Gauging Stuckelberg Axions: the Axi-Higgs

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Abstract

Variants of the usual Peccei-Quinn axion theory for the solution of the strong CP problem allows to generate more general axion-like terms in an effective Lagrangean beyond the Standard Model. One of these extensions involves Stuckelberg axions and (gauged) anomalous abelian symmetries. Similar interactions are generated by other methods, for instance by a decoupling of chiral fermions from the low energy spectrum in an anomaly-free theory. A third possibility is encoded in a scale invariant theory, where an axion, a dilaton and a dilatino are the anomaly multiplet of an N=1 Superconformal theory, in a nonlinear realization.

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General Results

Effective actions of Stuckelberg-type: **SU(3)xSU(2)XU(1)_Y x U(1)’**

Generalising a PQ global symmetry to a local U(1) symmetry
(Stuckelberg axion models). Predict a **fundamenta**l axion (**gauged axion**)
(the axi-Higgs) of a generic mass.

The mass is related to a misalignment potential which is generic.
It can cover the TeV region. Obviously, the misalignment has to be strong
For an axion at the Terascale.

Two models: MSLOM (Irges, Kiritsis, C.C.)
USSM-A (Lazarides, Irges, Mariano, C.C.) (Stuckelberg supermultiplet)

These models are built using a Wess-Zumino Lagrangean with an asymptotic and elementary axion

Decoupling of a Heavy fermion and a gauged (anomaly free U(1) symmetry
Can also also be described by this class of models

ALTERNATIVE PATHS

AXIONS, DILATONS AS COMPOSITE

Conformal/superconformal anomaly

Dilaton interactions and the anomalous breaking of scale invariance of the Standard Model
Delle Rose, Quintavalle, Serino, C.C.
JHEP 1306 (2013) 077

Superconformal sum rules and the spectral density flow of the composite dilaton (ADD) multiplet in N=1 theories
Delle Rose, Costantini, Serino, C.C.
JHEP 1406 (2014) 136

Work to appear soon: Bandyopadhyay, Irges, Guzzi, Delle Rose, C.C.
"Heavy Axions and Dilatons"

A superconformal theory can generate these states due to the alignment of
The anomaly multiplet.

Nonlinear realization of the superconformal symmetry

Axions emerge as a candidate solution of the strong CP problem

The well known solution of the strong CP problem is due to R. Peccei and H. Quinn (PQ)

It is based on the introduction of an extra U(1) global symmetry of the SM broken by an anomaly.

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$$E_8 \to M_8 \to E_6 \times SU(3)$$

$$E_6 \to M_6 \to SO(10) \times U(1)$$

$$SO(10) \to M_{10} \to SU(5) \times U(1)$$

$$SO(10) \to M'_{10} \to SO(6) \times SO(4)$$

$$SO(6) \sim SU(4) \qquad SO(4) \sim SU(2)_L \times SU(2)_R$$

$$SU(4) \to M_4 \to SU(3)_c \times U(1)_{B-L}$$

$$SU(5) \to M_5 \to SU(3) \times SU(2) \times U(1)$$

Various effective models

$$E_6 \to SM \times U(1)$$

$$E_6 \to M_6 \to M_{10} \to M_5 \to SM \times U(1)$$

$$E_6 \to M_{10} \to M'_{10} \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

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Generalization of the PQ proposal

Irges, Kiritsis, CC, 2005
U. of Crete, U. of Salento

Anomalous U(1) extension of the Standard Model
(N. Irges, S. Morelli, C.C.)

Phenomenology: **M. Guzzi** (Manchester U.), R. Armillis, C.C.

Susy extensions: Irges (Athens TU), **A. Mariano** (Salento U.), C.C.

Cosmology: **G. Lazarides** (Thessaloniki U.), A. Mariano (Salento), C.C.

The role played by anomalies and anomaly actions in QFT can be hardly underestimated.

Anomalies describe the radiative breaking of a certain classical symmetry and *theorists have tried to use anomaly actions as a way to show the effect of the anomaly (example: chiral dynamics and the pion, AVV anomaly)*
but also have tried to cancel anomalies when these symmetries are gauged

Anomaly cancellation (for a gauge symmetry):

1. by charge assignment in gauge theory (Standard Model):
in the exact (unbroken) phase of the theory, choose the representation
in such a way that anomalous chiral interactions cancel

2. by the introduction of extra sectors (axions, dilatons) in the form of local actions (Wess Zumino actions)

3. More complex mechanisms such as "anomaly inflows"

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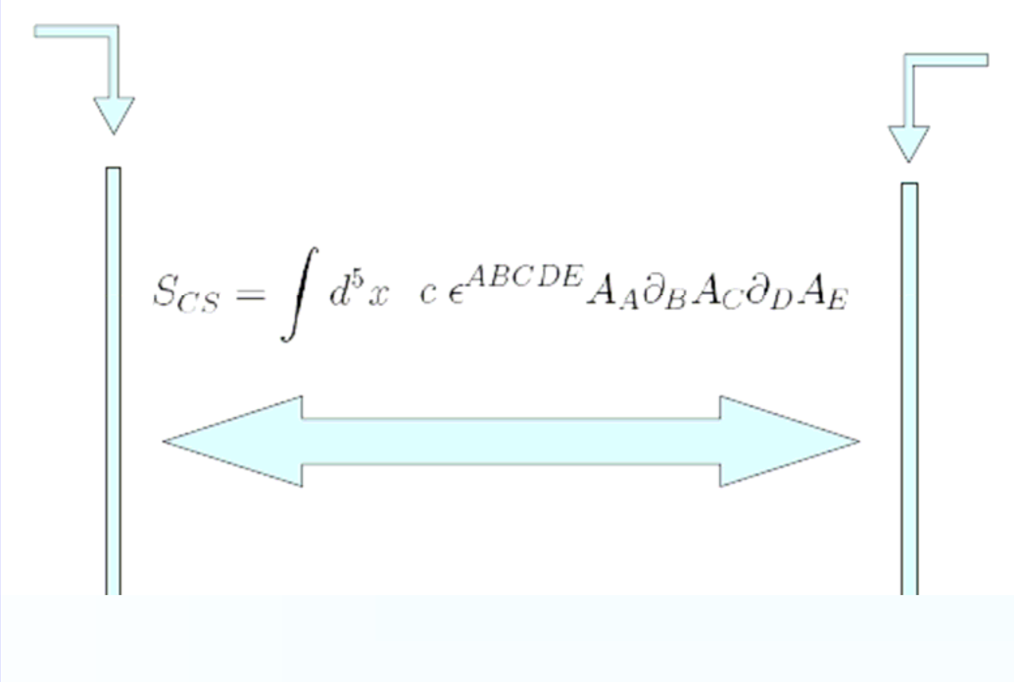
Anomaly inflow on branes

$$\int_I d^4x\, \overline{\psi}_L i \not{D}_L \psi_L$$



$$S_{CS} = \int d^5x \;\; c\, \epsilon^{ABCDE} A_A \partial_B A_C \partial_D A_E$$

$$\int_{II} d^4x\, \overline{\psi}_R i \not{D}_R \psi_R$$

Hill, Phys. Rev. D74 (2006)

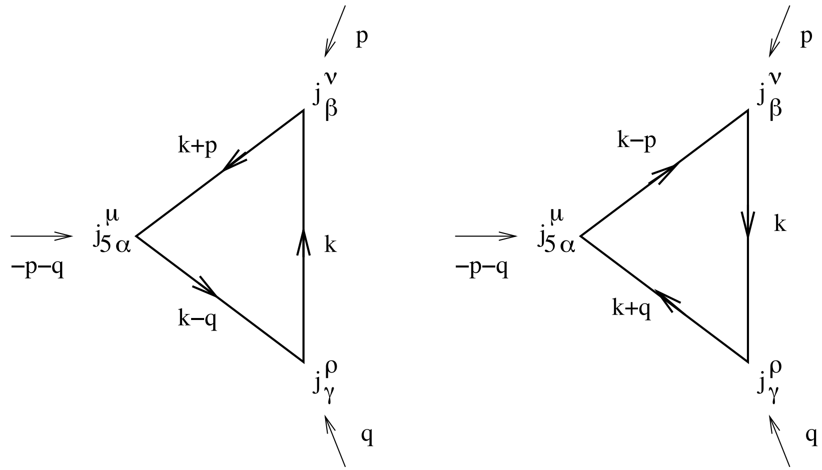
$$A_A(x_\mu, y) \rightarrow A_A(x_\mu, y) + \partial_A \theta(x_\mu, y)$$

$$S_{CS} \;\rightarrow\; S_{CS} + \frac{c}{4}\int_{II} d^4x\, \theta(R)\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(R) - \frac{c}{4}\int_I d^4x\, \theta(0)\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(0)$$

(Callan and Harvey)

	$SU(3)_c$	$SU(2)_w$	$U(1)_Y$	$U(1)'$
Q_L	3	2	1/6	z_Q
u_R	3	1	2/3	z_u
d_R	3	1	-1/3	$2z_Q - z_u$
L	1	2	-1/2	$-3z_Q$
e_R	1	1	-1	$-2z_Q - z_u$
H	1	2	1/2	z_H
$\nu_{R,k}$	1	1	0	z_k
χ	1	1	0	z_χ

Charge assignment of fermions and scalars in the $U(1)'$ SM extension.



Constraints on Abelian Extensions of the Standard Model from Two-Loop Vacuum Stability and U(1)B−L

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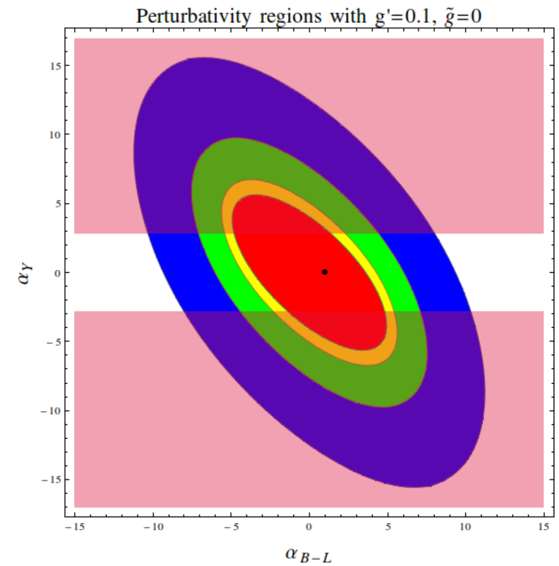
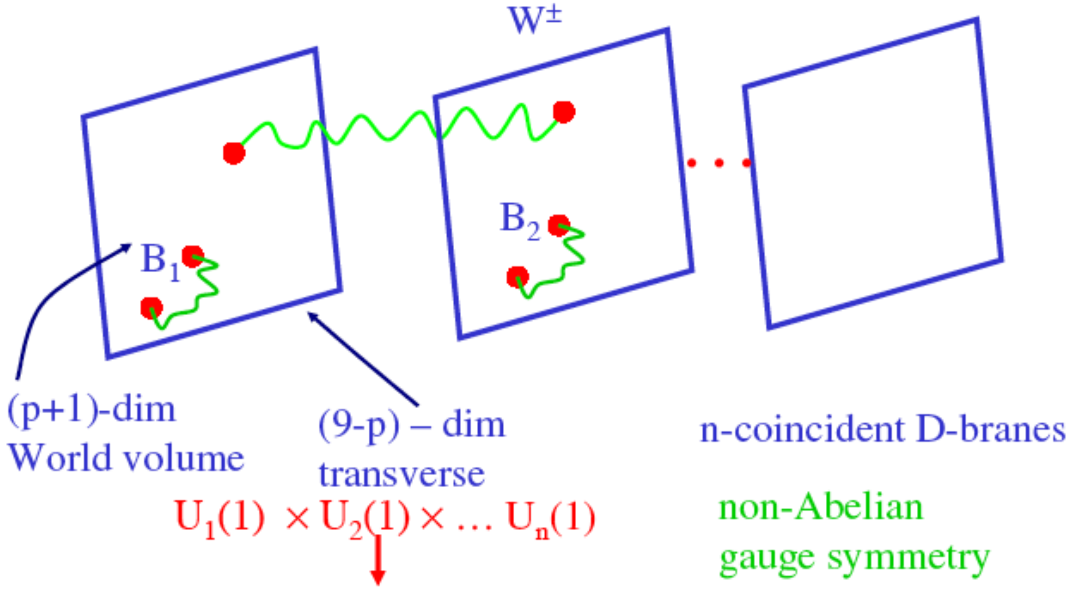


Delle Rose, Marzo, C.C.

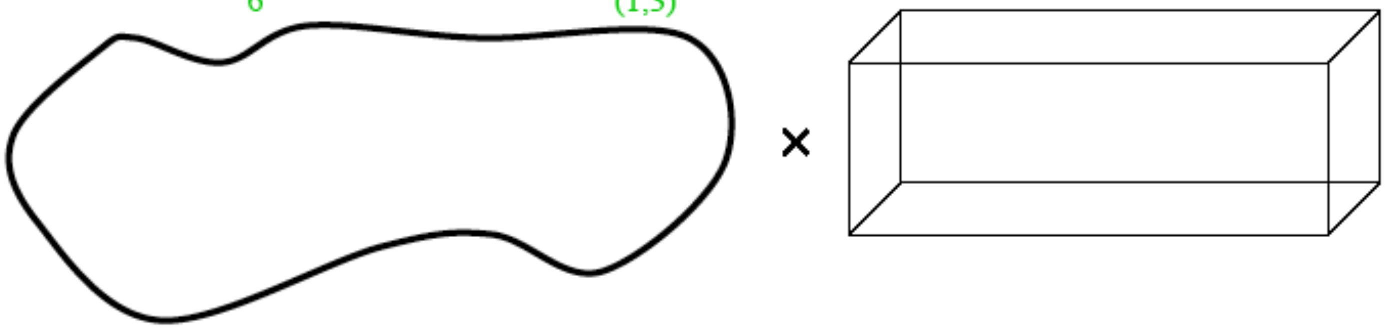
D p-branes



Compactification

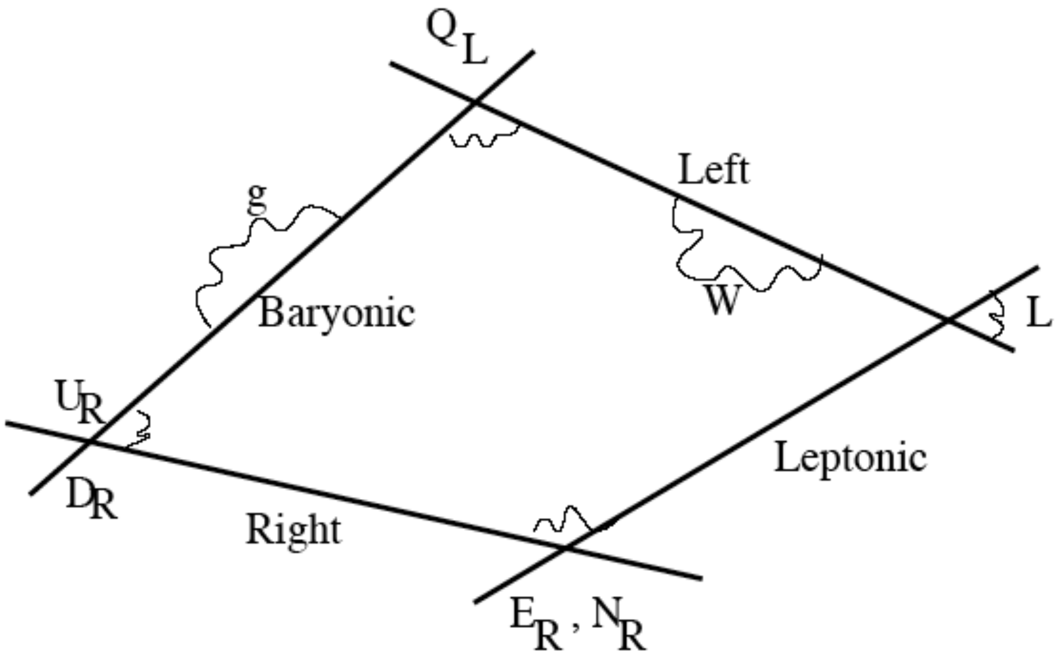
D=10 → D=4
&
N=1 supersymmetry

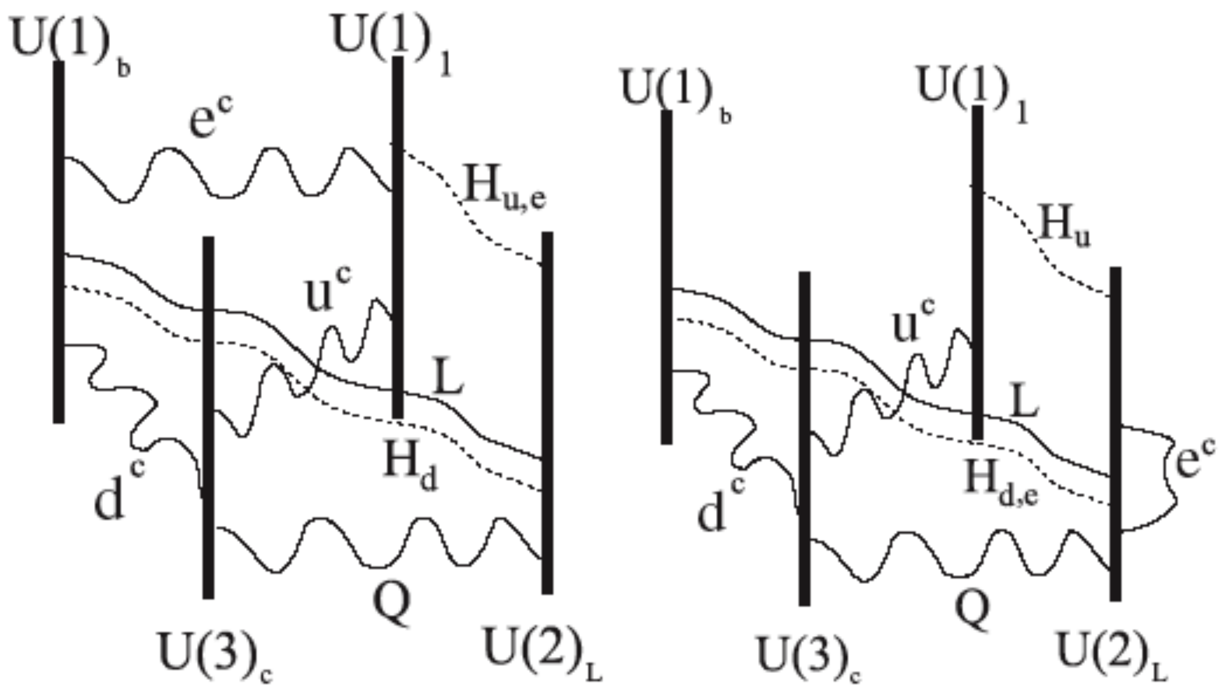
X_6- Calabi-Yau × $M_{(1,3)}$ Minkowski



Label	Multiplicity	Gauge Group	Name
stack a	$N_a = 3$	$SU(3) \times U(1)_a$	Baryonic brane
stack b	$N_b = 2$	$SU(2) \times U(1)_b$	Left brane
stack c	$N_c = 1$	$U(1)_c$	Right brane
stack d	$N_d = 1$	$U(1)_d$	Leptonic brane

Table 1: Brane content yielding the SM spectrum.





$$Y = -\frac{1}{3}\,Q_c - \frac{1}{2}\,Q_L + Q_1$$

$Q\,(\mathbf{3}, \mathbf{2}, +1, -1, 0, 0)$

$u^c(\bar{\mathbf{3}}, \mathbf{1}, -1, 0, -1, 0)$

$d^c(\bar{\mathbf{3}}, \mathbf{1}, -1, 0, 0, -1)$

$L(\mathbf{1}, \mathbf{2}, 0, +1, 0, -1)$

$e^c(\mathbf{1}, \mathbf{1}, 0, 0, +1, +1)$

$H_u(\mathbf{1}, \mathbf{2}, 0, +1, +1, 0)$

$H_d(\mathbf{1}, \mathbf{2}, 0, -1, 0, -1)$

Irges, Kiritsis, C.C.
“On the effective theory of low-scale Orientifold vacua“

The study the effective field theory of

a class of models containing a gauge structure of the form

SM x U(1) x U(1) x U(1)

$SU(3) \times SU(2) \times U(1)_Y \times U(1)$.....

from which the hypercharge is assigned to be anomaly free

These models are the object of an intense scrutiny by
many groups working on intersecting branes in the past.

Antoniadis, Kiritsis, Rizos, Tomaras
Antoniadis, Leontaris, Rizos
Ibanez, Marchesano, Rabadan,
Ghilencea, Ibanez, Irges, Quevedo
See. E. Kiritsis' review on Phys. Rep.

What happens if you to have an anomalous
U(1) at low energy? What is its signature?

Gauged Stuckelberg axions: field theory realization of the Green-Schwarz mechanism of string theory

The gauging procedure requires an anomalous abelian symmetry (an anomalous U(1)) and a periodic potential in order to make the axion physical.

But first we are going to review the PQ axion

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Axions and the Strong CP Problem

Axions have appeared in physics in an attempt to solve the strong CP problem of QCD.

Why is the $\theta G\tilde{G}$ term so small?
Consider an $SU(2)$ gauge theory

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad G_{\mu\nu} = G^a_{\mu\nu} T^a$$

$$A_\mu \to U A_\mu U^{-1} + U\partial_\mu U^{-1}$$

$$G_{\mu\nu} \to U G_{\mu\nu} U^{-1}$$

We look for minima of the Euclidean action

$$S = -\frac{1}{2g^2}\int d^4x Tr G_{\mu\nu} G_{\mu\nu}$$

In a nonabelian theory a vanishing field strength is possible with

$$A_\mu = U\partial_\mu U^{-1}$$

(pure gauge). Solutions of this condition are instanton configurations, characterised by a topological number.

$$-16\pi^2 Q(x) = Tr[G_{\mu\nu}\tilde{G}_{\mu\nu}] = Tr[\epsilon_{\mu\nu\alpha\beta}[2\partial_\mu(A_\nu\partial_\alpha A_\beta + \frac{2}{3}A_\nu A_\alpha A_\beta)]],$$

$$\tilde{G} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta},\ Q(x) = \partial_\mu J_\mu,\ J_\mu = -\frac{1}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}A_\nu(\partial_\alpha A_\beta + \frac{2}{3}A_\alpha A_\beta)$$

For an $SU(3)$ gauge theory such as QCD, similarly, the Lagrangean then allows a total derivative term $\theta G\tilde{G}$ which is a boundary term, but cannot be neglected. For instantons

$$G = \tilde{G}, \qquad \int d^4x G\tilde{G}(x) = 32\pi^2 n,$$

Therefore $\rightarrow$ *There is a dimension-4 operator that we can write down in the Standard Model (SM)*

$$\theta_0 G \tilde{G}$$

(violates Parity and Time reversal, CP is broken)

It is a total derivative term and as such it does not contribute *in perturbation theory*

Adding a total derivative term gives a zero momentum vertex in perturbation theory, but it contributes non-perturbatively

How?

If we consider an instanton (Euclidean) configuration, then the contribution to the path integral is

$$\sim e^{-\mathcal{S}_0} = e^{-\frac{1}{4g^2}\int d^4xFF} = e^{-\frac{8\pi^2}{g^2}}$$

- These configurations, at small coupling, give a negligible contribution
- They are solutions of the classical eq. of motion of QCD, which is scale invariant at classical level
  However, the solution of the equation $G = \tilde{G}$ involves an integration constant, the size of the instanton.
- The solution breaks scale invariance, because of the integration constant, which remains arbitrary.
  It tells us where the energy of the configuration is localized.
  At tree level g is constant, but at 1-loop it runs. Scale invariance is broken by renormalization.

In the functional integral we need to sum over all these configurations.

Small instantons (R)

- $\rightarrow$ large scale $\lambda \sim 1/R$
- $\rightarrow$ small coupling $g(\lambda) \ll 1$
- $\rightarrow$ large suppression in $e^{-\frac{8\pi^2}{g^2(\lambda)}}$. The contribution is perturbative, since g is small, but it is negligible.
  The instanton contribution to the QCD action is dominated by large instantons ($g(\lambda)$ large). Unfortunately the contribution is non-perturbative.

- The running is controlled by the size of the instanton, $g = g(\lambda)$

In the functional integral we need to sum over all these configurations.

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- The saddle point approximation is not valid any more since the action is O(1).

The partition function can be written in the form

$$\sim e^{-8\pi^2/g^2(\lambda)-i\theta_0}$$

and summing over instantons/anti instantons

$$\sum_{I\bar{I}} \sim e^{-8\pi^2/g^2(\lambda)} \cos\theta_0$$

θ_0 is not directly observable. One expects the energy density to dependen on θ_0 Notice, however, that QCD has a $U(1)_A$ anomaly, due to fermions. There is an axial symmetry

$$q \to q e^{i\gamma_5 \alpha}$$

and the integration measure is not invariant

$$DqD\bar{q} \to DqD\bar{q}e^{-\frac{i}{16\pi^2}\alpha \int F\tilde{F}d^4x}$$

Therefore θ_0 is not physical because it can be shifted by a field redefinition

$$\theta_0 \to \theta_0 + 2\alpha$$

But also the quark mass term gets a phase under the chiral transformation

$$\bar{q}_L M q_R + h.c. \to \bar{q}_L M q_R e^{2i\alpha} + h.c.$$

therefore

$$argM \to argM + 2\alpha$$

and

$$\theta \equiv \theta_0 - argM$$

is invariant under field redefinitions. *If we have fermions in complex representations of the gauge group, θ_0 is affected by field redefinitions and is not physical*, but θ is physical. This can be generalized to n_f fermions.

$$\theta_0 \to \theta_0 + 2n_f\alpha, \qquad ArgdetM \to ArgdetM + 2n_f\alpha$$

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$$\theta_0 \rightarrow \theta_0 + 2n_f\alpha, \qquad ArgdetM \rightarrow ArgdetM + 2n_f\alpha$$

$$\theta \equiv \theta_0 - ArgdetM$$

is physical.

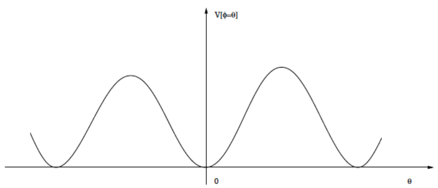
Experimentally θ is very small. We can set this value to zero assuming a cancellation between

- θ_0 (reated to gluon dynamics)
- *ArgDetM* (related to the electroweak sector, Yukawas and Higgs)

We can easily derive some properties of the vacuum energy as a function of θ.

$$e^{-VE(\theta)} = \left| \int D\Phi e^{-S[\Phi] - \frac{i}{32\pi^2}\theta \int F\tilde{F} d^4x} \right|$$

$$\leq \int D\Phi \left| e^{-S[\Phi] - \frac{i}{32\pi^2}\theta \int F\tilde{F} d^4x} \right| = e^{-VE(\theta=0)}$$



$$E(\theta) \geq E(0)$$

It is also even in θ: $E(\theta) = E(-\theta)$. Periodic of period 2π.

We can eliminate the θ_0 term and bring it completely into the fermion Mass matrix.

$$q_L \to q_L e^{+i\theta_0/2} \qquad q_R \to q_R e^{-i\theta_0/2}$$

Then

$$M \to e^{-i\theta_0/2} M e^{-i\theta_0/2}$$

It can be generalized to

$$q_L^f \to q_L e^{+iQ_f\theta_0/2} \qquad q_R^f \to q_R e^{-iQ_f\theta_0/2}$$

as far as

$$Tr Q_f = 1$$

(global phase is θ_0).

QCD with light quarks has a chiral symmetry (u,d)

$$U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

broken by quark condensates and anomalies to

$$SU(2)_V \times U(1)_V$$

with $U(1)_V$=baryon number. Three NG-models $\pi^{\pm}, \pi^0$ of the broken chiral symmetry. We try to fix the low energy effective action using the left-over global symmetries

$$\mathcal{Z}[J] = \int D\Phi e^{iS_{QCD}(\Phi)+J\Phi} = \int D\pi e^{iS(\pi,J)}$$

$$\begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ -\sqrt{2}\pi^- & -\pi^0 \end{bmatrix}$$

$$U = e^{i\pi\cdot T/f_\pi}$$

$$\mathcal{L} = \frac{{f_\pi}^2}{4}\left(Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + 2B_0\, Tr\left[MU^\dagger + M^\dagger U\right]\right)$$

$$E(\theta,\pi) = -\frac{{f_\pi}^2}{4} 2B_0\, 2 Re Tr\left(\begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} e^{i\theta/2} Exp\, \frac{i}{f_\pi} \begin{bmatrix} \pi^0 & 0 \\ 0 & -\pi^0 \end{bmatrix} \right)$$

$$= -m_\pi^2 f_\pi^2 \sqrt{\cos^2\frac{\theta}{2} + \left(\frac{m_u - m_d}{m_u + m_d}\right)^2 \sin^2\frac{\theta}{2}}\cos\left(\frac{\pi^0}{f_\pi} - \phi(\theta)\right)$$

where

$$\sin(\phi) = \frac{m_d - m_u}{m_d + m_u}\sin^2\frac{\theta}{2}$$

A minimum is obtained for (vev) $\pi^0 = f_\pi \phi(\theta)$ (with $m_\pi^2 = B_0(m_u + m_d)$) Then

$$E(\theta) = -m_\pi^2 {f_\pi}^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}\sin^2\frac{\theta}{2}}$$

when the mass of any of the quarks goes to zero, the θ-dependence disappears.

For $\theta = 0 \quad E(0) = -m_\pi^2 f_\pi^2$

Possible solutions. Can we use any existing SM symmetry?

After we turn on the Yukawa's only B and L are left as global symmetries of the SM.

In the SM we have an anomalous symmetry B, baryon number and L, lepton number (B-L is anomaly free).

But B is not anomalous respect to $SU(3)_c$, whence it cannot produce a $F_g \tilde{F}_g$ (gluon).

We then require an extra $U(1)_{PQ}$ global symmetry.

There is another solution: if $Y_u = 0$ then we could rotate:

$$u_R \to e^{i\alpha} u_R$$

.

This symmetry would be anomalous under $SU(3)_c$ and we could erase the $\theta F_g \tilde{F}_g$ term.

Notice that in the electroweak case we could also consider a "weak CP" problem $\sim \theta_W F_W \tilde{F}_W$

In fact B is anomalous under $SU(2)_L$, electroweak quark doublets therefore could be redefined under $U(1)_{baryon}$, canceling the corresponding weak-CP violating term.
A second type of protection from θ_W contributions come from the fact that the theory is in a Higgs phase. The contribution is $e^{\frac{-8\pi^2}{g_W(W)^2}}$ which are screened due to the masses of the W'sand Z.

KSVZ axion(Kim, Shifman, Vainshtein, Zakharov)
A pseudoscalar $a(x)$ that shifts under a global $U(1)_{PQ}$ symmetry (NG mode) $a(x) \to a(x) + \alpha f_a$ can do the job.
Use the Lagrangean

$$\frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{32\pi^2} F\tilde{F} + i\bar{Q}\gamma^\mu \partial_\mu Q + \lambda \phi \bar{Q}_L Q_R$$

ϕ has a typical mexican-hat potential, with $\langle \phi \rangle = v_{PQ}$. Then $\phi(x) = \frac{v_{PQ}+\rho}{\sqrt{2}} e^{i\frac{a(x)}{v_{PQ}}}$ and

$$\frac{\lambda}{\sqrt{2}} \phi \bar{Q}_L Q_R \sim \lambda v_{PQ} e^{i\frac{a}{v_{PQ}}} \bar{Q}_L Q_R$$

We perform now a chiral field redefinition

$$Q \to Q' = e^{-i\frac{a}{2v_{PQ}}\gamma_5} Q \qquad \frac{\lambda}{\sqrt{2}} v_{PQ} \bar{Q}'_L Q'_R$$

. We will generate a term $\delta\mathcal{S} = \frac{a}{32\pi^2 v_{PQ}} F\tilde{F}$, since the field redefinition is anomalous under $U(1)_{PQ}$.

Now we can integrate out Q and ρ. We are left with an interaction

$$\frac{a(x)N}{32\pi^2 v_{PQ}} F\tilde{F} = \frac{a(x)}{32\pi^2 f_a} F\tilde{F}$$

for N quarks Q, with $\frac{v_{PQ}}{N} = f_a$.

DFSZ axion (PQ), (WW). This is generated using only scalars.

$$H_u, H_d, \phi$$

Up to dimension-4 involves three mexican-hat types of potentials for H_u, H_d and ϕ, and an extra contribution
V' which depends on
$|H_u|^2, |H_d|^2, |\phi|^2, |H_u H_d^\dagger|^2, |H_u \cdot H_d|^2, H_u \cdot H_d \phi^2$.
Collecting the phases, one can identify the NG mode of the $U(1)_{PQ}$ using the condition that it has to be orthogonal to the hypercharge
There are 3 phases. One of them will identify the Goldstone mode. Orthogonality respect to the Goldstone of the Z boson is found by looking at the bilinear mixing $M_Z Z_\mu \partial^\mu G_Z$

$$H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{q_u a(x)}{v_{PQ}}} \qquad H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{q_d a(x)}{v_{PQ}}} \qquad \phi = \frac{v_\phi}{\sqrt{2}} e^{i\frac{q_\phi a(x)}{v_{PQ}}}$$

$$q_\phi = -1 \qquad q_u = 2\frac{v_d^2}{v^2} \qquad q_d = 2\frac{v_d^2}{v^2}$$

$$v^2 = v_u^2 + v_d^2$$

absence of mixing with G_Z: $q_u^2 v_u^2 - q_d^2 v_d^2 = 0$. v is the electroweak vev (246 GeV).
By requiring that $a(x)$ is canonically normalized:
$v_{PQ} = v_\phi^2 + v^2 \sin 2\beta$, with $\sin\beta = \frac{v_u}{v}$ and $\cos\beta = \frac{v_d}{v}$. Notice that $a(x)$ is associated mostly to ϕ.
From the Yukawa couplings one gets

$$-Y_u \bar{q}_L H_u q_R - Y_d \bar{q}_L H_d d_R$$

$$-Y_u \bar{u}_L \frac{v_u}{\sqrt{2}} e^{2ia\sin^2\beta \frac{a}{v_{PQ}}} u_R - Y_d \bar{d}_L \frac{v_d}{\sqrt{2}} e^{2ia\cos^2\beta \frac{a}{v_{PQ}}} d_R$$

Doing a chiral redefinition

$$\delta\mathcal{L} = \frac{6}{32\pi^2 v_{PQ}} aF\tilde{F} \qquad \bar{q}_L \gamma^\mu D_\mu q_L \to \frac{c}{v_{PQ}} \partial_\mu a \bar{q} \gamma^\mu \gamma_5 q$$

$$\mathcal{L} = \mathcal{L}_{QCD}(\theta = 0) + \frac{1}{f_a}\partial_\mu J^\mu + \left(\frac{a}{f_a} - \theta\right)\frac{1}{32\pi^2}F\tilde{F} + \frac{1}{2}\partial_\mu a \partial^\mu a$$

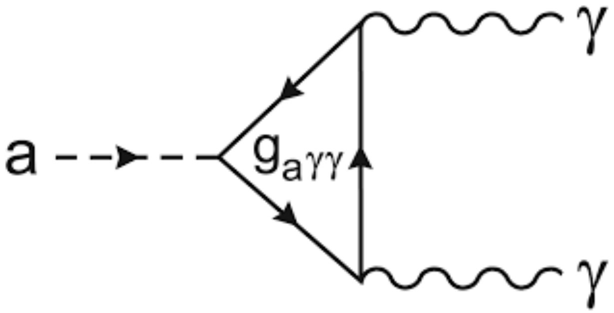
we can clearly redefine $a(x)$ in order to absorbe θ.
Since f_a is very large, then we can treat $a(x)$ as an external source.
To determine its potential, we can then take $V(\theta)$ with $\theta \to a/f_a$

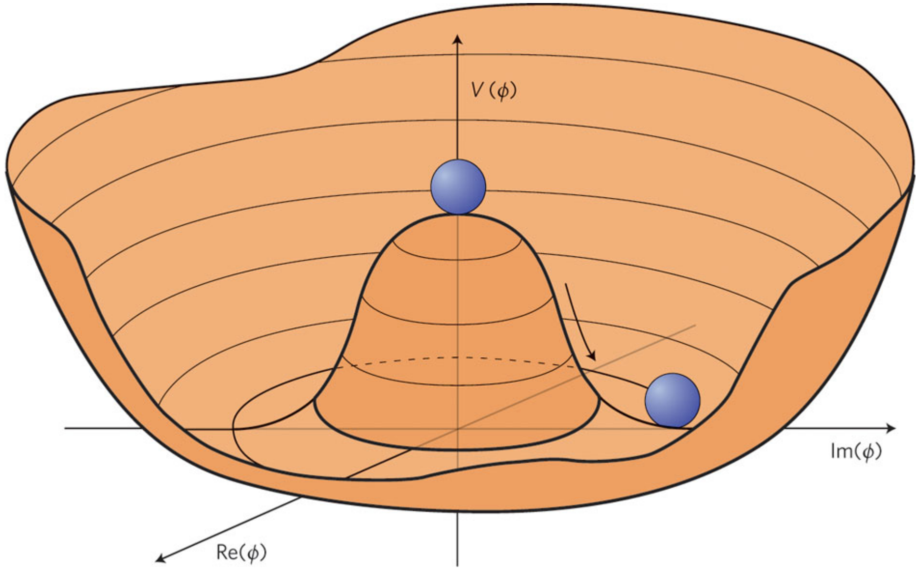
$$V(\frac{a}{f_a}) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{m_u m_d}{(m_u + m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$

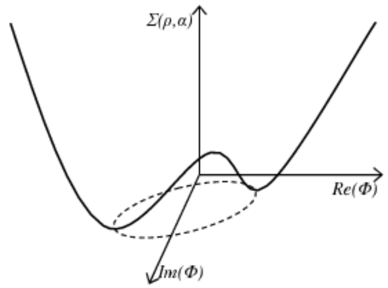
from which one can extract the axion mass

$$m_a^2 = \frac{m_\pi^2}{f_a^2} f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

The breaking of the PQ symmetry
takes place at a large scale f_a, but
The wiggling of the PQ potential
Occurs much later, at the QCD phase
transition

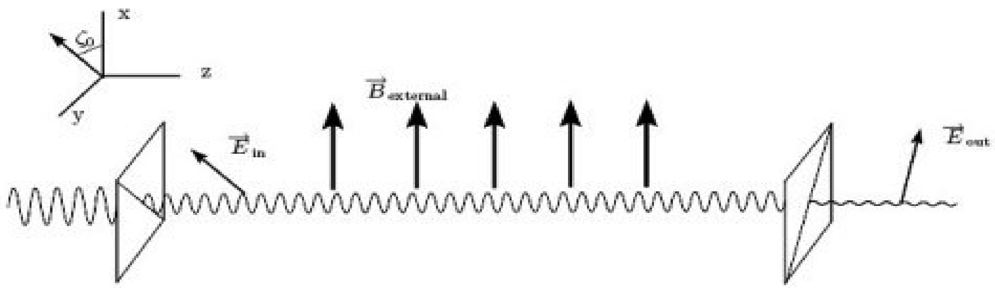




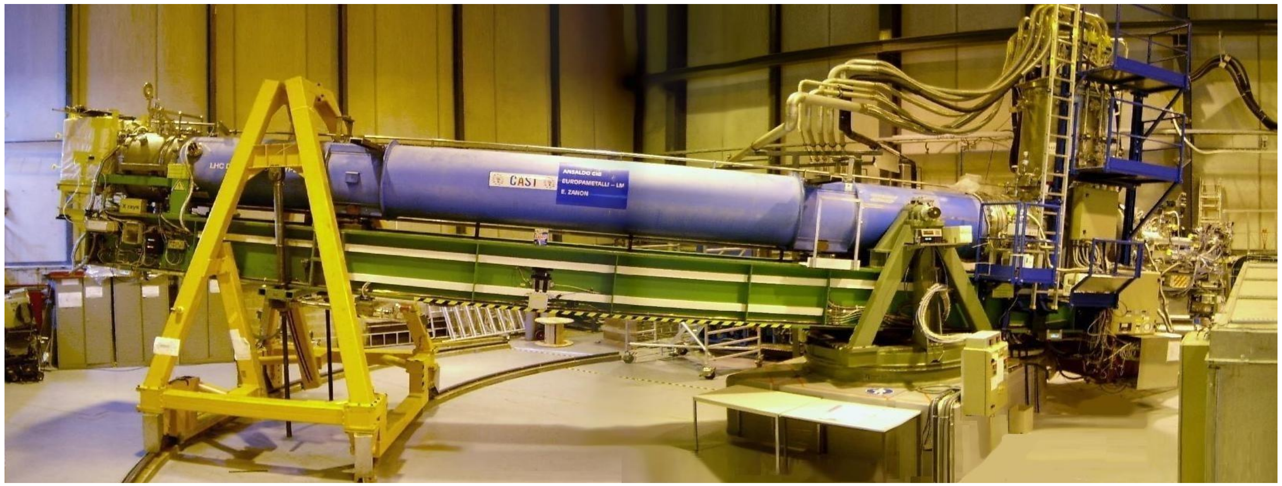


For a PQ axion a: $m = C/f_a$, while the aFF interaction is also suppressed by : a/f_a FF with f_a = 10^9 GeV

Experimental signatures



PVLAS (INFN)



CAST (Cern)

Optical activity

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{4}\widetilde{g}\,\varphi\,F_{\mu\nu}\widetilde{F}^{\mu\nu},$$

$$\begin{cases} \nabla\cdot\boldsymbol{B} \;=\; 0, \\ \frac{\partial\boldsymbol{B}}{\partial t} + \nabla\times\boldsymbol{E} =\; 0, \\ \Box\varphi \;=\; -\widetilde{g}\,\boldsymbol{E}\cdot\boldsymbol{B}, \\ \nabla\cdot\boldsymbol{E} = \widetilde{g}\,\nabla\varphi\cdot\boldsymbol{B}, \\ \nabla\times\boldsymbol{B} - \frac{\partial\boldsymbol{E}}{\partial t} = -\widetilde{g}\boldsymbol{B}\frac{\partial\varphi}{\partial t} + \widetilde{g}\boldsymbol{E}\times\nabla\varphi, \end{cases}$$

PVLAS-type

L. Carcagni', C.C.

$$\Box(\boldsymbol{E} - \frac{1}{2}\widetilde{g}\varphi\boldsymbol{B}) = -\frac{1}{2}\widetilde{g}\varphi\,\Box\boldsymbol{B},$$

$$\Box(\boldsymbol{B} + \frac{1}{2}\widetilde{g}\varphi\boldsymbol{E}) = \frac{1}{2}\widetilde{g}\varphi\,\Box\boldsymbol{E}.$$

$$\boldsymbol{D} \equiv \boldsymbol{E} - \frac{1}{2}\widetilde{g}\varphi\boldsymbol{B},$$

$$\boldsymbol{H} \equiv \boldsymbol{B} + \frac{1}{2}\widetilde{g}\varphi\boldsymbol{E}.$$

$$\Delta\boldsymbol{E} \equiv \boldsymbol{E}(L) - \boldsymbol{E}(0) = \frac{1}{2}\widetilde{g}\Delta\varphi\boldsymbol{H}(0).$$

Optical activity

Gauging axionic symmetries

The chain of anomalous U(1) symmetries require

- **One Stuckelberg term for each anomalous symmetry**
- The U(1)'s are in a massive (Stuckelberg phase)
- One linear combination of them generates the anomaly free hypercharge

Possibility of describing axion-like particles.

Such types of particles have been conjectured in several phenomenological analysis.
The mass of the particle and its interactions with the photons are independent quantities.

This brings us to a mechanism of cancelation of the gauge anomalies of "Green-Schwarz" type

Compared to a Peccei-Quinn axion, the new axion is gauged

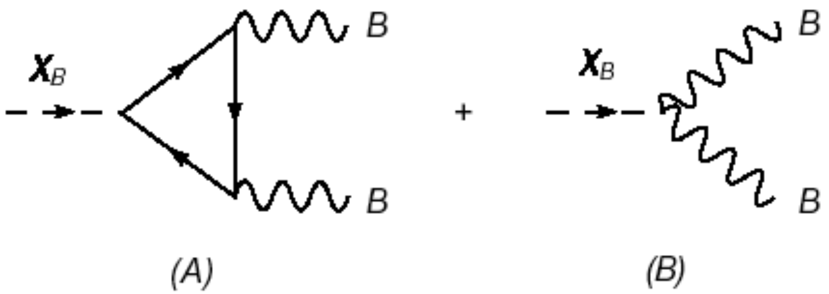
For a PQ axion a: $m = C/f_a$, while the aFF interaction is also suppressed by : $a/f_a\, FF$ with $f_a = 10^9$ GeV

In the case of these models, the mass of the axion and its gauge interactions are unrelated

the mass is generated by the combination of the Higgs and the Stuckelberg mechanisms combined

The interaction is controlled by the Stuckelberg mass (M_1)

The axion shares the properties of a CP odd scalar



Asymptotic axions for Wess Zumino actions and gauge invariance

$$\mathcal{L} = -\frac{1}{4}F_B^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_B\gamma_5 B_\mu)\psi$$

$$\mathcal{L} = -\frac{1}{4}F_B^2 - \frac{1}{4}F_A^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_A A_\mu + ig_B\gamma^5 B_\mu)\psi$$

Using a Stuckelberg axion and the inclusion of local counterterms

$$B_\mu \to B_\mu - \partial_\mu\theta$$

$$a_{BBB}\frac{b}{M}F_B \wedge F_B + a_{BAA}\frac{b}{M}F_A \wedge F_A$$

$$b \to b + M\theta$$

$$\frac{1}{2}(\partial_\mu b + MB_\mu)^2$$

One then considers the effective action

$$\mathcal{L} = -\frac{1}{4}F_B^2 + \frac{1}{2}(B_\mu + \frac{1}{M}\partial_\mu b)^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + ig_B\gamma_5)\psi + a_n\frac{b}{M}F_B \wedge F_B$$

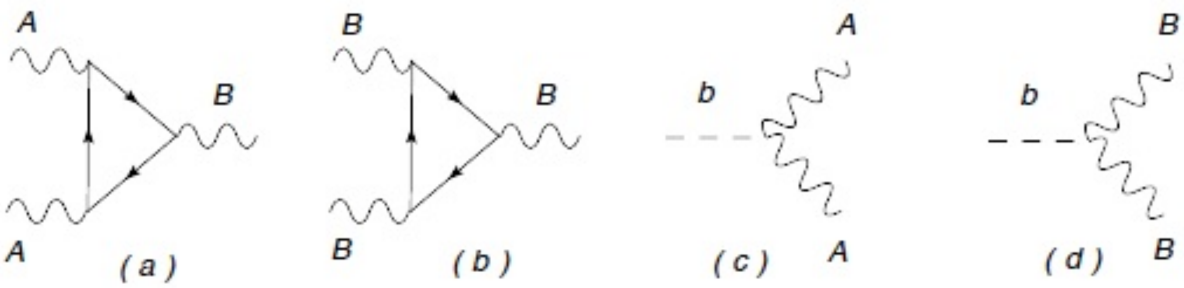
where the anomaly generated at one loop level by the fermion is removed by the Wess-Zumino counterterm

$$a_n\frac{b}{M}F_B \wedge F_B$$

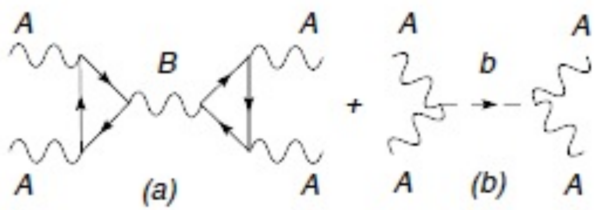
Somehow, this mechanism is viewed, from the point of view of QFT, as the mechanism of "Anomaly Cancellation"

But anomalies are not cancelled by local counterterms. One should notice that the mechanism of "anomaly cancellation", in this case, is based on introducing an extra field degree of freedom (b(x))

One could go to a gauge where b(x)=0. In what sense, then we cancel the anomaly?



One loop vertices and counterterms in the R_ξ gauge for the $A-B$ model for the WZ case.



A typical Bouchiat-Iliopoulos-Meyer amplitude and the axion counterterm to restore gauge invariance in the R_ξ gauge in the WZ effective action.

$$\mathcal{L}_{WZ} = \frac{C_{AA}}{2!M_1} bF_A \wedge F_A + \frac{C_{BB}}{2!M_1} bF_B \wedge F_B,$$

Variants: Higgs-axion mixing

There are some variants of this Lagrangian which may help us clarify this issue

$$\mathcal{L}_0 = |(\partial_\mu + i g_B q_B B_\mu)\phi|^2 - \frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{1}{2}(\partial_\mu b + M_1\ B_\mu)^2 - \lambda(|\phi|^2 - \frac{v^2}{2})^2$$
$$+\overline{\psi} i\gamma^\mu(\partial_\mu + ieA_\mu + ig_B\gamma^5 B_\mu)\psi - \lambda_1\overline{\psi}_L\phi\psi_R - \lambda_1\overline{\psi}_R\phi^*\psi_L$$

In this case we consider a model with 2 U(1)'s. The two gauge fields are A and B. The fermion has axial vector couplings to B and is vector coupled to A.

We have BBB and BAA anomalies. Vector field B is massive, A is massless

B mass generated via a combination of the Stuckelberg + Higgs mechanisms.

$$|(\partial_\mu + i g_B q_B B_\mu)\phi|^2$$
$$+\frac{1}{2}(\partial_\mu b + M_1\ B_\mu)^2 - \lambda(|\phi|^2 - \frac{v^2}{2})^2$$

ϕ is the Higgs field

$$\mathcal{L}_b = \frac{C_{AA}}{M} b\, F_A \wedge F_A + \frac{C_{BB}}{M} b\, F_B \wedge F_B.$$

$$\delta_B\left(\mathcal{L}_b + \mathcal{L}_{an}\right) = 0$$

B field massive by the Higgs and Stuckelberg mechanism

Higgs-Axion Mixing in $U(1)$ Models: massless axi-Higgs

$$\mathcal{L}_q = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2}\left(M_1^2 + (q_B g_B v)^2\right) B_\mu B^\mu - \frac{1}{2}m_1^2\phi_1^2$$
$$+B_\mu\partial^\mu\left(M_1 b + v g_B q_B \phi_2\right),$$

$$\phi = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2),$$

Goldstone mode is a combination of Stuckelberg field and CP odd part of the Higgs

$$\mathcal{L}_q = \frac{1}{2}(\partial_\mu \chi_B)^2 + \frac{1}{2}(\partial_\mu G_B)^2 + \frac{1}{2}(\partial_\mu h_1)^2 + \frac{1}{2}M_B^2 B_\mu B^\mu - \frac{1}{2}m_1^2 h_1^2 + M_B B^\mu \partial_\mu G_B$$

$$m_1 = v\sqrt{2\lambda},$$

$$M_B = \sqrt{M_1^2 + (q_B g_B v)^2}.$$

The mass of the B gauge boson is a combination of the Higgs and the Stuckelberg mechanism

1 physical axion (axi-Higgs) χ_B

1 Higgs h_1

1 massive gauge boson B_μ

$$\theta_B = \arccos(M_1/M_B)$$

$$\chi_B = \frac{1}{M_B}(-M_1\phi_2 + q_B g_B v\, b),$$

$$G_B = \frac{1}{M_B}(q_B g_B v\,\phi_2 + M_1 b),$$

$$(\phi_2, b) \rightarrow (\chi_B, G_B)$$

$$U = \begin{pmatrix} -\cos\theta_B & \sin\theta_B \\ \sin\theta_B & \cos\theta_B \end{pmatrix}$$

$$b = \alpha_1 \chi_B + \alpha_2 G_B = \frac{q_B g_B v}{M_B}\chi_B + \frac{M_1}{M_B}G_B,$$

The Stuckelberg has a gauge invariant physical component, χ_B

A massive axi-Higgs (periodic potential)

ordinary Higgs potential $V = \mu^2 \phi^* \phi + \lambda(\phi^*\phi)^2$

$$V' = b_1\left(\phi\, e^{-i q_B g_B \frac{b}{M_1}}\right) + \lambda_1\left(\phi\, e^{-i q_B g_B \frac{b}{M_1}}\right)^2 + 2\lambda_2(\phi^*\phi)\left(\phi\, e^{-i q_B g_B \frac{b}{M_1}}\right) + \text{c.c.}$$

extra potential allowed by the symmetry

$$c_\chi = 4\left(\frac{b_1}{v^3} + \frac{4\lambda_1}{v^2} + \frac{2\lambda_2}{v}\right).$$

$$m_\chi^2 = -\frac{1}{2}c_\chi v^2 \frac{M_B^2}{M_1^2}.$$

massive axi-Higgs

$$
\begin{aligned}
\mathcal{L} \;=\; & - \frac{1}{2} tr\, G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} tr\, W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} F^{l}_{\mu\nu} F^{\mu\nu,l} \\
& - |(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q_l^{(H_u)} g_l A^l_\mu) H_u|^2 - |(\partial_\mu + i\frac{g_2}{2}\tau^a W^a_\mu + i q_l^{(H_d)} g_l A^l_\mu) H_d|^2 \\
& + Q^\dagger_{Li} \sigma^\mu \mathcal{D}_\mu Q_{Li} + u^\dagger_{Ri} \overline{\sigma}^\mu \mathcal{D}_\mu u_{Ri} + d^\dagger_{Ri} \overline{\sigma}^\mu \mathcal{D}_\mu d_{Ri} \\
& + L^\dagger_{Li} \sigma^\mu \mathcal{D}_\mu L_{Li} + e^\dagger_{Ri} \overline{\sigma}^\mu \mathcal{D}_\mu e_{Ri} + \nu^\dagger_{Ri} \overline{\sigma}^\mu \mathcal{D}_\mu \nu_{Ri} \\
& + \gamma^u_{ij} H^T_u \tau^2 \, (Q^t_{Li} \sigma^2 u_{Rj}) + \gamma^d_{ij} H^\dagger_d \, (Q^t_{Li} \sigma^2 d_{Rj}) + c.c. \\
& + \gamma^e_{ij} H^\dagger_u \, (L^t_{Li} \sigma^2 e_{Rj}) + \gamma^\nu_{ij} H^T_d \tau^2 \, (L^t_{Li} \sigma^2 \nu_{Rj}) + c.c. \\
& - \frac{1}{2} \sum (\partial_\mu a^I + g_l \mathcal{M}^I_l A^l_\mu)^2 + E_{lmn}\, \epsilon^{\mu\nu\rho\sigma}\, A^l_\mu\, A^m_\nu\, F^n_{\rho\sigma} \\
+ & \sum_I (D_I\, a^I\, tr\,\{G \wedge G\} + F_I\, a^I\, tr\,\{W \wedge W\} + C_{Imn}\, a^I\, F^m \wedge F^n) \\
+ & \; V(H_u, H_d, a^I).
\end{aligned}
$$

Generic extension

The gauge symmetry under which this Lagrangian is invariant is

$$
SU(3)_c \times SU(2)_W \times G_1, \qquad G_1 = \prod_{l=1}^{4} U(1)_l.
$$

$$
SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d.
$$

Gauge kinetic

Stuckeberg mass terms

Chern Simons abelian interactions

$a_I, \qquad I = 1, 2, \ldots n$ Stuckelberg axions

F_I

$H_u \qquad H_d$

$E_{lmn} \epsilon^{\mu\nu\rho\sigma} A^l_\mu A^m_\nu F^n_{\rho\sigma}$ Abelian CS terms

Higgs sector

$$|\mathcal{D}_\mu H_u|^2 + |\mathcal{D}_\mu H_d|^2 + \frac{1}{2}\sum_I (\partial a'_I + M_I A^I)^2$$

$$\begin{aligned}
\mathcal{D}_\mu H_u &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + \frac{i}{2} g_2 \tau_3 W_{3\mu} + \frac{i}{2} g_Y A^Y_\mu + \frac{i}{2} \sum_I q^I_u g_I\, A^I_\mu\right) H_u \\
\mathcal{D}_\mu H_d &= \left(\partial_\mu + \frac{i}{\sqrt{2}} g_2 \left(T^+ W^+ + T^- W^-\right) + i \frac{g_2}{2} \tau_3 W_{3\mu} + \frac{i}{2} g_Y A^Y_\mu + \frac{i}{2} \sum_I q^I_d g_I\, A^I_\mu\right) H_d
\end{aligned}$$

Typical mass terms for the gauge bosons are generated both from the Higgs and the Stuckleberg contributions

$$\frac{1}{2}\sum_I M_I^2 (A^I_\mu)^2 + \frac{1}{4}(-g_2 W_{3\mu} + g_Y A^Y_\mu + \sum_I q^I_u\, g_I\, A^I_\mu)^2 v_u^2$$
$$+\frac{1}{4}(-g_2 W_{3\mu} + g_Y A^Y_\mu + \sum_I q^I_d\, g_I\, A^I_\mu)^2 v_d^2,$$

There will be bilinear mixings in the broken (electroweak) phase

$$Z^\mu\,\partial_\mu\left\{f_u C^u + f_d C^d + \sum_I g_I M_I O^A_{ZI} a'_I\right\} + \sum_J Z'^{\mu}_J\,\partial_\mu\left\{f_{u,J} C^u + f_{d,J} C^d + \sum_I g_I M_I O^A_{Z'_J I} a'_I\right\},$$

We can extract the NG modes by a rotation, identifying 1 single physical axion

$$\begin{pmatrix} \mathrm{Im} H_u^0 \\ \mathrm{Im} H_d^0 \\ . \\ a'_I \\ . \end{pmatrix} = O^\chi \begin{pmatrix} \chi \\ G_1^0 \\ G_2^0 \\ . \\ . \end{pmatrix}$$

The scalar potential has an ordinary 2-Higgs doublet part and an extra contribution

$$V_{PQ} = \sum_{a=u,d}\left(\mu_a^2 H_a^\dagger H_a + \lambda_{aa}(H_a^\dagger H_a)^2\right) - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2$$

$$\begin{aligned} V_{\not{P}\not{Q}} = \quad & b\,(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + \lambda_1 (H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}})^2 \\ + \quad & \lambda_2 (H_u^\dagger H_u)(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_d e^{-i\sum_I (q_u^I - q_d^I)\frac{a'_I}{M_I}}) + c.c. \end{aligned}$$

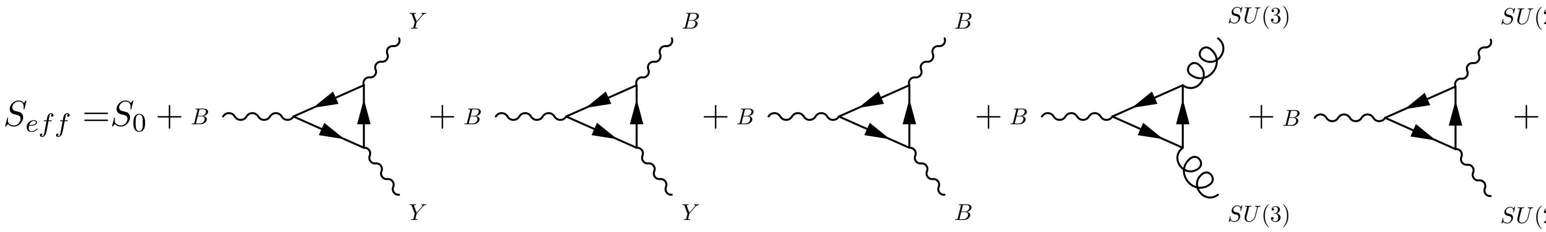
The Standard Model with 1 extra anomalous U(1) and an axion

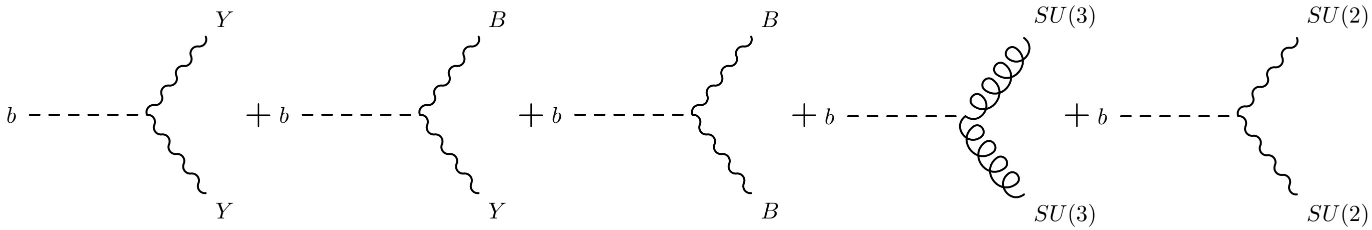
f	Q	u_R	d_R	L	e_R
q^B	q^B_Q	$q^B_{u_R}$	$q^B_{d_R}$	q^B_L	$q^B_{e_R}$

f	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
Q	3	2	$1/6$	q^B_Q
u_R	3	1	$2/3$	$q^B_Q + q^B_u$
d_R	3	1	$-1/3$	$q^B_Q - q^B_d$
L	1	2	$-1/2$	q^B_L
e_R	1	1	-1	$q^B_L - q^B_d$
H_u	1	2	$1/2$	q^B_u
H_d	1	2	$1/2$	q^B_d

The effective action has the structure given by

$$\mathcal{S} \;=\; \mathcal{S}_0 + \mathcal{S}_{Yuk} + \mathcal{S}_{an} + \mathcal{S}_{WZ} + \mathcal{S}_{CS}$$





Axionic contributions

$$
\begin{aligned}
\mathcal{S}_{WZ} &= C_{BB}\langle b\,F_B \wedge F_B\rangle + C_{YY}\langle b\,F_Y \wedge F_Y\rangle + C_{YB}\langle b\,F_Y \wedge F_B\rangle \\
&\quad + F\langle b\,Tr[F^W \wedge F^W]\rangle + D\langle b\,Tr[F^G \wedge F^G]\rangle,
\end{aligned}
$$

Abelian/non-abelian Chern Simons terms

$$
\begin{aligned}
\mathcal{S}_{CS} &= +d_1\langle BY \wedge F_Y\rangle + d_2\langle YB \wedge F_B\rangle \\
&\quad + c_1\langle \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(2)}_{\nu\rho\sigma}\rangle + c_2\langle \epsilon^{\mu\nu\rho\sigma} B_\mu C^{SU(3)}_{\nu\rho\sigma}\rangle.
\end{aligned}
$$

$$
\begin{aligned}
C^{SU(2)}_{\mu\nu\rho} &= \frac{1}{6}\left[W^i_\mu\left(F^W_{i,\nu\rho} + \frac{1}{3}\,g_2\,\varepsilon^{ijk}W^j_\nu W^k_\rho\right) + cyclic\right], \\
C^{SU(3)}_{\mu\nu\rho} &= \frac{1}{6}\left[G^a_\mu\left(F^G_{a,\nu\rho} + \frac{1}{3}\,g_3\,f^{abc}G^b_\nu G^c_\rho\right) + cyclic\right].
\end{aligned}
$$

With a single anomalous U(1) these terms care not essential.

$$V = V_{PQ}(H_u, H_d) + V_{\not{P}\not{Q}}(H_u, H_d, b).$$

$$V_{PQ} = \mu_u^2 H_u^\dagger H_u + \mu_d^2 H_d^\dagger H_d + \lambda_{uu}(H_u^\dagger H_u)^2 + \lambda_{dd}(H_d^\dagger H_d)^2 - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2$$

$$V_{\not{P}\not{Q}} = \lambda_0(H_u^\dagger H_d e^{-ig_B(q_u-q_d)\frac{b}{2M}}) + \lambda_1(H_u^\dagger H_d e^{-ig_B(q_u-q_d)\frac{b}{2M}})^2 + \lambda_2(H_u^\dagger H_u)(H_u^\dagger H_d e^{-ig_B(q_u-q_d)\frac{b}{2M}}) +$$

$$\lambda_3(H_d^\dagger H_d)(H_u^\dagger H_d e^{-ig_B(q_u-q_d)\frac{b}{2M}}) + \text{h.c.},$$

$$H_u = \begin{pmatrix} H_u^+ \\ v_u + H_u^0 \end{pmatrix} \qquad H_d = \begin{pmatrix} H_d^+ \\ v_d + H_d^0 \end{pmatrix}.$$

This potential is characterized by two null eigenvalues corresponding to two neutral Goldstone modes (G_0^1, G_0^2) and an eigenvalue corresponding to a massive state with an axion component (χ). In the $(\mathrm{Im}H_d^0, \mathrm{Im}H_u^0, b)$ CP-odd basis we get the following normalized eigenstates

$$G_0^1 = \frac{1}{\sqrt{v_u^2 + v_d^2}}(v_d, v_u, 0)$$

$$G_0^2 = \frac{1}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2 + 2M^2\left(v_d^2+v_u^2\right)}}\left(-\frac{g_B(q_d-q_u)v_d v_u^2}{\sqrt{v_u^2+v_d^2}}, \frac{g_B(q_d-q_u)v_d^2 v_u}{\sqrt{v_d^2+v_u^2}}, \sqrt{2}M\sqrt{v_u^2+v_d^2}\right)$$

$$\chi = \frac{1}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2 + 2M^2(v_d^2+v_u^2)}}\left(\sqrt{2}Mv_u, -\sqrt{2}Mv_d, g_B(q_d-q_u)v_d v_u\right) \qquad (14)$$

$$\begin{pmatrix} G_0^1 \\ G_0^2 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \mathrm{Im}H_d^0 \\ \mathrm{Im}H_u^0 \\ b \end{pmatrix};$$

$$
O^{\chi} = \begin{pmatrix} \frac{v_d}{v} & \frac{v_u}{v} & 0 \\ -\frac{g_B(q_d-q_u)v_d v_u^2}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d^2 v_u}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{\sqrt{2}Mv}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} \\ \frac{\sqrt{2}Mv_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & -\frac{\sqrt{2}Mv_d}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d v_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} \end{pmatrix}
$$

where $v = \sqrt{v_u^2 + v_d^2}$.

χ inherits WZ interaction since b can be related to the physical axion χ and to the Goldstone modes via this matrix

$$
b = O^{\chi}_{13} G^1_0 + O^{\chi}_{23} G^2_0 + O^{\chi}_{33} \chi,
$$

Stuckelberg axion

Physical axi-Higgs (gauged axion)

$$
\chi = O^{\chi}_{31} \mathrm{Im} H_d + O^{\chi}_{32} \mathrm{Im} H_u + O^{\chi}_{33} b.
$$

The phase-dependent potential has a well-defined periodicity. To identify the corresponding phase in the Higgs-neutral CP-odd sector we introduce a polar parametrization of the neutral components in the broken electroweak phase

$$H_u^0 = \frac{1}{\sqrt{2}}\left(\sqrt{2}v_u + \rho_u^0(x)\right) e^{i\frac{F_u^0(x)}{\sqrt{2}v_u}} \qquad H_d^0 = \frac{1}{\sqrt{2}}\left(\sqrt{2}v_d + \rho_d^0(x)\right) e^{i\frac{F_d^0(x)}{\sqrt{2}v_d}}, \tag{22}$$

where we have introduced the two phases F_u and F_d of the two neutral Higgs fields. The potential is periodic with respect to the linear combination of fields

$$\theta(x) \equiv \frac{g_B(q_d - q_u)}{2M} b(x) - \frac{1}{\sqrt{2}v_u} F_u^0(x) + \frac{1}{\sqrt{2}v_d} F_d^0(x), \tag{23}$$

and using the matrix O^χ to rotate on the physical basis, the phase describing the periodicity of the potential turns out to be proportional to the physical axion, modulo a dimensionful constant (σ_χ)

$$\theta(x) \equiv \frac{\chi(x)}{\sigma_\chi}, \tag{24}$$

$$\sigma_\chi \equiv \frac{2v_u v_d M}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2 + v_u^2)}}.$$

Replaces f_a of Peccei Quinn

Notice that χ (or, equivalently, θ) is gauge invariant as one can check quite directly. In fact a $U(1)_B$

PQ axion. Vacuum misalignment at the QCD phase transition

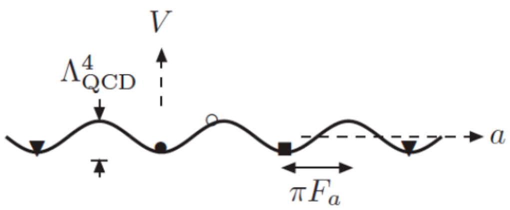
The PQ axion feels the QCD vacuum via the interaction $\frac{a}{f_a}G\tilde{G}$

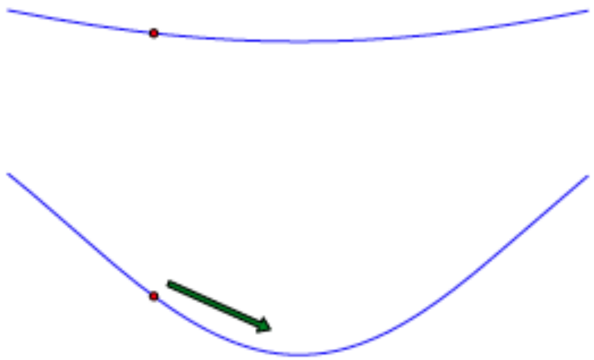
The angle of misalignment is

$$\theta = \frac{a(x)}{f_a}$$

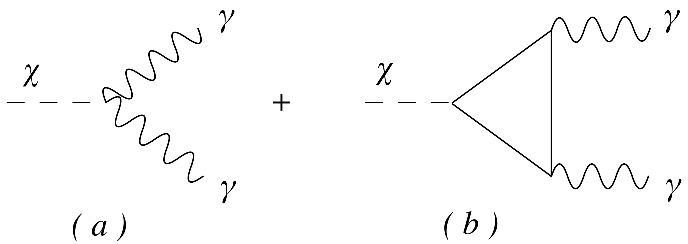
The mass is sizeable

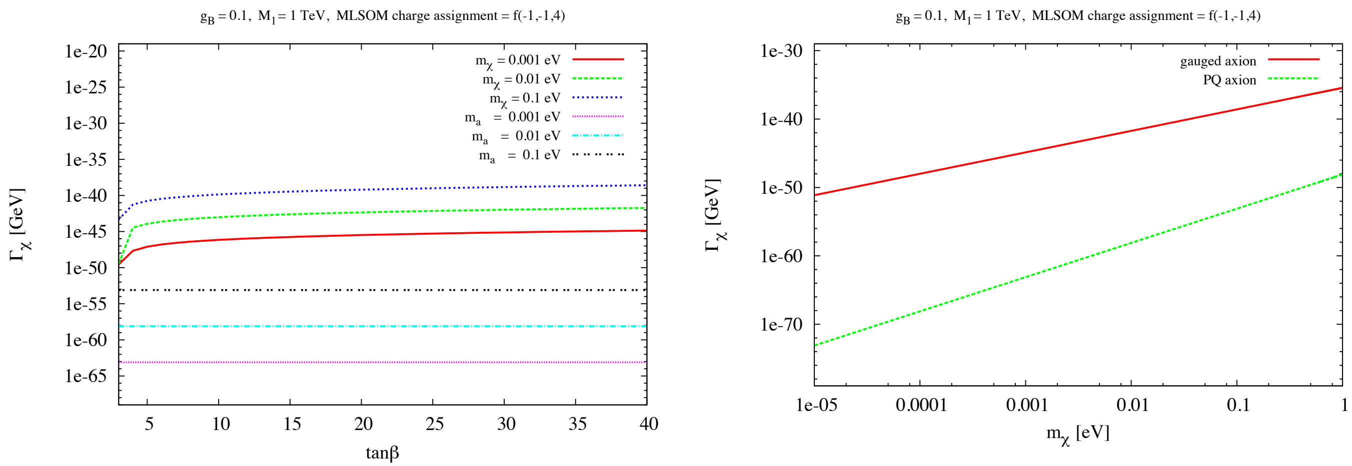
$$10^{-3} - 10^{-4} eV$$





If an axion has charges both under SU(3) and SU(2) we could consider the possibility of sequential misalignments. The dominant misalignment clearly comes from the largest potential



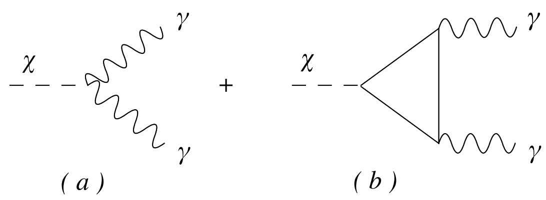


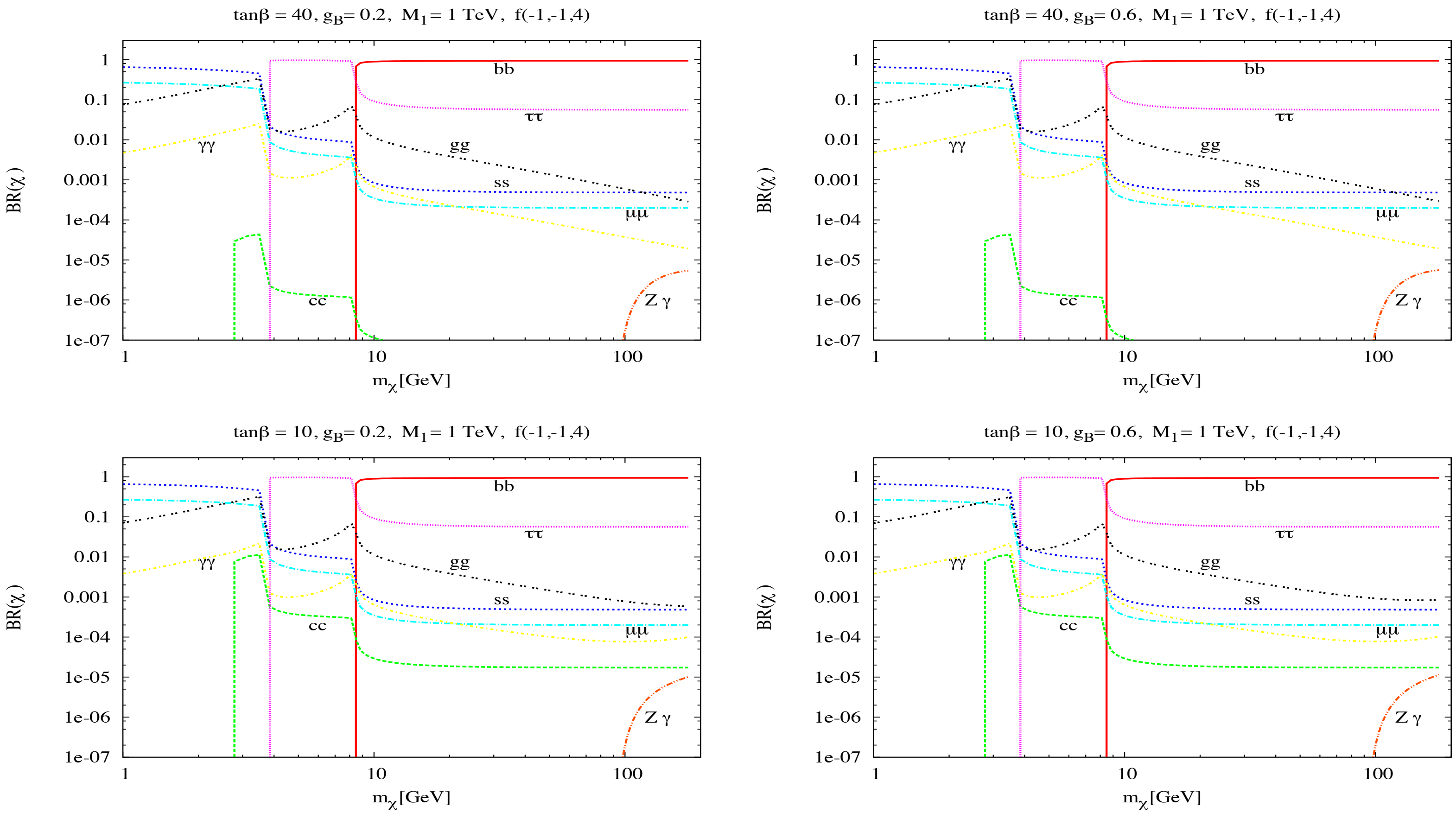
Total decay rate of the axi-Higgs for several mass values. Here, for the PQ axion, we have chosen $f_a = 10^{10}$ GeV.

$$m_\chi^2 = -\frac{1}{2}\, c_\chi\, v^2 \left[1 + \left(\frac{q_u^B - q_d^B}{M_1}\, \frac{v \sin 2\beta}{2}\right)^2\right] = -\frac{1}{2}\, c_\chi\, v^2 \left[1 + \frac{(q_u^B - q_d^B)^2}{M_1^2}\, \frac{v_u^2 v_d^2}{v^2}\right],$$

G. Lazarides, A.Mariano, C.C.

Since themass is an independent parameter, you can also Consider the axi-Higgs tobe in the GeV range.





Study of the branching ratios of the axi-Higgs. We analyze the dependence on the free parameters $g_B, \tan\beta$.

Axions from Intersecting Branes and Decoupled Chiral Fermions at the Large Hadron Collider

M. Guzzi, C.C.

Anomalous extra Z prime

$$\hat{D}_\mu = \left[\partial_\mu - i g_2 \left(W^1_\mu T^1 + W^2_\mu T^2 + W^3_\mu T^3\right) - i\frac{g_Y}{2}\hat{Y} B^\mu_Y - i\frac{g_z}{2}\hat{z} B^\mu_z\right]$$

$$\tan\theta_W = g_Y/g_2.$$

$$\varepsilon = \frac{\delta M^2_{ZZ'}}{M^2_{Z'} - M^2_Z}$$

$$M^2_Z = \frac{g_2^2}{4\cos^2\theta_W}(v^2_{H_1} + v^2_{H_2})\left[1 + O(\varepsilon^2)\right]$$

$$M^2_{Z'} = \frac{g_z^2}{4}(z^2_{H_1}v^2_{H_1} + z^2_{H_2}v^2_{H_2} + z^2_\phi v^2_\phi)\left[1 + O(\varepsilon^2)\right]$$

$$\delta M^2_{ZZ'} = -\frac{g_2 g_z}{4\cos\theta_W}(z^2_{H_1}v^2_{H_1} + z^2_{H_2}v^2_{H_2}).$$

$$M^2_Z = \frac{1}{4}\left(2M_1^2 + g^2v^2 + N_{BB} - \sqrt{\left(2M_1^2 - g^2v^2 + N_{BB}\right)^2 + 4g^2x_B^2}\right)$$

$$\simeq \frac{g^2v^2}{2} - \frac{1}{M_1^2}\frac{g^2x_B^2}{4} + \frac{1}{M_1^4}\frac{g^2x_B^2}{8}(N_{BB} - g^2v^2),$$

$$M^2_{Z'} = \frac{1}{4}\left(2M_1^2 + g^2v^2 + N_{BB} + \sqrt{\left(2M_1^2 - g^2v^2 + N_{BB}\right)^2 + 4g^2x_B^2}\right)$$

$$\simeq M_1^2 + \frac{N_{BB}}{2}.$$

$$\begin{pmatrix} A_\gamma \\ Z \\ Z' \end{pmatrix} = O^A \begin{pmatrix} W_3 \\ A^Y \\ B \end{pmatrix}$$

$$N_{BB} = \left(q_u^{B\,2}\, v_u^2 + q_d^{B\,2}\, v_d^2\right)\, g_B^2, \qquad x_B = \left(q_u^B v_u^2 + q_d^B v_d^2\right)\, g_B.$$

$$O^A \simeq \begin{pmatrix} \frac{g_Y}{g} & \frac{g_2}{g} & 0 \\ \frac{g_2}{g} + O(\epsilon_1^2) & -\frac{g_Y}{g} + O(\epsilon_1^2) & \frac{g}{2}\epsilon_1 \\ -\frac{g_2}{2}\epsilon_1 & \frac{g_Y}{2}\epsilon_1 & 1 + O(\epsilon_1^2) \end{pmatrix}$$

M. Guzzi, C.C.

The UV/IR conspiracy of the anomaly

The possibility that axions are associated to a rearrangement of the vacuum of a gauge theory cannot be excluded.

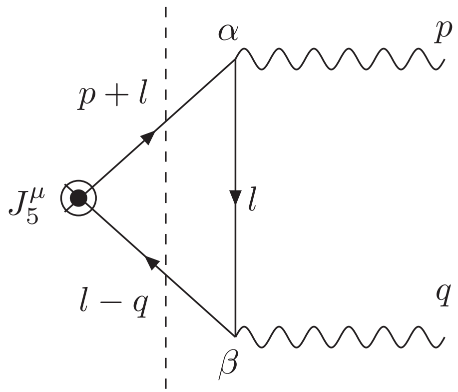
A similar rearrangement is possible for the conformal anomaly.

Follow the case of the QCD dynamics

The QCD U(1) anomaly responsible for the pion→ gamma gamma decay

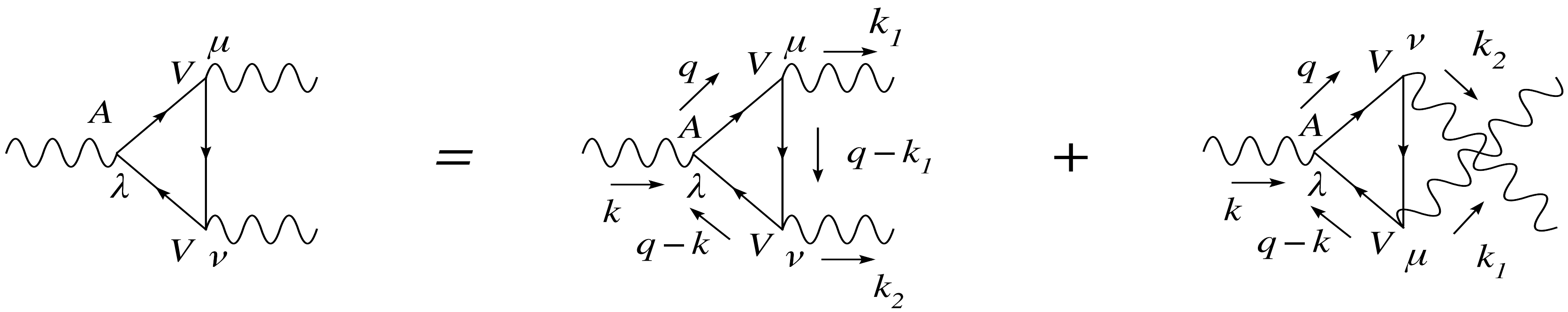
Nonlinearly realised Lagrangean in the IR.

In the UV the action exhibits an anomaly pole



Armillis, Delle Rose, Guzzi, C.C.

Anomalous $U(1)$ Models in Four and Five Dimensions and their Anomaly Poles



$$\Delta_0^{\lambda\mu\nu} = A_1(k_1,k_2)\varepsilon[k_1,\mu,\nu,\lambda] + A_2(k_1,k_2)\varepsilon[k_2,\mu,\nu,\lambda] + A_3(k_1,k_2)\varepsilon[k_1,k_2,\mu,\lambda]k_1^{\nu} + A_4(k_1,k_2)\varepsilon[k_1,k_2,\mu,\lambda]k_2^{\nu} + A_5(k_1,k_2)\varepsilon[k_1,k_2,\nu,\lambda]k_1^{\mu} + A_6(k_1,k_2)\varepsilon[k_1,k_2,\nu,\lambda]k_2^{\mu}.$$

$$A_1(k_1,k_2) = k_1\cdot k_2\, A_3(k_1,k_2) + k_2^2\, A_4(k_1,k_2),$$
$$A_2(k_1,k_2) = k_1^2\, A_5(k_1,k_2) + k_1\cdot k_2\, A_6(k_1,k_2),$$

$$A_5(k_1,k_2) = -A_4(k_2,k_1)$$
$$A_6(k_1,k_2) = -A_3(k_2,k_1).$$

Rosenberg, 1963

$$A_1(s,s_1,s_2) = -\frac{i}{4\pi^2}+\frac{i}{8\pi^2\sigma}\left\{\Phi(s_1,s_2)\frac{s_1 s_2\,(s_2-s_1)}{s}+s_1\,(s_2-s_{12})\log\left[\frac{s_1}{s}\right]\right.$$
$$\left.-s_2\,(s_1-s_{12})\log\left[\frac{s_2}{s}\right]\right\},$$

.

Nothing specific emerges from this computation in this parameterization

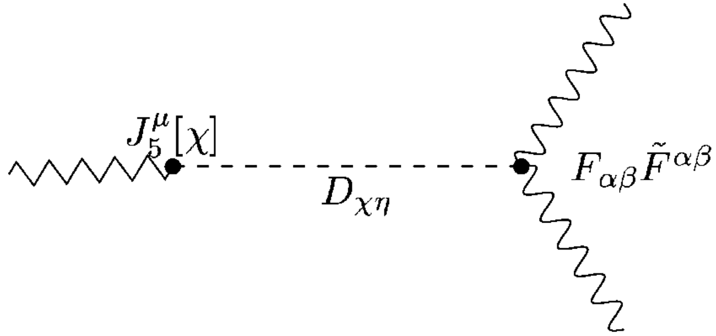
$$\Phi(x,y) = \frac{1}{\lambda}\Big\{2[Li_2(-\rho x)+Li_2(-\rho y)]+\ln\frac{y}{x}\ln\frac{1+\rho y}{1+\rho x}+\ln(\rho x)\ln(\rho y)+\frac{\pi^2}{3}\Big\},$$

where $s = k^2,\ s_1 = k_1^2,\ s_2 = k_2^2,\ s_{12} = k_1\cdot k_2$ with $\sigma = s_{12}^2 - s_1 s_2$

$$\lambda(x,y)=\sqrt{\Delta}, \qquad \Delta=(1-x-y)^2-4xy,$$
$$\rho(x,y)=2(1-x-y+\lambda)^{-1}, \qquad x=\frac{s_1}{s}, \qquad y=\frac{s_2}{s}.$$

The vertex in the longitudinal/transverse (L/T) formulation and comparisons

$$W^{\lambda\mu\nu} = \frac{1}{8\pi^2}\left[W^{L\,\lambda\mu\nu} - W^{T\,\lambda\mu\nu}\right],$$



(with $w_L = -4i/s$)

$$\begin{aligned} {W^T}_{\lambda\mu\nu}(k_1,k_2) &= w_T^{(+)}\left(k^2,k_1^2,k_2^2\right)\, t^{(+)}_{\lambda\mu\nu}(k_1,k_2) + w_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, t^{(-)}_{\lambda\mu\nu}(k_1,k_2) \\ &+ \widetilde{w}_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, \widetilde{t}^{(-)}_{\lambda\mu\nu}(k_1,k_2), \end{aligned}$$

The anomaly is associated to a longitudinal component, which has a pole: the anomaly pole (1/s). The transverse sector does not contribute to the anomaly.

In the on-shell case (two photons on shell) $\Delta^{\lambda\mu\nu}(s,0,0) = W_{\mu\nu\lambda}(s,0,0) = -\frac{i}{2\pi^2}\frac{k^\lambda}{s}\,\varepsilon[k_1,k_2,\mu,\nu].$

Conformal anomalies share the same behaviour

✪ Anomaly poles as the common signature of chiral and conformal anomalies

Phys.Lett. B682 (2009) 322-327 Armillis, Delle Rose, C.C.

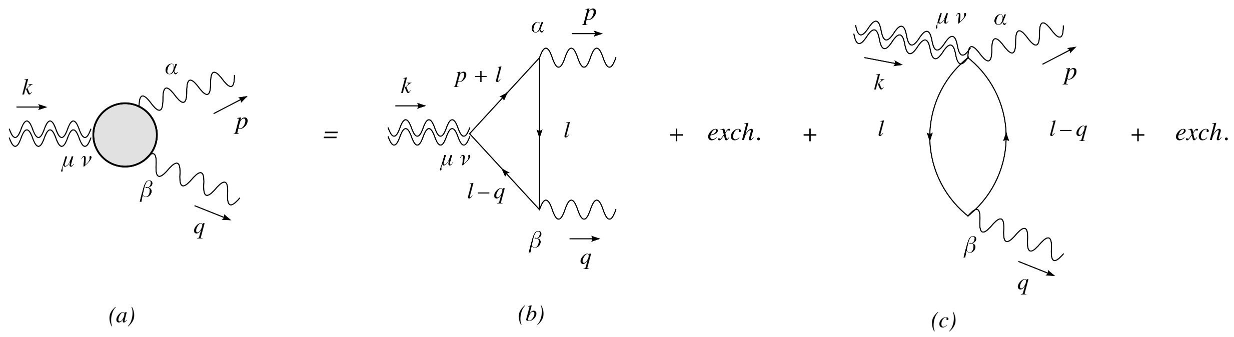
Conformal Anomalies and the Gravitational Effective Action:
The TJJ Correlator for a Chiral Fermion ✪

Phys.Rev. D81 (2010) 085001

Giannotti and Mottola Phys.Rev. D79 (2009) 045014

The Trace Anomaly and Massless Scalar Degrees of Freedom in Gravity ✪

❖ The general conformal bootstrap in momentum space for 3-point functions **confirms** these results (**Bzowski, McFadden, Skenderis**)



$$T^{\mu}_{\mu} = -\frac{1}{8}\left[2b\,C^2 + 2b'\left(E - \frac{2}{3}\Box R\right) + 2c\,F^2\right]$$

Armillis et al
Giannotti and Mottola

$$S_{anom}[g,A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E - \frac{2}{3}\Box R\right)_x G_4(x,x')\left[2b\,C^2 + b'\left(E - \frac{2}{3}\Box R\right) + 2c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}$$

$G_4(x,x')$ denotes the Green's function of the differential operator defined by

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3}R\Box$$

$$S_{anom}[g,A] = -\frac{c}{6}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\,R^{(1)}_x\,\Box^{-1}_{x,x'}\,[F_{\alpha\beta}F^{\alpha\beta}]_{x'}\,,$$

Exchange of an anomaly pole

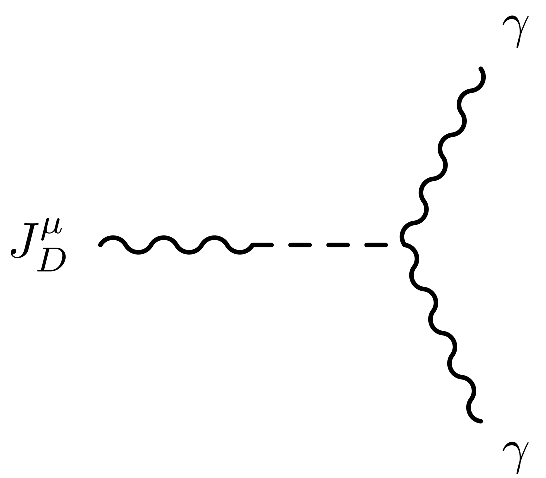
1 form factor in the TVV is responsible for the anomaly

Appearance of sum rules as a signature of the anomaly away from the conformal point

This is a strong indication of compositeness in the IR realization of the anomaly action

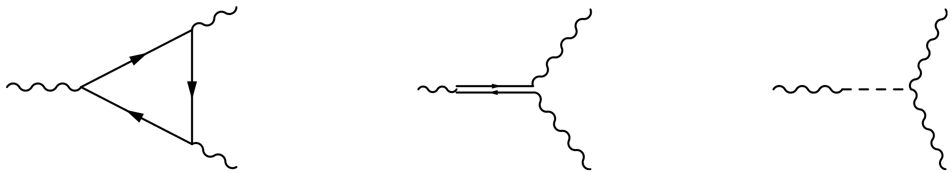
(Serino, Marzo, Delle Rose, C C.)

Dilaton Interactions and the Anomalous Breaking of Scale Invariance of the Standard Model



Superconformal Sum Rules and the Spectral Density Flow of the Composite Dilaton (ADD) Multiplet in $\mathcal{N} = 1$ Theories

Delle Rose, Costantini, Serino, C.C. JHEP 2014



(a) (b) (c)

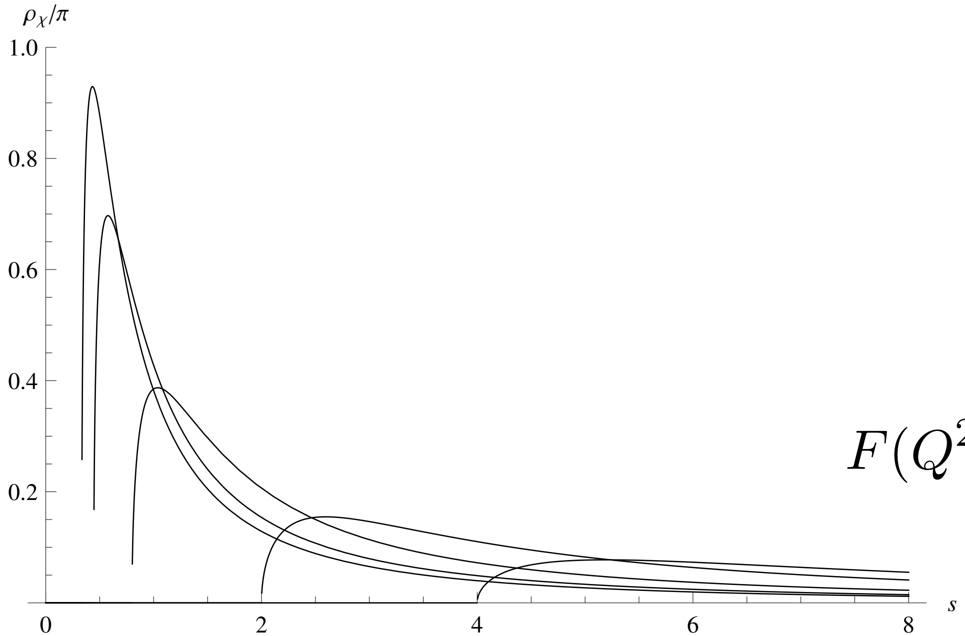
$$
\begin{aligned}
R^\mu &= \bar{\lambda}^a \bar{\sigma}^\mu \lambda^a + \frac{1}{3}\left(-\bar{\chi}_i \bar{\sigma}^\mu \chi_i + 2i\phi_i^\dagger \mathcal{D}^\mu_{ij}\phi_j - 2i(\mathcal{D}^\mu_{ij}\phi_j)^\dagger \phi_i\right) , \\
S^\mu_A &= i(\sigma^{\nu\rho}\sigma^\mu \bar{\lambda}^a)_A F^a_{\nu\rho} - \sqrt{2}(\sigma_\nu \bar{\sigma}^\mu \chi_i)_A (\mathcal{D}^\nu_{ij}\phi_j)^\dagger - i\sqrt{2}(\sigma^\mu \bar{\chi}_i)\mathcal{W}^\dagger_i(\phi^\dagger) \\
&- ig(\phi_i^\dagger T^a_{ij}\phi_j)(\sigma^\mu \bar{\lambda}^a)_A + S^\mu_{I\,A} , \\
T^{\mu\nu} &= -F^{a\,\mu\rho}F^{a\,\nu}{}_\rho + \frac{i}{4}\left[\bar{\lambda}^a \bar{\sigma}^\mu (\delta^{ac} \overrightarrow{\partial^\nu} - g\, t^{abc} A^{b\,\nu})\lambda^c + \bar{\lambda}^a \bar{\sigma}^\mu(-\delta^{ac} \overleftarrow{\partial^\nu} - g\, t^{abc}A^{b\,\nu})\lambda^c + (\mu \leftrightarrow \nu)\right] \\
&+ (\mathcal{D}^\mu_{ij}\phi_j)^\dagger (\mathcal{D}^\nu_{ik}\phi_k) + (\mathcal{D}^\nu_{ij}\phi_j)^\dagger(\mathcal{D}^\mu_{ik}\phi_k) + \frac{i}{4}\Big[\bar{\chi}_i \bar{\sigma}^\mu(\delta_{ij} \overrightarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j \\
&+ \bar{\chi}_i \bar{\sigma}^\mu(-\delta_{ij} \overleftarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j + (\mu \leftrightarrow \nu)\Big] - \eta^{\mu\nu}\mathcal{L} + T^{\mu\nu}_I ,
\end{aligned}
$$

$$\partial_\mu R^\mu = \frac{g^2}{16\pi^2}\left(T(A) - \frac{1}{3}T(R)\right) F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu}\,,$$

$$\bar{\sigma}_\mu S^\mu_A = -i\frac{3\,g^2}{8\pi^2}\left(T(A) - \frac{1}{3}T(R)\right)\left(\bar{\lambda}^a\bar{\sigma}^{\mu\nu}\right)_A F^a_{\mu\nu}\,,$$

$$\eta_{\mu\nu}T^{\mu\nu} = -\frac{3\,g^2}{32\pi^2}\left(T(A) - \frac{1}{3}T(R)\right) F^{a\,\mu\nu}F^a_{\mu\nu}\,.$$

Suoerconformal anomaly multiplet



$$\frac{1}{\pi}\int_0^\infty \rho(s,m^2)ds = f,$$

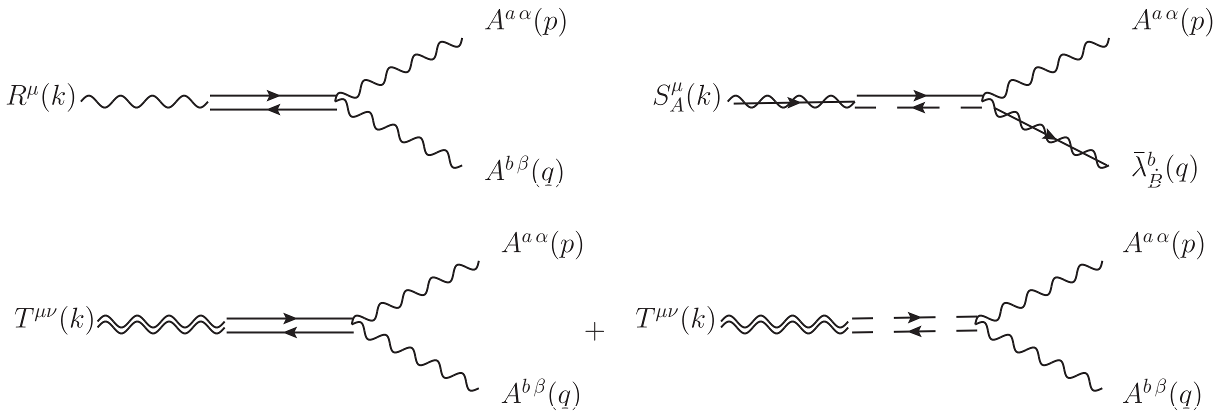
$$F(Q^2,m^2) = \frac{1}{\pi}\int_0^\infty ds\frac{\rho(s,m^2)}{s+Q^2}\,,$$

Anomaly form factors

3 SUM RULES for the anomalies (chiral and conformal)

Away from the conformal limit (nonzero mass)
Appearance of sum rules and spectral density flows

$$
\begin{aligned}
S_{\rm axion} &= -\frac{g^2}{4\pi^2}\left(T(A)-\frac{T(R)}{3}\right)\int d^4z\, d^4x\, \partial^\mu B_\mu(z)\, \frac{1}{\square_{zx}}\, \frac{1}{4}F_{\alpha\beta}(x)\tilde{F}^{\alpha\beta}(x) \\
S_{\rm dilatino} &= \frac{g^2}{2\pi^2}\left(T(A)-\frac{T(R)}{3}\right)\int d^4z\, d^4x\left[\partial_\nu \Psi_\mu(z)\sigma^{\mu\nu}\sigma^\rho \frac{\overleftarrow{\partial}_\rho}{\square_{zx}}\,\bar{\sigma}^{\alpha\beta}\bar{\lambda}(x)\frac{1}{2}F_{\alpha\beta}(x) + h.c.\right] \\
S_{\rm dilaton} &= -\frac{g^2}{8\pi^2}\left(T(A)-\frac{T(R)}{3}\right)\int d^4z\, d^4x\, \left(\square h(z) - \partial^\mu\partial^\nu h_{\mu\nu}(z)\right)\, \frac{1}{\square_{zx}}\, \frac{1}{4}F_{\alpha\beta}(x)F^{\alpha\beta}(x).
\end{aligned}
$$



The IR stricture of the N=1

Resort to the anomaly action in the Wess Zumino form

$$\Gamma_{WZ}[g,\tau] = \int d^4x\,\sqrt{g}\,\Bigg\{\beta_a\left[\frac{\tau}{\Lambda}\left(F-\frac{2}{3}\Box R\right)+\frac{2}{\Lambda^2}\left(\frac{R}{3}\,\partial^\lambda\tau\,\partial_\lambda\tau+(\Box\tau)^2\right)-\frac{4}{\Lambda^3}\,\partial^\lambda\tau\,\partial_\lambda\tau\,\Box\tau+\frac{2}{\Lambda^4}\,\left(\partial^\lambda\tau\,\partial_\lambda\tau\right)^2\right] + \beta_b\left[\frac{\tau}{\Lambda}\,G-\frac{4}{\Lambda^2}\left(R^{\alpha\beta}-\frac{R}{2}\,g^{\alpha\beta}\right)\partial_\alpha\tau\,\partial_\beta\tau-\frac{4}{\Lambda^3}\,\partial^\lambda\tau\,\partial_\lambda\tau\,\Box\tau+\frac{2}{\Lambda^4}\,\left(\partial^\lambda\tau\,\partial_\lambda\tau\right)^2\right]\Bigg\}. \quad (49)$$

Conformal Trace Relations from the Dilaton Wess-Zumino Action

Delle Rose, Marzo, Serino, C.C.

$$
\begin{aligned}
\langle T(k_1)T(-k_1)\rangle &= -4\,\beta_a\,{k_1}^4\,,\\
\langle T(k_1)T(k_2)T(k_3)\rangle &= 8\Big[-\Big(\beta_a+\beta_b\Big)\Big(f_3(k_1,k_2,k_3)+f_3(k_2,k_1,k_3)+f_3(k_3,k_1,k_2)\Big)+\beta_a\sum_{i=1}^{3}k_i^4\Big]\,,\\
\langle T(k_1)T(k_2)T(k_3)T(k_4)\rangle &= 8\Big\{6\Big(\beta_a+\beta_b\Big)\Big[\sum_{\mathcal{T}\{4,[(k_{i_1},k_{i_2}),(k_{i_3},k_{i_4})]\}}k_{i_i}\cdot k_{i_2}\,k_{i_3}\cdot k_{i_4}\\
&\quad+f_4(k_1\,k_2,k_3,k_4)+f_4(k_2\,k_1,k_3,k_4)+f_4(k_3\,k_1,k_2,k_4)+f_4(k_4\,k_1,k_2,k_3)\Big]\\
&\quad-\beta_a\Big(\sum_{\mathcal{T}\{4,(k_{i_1},k_{i_2})\}}(k_{i_1}+k_{i_2})^4+4\sum_{i=1}^{4}k_i^4\Big)\Big\}\,, \qquad (
\end{aligned}
$$

$$
\begin{aligned}
f_3(k_a,k_b,k_c) &= k_a^2\,k_b\cdot k_c\,,\\
f_4(k_a,k_b,k_c,k_d) &= k_a^2\,\left(k_b\cdot k_c+k_b\cdot k_d+k_c\cdot k_d\right)\,.
\end{aligned}
$$

Delle Rose, Marzo, Serino

Conclusions

Stuckelberg models have introduced the concept of
gauging of an anomalous U(1) symmetry.

They predict an axion and an anomalous U(1)
In their susy version they predict a supersymmetric multiplet.
(fundammental)

But there are variants

Anomaly actions seem to indicate that we coudl also take a different route.
All the states that we come from the breaking of a dilatation current, assuming
A conformal symmetry in the UV are the anomaly poles of the
supersymmetric current. The presence of sum rules in N=1 theories away from
the conformal point seem to provide this indication

The symmetry should also appear as broken (spontaneously?)