Rayleigh Operators

from muon conversion to shining dark matter

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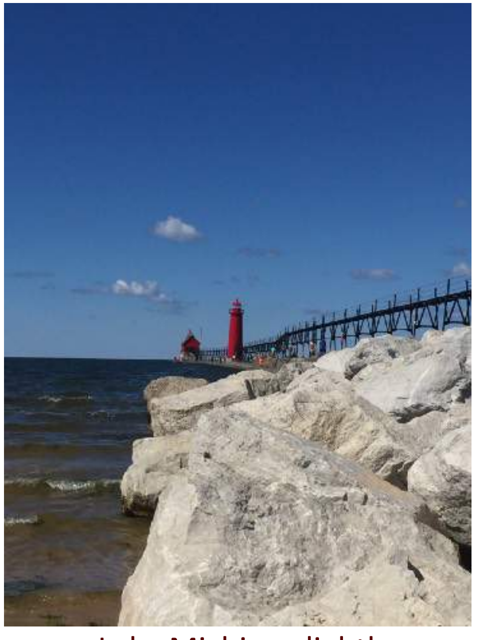
- Why is the sky blue?
  - stronger scattering of shorter wavelengths (blue) compared to longer wavelengths (red)

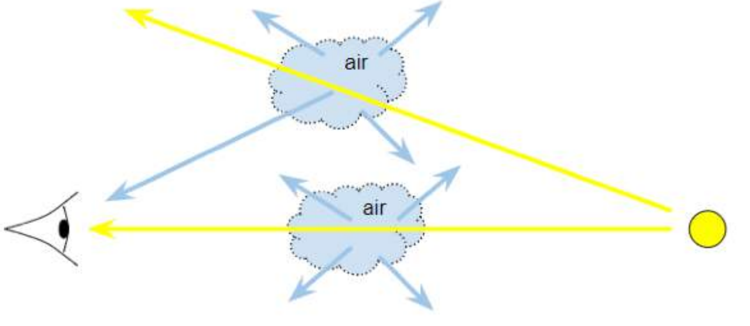
$$\sigma_{\rm R} = \frac{2\pi^5}{3}\frac{d^6}{\lambda^4}\left(\frac{n^2-1}{n^2+2}\right)^2$$

where n is the refractive index,
d is the scatterer size, and
λ is the wavelength

  - this is the so-called Rayleigh scattering cross section
  - blue (430 nm) scatters about a factor of 6 more efficient than red (680 nm)!



Lake Michigan lighthouses



Why is the sky blue (EFT version)?

- Let's construct EFT for low-energy elastic photon-atom scattering
  - photon energy E_γ is smaller that atom's excitation energy ΔE and (inverse) atomic Bohr radius a^{-1}

$$E_\gamma \ll \Delta E \ll a^{-1} \ll M_{\rm atom}$$

  - $\mathscr{L}_{\rm eff}$ should respect Lorentz, gauge, etc. invariance; need:
    - two photons (from $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) with $\left[A_\mu\right] = 1$
    - two atomic fields ϕ_v with NR scaling $\left[\phi\right] = 3/2$

$$\mathcal{L}_{\rm eff} = \frac{c_1}{\Lambda^3}\ \phi_v^\dagger \phi_v\ F_{\mu\nu} F^{\mu\nu} + \frac{c_2}{\Lambda^3}\ \phi_v^\dagger \phi_v\ v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + \dots$$

where c_i are $\mathcal{O}(1)$ and $\Lambda \sim a^{-1}$ (as low energy photons cannot feel inside the atom)

also, $v_\mu v^\mu = 1,\ v^\mu = (1,0,0,0)$ in ϕ_v rest frame

Why is the sky blue (EFT version)?

- Let's estimate the cross section ($\sigma \sim \Lambda^{-6}$)
  - in principle, we can match to QED to obtain c_i
  - order-of-magnitude: as $[\sigma] = -2$ and $\Lambda \sim a^{-1}$ we expect

$$\sigma \sim E_\gamma^4 a^6 \left[1 + \mathcal{O}\left(E_\gamma / \Delta E\right)\right]$$

  - this is Rayleigh scattering: higher energy blue light scatters more than the low energy red light
  - expect corrections $\mathcal{O}(E_\gamma / \Delta E)$
  - results from Rayleigh operators

- Why is the sky red on Mars? Is it red?
- What about the night sky?

Mars Pathfinder picture

- Rayleigh operators are built out of singlets (SM gauge + others)
  - ideal for studies of portal-type of SM-NP interaction models

SM FCNC

$$\overline{\psi}_a \psi_b$$

$$\phi^\dagger \phi \qquad \overline{\chi}\chi \qquad \overline{\psi}\psi$$

dark sector

$$\mathcal{L}_{\text{eff}}$$

$$F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu}\widetilde{F}^{\mu\nu} \qquad G^a_{\mu\nu}G^{\mu\nu}_a \qquad G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a$$

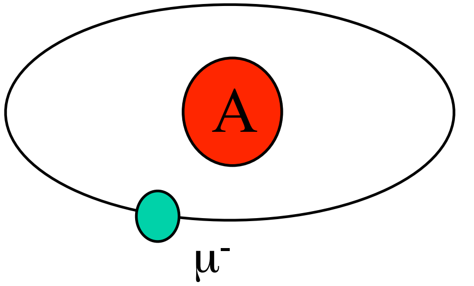
SM gauge fields

  - also ideal for studies of FCNC effects in the SM
  - can represent leading effect for NP interactions with SM gauge fields

We will consider two such effects today: lepton FCNC/gluons and DM/photons interactions

2. Rayleigh operators and muon conversion

★ Basic idea for the muon conversion experiment



★ take low energy muons (~ 30 MeV) a stop them in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$

overlap probability: $p \sim r^{2\ell} Z^3$

large overlap for an s-wave and high-Z nucleus

Measure $R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$ to probe NP

Experimental ideas

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

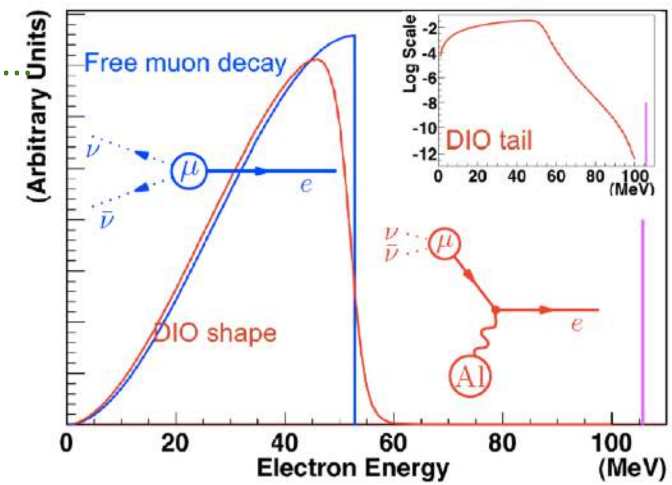
★ The experiment is tricky

- ✓ Muon conversion gives monoenergetic electrons...
- ✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} <$ a few $\times 10^{-17}$

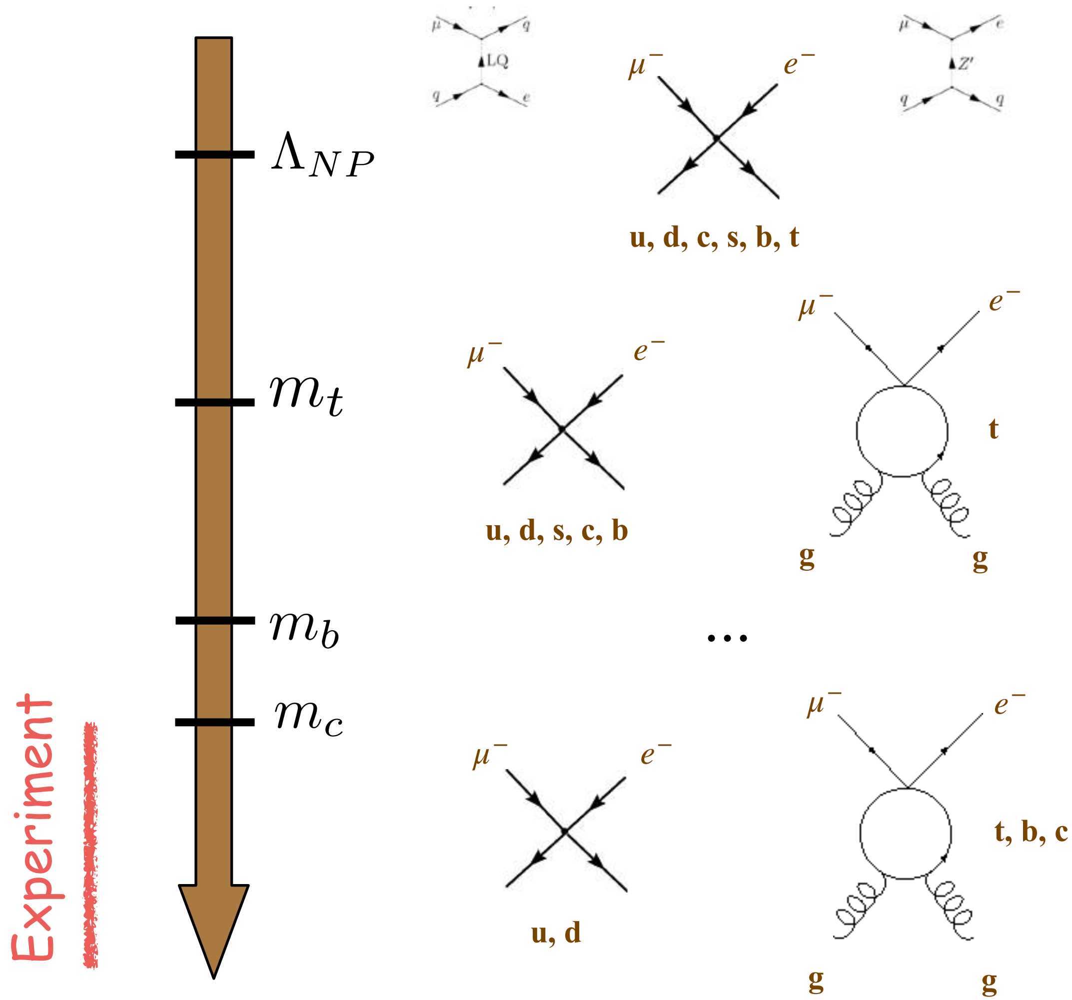


Czarnecki, Marciano, Tormo

Muon conversion: EFT approach

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

Effective Lagrangian for muon conversion

★ Naive power counting: largest contribution from lowest dimensional operators

★ Can write the most general LFV Lagrangian $\mathcal{L}_{LFV} = \mathcal{L}_D + \mathcal{L}_{lq} + \mathcal{L}_G + \ldots$

– dipole operators

$$\mathcal{L}_D = -\frac{m_2}{\Lambda^2}\left[\left(C_{DR}\bar{\ell}_1\sigma^{\mu\nu}P_L\ell_2 + C_{DR}\bar{\ell}_1\sigma^{\mu\nu}P_R\ell_2\right)F_{\mu\nu} + h.c.\right]$$

– four-fermion operators

$$\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2}\sum_q \Bigg[& \left(C_{VR}^{q\ell_1\ell_2}\,\bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{VL}^{q\ell_1\ell_2}\,\bar{\ell}_1\gamma^\mu P_L\ell_2\right)\,\bar{q}\gamma_\mu q \\
& + \left(C_{AR}^{q\ell_1\ell_2}\,\bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{AL}^{q\ell_1\ell_2}\,\bar{\ell}_1\gamma^\mu P_L\ell_2\right)\,\bar{q}\gamma_\mu\gamma_5 q \\
& + m_2 m_q G_F\left(C_{SR}^{q\ell_1\ell_2}\,\bar{\ell}_1 P_L\ell_2 + C_{SL}^{q\ell_1\ell_2}\,\bar{\ell}_1 P_R\ell_2\right)\,\bar{q}q \\
& + m_2 m_q G_F\left(C_{PR}^{q\ell_1\ell_2}\,\bar{\ell}_1 P_L\ell_2 + C_{PL}^{q\ell_1\ell_2}\,\bar{\ell}_1 P_R\ell_2\right)\,\bar{q}\gamma_5 q \\
& + m_2 m_q G_F\left(C_{TR}^{q\ell_1\ell_2}\,\bar{\ell}_1\sigma^{\mu\nu} P_L\ell_2 + C_{TL}^{q\ell_1\ell_2}\,\bar{\ell}_1\sigma^{\mu\nu} P_R\ell_2\right)\,\bar{q}\sigma_{\mu\nu} q \; + \; h.c. \Bigg].
\end{aligned}$$

– gluonic (Rayleigh) operators

$$\begin{aligned}
\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[& \left(C_{GR}\bar{\ell}_1 P_R\ell_2 + C_{GL}\bar{\ell}_1 P_L\ell_2\right)G^a_{\mu\nu}G^{a\mu\nu} \\
& + \left(C_{\tilde{G}R}\bar{\ell}_1 P_R\ell_2 + C_{\tilde{G}L}\bar{\ell}_1 P_L\ell_2\right)G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} + h.c.\Big]
\end{aligned}$$

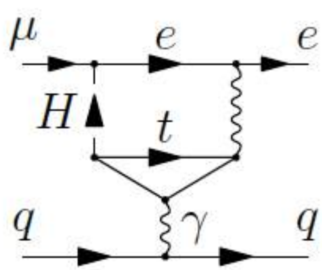
Effective Lagrangian for muon conversion

★ Wilson coefficients of $\mathscr{L}_{\text{eff}}$ for muon conversion and New Physics models

★ E.g. FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$ Harnik, Kopp, Zupan

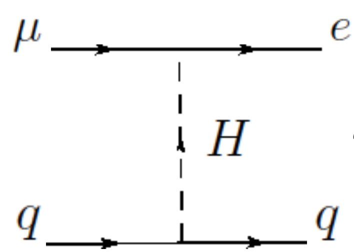
★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level

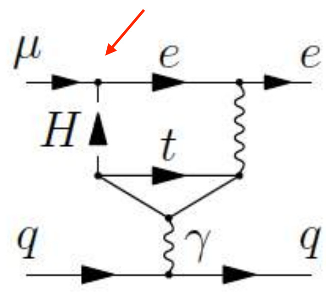
★ … but note: here couplings of new physics to light quarks are suppressed

can leptons interact with gluons instead?

Effective Lagrangian for muon conversion

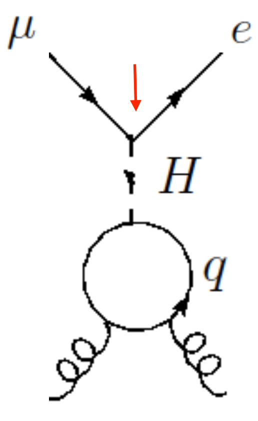
★ Contribution of heavy quarks can, in principle, be large even at low energies

★ Two-loop sensitivity to NP in muon conversion experiment...



$$\sim \mathcal{O}(\alpha^2)$$

★ ... becomes one-loop!



➡ gluonic couplings to hadrons are not (always) suppressed!
➡ NP couplings to heavy quarks are not well constrained and could be large

Effective Lagrangian for muon conversion

★ Contribution of heavy quarks can, in principle, be large even at low energies

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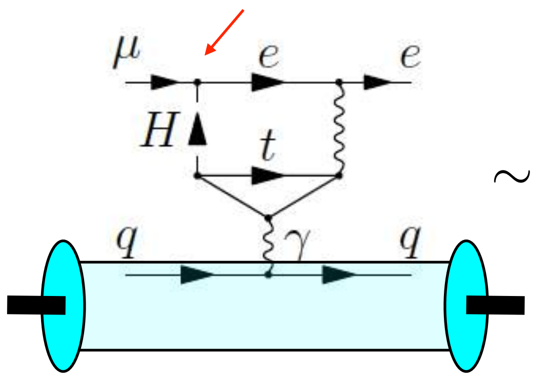
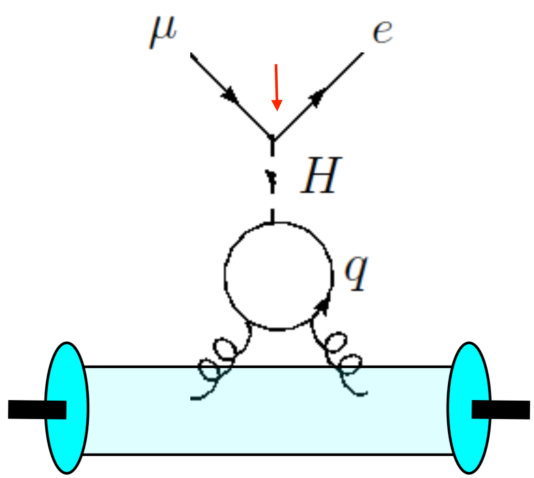


$$\sim \mathcal{O}(\alpha^2)$$

★ ... becomes one-loop!

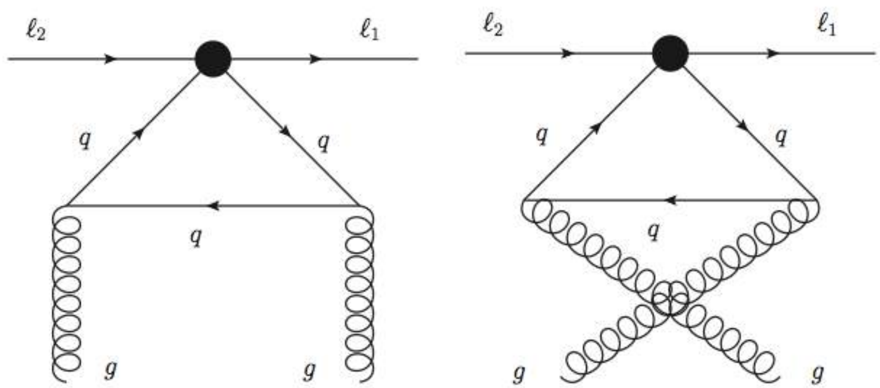


➡ gluonic couplings to hadrons are not (always) suppressed!

➡ NP couplings to heavy quarks are not well constrained and could be large

Rayleigh operators and muon conversion

★ Coefficients of gluonic operators depend on the number of active flavors



$$\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[\Big(C_{GR}\overline{\ell}_1 P_R \ell_2 + C_{GL}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu}G^{a\mu\nu} + \Big(C_{\tilde{G}R}\overline{\ell}_1 P_R \ell_2 + C_{\tilde{G}L}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} + h.c.\Big]$$

★ we can calculate their contribution to muon conversion experiments
★ also relevant for meson or tau decay rates!
★ c_i probe couplings of heavy quarks to New Physics

AAP and D. Zhuridov
PRD89 (2014) 3, 033005

Rayleigh operators and muon conversion

★ ... get an effective Lagrangian

$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \text{H.c.},$$

AAP and D. Zhuridov
PRD89 (2014) 3, 033005

...where we defined operators

$$O_1^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_2^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

$$O_3^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_4^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

...and Wilson coefficients

$$c_1^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} + C_2^{q\ell_1\ell_2}\right),$$

$$c_2^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} - C_2^{q\ell_1\ell_2}\right),$$

$$c_3^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} + C_4^{q\ell_1\ell_2}\right),$$

$$c_4^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} - C_4^{q\ell_1\ell_2}\right),$$

$$I_1 = \frac{1}{3}, \qquad I_2 = \frac{1}{2}.$$

Need matrix elements to convert experimental results into constraints on $c_i^{\ell_1\ell_2}$

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics - and QED!

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Lepton wave functions are taken as solutions of Dirac equation
- with usual substitutions $u_1(r) = r\,g(r)$ and $u_2(r) = r\,f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi^\mu_\kappa = \begin{pmatrix} g(r)\chi^\mu_\kappa(\theta,\phi) \\ if(r)\chi^\mu_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

$$\text{SINDRUM II (PSI), 2006}: \quad R_{\mu e} < 7\times 10^{-13}$$

$$\text{M2e goal}: \quad R_{\mu e} < \text{a few}\times 10^{-17}$$

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a general quark operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z\rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle \right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \qquad \int d^3\rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) \ll m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

★ Gluonic contribution can be removed removed using anomaly equation or can be computed

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a gluonic Rayleigh operator

$$\langle N|\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu}|N\rangle = -\frac{9}{2}\left[ZG^{(g,p)}\rho^{(p)} + (A-Z)G^{(g,n)}\rho^{(n)}\right],$$

where $G^{(g,\mathcal{N})} = \langle\mathcal{N}|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}G^{a\mu\nu}|\mathcal{N}\rangle \approx -189 \text{ MeV}$

★ The (coherent) conversion rate is

$$\Gamma_{(}\mu^- + (A,Z) \to e^- + (A,Z)) = \frac{4a_N^2}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right)$$

with $a_N = G^{(g,p)}S^{(p)} + G^{(g,n)}S^{(n)}$

The overlap integrals $S^{(p,n)}$ with muon and electron wave functions are

$$S^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 Z\rho^{(p)}(g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 (A-Z)\rho^{(n)}(g_e^- g_\mu^- - f_e^- f_\mu^-).$$

Constraints on Wilson coefficients

★ Conversion probability

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

Nucleus	$\Gamma_{\rm capt}(\mu^- N)$, s^{-1}	$\Gamma(\mu e)$(90% C.L.)	E_b, MeV
$^{48}_{22}$Ti	2.59×10^6	4.3×10^{-12} Ref. [28]	1.25 Ref. [29]
$^{197}_{79}$Au	13.07×10^6	7×10^{-13} Ref. [19]	10.08 Ref. [19]

★ ... results in constraints on scale/Wilson coefficient

Coefficient	Bound on $\lvert c_i^{e\mu}\rvert/\Lambda^2$, GeV^{-3}	
	conversion on $^{48}_{22}$Ti	conversion on $^{197}_{79}$Au
c_1	2.5×10^{-11}	1.2×10^{-11}
c_2	—	—
c_3	2.5×10^{-11}	1.2×10^{-11}
c_4	—	—

★ Important: muon conversion can only probe parity-conserving operators

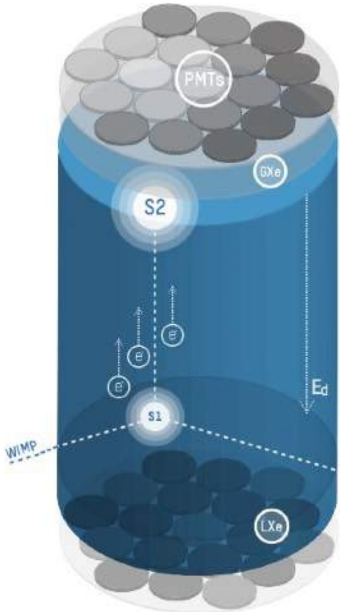
- Rayleigh operators might represent the leading interactions between dark sector and the Standard Model

Weiner, Yavin (2013); Kavanagh, Panci, Ziegler (2019)

  - for scalar or Majorana fermion DM the lowest dimensional operators coupling DM and SM gauge fields are Rayleigh operators
  - this talk: assume that DM is a real scalar φ which is Z_2-odd (thus stable); SM fields are Z_2-even

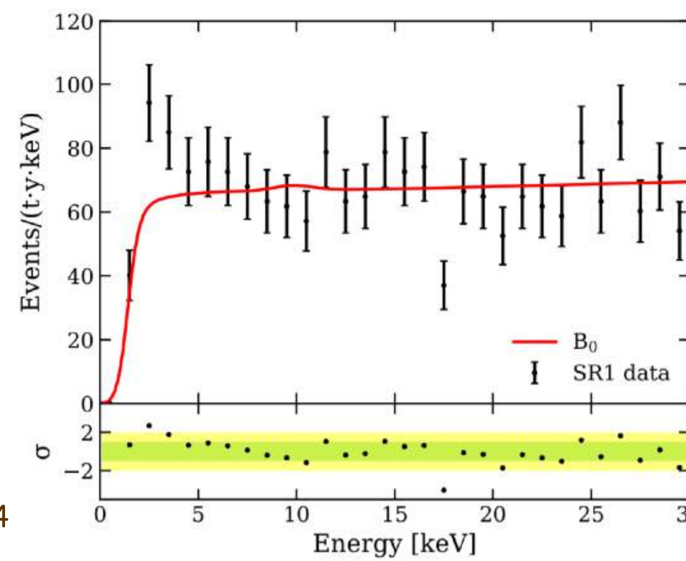
$$\mathcal{L}_{\rm int} = \frac{\alpha}{12\pi}\frac{1}{\Lambda^2}\varphi^2\Big(C_\gamma F_{\mu\nu}F^{\mu\nu} + \widetilde{C}_\gamma F_{\mu\nu}\widetilde{F}^{\mu\nu}\Big),$$

  - note: $C_\gamma, \tilde{C}_\gamma \sim \mathcal{O}(1)$ are generated at one loop, if DM couples to heavier states of mass $\mathcal{O}(\Lambda)$ charged under the SM electroweak group



- The XENON1T experiment operates a time projection chamber (TPC), which utilizes a liquid xenon target (LXe) [3.2 tonnes with 2 tonnes of fiducial volume].
- Particle interactions in LXe produce scintillations and ionizations, which are detected by PMTs

- In June 2020, the XENON1T collaboration reported an excess of electron recoils: 285 events, 53 more than the expected 232
- Statistical significance of the excess is 3.5 sigma



XENON Collaboration
E. Aprile et al. PRD 102 (2020) 7, 072004
e-Print: 2006.09721

Xenon1T anomaly: explanation?

- There are many possible explanations of the signal
  - tritium decays (background): need 3 tritium atoms per 1 kg of LXe
  - some NP interacting with electrons: solar axions, large magnetic moment of neutrinos, boson DM/ALP absorption on electrons, hidden photons, etc.

See references to 2006.09721

- Observation: XENON1T cannot discriminate between e^- and γ
  - electromagnetic interactions of non-relativistic DM $\mathcal{O}$(GeV) with Xe nuclei

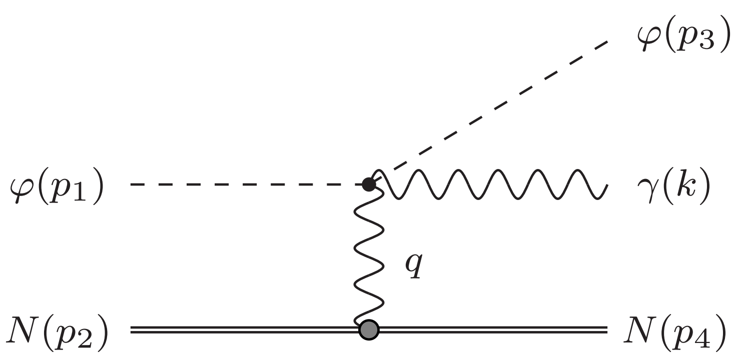
$$\mathcal{L}_{\rm int} = \frac{\alpha}{12\pi}\frac{1}{\Lambda^2}\varphi^2\left(C_\gamma F_{\mu\nu}F^{\mu\nu} + \widetilde{C}_\gamma F_{\mu\nu}\widetilde{F}^{\mu\nu}\right),$$

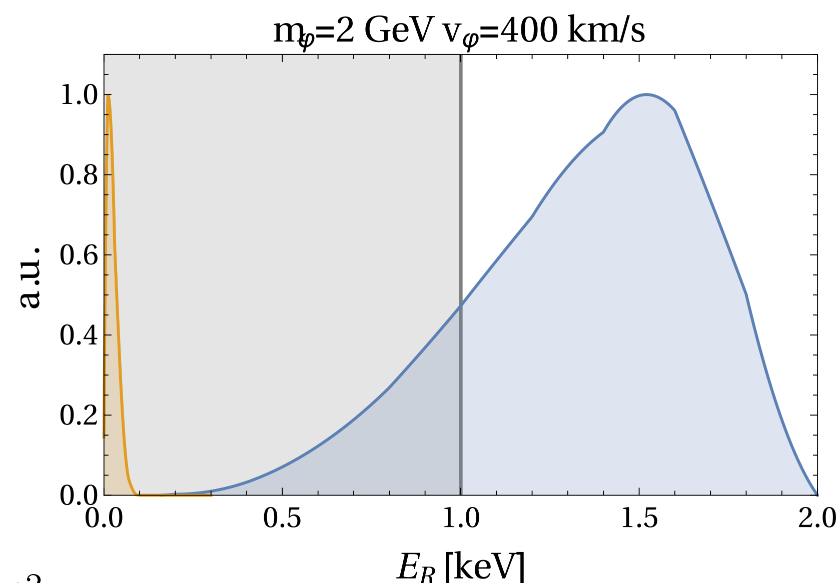
  - this can be realized by Rayleigh scattering of DM with possible signatures of $\varphi N \to \varphi N\gamma$ and nuclear recoil $\varphi N \to \varphi N$

Paz, AAP, Tammaro, Zupan
arXiv:2006.12462

Xenon1T anomaly: shining DM

- Experimental signature: Xenon1T sees photon instead of an electron





event simulated with FeynRules and MadGraph

Cross section:
$$\frac{d\sigma}{dE_{\rm NR}dE_\gamma} = \frac{1}{16}\frac{1}{(2\pi)^3}\frac{|\mathcal{M}|^2}{m_\varphi m_N v}$$

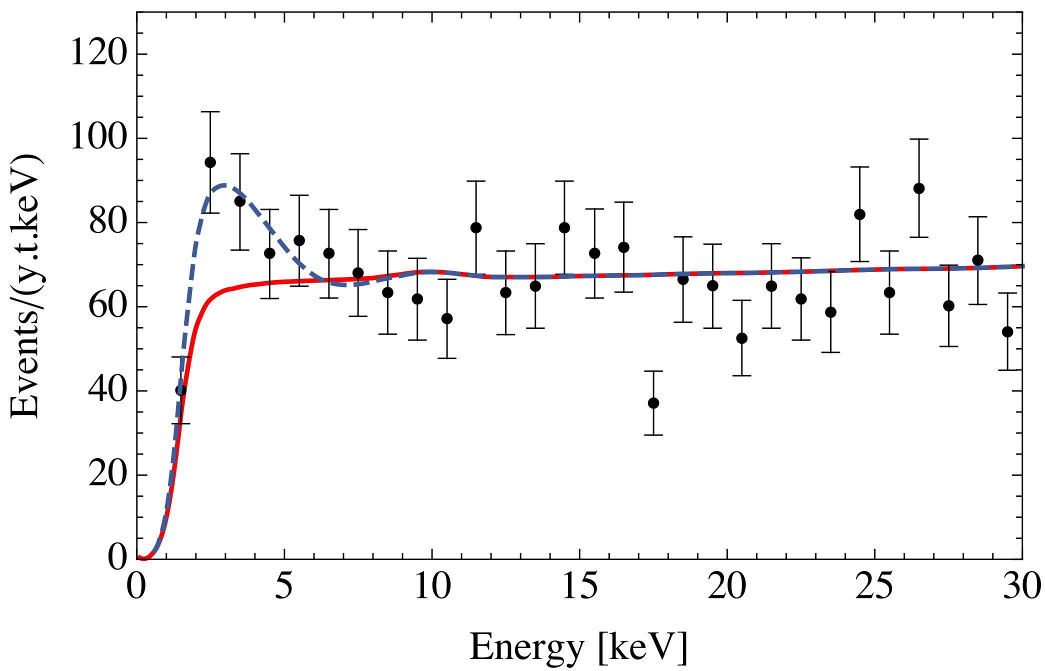
$$|\mathcal{M}|^2 = \left(\frac{2\sqrt{2}}{3\pi}\frac{\alpha Z e C_\gamma}{\Lambda^2}\right)^2 \frac{1}{(Q^2)^2}\Big[Q^2\times$$
$$\times\left((k\cdot p_2)^2 + (k\cdot p_4)^2\right) - 2m_N^2\,(k\cdot q)^2\Big],$$ (set $\widetilde{C}_\gamma = 0$ for simplicity)

Xenon1T anomaly: shining DM

- The signal rate was computed with the standard model DM halo type velocity distribution $f_\odot(v)$ [$v_{\rm esc} = 550$ km/s]

$$dR/dE_\gamma = \rho_0 \int_{v>v_{\rm min}} d^3v\, v f_\odot(\vec{v})(d\sigma/dE_\gamma)/(m_\varphi m_N)$$

with $v_{\rm min} \simeq \sqrt{2E_\gamma/m_\varphi}$



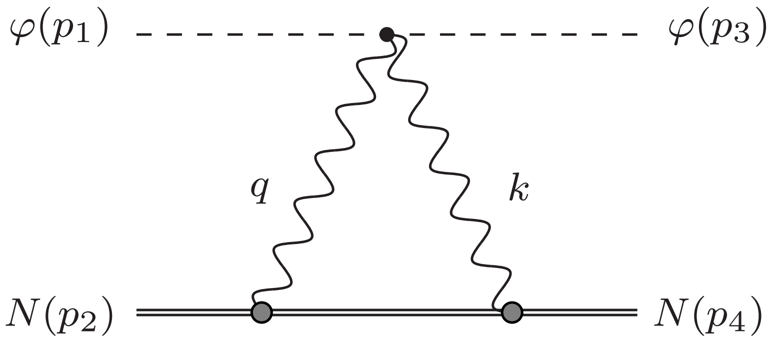
Best fit is obtained for:

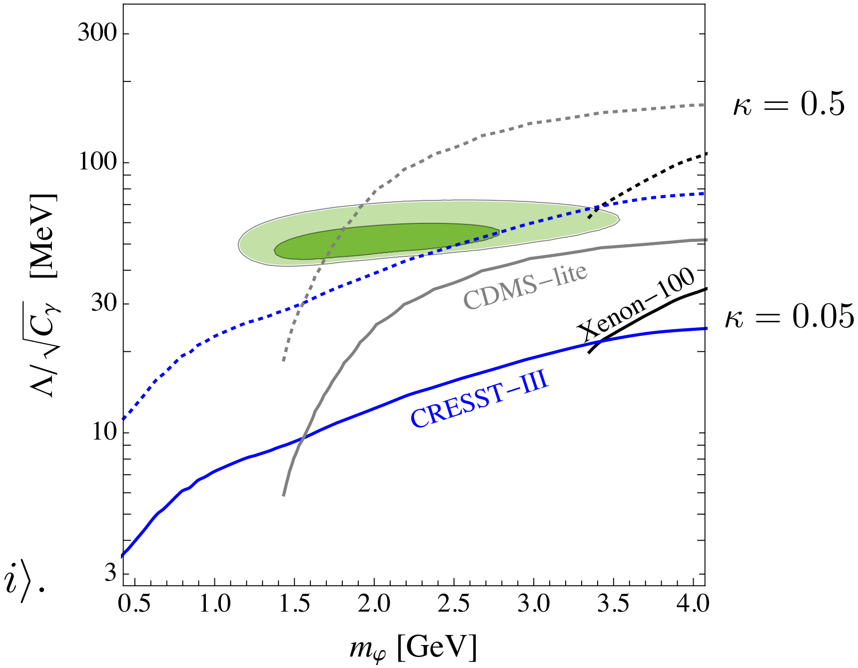
$$m_\varphi = 1.9 \text{ GeV}$$

$$C_\gamma/\Lambda^2 = 1/(f_\varphi(50 \text{ MeV})^2)$$

Paz, AAP, Tammaro, Zupan
arXiv:2006.12462

- Note that RO would also lead to spin-independent nuclear scattering $\varphi N \to \varphi N$: does it lead to visible effects?





This diagram depends on a NP matrix element

$$\langle f|(\varphi\varphi)F_{\mu\nu}F^{\mu\nu}|i\rangle = \frac{\alpha Z^2}{4\pi}\widetilde{Q}_0\langle f|(\varphi\varphi)\bar{u}_N u_N|i\rangle.$$

naively, $\widetilde{Q}_0 = \kappa/\sqrt{\langle r^2\rangle}$

Cross section: $\sigma_N = A^2(\mu_{\varphi N}^2/\mu_{\varphi n}^2)\sigma_n$ with $\sigma_n = \frac{1}{4\pi}\left(\frac{\alpha}{12\pi}\frac{C_\gamma}{\Lambda^2}\right)^2\left(\frac{\alpha Z^2}{2\pi}\tilde{Q}_0\right)^2\frac{\mu_{\varphi n}^2}{\mu_{\varphi N}^2}\frac{1}{A^2},$

Example: secluded DM

- Should we be concerned about low effective scale?
  - such situation arises in secluded models of DM (no direct interaction between visible matter and DM: a mediator)
  - imagine light (pseudo)scalar mediator $m_a \sim \mathcal{O}(1-10$ MeV)

$$\mathcal{L}_a \supset \mu_\varphi \varphi^2 a + \frac{\alpha}{12\pi} \frac{a}{\Lambda_{\rm UV}} \left(C_{a\gamma} F F + \widetilde{C}_{a\gamma} F \tilde{F} \right),$$

  - for momentum exchanges below m_a it can be integrated out resulting in the CP-even Rayleigh operator with

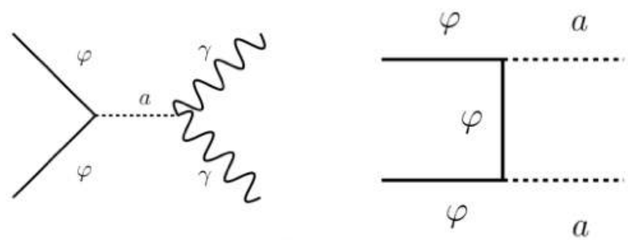
$$\frac{C_\gamma}{\Lambda^2} = \frac{C_{a\gamma}}{\Lambda_{\rm UV}} \frac{\mu_\varphi}{m_a^2},$$

  - the best point is obtained for $\frac{\Lambda_{\rm UV}}{C_{a\gamma}} = 1 \text{ TeV} \left(\frac{f_\varphi}{0.2} \right) \left(\frac{1 \text{ MeV}}{m_a} \right)^2 \left(\frac{\mu_\varphi}{2 \text{ GeV}} \right),$

Note that large cross section for $\varphi\varphi \to \varphi\varphi$ mediated by a is mitigated by small f_φ

Indirect DM constraints

- Are there other constraints on the model?
  - there are two important processes, $\varphi\varphi \to aa$ and $\varphi\varphi \to a^* \to \gamma\gamma$
  - both processes constrain the model is a decays to $\gamma\gamma$ appreciably

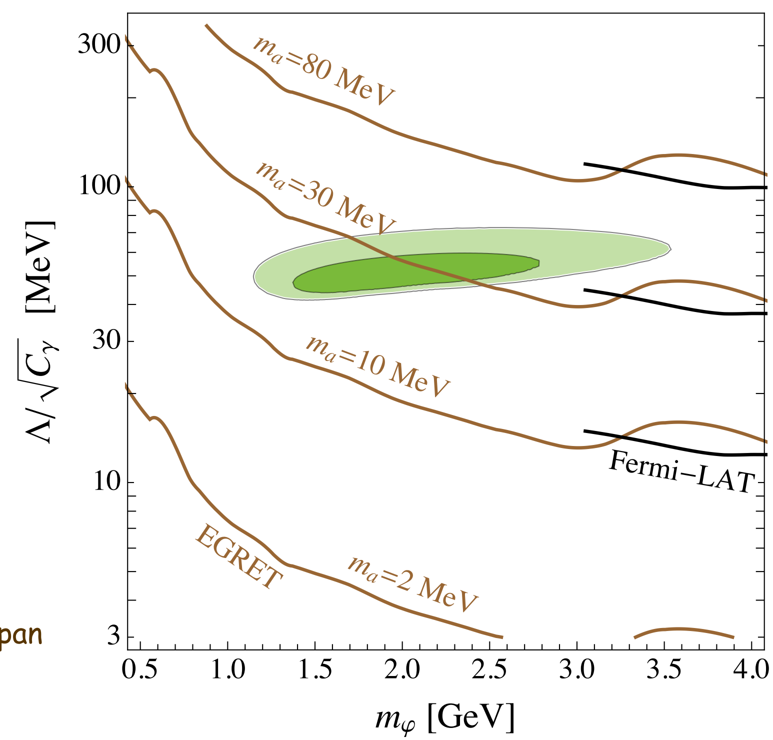


$$(\sigma v)_{\varphi\varphi\to 2\gamma} = \left[\mu_\varphi \alpha C_{a\gamma}/(12\pi\Lambda_{\rm UV})\right]^2/(4\pi m_\varphi^2)$$

$$(\sigma v)_{\varphi\varphi\to 2a} = \mu_\varphi^4/(8\pi m_\varphi^6)$$

  - to avoid constraints, assume that a mainly decays invisibly (neutrinos)

Paz, AAP, Tammaro, Zupan
arXiv:2006.12462



4. Conclusions and takeaways

- Rayleigh operators provide interesting tools for BSM studies
  - they are known to be important for explaining SM phenomena
  - could be seen to give leading effects in BSM-SM couplings
- Gluonic Rayleigh operators in muon conversion
  - affect muon conversion experiment, important for some models
  - possible effects from $gg \to \tau\mu$ due to large gluon luminosity of LHC
  - more data from ATLAS/CMS/(LHCb?) on $pp \to \tau\mu + X$

Bhattacharya, Morgan, Osborne, AAP

- Electromagnetic Rayleigh operators could explain puzzles in DM physics
  - NR shiny DM can explain XENON1T excess (photon detection)
  - viable (secluded DM) models exist with light (pseudo)scalar mediators
  - models can be better constrained in direct detections experiments

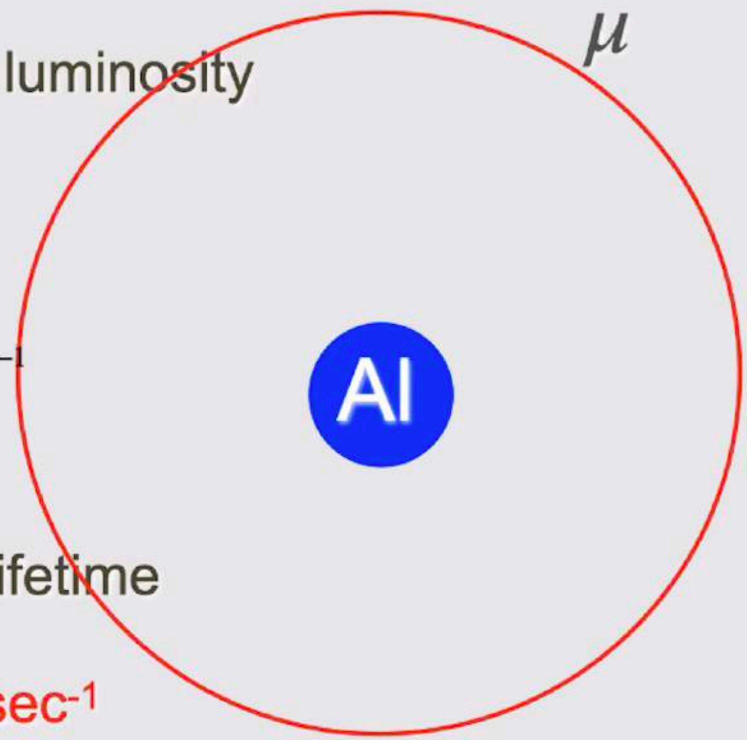
… but need better estimates of nonperturbative matrix elements


EXTRA!

Effective luminosity of Mu2E experiment

- The captured muon is in a 1s state and the wave function overlaps the nucleus (*picture ~ to scale*)
- We can turn this into an effective luminosity
- Luminosity = density x velocity

$$|\psi(0)|^2 \times \alpha Z = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} = 8 \times 10^{43}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$$



- Times 10^{10} muons/sec X 2 μsec lifetime
- Effective Luminosity of 10^{48} cm^{-2}sec^{-1}

Andrzej Czarnecki

R. Bernstein, FNAL 11 Mu2e

Matching: Wilson coefficients for leptoquarks

★ Consider example: leptoquark interactions

- scalar leptoquarks

$$\mathcal{L}_S = \left(\lambda_{LS_0} \bar{q}_L^c i\tau_2 \ell_L + \lambda_{RS_0} \bar{u}_R^c e_R\right) S_0^\dagger + \left(\lambda_{LS_{1/2}} \bar{u}_R \ell_L + \lambda_{RS_{1/2}} \bar{q}_L i\tau_2 e_R\right) S_{1/2}^\dagger + \text{H.c.},$$

- vector leptoquarks

$$\mathcal{L}_V = \left(\lambda_{LV_0} \bar{q}_L \gamma_\mu \ell_L + \lambda_{RV_0} \bar{d}_R \gamma_\mu e_R\right) V_0^{\mu\dagger} + \left(\lambda_{LV_{1/2}} \bar{d}_R^c \gamma_\mu \ell_L + \lambda_{RV_{1/2}} \bar{q}_L^c \gamma_\mu e_R\right) V_{1/2}^{\mu\dagger} + \text{H.c.},$$

Davidson, Bailey, Campbell

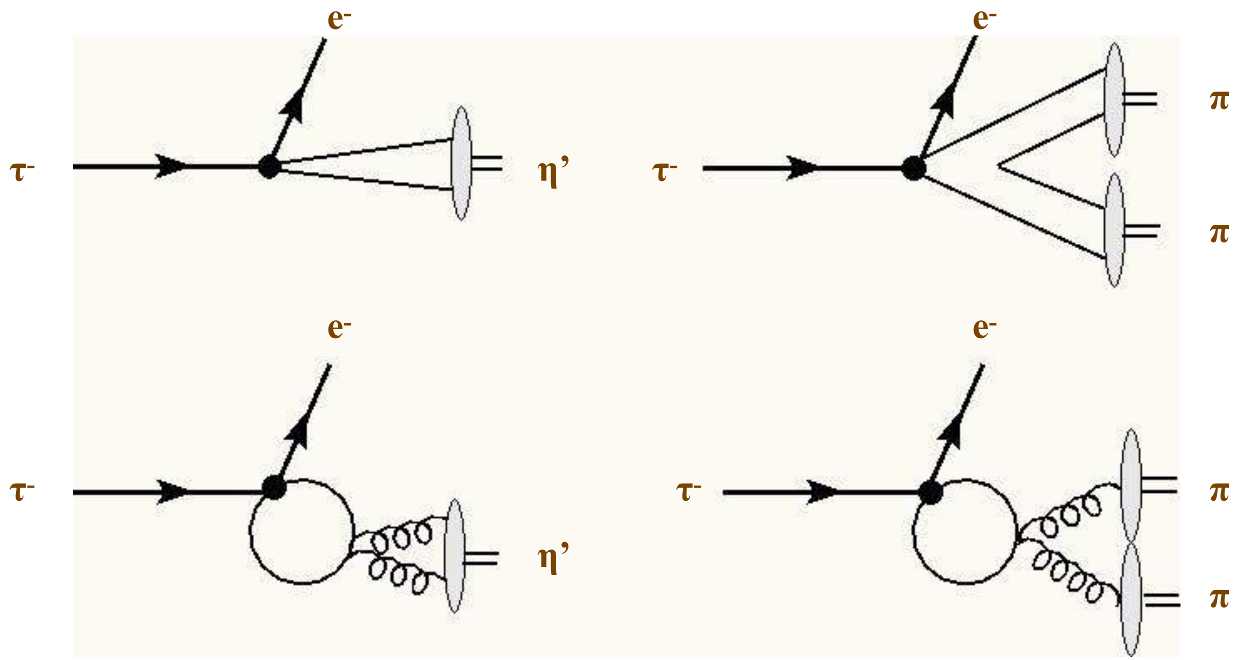
★ Matching to the general result above, get

C_i^u/Λ^2	Expression	C_i^d/Λ^2	Expression
$\frac{C_1^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_1 u}\lambda_{LS_{1/2}}^{\ell_2 u}}{2M_{S_{1/2}}^2}$	$\frac{C_1^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_2 b}\lambda_{RV_{1/2}}^{\ell_1 b}}{M_{V_{1/2}}^2}$
$\frac{C_2^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_1 u}\lambda_{LS_0}^{\ell_2 u}}{2M_{S_0}^2}$	$\frac{C_2^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_2 b}\lambda_{RV_0}^{\ell_1 b}}{M_{V_0}^2}$
$\frac{C_3^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_2 u}\lambda_{LS_0}^{\ell_1 u}}{2M_{S_0}^2}$	$\frac{C_3^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_1 b}\lambda_{RV_0}^{\ell_2 b}}{M_{V_0}^2}$
$\frac{C_4^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_2 u}\lambda_{LS_{1/2}}^{\ell_1 u}}{2M_{S_{1/2}}^2}$	$\frac{C_4^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_1 b}\lambda_{RV_{1/2}}^{\ell_2 b}}{M_{V_{1/2}}^2}$

Tau decays as probes for gluonic Rayleigh operators

★ Can compute FCNC tau decays, as they are sensitive to gluonic operators

AAP and D. Zhuridov
PRD89 (2014) 3, 033005



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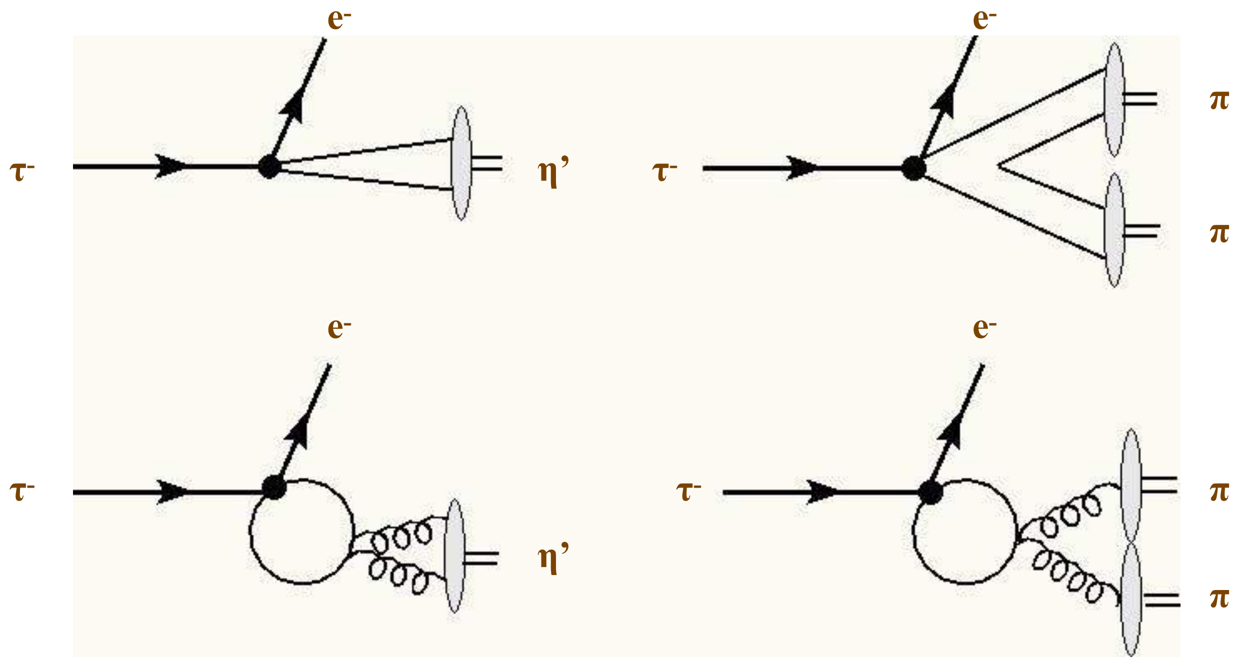

Partity-violating operators

Parity-conserving operators

Example: explicit secluded DM

- Explicit complete models can be constructed
  - natural models with large neutrino decay widths for the mediator a decays naturally appear if neutrino masses are generated due to inverse see-saw mechanics and a participates in this process

$$-\mathcal{L}_Y = y_\nu \bar{L}\tilde{H}^\dagger N_R + \bar{N}_L M_N N_R + \frac{1}{2}\bar{N}^c_R \mu_R N_R + \frac{1}{2}\bar{N}_L(\mu_L + \lambda_L a)N^c_L + \text{h.c.},$$

$$-\mathcal{L}_Y \supset \frac{1}{2}s^2_\alpha \lambda_L \left(\bar{\nu}^c \nu\right) a + \cdots .$$

Dev, Pilaftsis
arXiv:1209.4051