

- NDA: $|c|, |\tilde{c}| \sim g_*^2$
- Dispersion relations: $c, \tilde{c} > 0$

Theories with wrong-sign coefficients live in the *swampland*.

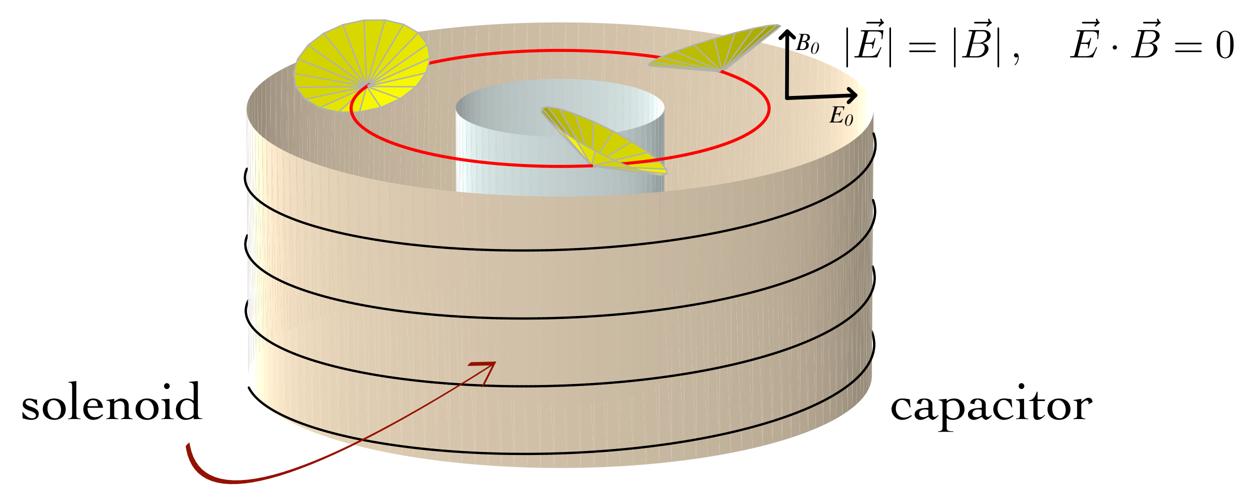
These constraints are verified in explicit examples, e.g. QED.



$$\frac{c}{\Lambda^4} = \frac{\alpha^2}{90 m_e^4}\,, \qquad \frac{\tilde{c}}{\Lambda^4} = \frac{7\alpha^2}{360 m_e^4}$$

Theories with wrong-sign coefficients lead to superluminality and violation of causality.

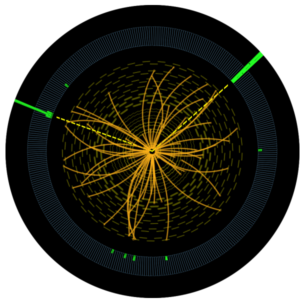
(Adams et al. '06)



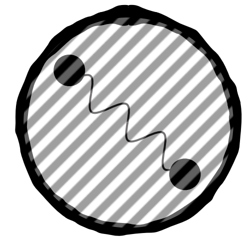
$$v = \frac{1 - c\frac{32}{\Lambda^4}|\vec{E}|^2}{1 + c\frac{32}{\Lambda^4}|\vec{E}|^2}$$

Applications

Composite Higgs



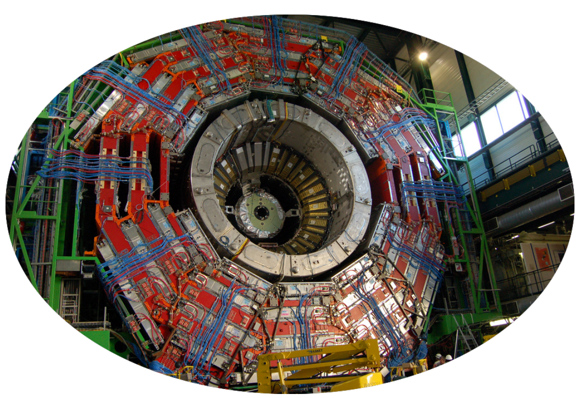
New strong force as a solution to the electroweak hierarchy problem.



H

experiment

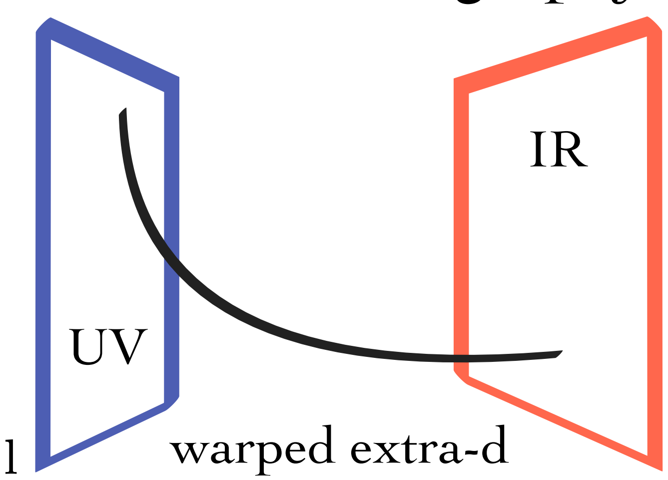
LHC-era



$$\Lambda \sim \text{TeV} \ll M_{\text{Pl}}$$

theory

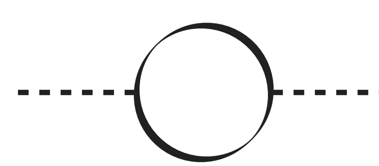
AdS/CFT-holography



Light Higgs as a pseudo-Nambu-Goldstone boson from symmetry breaking.

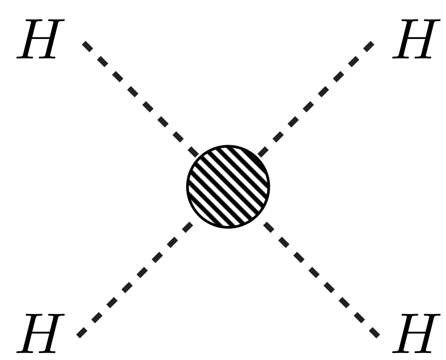
$$H \to H + \Theta$$

(Kaplan, Georgi, Dimopoulos '84)

m_H^2  $\ll \Lambda^2$

Strongly interacting light Higgs

$$\mathcal{L}_{\rm CH} = \mathcal{L}_{\rm SM} + \frac{c}{\Lambda^2}\left(\partial H^\dagger H\right)^2 + \cdots$$



$$\mathcal{M} \sim c\,\frac{s}{\Lambda^2}$$

Compositeness gives rise to scattering amplitudes that grow with energy.

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty}\frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

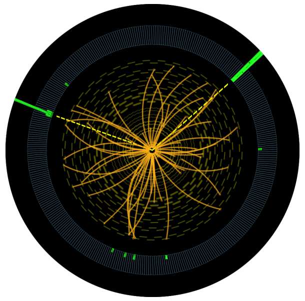
(Low, Rattazzi, Vichi '09)

No significant information on leading higher-dimensional operators: $|c| \sim g_*^2 \gtrsim 1$.

Experiments crucial to learn: *WW*-scattering, double-*H* production, *S*-parameter, etc.

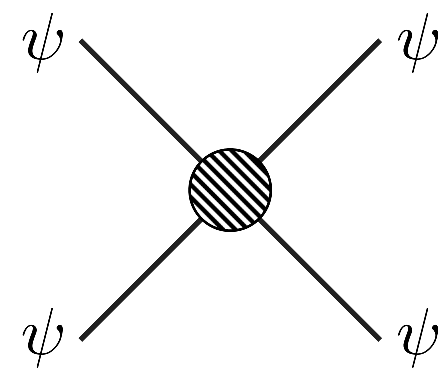
CMS ATLAS EXPERIMENT FCC hh ee he ilc clic

Fermion Compositeness



Strongly interacting light Fermions

Compositeness gives rise to scattering amplitudes that grow with energy.



$$\mathcal{L}_{\rm CF} = i\bar\psi\gamma\partial\psi + \left\{ \frac{c_0}{\Lambda^2}\left(\bar\psi\gamma\psi\right)^2 , \quad \frac{c_1}{\Lambda^4}\left(\bar\psi\gamma\partial\psi\right)^2 , \quad \frac{c_2}{\Lambda^6}\left(\partial\bar\psi\gamma\partial\psi\right)^2 , \quad \cdots \right\} + \cdots$$

$\mathcal{M} \sim c_0 \dfrac{s}{\Lambda^2}$	$\mathcal{M} \sim c_1 \dfrac{s^2}{\Lambda^4}$	$\mathcal{M} \sim c_2 \dfrac{s^3}{\Lambda^6}$
$\psi \to e^{i\alpha}\psi$	$\psi \to \psi + \xi + i(\bar\psi\gamma\xi - \bar\xi\gamma\psi)\partial\psi$	$\psi \to \psi + \xi$
chiral compositeness	goldstino compositeness	not consistent

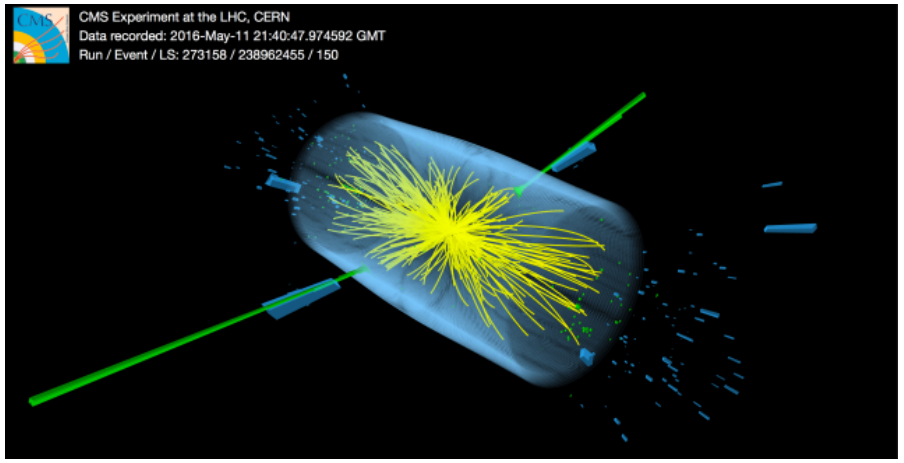
$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

There are only two consistent types of composite fermions.

These two types of compositeness, and principles of QFT (positivity) can be probed at colliders.

$c_1 > 0$

$pp \to jj$



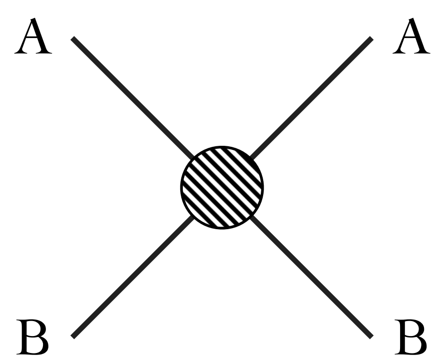
	goldstino	chiral
composites	$\sqrt{F}$ $[\,\Lambda\,]$ (TeV)	f $[\,\Lambda\,]$ (TeV)
d_R	2.6 [9.4]	2.9 [36]
u_R	3.8 [13.5]	4.7 [59]
u_R, d_R	3.9 [13.7]	4.9 [62]
q_L	3.9 [13.7]	4.9 [62]
q_L, d_R	4.0 [14.2]	5.0 [63]
q_L, u_R	4.5 [16.1]	5.7 [72]
q_L, u_R, d_R	4.6 [16.2]	5.8 [73]
	$\Lambda = \sqrt{4\pi F}$	$\Lambda = 4\pi f$

(Bellazzini, Riva, JS, Sgarlata '17)

(Bellazzini '16)

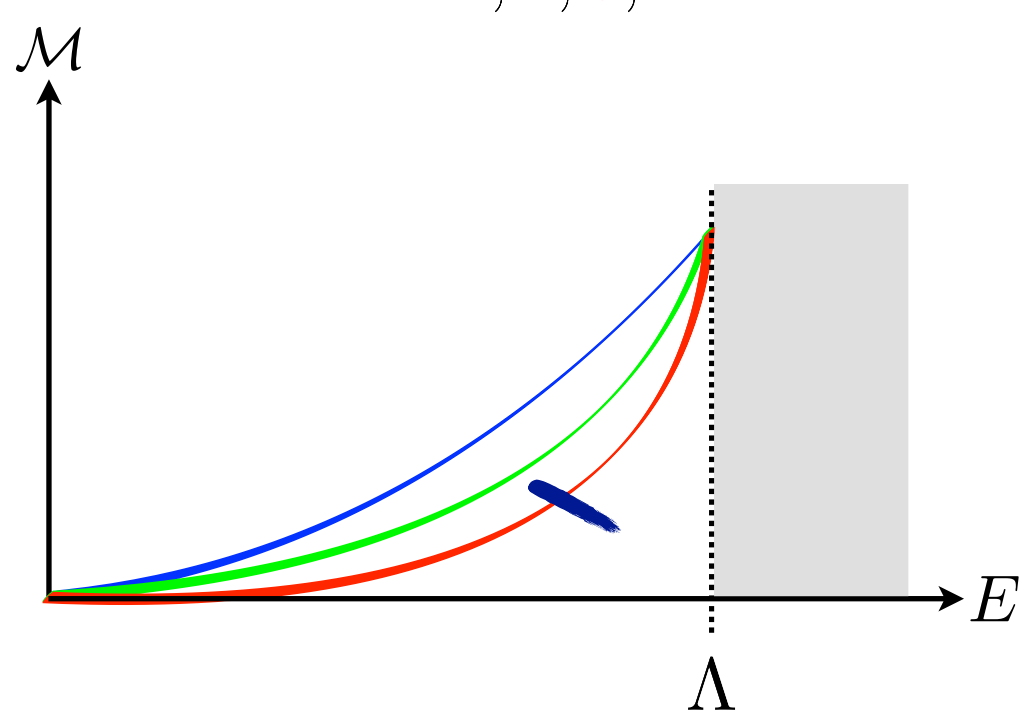
$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$

Universal statement irrespective of spins.



$$\mathcal{M} \sim \left(\frac{s}{\Lambda^2}\right)^n$$

$n = 1, 2,$ ~~$3, \ldots$~~

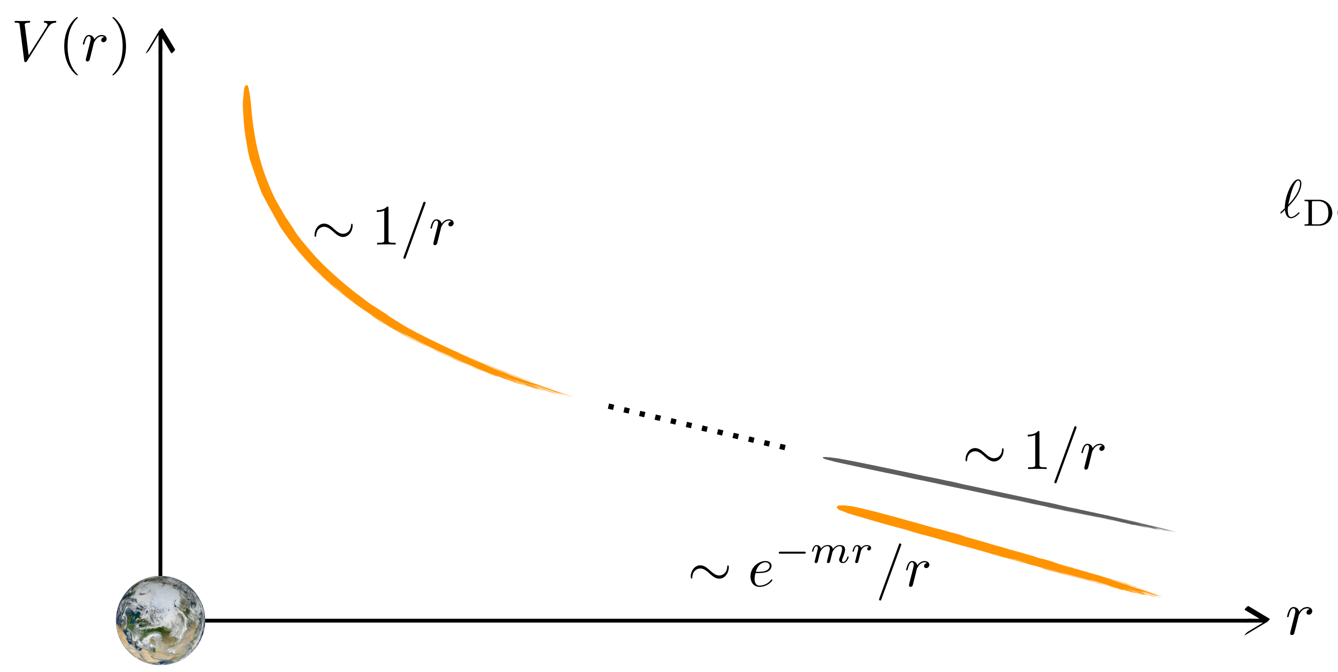


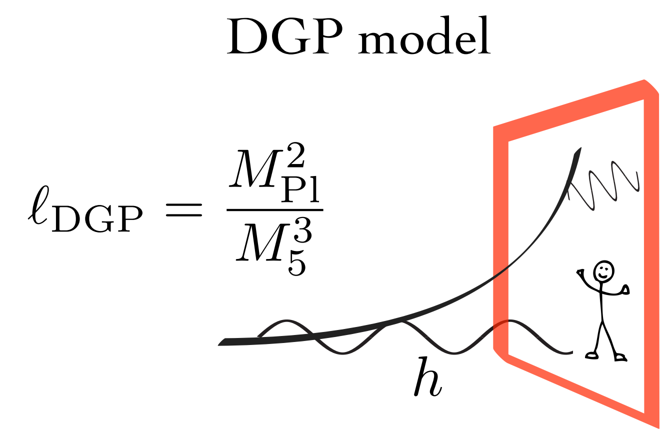
Massive Gravity

IR modification of gravity such that cosmology is modified.

$$m \approx H_0 \approx (10^{28}\mathrm{cm})^{-1} \approx 2 \times 10^{-33}\mathrm{eV}$$

(inverse) size of universe





MG can reproduce late-time cosmic acceleration without Λ_4; however Λ_4 still gravitates.

MG as very interesting modification of gravity; it helped us *understand* gravity.

Massive spin-2: dRGT- or Λ_3-theory

(5 d.o.f.)

(de Rham, Gabadadze, Tolley '10)

$$\mathcal{S}_{\rm EH} = \tfrac{1}{2} M_{\rm Pl}^2 \int d^4x \sqrt{-g}\, R$$

$$\mathcal{S}_{\rm matter} = \tfrac{1}{2} \int d^4x \sqrt{-g}\, h_{\mu\nu} T_{\rm m}^{\mu\nu} + O(h^2)\,, \quad \partial_\mu T_{\rm m}^{\mu\nu} \overset{*}{=} 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

MG has same action as GR but with a potential: mass + cubic + quartic terms.

$$\mathcal{S}_{\rm mass} = -\tfrac{M_{\rm Pl}^2}{2} m^2 \int d^4x \sqrt{-g} \left[h_{\mu\nu}^2 - h^2 + O(h^3) + O(h^4) \right]$$

Stueckelberg fields of Massive Gravity

(Arkani-Hamed, Georgi, Schwartz '02)

Goldstone formalism introduces the d.o.f.'s that identify the cutoff of the EFT.

$$h_{\mu\nu} \to H_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu \pi_\nu + \partial_\nu \pi_\mu - \partial_\mu \pi^\alpha \partial_\nu \pi_\alpha$$

$$\pi_\mu \equiv A_\mu + \partial_\mu \phi$$

5 ~ 2 (h=±2) + 2 (h=±1) + 1 (s=0) d.o.f.

manifest gauge invariance

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$A_\mu \to A_\mu - \xi_\mu + \partial_\mu \lambda$$

$$\phi \to \phi - \lambda$$

$$\mathcal{S}_{\rm EH} = \mathcal{S}_{\rm EH}[h]\,, \quad \mathcal{S}_{\rm matter} = \mathcal{S}_{\rm matter}[h]$$

$$\mathcal{S}_{\rm mass} = \mathcal{S}_{\rm mass}[h, A, \phi]$$

van Dam-Veltman-Zakharov discontinuity

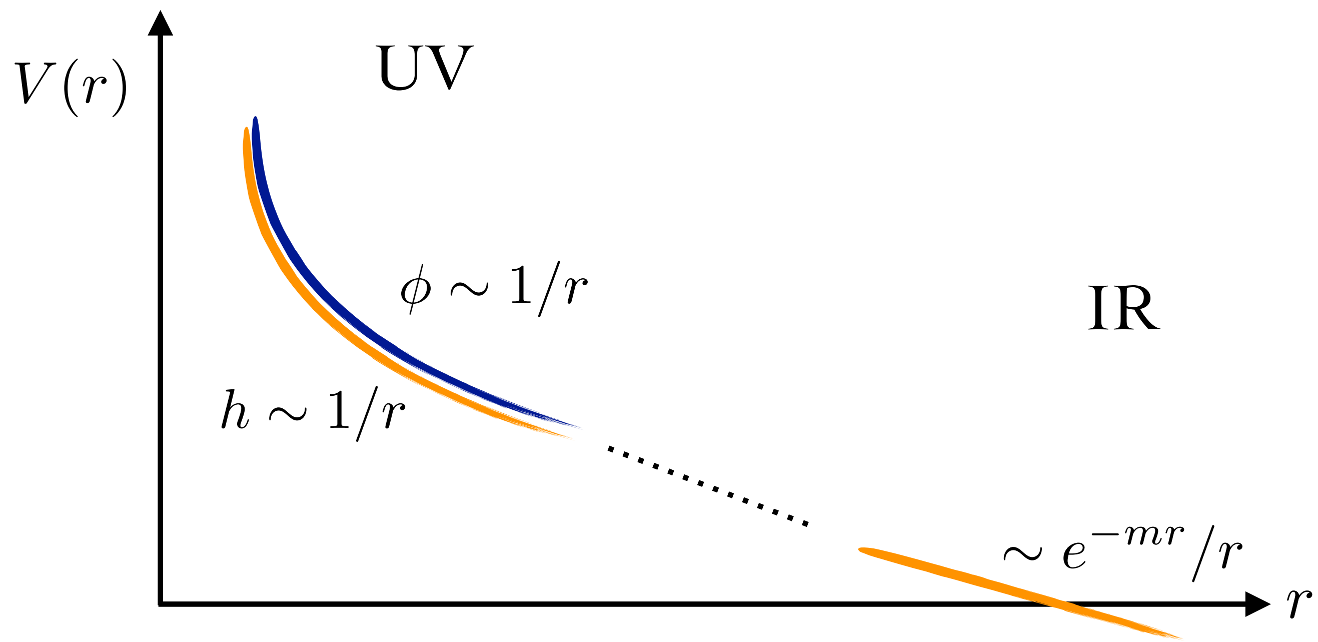
$$\mathcal{S}_{\text{mass}} = \mathcal{S}_{\text{mass}}[h, A, \underline{\phi}]$$

$$h_{\mu\nu} \to h_{\mu\nu} + m^2 \eta_{\mu\nu} \phi$$

$$\frac{1}{M_{\text{Pl}}} \phi \, T_{\text{m}\,\mu}^{\ \ \mu}$$

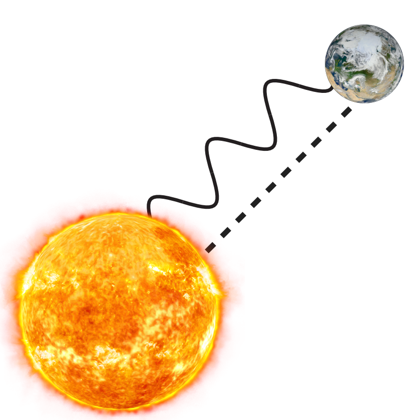
non-decoupling fifth-force

MG is not only an IR modification of GR, but also UV.

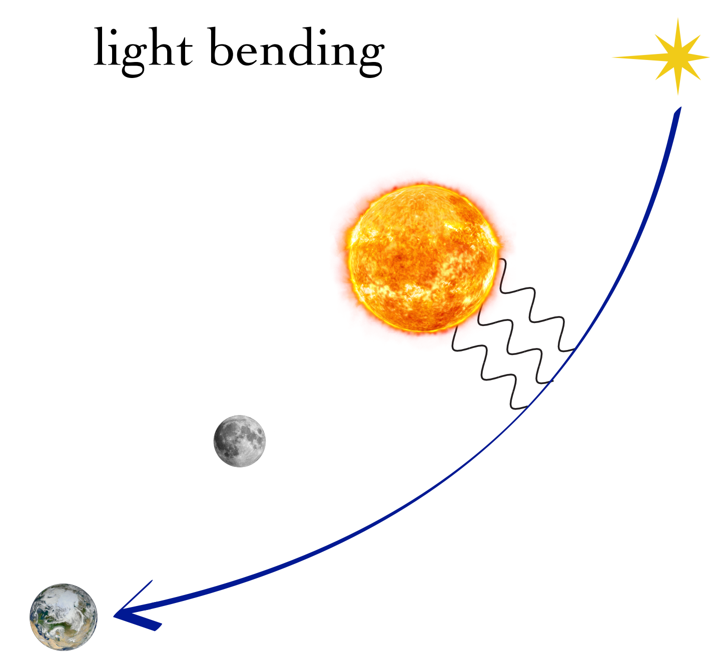


Scalar mode couples to non-relativistic matter but not to radiation.

Earth-Moon system



light bending



Scalar amplitudes of Massive Gravity

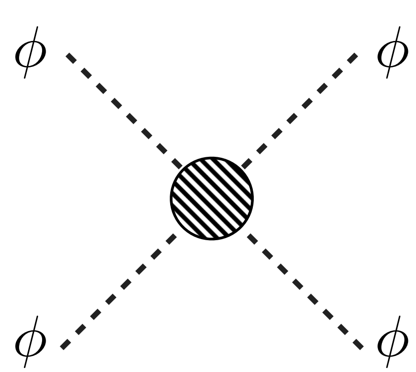
(Nicolis, Rattazzi, Trincherini '08)

$$\mathcal{S}_{\text{mass}} = \mathcal{S}_{\text{mass}}[h, A, \phi]$$

$$h_{\mu\nu} \to h_{\mu\nu} + m^2 \eta_{\mu\nu} \phi$$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda_3^6}(\partial\phi)^2(\partial^2\phi)^2 + \cdots$$

$$\Lambda_3 = (m^2 M_{\text{Pl}})^{1/3} \approx (10^3\,\text{km})^{-1}$$



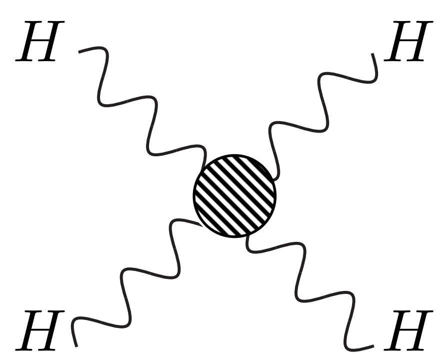
$$\mathcal{M} \sim \left(\frac{E}{\Lambda_3}\right)^6 \xrightarrow{\text{NDA}} \Lambda = g_*^{1/3}\Lambda_3$$

Theoretical constraint on Massive Gravity

(Bellazzini, Riva, JS, Sgarlata '17)

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

$H = h, A, \phi$

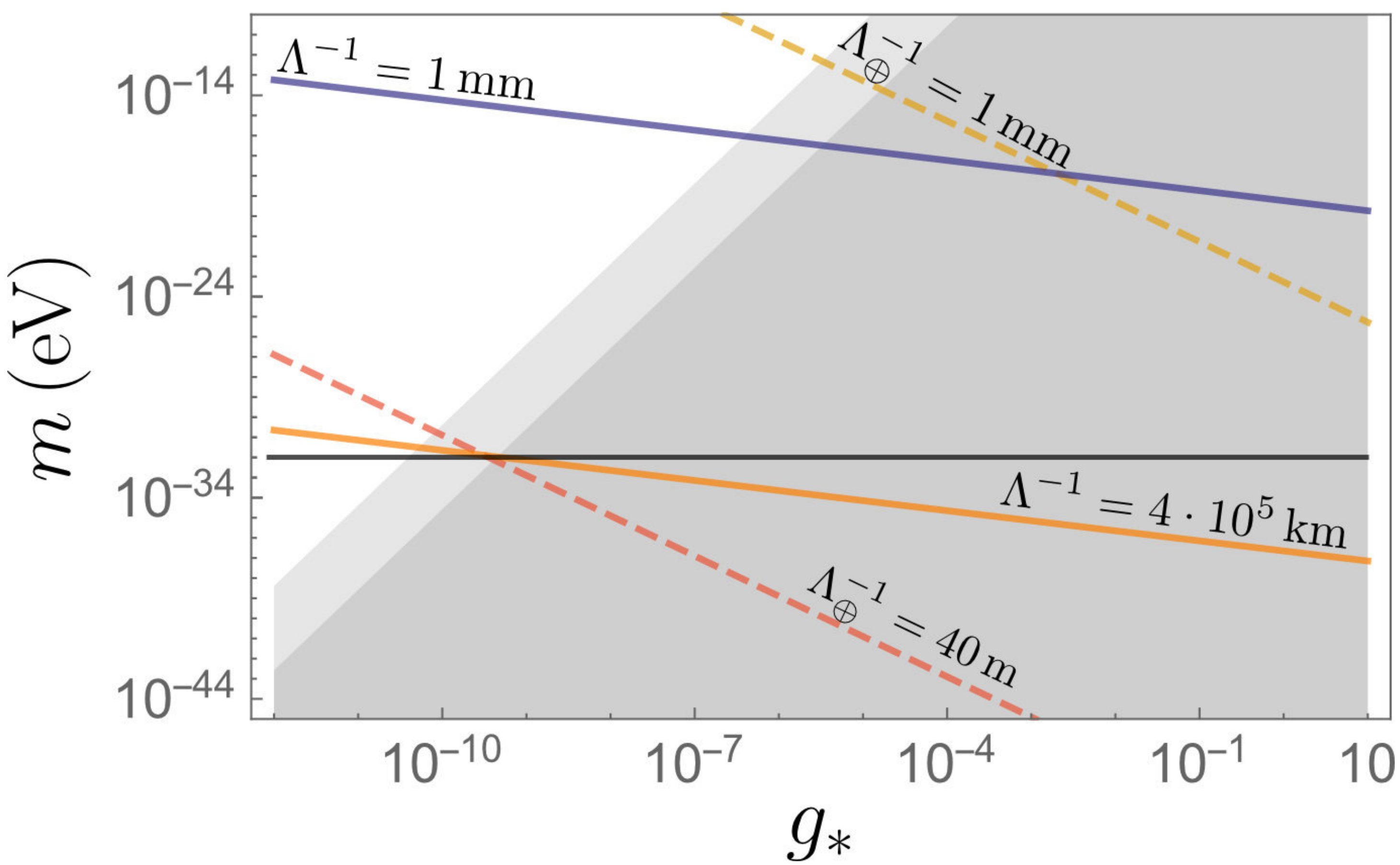


$$\mathcal{M} = \#\frac{stu}{\Lambda_3^6} + \#'\, m^2 \frac{s^2}{\Lambda_3^6} + \cdots$$

$$m > 4\pi M_{\rm Pl}\,\#\left(\frac{g_*}{4\pi}\right)^4\left(\frac{\mu}{\Lambda}\right)^{12}\cdot\left[1+O\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

mass > **max-cutoff** × O(1/10⁶) × 2-loop × **accuracy**^{6}

Cutoff of Massive Gravity



$$m \approx H_0 \approx (10^{28}\mathrm{cm})^{-1} \approx 2 \times 10^{-33}\mathrm{eV}$$

$$\Lambda \approx (1 \times 10^6\,\mathrm{km})^{-1}$$

Light graviton at the expense of EFT validity and predictivity.

$$r_{\mathrm{Moon}} \approx 4 \times 10^5\,\mathrm{km}$$

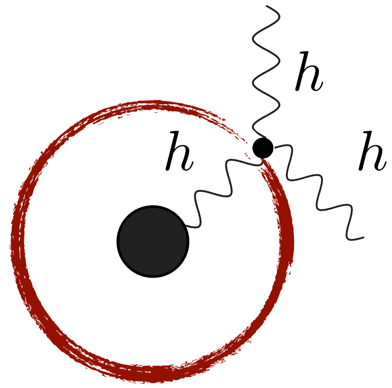
Vainshtein screening

- Schwarzschild radius:

where graviton non-linearities become O(1).

$$\mathcal{L}_{\rm EH} = \frac{1}{2}(\partial h)^2\left[1+\frac{h}{M_{\rm Pl}}+\cdots\right]+\frac{h_{\mu\nu}}{M_{\rm Pl}}T^{\mu\nu}$$

fixed by gauge invariance



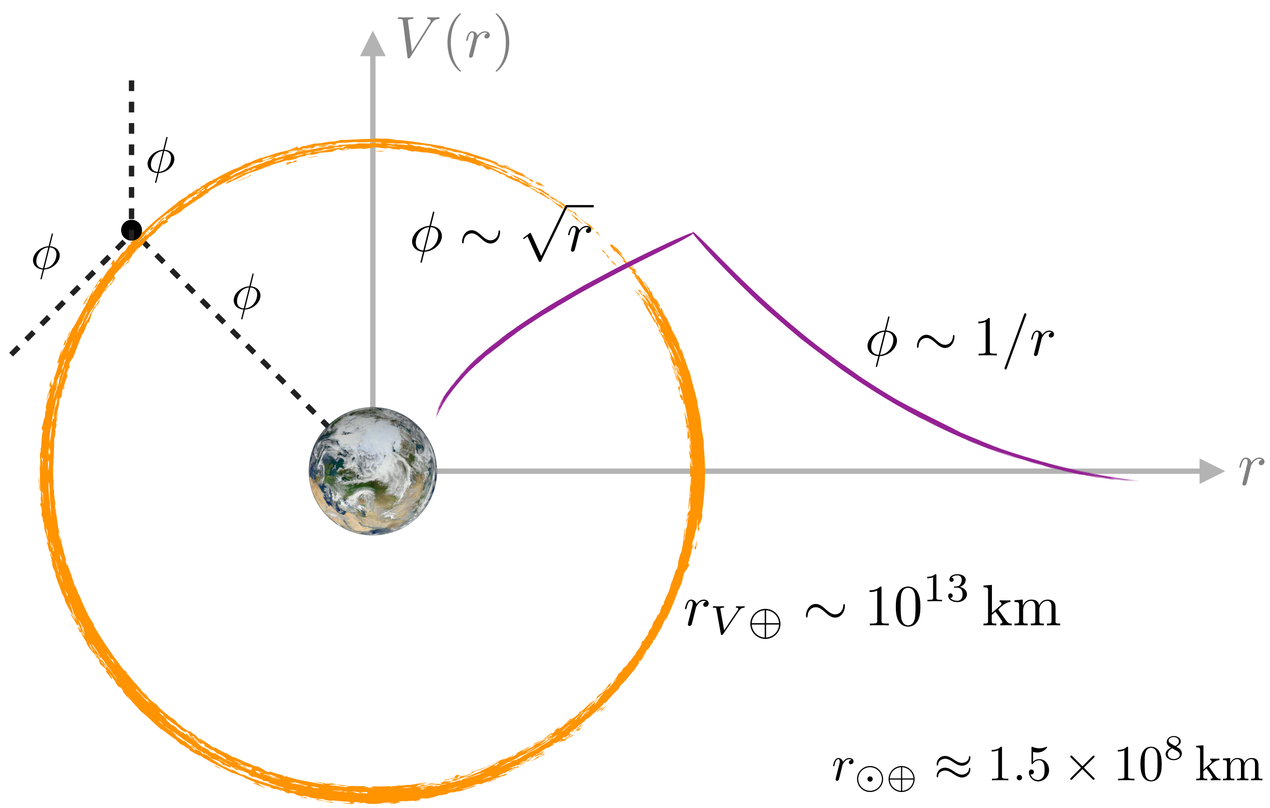
$$r_S \sim \frac{M_\bullet}{M_{\rm Pl}^2}$$

- Vainshtein radius:

where scalar non-linearities become O(1).

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2\left[1+\hat{c}\frac{(\partial^2\phi)}{\Lambda_3^3}+\cdots\right]+\frac{\phi}{M_{\rm Pl}}T^\mu_\mu$$

$$r_V \sim \left(\frac{M_\bullet}{m^2 M_{\rm Pl}^2}\right)^{1/3}$$



$r_{\odot\oplus} \approx 1.5 \times 10^8$ km

Non-linearities are crucial to evade fifth-force experimental constraints.

Fifth-force experiments / Moon's orbital precession

(Dvali, Gruzinov, Zaldarriaga '02)

Very sensitive experimental probe of gravity.

$$(\delta\theta)_{\rm exp} \approx 10^{-11}$$

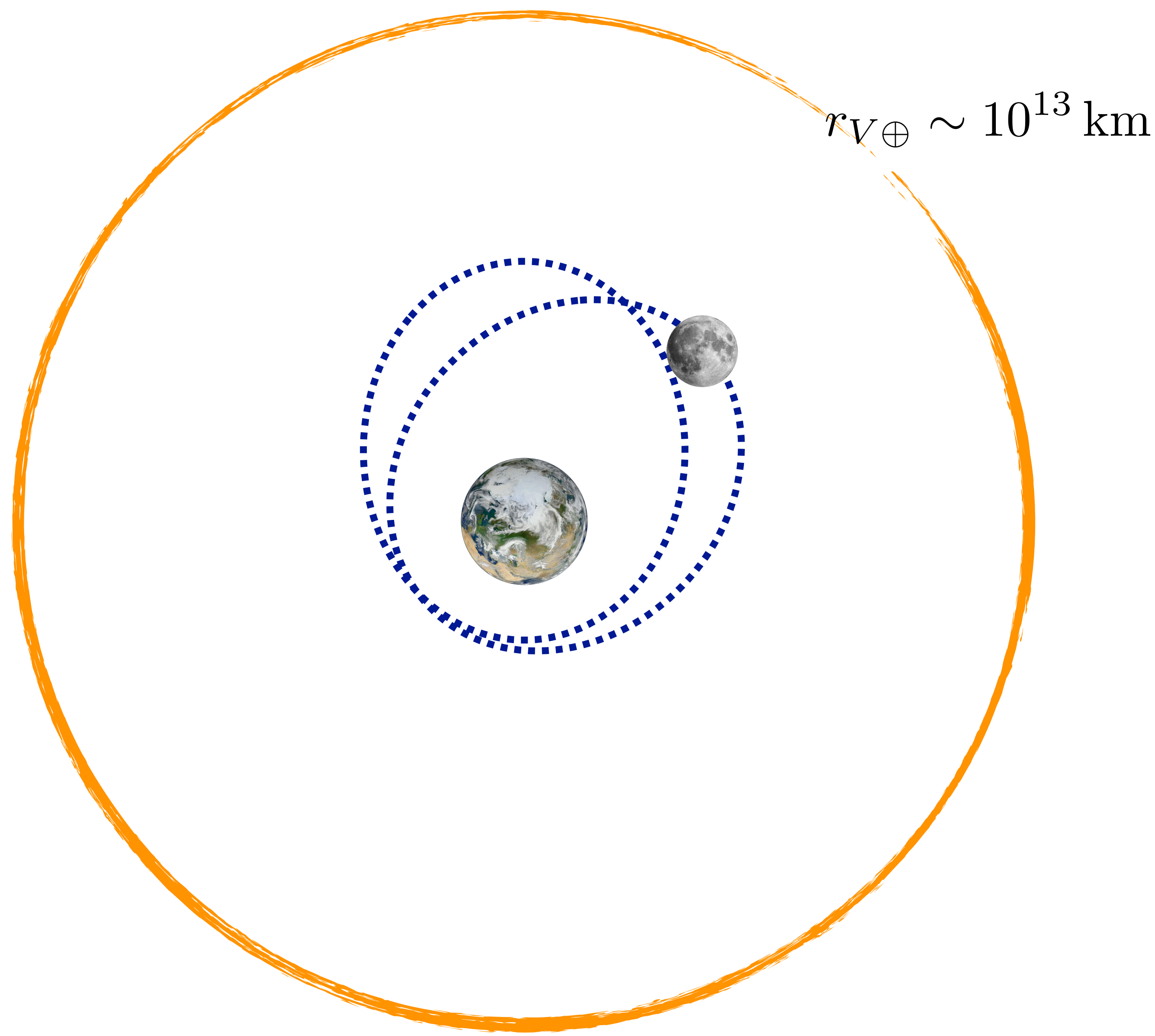
Within Vainshtein region:

$$(\delta\theta) \sim \pi \frac{F_\phi}{F_{h_{\rm GR}}} \sim \left(\frac{r}{r_V}\right)^{3/2}$$

$r = r_{\rm Moon} \approx 4 \times 10^5\,{\rm km}$

$m \approx 2 \times 10^{-33}{\rm eV}$

$$(\delta\theta)_{\rm th} \approx 10^{-11}$$



Currently a very interesting probe of MG.

$$\Lambda^{-1} \approx 1 \times 10^6\,\mathrm{km}$$

Why should Vainshtein screening be operative close and beyond the EFT cutoff?

$$(\delta\theta)_{\mathrm{exp}} \approx 10^{-11}$$

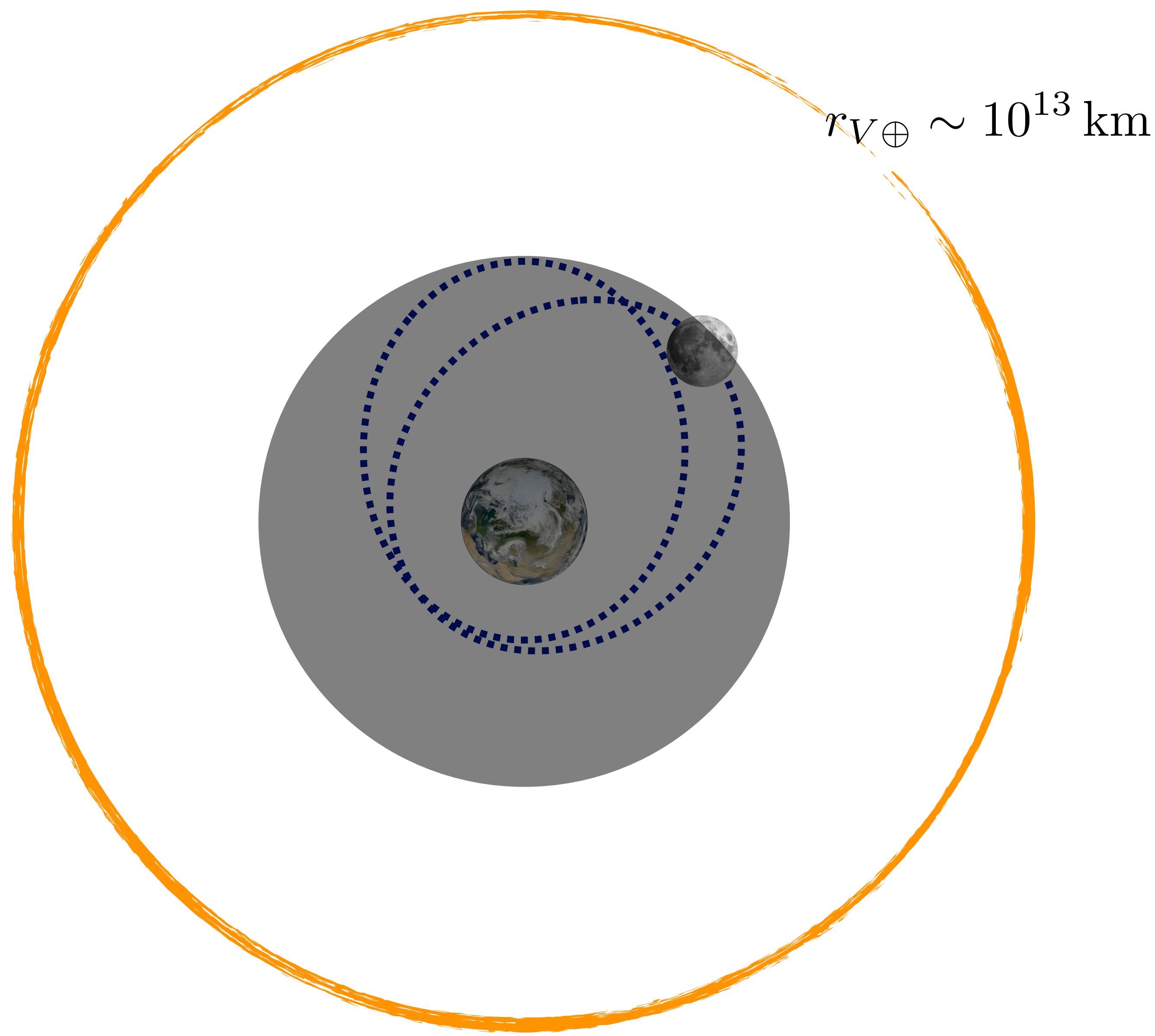
Within Vainshtein region:

$$(\delta\theta) \sim \pi \frac{F_\phi}{F_{h_{\mathrm{GR}}}} \sim \left(\frac{r}{r_V}\right)^{3/2}$$

$$r = \Lambda^{-1} \approx 1 \times 10^6\,\mathrm{km}$$

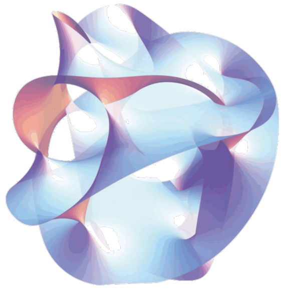
$$m \approx 2 \times 10^{-33}\mathrm{eV}$$

$$(\delta\theta)_{\mathrm{th}}^{\Lambda^{-1}} \approx 10^{-10}$$



MG (dRGT-theory) is experimentally compromised.*

Weak Gravity Conjecture



(Arkani-Hamed et al. ’06)

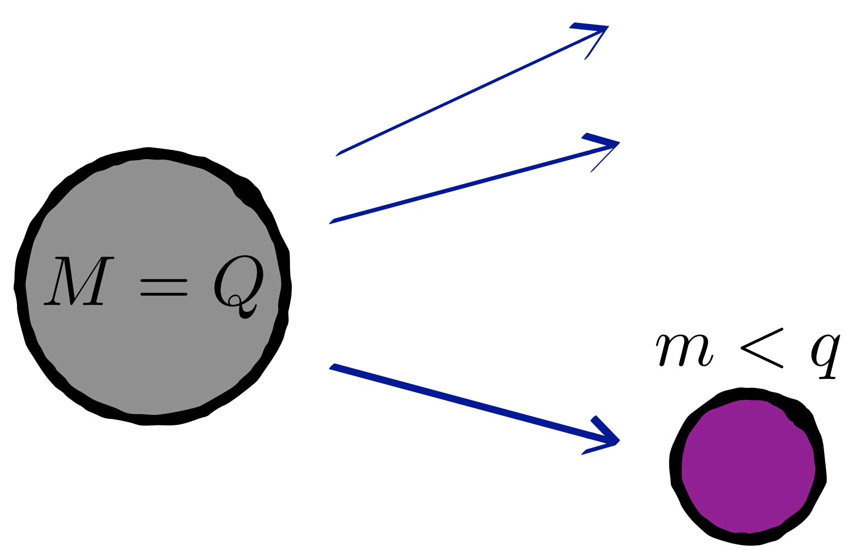
Extremal black holes

Mild form of the WGC: There exist states for which gravity is the weakest force.

$$\left.\frac{\sqrt{2}q}{m/M_{\mathrm{Pl}}}\right|_{\mathrm{min}} > 1$$

suitable units

Extremal BHs are kinematically allowed to decay.



(Arkani-Hamed et al. '06)

Extremal black holes in Einstein-Maxwell EFT

Effects of leading higher-dimensional operators on BH extremality.

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}M_{\rm Pl}^2 R - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \alpha_1 \tfrac{1}{4M_{\rm Pl}^4}\left(F^{\mu\nu}F_{\mu\nu}\right)^2 + \alpha_2 \tfrac{1}{4M_{\rm Pl}^4}\left(\widetilde{F}^{\mu\nu}F_{\mu\nu}\right)^2 + \alpha_3 \tfrac{1}{2M_{\rm Pl}}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\right]$$

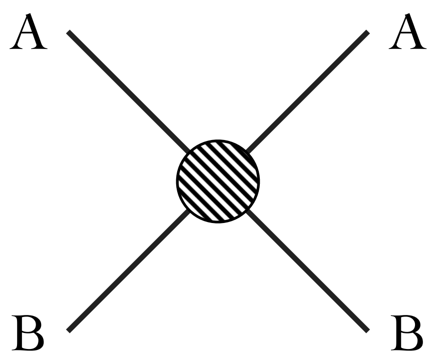
↓ (Kats, Motl, Padi '06)

$$\left(\frac{\sqrt{2}|Q|}{M/M_{\rm Pl}}\right)_{\rm extr.} = 1 + \frac{4}{5}\frac{(4\pi)^2 M_{\rm Pl}^2}{M^2}(2\alpha_1 - \alpha_3)$$

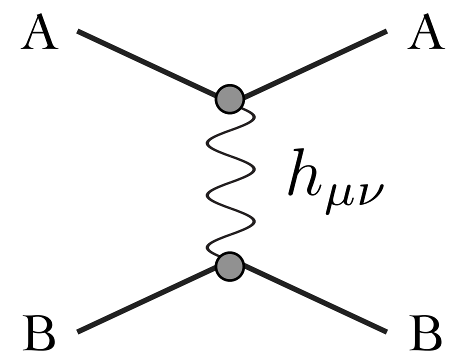
If $2\alpha_1 - \alpha_3 > 0$, extremal black holes are in fact the required states (gravity is weakest).

Gravitational Coulomb singularity

Turning on gravity gives rise to a universal forward singularity.



$\supset$

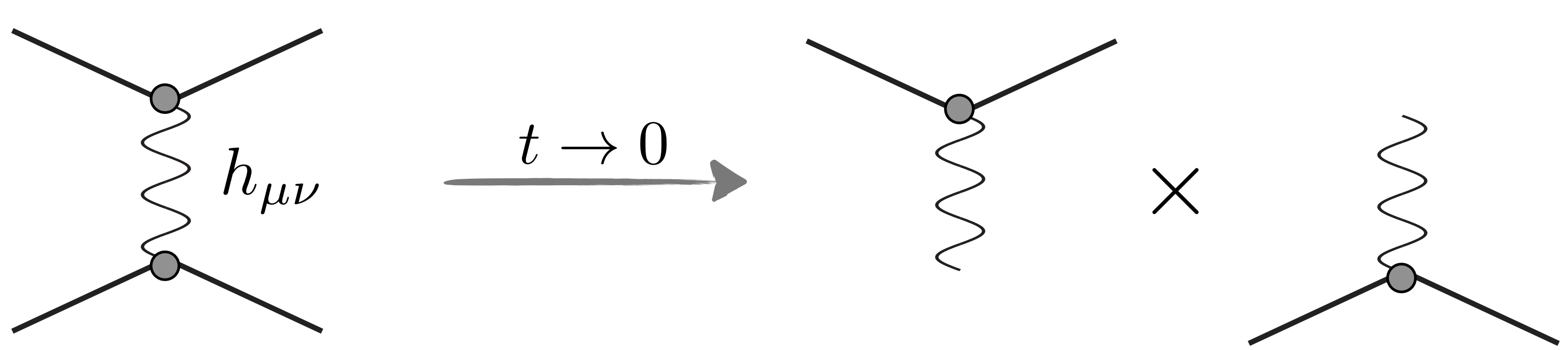


$$\mathcal{M}(t \to 0) = -\tfrac{1}{M_{\mathrm{Pl}}} \frac{s^2}{t} + O(s)$$

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi} \int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB \to X}(s) > 0$$

$$\infty = \infty$$

Factorization in the forward limit: soft graviton probes arbitrarily large distances.



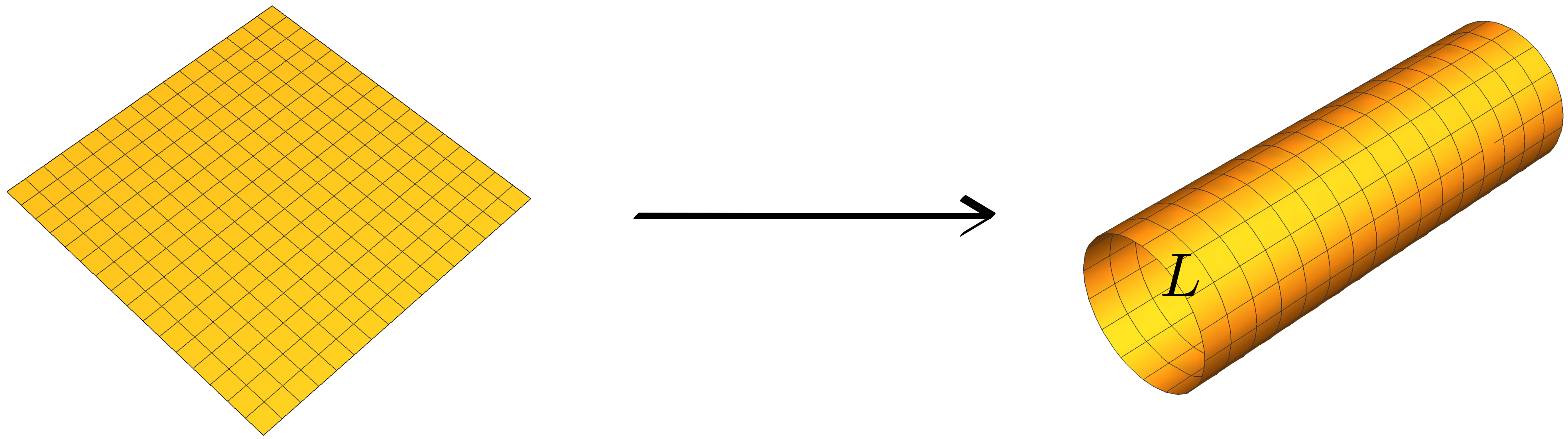
(Bellazzini, Lewandowski, JS '19)

Compactifying to 3D

To regulate the Coulomb singularity while retaining Lorentz invariance.

4D flat space

3D flat space



$\hat{g}_{MN}$ $\longrightarrow$ $g_{\mu\nu}\,,\ \sigma\,,\ V_\mu\,,$ KK-modes

- $g_{\mu\nu}$: non-dynamical metric
- σ: dilaton
- V_μ: graviphoton
- KK-modes: $m_n^2 \sim \frac{n^2}{L^2}$

$\hat{A}_M$ $\longrightarrow$ $A_\mu\,,\ \Phi\,,$ KK-modes

- A_μ: photon
- Φ: scalarphoton

Einstein-Maxwell EFT from 4D to 3D

$$\mathcal{S} = L \int d^3x \sqrt{-g} \times$$

$$R - \tfrac{1}{2}(\partial\sigma)^2 - \tfrac{1}{4} V^{\mu\nu} V_{\mu\nu}$$

$$-\tfrac{1}{4}(1-\sigma) F^{\mu\nu} F_{\mu\nu} - (1+\sigma)\tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{2} F_{\mu\nu} V^{\mu\nu} \Phi$$

$$\alpha_1 \tfrac{1}{4M_{\rm Pl}^4} \left(F^{\mu\nu} F_{\mu\nu} + 2(\partial\Phi)^2\right)^2 + \alpha_2 \tfrac{1}{M_{\rm Pl}^4} \left(\epsilon^{\mu\nu\rho} F_{\mu\nu} \partial_\rho \Phi\right)^2$$

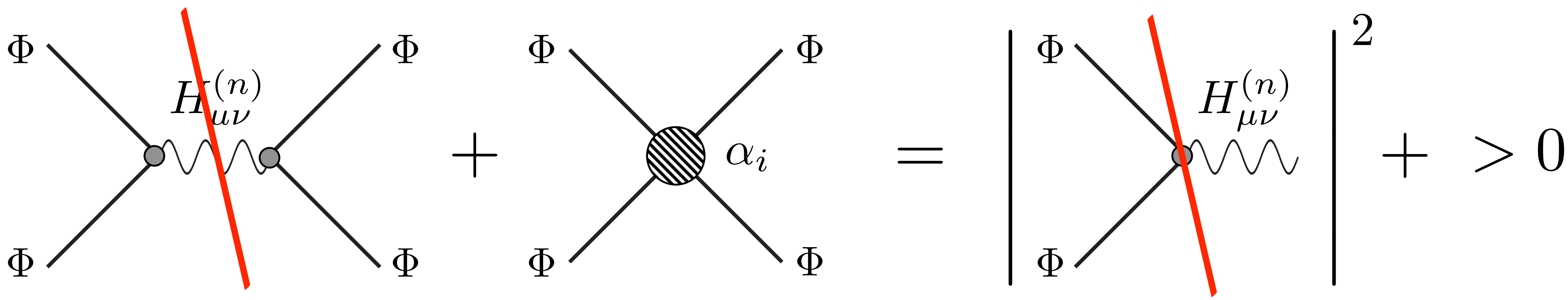
$$-\alpha_3 \tfrac{1}{M_{\rm Pl}^2} \left[\left(F_{\rho\mu} F^{\rho}{}_{\nu} - \partial_\mu \Phi \partial_\nu \Phi\right) \nabla^\mu \nabla^\nu \sigma + F_{\mu\nu} \partial_\rho \Phi \left(\nabla^\rho V^{\mu\nu} + g^{\mu\rho} \nabla_\alpha V^{\nu\alpha}\right)\right]$$

$$\alpha_3 \tfrac{1}{M_{\rm Pl}^4} \left[F_{\rho\mu} F^{\rho\nu} F^{\mu\sigma} F_{\nu\sigma} - \tfrac{1}{2} F^4 - (\partial\Phi)^4 + \tfrac{1}{2} F^2 (\partial\Phi)^2\right]$$

$$+\,\text{KK-modes}$$

Cancelling "Coulomb" singularity

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$



Same procedure holds with loops of KK-modes (the ones that actually matter).

$$\hat{\mathcal{M}}(\Phi\Phi \to \Phi\Phi) = \frac{2s^2}{M_{\rm Pl}^4 L}\left(2\alpha_1 - \alpha_3\right) > 0$$

$$\hat{\mathcal{M}}(AA \to AA) = \frac{2s^2}{M_{\rm Pl}^4 L}\left(2\alpha_1 + \alpha_3\right) > 0$$

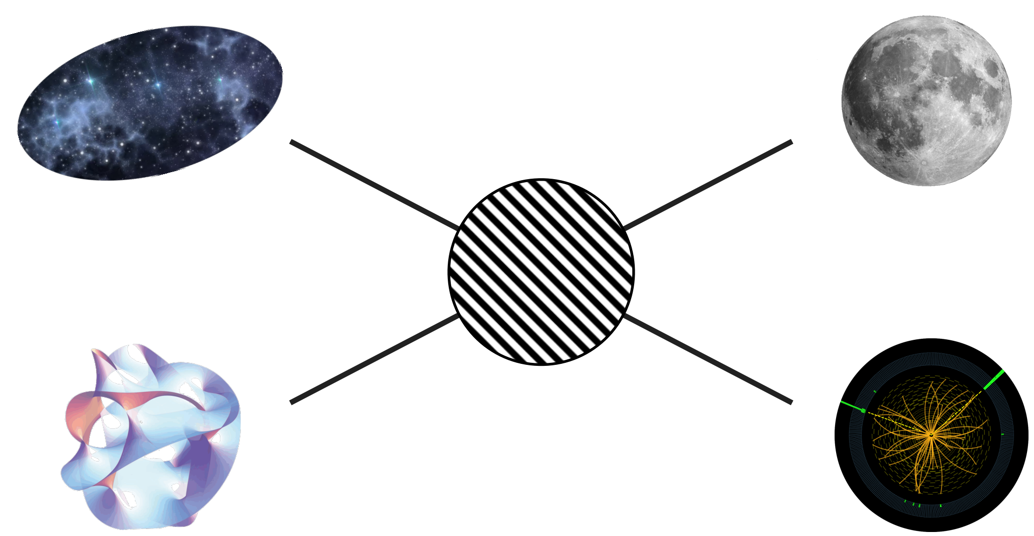
$$\hat{\mathcal{M}}(\Phi A \to \Phi A) = \frac{4s^2}{M_{\rm Pl}^4 L}\alpha_2 > 0$$

The mild form of the WGC follows from prime principles of the S-matrix.

Similar consequences on scalar theories with gravity: *P(X)* theories, galileons.

Conclusions

- Not everything goes in Quantum (Effective) Field Theory.
  - Fundamental properties of the microscopic world are very powerful and restrictive.
  - Unitarity, locality, and causality of the S-matrix implies dispersion relations.



- Far-reaching implications in many fields of fundamental physics.
  - Understanding of the world we see and of its conservations laws.
  - There exist only two consistent types of composite fermions.
  - dRGT-theory in need of a sensible UV completion at macroscopic distances.
  - Tentative S-matrix proof of the weak gravity conjecture.

Thank you.

Other interesting applications:

a-theorem (4D)
Komargodski, Schwimmer '11
Luty, Polchinski, Rattazzi '12
...

SUSY + R-symmetry breaking
Dine, Festuccia, Komargodski '09
Bellazzini, Mariotti, Redigolo, Sala, JS '17
...

composite EWSB
Distler, Grinstein, Porto, Rothstein '06
Vecchi '07
Low, Rattazzi, Vichi '09
...

quantum gravity
Adams et al.'06
Bellazzini, Cheung, Remmen '15
...

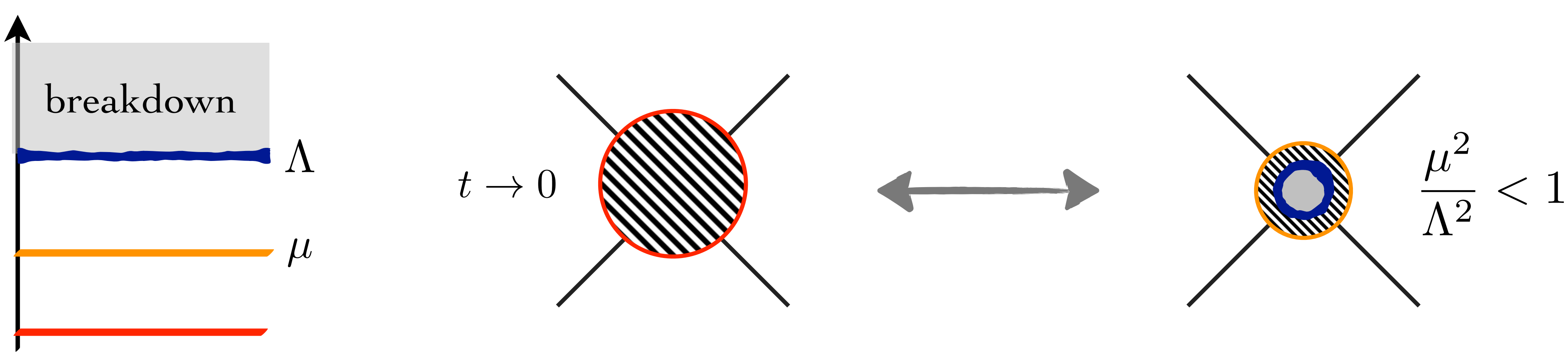
Beyond Positivity Constraints

(Bellazzini, Riva, JS, Sgarlata '17)

$$\hat{\mathcal{M}}''_{AB} \sim \underbrace{\int^{\mu^2} \frac{ds}{s^2}\sigma}_{>0} + \underbrace{\int_{\mu^2}^{\infty} \frac{ds}{s^2}\sigma}_{>0}$$

$$\hat{\mathcal{M}}''_{AB}(s=0) > \frac{2}{\pi}\int^{\mu^2} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) \times \left[1 + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] > 0$$

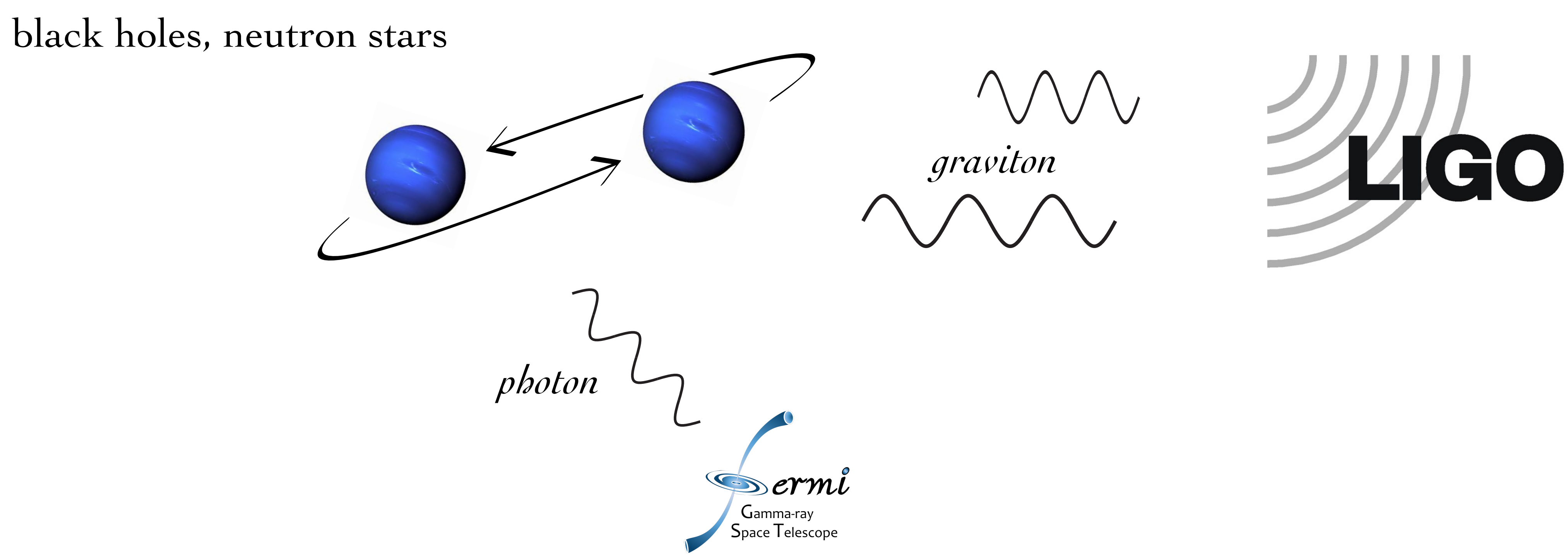
Calculable in the EFT as long as within EFT validity: $\mu^2 < \Lambda^2$.



Modified dispersion relation / Gravitational waves

$$v^2(E) = 1 - \frac{m^2}{E^2}$$

Distortions in the shape of the gravitational waveform can now be probed.
Likewise departures from the speed of light.

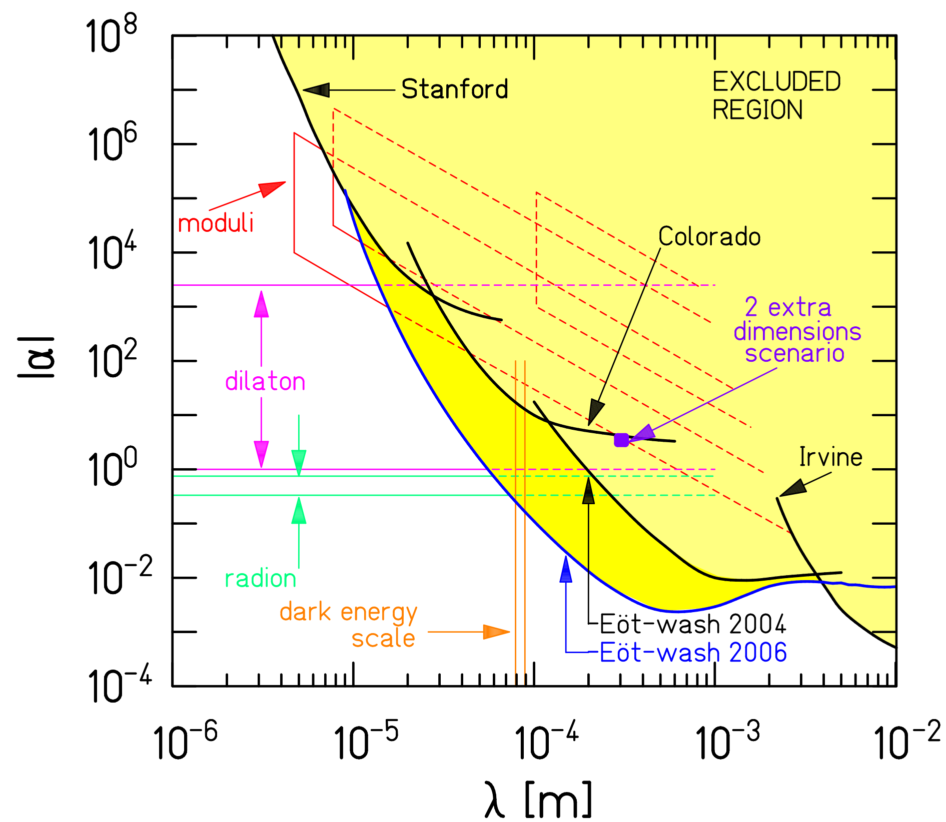


(Abbott et al. ’16, Abbott et al. ’17)

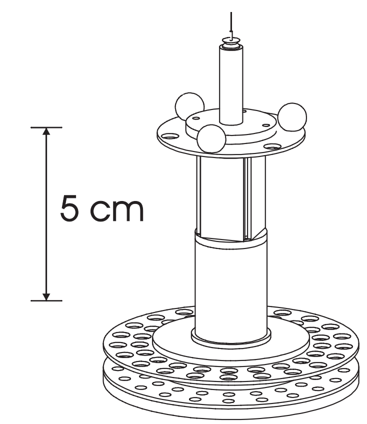
$$m \lesssim 10^{-22}\,\mathrm{eV}$$

Yukawa potential / Torsion balance experiments

$$V(r) = -\frac{m_1 m_2}{8\pi M_{\rm Pl}^2}\left(1+\alpha\, e^{-r/\lambda}\right)$$



(Kapner et al. '06)



GR has been successfully tested down to sub-millimeter scales.

$$\Lambda^{-1} \approx 1\times 10^6\,\mathrm{km}$$

MG (dRGT-theory) cannot predict how gravity behaves at these distances.

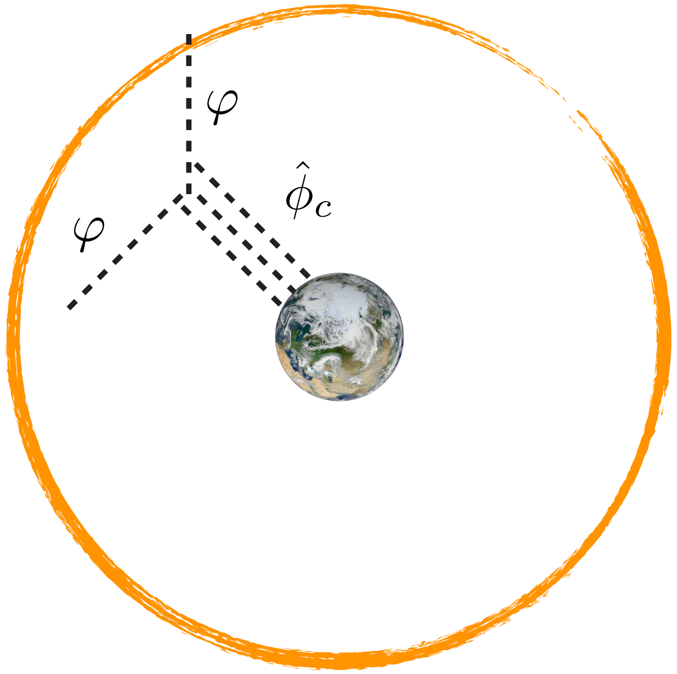
Vainshtein cutoff redressing

(Nicolis, Rattazzi '04)

Once in the Vainshtein region…

$$\mathcal{L}_{\hat\phi} = -\frac{1}{2}(\partial\hat\phi)^2\left[1+\hat c_{i=3,4,5}\left(\frac{\partial^2\hat\phi}{\Lambda_3^3}\right)^{i-2}\right]+\frac{\hat\phi}{M_{\rm Pl}}T^\mu_\mu$$

$$\equiv Z_{\mu\nu} \gg 1$$



$$\hat\phi = \hat\phi_c + \varphi$$

$$Z(\partial\varphi)^2 + \frac{\#}{\Lambda_3^3}(\partial\varphi)^2\Box\varphi \longrightarrow \Lambda_3^{\rm back} \sim \sqrt{Z}\Lambda_3 \gg \Lambda_3$$

Vainshtein cutoff redressing

(Nicolis, Rattazzi '04)

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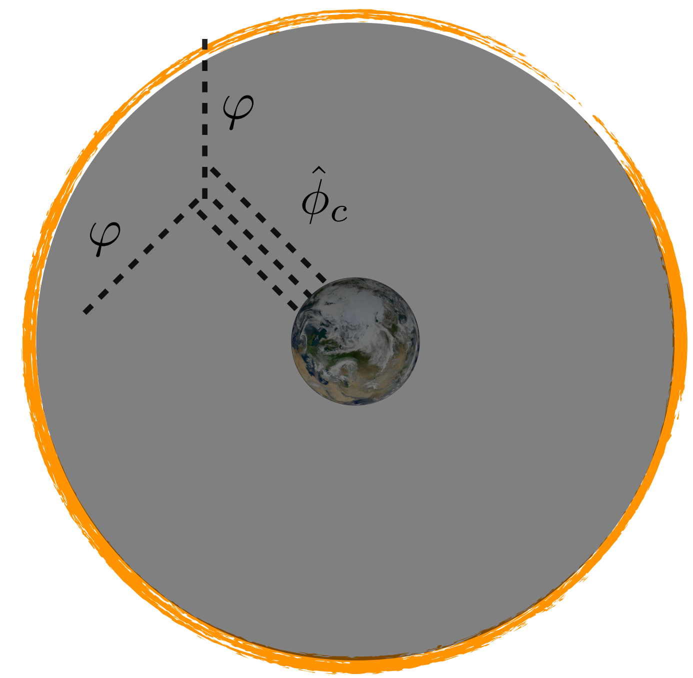


$$\hat\phi = \hat\phi_c + \varphi$$

$$Z(\partial\varphi)^2 + \frac{\#}{\Lambda_3^3}(\partial\varphi)^2\Box\varphi \longrightarrow \Lambda_3^{\rm back} \sim \sqrt{Z}\Lambda_3 \gg \Lambda_3$$

but our bounds *beyond* positivity require, e.g.:

$$\mathcal{L}^{\rm full}_{\hat\phi} = \mathcal{L}_{\hat\phi} + \frac{1}{\Lambda^2}(\Box\hat\phi)^2 \underbrace{\left[\sum^{\infty} \tilde c_n \left(\frac{\partial^2\hat\phi}{\Lambda_3^3}\right)^n\right]}_{\gg 1} \longrightarrow \Lambda^{\rm back} \ll \Lambda \to r_V^{-1}$$

(Arkani-Hamed et al. '03)

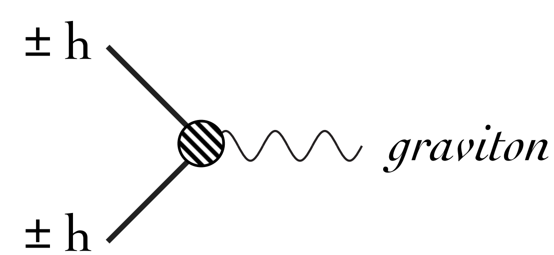
No massless interacting Higher-Spins

Starting point is conceptually/fundamentally different.*

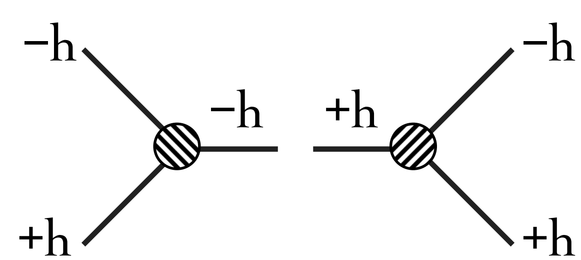
- Weinberg: $\mathcal{M}_{i=n} \approx -\kappa_n \frac{p_n^\mu \cdots p_n^\nu \, \epsilon_{\mu\cdots\nu}}{p_n \cdot q} \mathcal{M}_0 \longrightarrow \kappa_{q=0} = 0$ No long-range force.

- Weinberg-Witten, Porrati: $\langle p+q, \pm h | \Theta_{\mu\nu} | p, \pm h \rangle = e^{\pm i\theta(2h-m)} \langle p+q, \pm h | \Theta_{\mu\nu} | p, \pm h \rangle \quad {}_{m\,=\,0,1,2}$

  No minimal coupling to gravity.



- Arkani-Hamed - Huang[2]:



$$\mathcal{M}(1^{-h}2^{+h}3^{-h}4^{+h}) = \langle 13 \rangle^{2h} [24]^{2h} F(s,t,u)$$

$$R_s = \left(\frac{\langle 13 \rangle^2 [24]^2}{u} \right)^h$$

No 3-particle vertex leading to consistent 4-particle amplitude.*

Massive interacting Higher-Spins

- Coleman-Mandula: $[Q, S] = 0 \longrightarrow [Q, J_{\mu\nu}] = [Q, P_\mu] = 0 \longrightarrow G_{\text{Poincaré}} \times G_{\text{internal}}$

No conserved HS charges.

↓

HS symmetries must be non-linearly realized, aka spontaneously broken.*

and

HS particles must be *composite*.

No-go theorems for light interacting Higher-Spins

U(1) charged

Porrati '08

Arkani-Hamed, Huang, Huang '17

...

$$\Lambda \lesssim m/g^{1/(2s-1)}$$

gravitational

Carot-Huot et al. '16

Bonifacio, Hinterbichler '18

Afkhami-Jeddi et al. '18

...

$$\Lambda \sim m$$

Qualitative bounds on interactions with *external* currents.