QED effects in rare exclusive B decays

M. Beneke (TU München)

Oxford, November 26, 2020

1708.09152, 1908.07011, with C. Bobeth and R. Szafron [$B_s \to \mu^+\mu^-$]
2008.12494, with C. Bobeth and Y. Wang [$B_s \to \mu^+\mu^-\gamma$]
2008.10615, with P. Böer, J. Toelstede and K. Vos [$B \to \pi K$, charmless]

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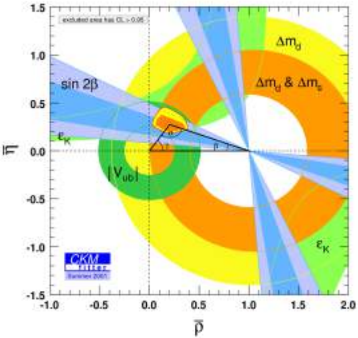
Motivation: Precision

Flavour physics: search for new physics in small quantum fluctuations in an intrinsically hadronic environment

2001 (B factory turn-on)



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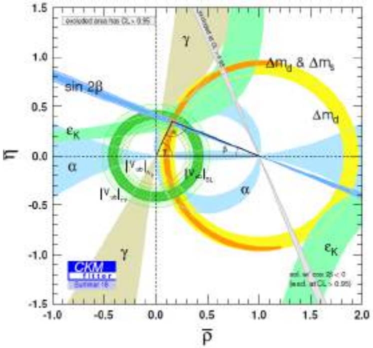

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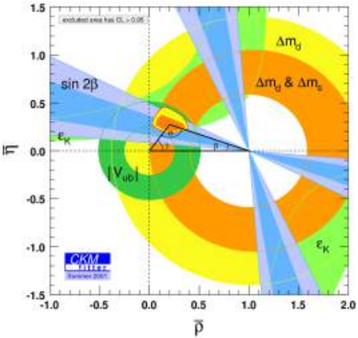

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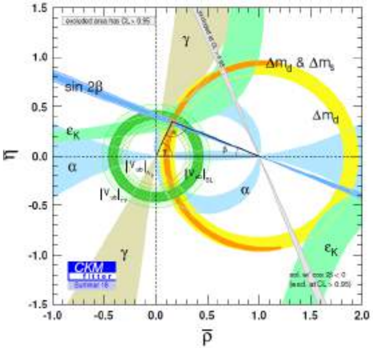

Traditionally focus on hadronic uncertainties. Time to look at QED.
QED effects violate isospin symmetry and can cause large "lepton-flavour violating" logarithms, $\log m_\ell$.

Motivation: Theory

- Photons couple weakly to strongly interacting quarks $\rightarrow$ probe of hadronic physics, requires factorization theorems.

- Photons have long-range interactions with the charged particles in the initial/final state $\rightarrow$ QED factorization is more complicated than QCD factorization.

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Observables

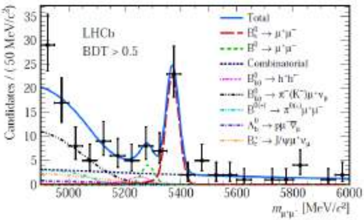
IR finite observable is

$$\Gamma_{\text{phys}} = \sum_{n=0}^{\infty} \Gamma(B \to f + n\gamma, \sum_n E_{\gamma,n} < \Delta E)$$
$$\equiv \omega(\Delta E) \times \Gamma_{\text{non-rad.}}(B \to f)$$

Signal window $|m_B - m_f| < \Delta \quad \Longrightarrow \quad \Delta E = \Delta$

Assume $\Delta \ll \Lambda_{\text{QCD}} \sim$ size of hadrons

Large $\ln \Delta E$.

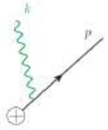


[LHCb, $B_s \to \mu^+\mu^-$, 1703.05748]

Ultrasoft photons and the point-like approximation

Universal soft radiative amplitude

$$A^{i\to f+\gamma}(p_j,k) = A^{i\to f}(p_j) \times \sum_{j=\text{legs}} \frac{-eQ_j p_j^\mu}{\eta_j p_j\cdot k + i\epsilon}$$



Exponentiates for the decay rate, but the virtual correction is UV divergent in the soft limit. Cut-off Λ. The amplitude implies that the charged particles (B-meson, pion, lepton, ...) are treated as point-like.

$$\Gamma = \Gamma^{i\to f}_{\text{tree}} \times \left(\frac{2\Delta E}{\Lambda}\right)^{-\frac{\alpha}{\pi}\sum_{i,j} Q_i Q_j f(\beta_{ij})}$$

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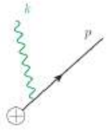

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What is Λ?

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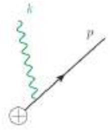

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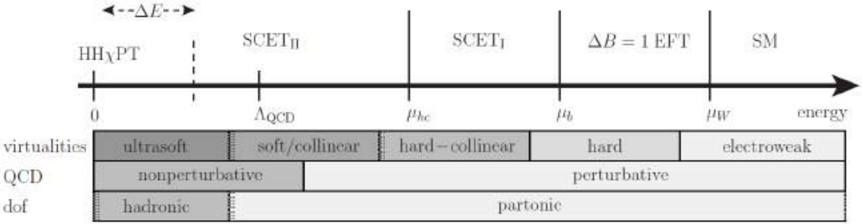
What is Λ?

- Present treatment of QED effects sets $\Lambda = m_B$ (e.g. using a theory of point-like mesons)
- Experimental analyses uses the PHOTOS Monte Carlo [Golonka, Was, 2005], which in addition neglects radiation from charged initial state particles.

However, the derivation implies that $\Lambda \ll \Lambda_{\text{QCD}} \sim$ size of the hadron (B-meson). Otherwise virtual corrections resolve the structure of the hadron and higher-multipole couplings are unsuppressed.

Scales and Effective Field theories (EFTs)

Multiple scales: m_W, m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, m_μ, ΔE



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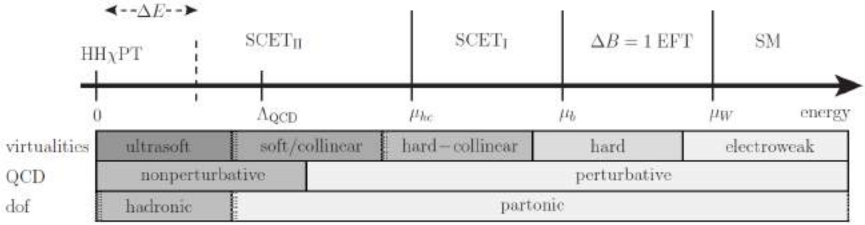
Short-distance QED at $\mu \gtrsim m_b$ can be included in the usual weak effective Lagrangian (extended Fermi theory) + renormalization group.



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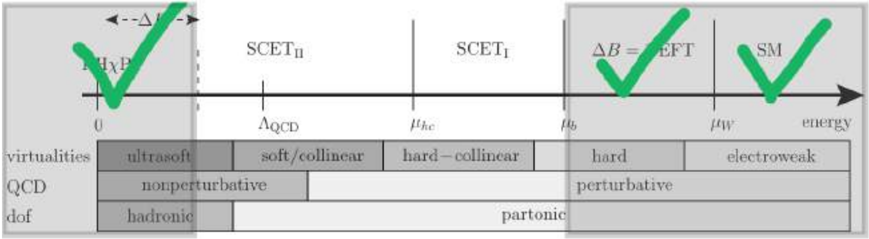
Far IR (ultrasoft scale) described by theory of point-like hadrons.



Goal: Theory for QED corrections between the scales m_b and $\Lambda_{\rm QCD}$ (structure-dependent effects).

$B_s \to \mu^+\mu^-$

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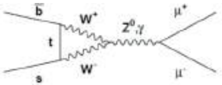
Status of $B_s \to \mu^+\mu^-$

"Instantaneous", "non-radiative" branching fraction



$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{m_{B_s}^2}}$$
$$\times \left\{ \left| \frac{2m_\mu}{m_{B_s}} (C_{10} - C_{10}') + (C_P - C_P') \right|^2 + \left(1-\frac{4m_\mu^2}{m_{B_s}^2}\right) |C_S - C_S'|^2 \right\}$$

- SM only $C_{10}[\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu\gamma_5\ell] \Rightarrow$ helicity suppression. Sensitive to scalar couplings.
- SM C_{10} calculations includes NNLO QCD, NLO EW matching corrections at EW scale, NNLL renormalization-group evolution to the b-quark mass scale including QED logarithms
- LHCb [1703.05747] $(3.0^{+0.7}_{-0.6}) \times 10^{-9}$ vs. Theory [Bobeth et al., 1311.0903] $(3.65 \pm 0.23) \times 10^{-9}$

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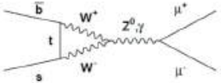

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Theory uncertainties [Bobeth et al., 1311.0903]

- Parametric: f_B (4.0%), CKM (4.3%), m_t (1.6%), $\tau_{B_s^H}$ (1.3%), $\alpha_s(0.1\%)$
- Non-parametric: Higher-order corrections at m_W (0.4%), QED scale variation (0.3%), m_t pole-$\overline{\mathrm{MS}}$ conversion (0.3%), other (0.5%) [e.g. dim-8 operators] – total of 1.5%

Some facts about $B_q \to \ell^+\ell^-$

- Long-distance QCD effects are very simple. Local annihilation. Only

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle = i f_{B_q} p^\mu$$

Task for lattice QCD (1.5% [Aoki et al. 1607.00299], 0.5% [FNAL/MILC 1712.09262]).

- Only the operator Q_{10} from the weak effective Lagrangian enters.

- No scalar lepton current $\bar{\ell}\ell$, only $\bar{\ell}\gamma_5\ell \Longrightarrow$

$$\mathcal{A}^\lambda_{\Delta\Gamma} = 1 \qquad C_\lambda = S_\lambda = 0$$

$$\frac{\Gamma(B_s(t) \to \mu^+_\lambda \mu^-_\lambda) - \Gamma(\bar{B}_s(t) \to \mu^+_\lambda \mu^-_\lambda)}{\Gamma(B_s(t) \to \mu^+_\lambda \mu^-_\lambda) + \Gamma(\bar{B}_s(t) \to \mu^+_\lambda \mu^-_\lambda)} = \frac{C_\lambda \cos(\Delta M_{B_s} t) + S_\lambda \sin(\Delta M_{B_s} t)}{\cosh(y_s t/\tau_{B_s}) + \mathcal{A}^\lambda_{\Delta\Gamma} \sinh(y_s t/\tau_{B_s})}$$

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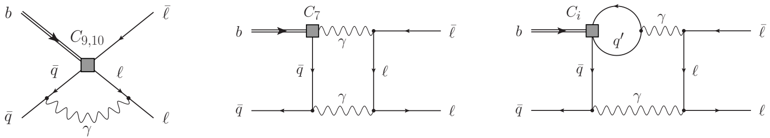
None of these are exactly true in the presence of electromagnetic corrections

Enhanced electromagnetic effect

Surprise: m_B/Λ power-enhanced and logarithmically enhanced, purely virtual correction



$$i\mathcal{A} = m_\ell f_{B_q}\mathcal{N}\, C_{10}\,\bar\ell\gamma_5\ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell f_{B_q}\mathcal{N}\,\bar\ell(1+\gamma_5)\ell$$

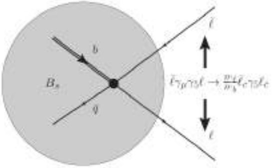
$$\times\left\{\int_0^1 du\,(1-u)\,C_9^{\rm eff}(um_b^2)\, m_B\int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega)\left[\ln\frac{m_b\omega}{m_\ell^2}+\ln\frac{u}{1-u}\right]\right.$$

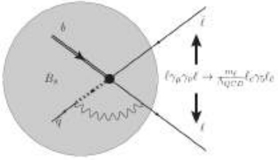
$$\left. - Q_\ell C_7^{\rm eff} m_B\int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega)\left[\ln^2\frac{m_b\omega}{m_\ell^2} - 2\ln\frac{m_b\omega}{m_\ell^2}+\frac{2\pi^2}{3}\right]\right\} + \ldots$$

The virtual photon probes the B meson structure. B-meson LCDA and $1/\lambda_B$ enters.

$$\frac{m_B}{\lambda_B} \equiv m_B\int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \sim 20 \qquad \ln\frac{m_b\omega}{m_\mu^2}\sim 6$$

Interpretation of the enhanced correction





$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$$

Local annihilation and helicity flip.

$$\langle 0|\int d^4x\, T\{j_{\mathrm{QED}}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

Helicity-flip and annihilation delocalized by a hard-collinear distance

The virtual photon probes the B meson structure. Annihilation/helicity-suppression is "smeared out" over light-like distance $1/\sqrt{m_B\Lambda}$ [→ B-LCDA]. Still short-distance.

Logarithms are not the standard soft logarithms, but due to hard-collinear, collinear and soft regions.

Numerical size of the correction

Include through the substitution

$$\overline{\mathcal{B}}(B_s \to \ell^+\ell^-) = \frac{\tau_{B_q} m_{B_q}^3 f_{B_q}^2}{8\pi} |\mathcal{N}|^2 \frac{m_\ell^2}{m_{B_q}^2} \sqrt{1 - \frac{4m_\ell^2}{m_{B_q}^2}} |C_{10}|^2, \qquad C_{10} \to C_{10} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \Delta_{\rm QED}$$

where

$$\Delta_{\rm QED} = (33 \ldots 119) + i\,(9 \ldots 23)$$

- Reduction of the branching fraction by 0.3–1.1 %
  Uncertainty entirely due to B-meson LCDA.
- Cancellation of a factor of three between the $C_9^{\rm eff}(um_b^2)$ and double-log enhanced $C_7^{\rm eff}$ term:
  $$-0.6\% = 1.1\%\,(C_9^{\rm eff}) - 1.7\%\,(C_7^{\rm eff})$$
- Significantly larger than previously estimated QED correction.
  QED uncertainty almost as large as other non-parametric uncertainties (1.2%)
- Small time-dependent rate asymmetries are generated.
  $[C_\lambda = -\eta_\lambda\, 2r\, {\rm Re}(\Delta_{\rm QED}) \approx \eta_\lambda\, 0.6\%]$

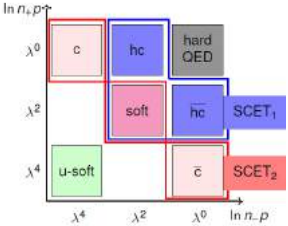
All orders, EFT, summation of logarithms

Back-to-back energetic lepton pair
Collinear (lepton $n_+ p_\ell$ large) and anti-collinear (anti-lepton $n_- p_{\bar{\ell}}$ large) modes

$$n_+^2 = n_-^2 = 0, \quad n_+ \cdot n_- = 2, \qquad p^\mu = n_+ p \, \frac{n_-^\mu}{2} + n_- p \, \frac{n_+^\mu}{2} + p_\perp^\mu$$

$$p = (n_+ p, p_\perp, n_- p), \qquad \lambda \sim \frac{\Lambda_{\rm QCD}}{m_b} \sim \frac{m_\mu}{m_b}$$

- Modes in the EFT classified by virtuality and rapidity
- Matching QCD+QED $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$



mode	relative scaling	absolute scaling	virtuality k^2
hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
hard-collinear	$(1,\lambda,\lambda^2)$	$(m_b, \sqrt{m_b\Lambda_{\rm QCD}}, \Lambda_{\rm QCD})$	$m_b\Lambda_{\rm QCD}$
anti-hard-collinear	$(\lambda^2,\lambda,1)$	$(\Lambda_{\rm QCD}, \sqrt{m_b\Lambda_{\rm QCD}}, m_b)$	$m_b\Lambda_{\rm QCD}$
collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
anticollinear	$(\lambda^4,\lambda^2,1)$	$(m_\mu^2/m_b, m_\mu, m_b)$	m_μ^2
soft	$(\lambda^2,\lambda^2,\lambda^2)$	$(\Lambda_{\rm QCD}, \Lambda_{\rm QCD}, \Lambda_{\rm QCD})$	$\Lambda_{\rm QCD}^2$

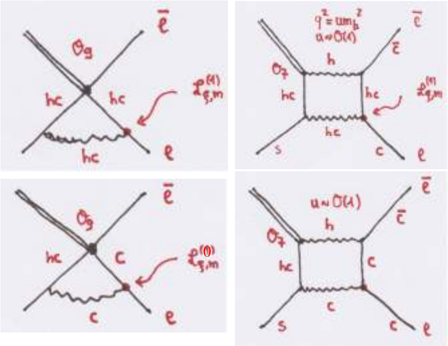
SCET interpretation of the one-loop result

- Typical $\mathrm{SCET_{II}}$ problem

  - hard-collinear $p_{hc}^2 \sim m_b\Lambda$
  - collinear $p_c^2 \sim \Lambda^2, m_\mu^2$
  - soft $p_s^2 \sim p_c^2$

- Matching to $\mathrm{SCET_{II}}$ non-zero only at sub-leading power (helicity-flip required)
  NLP SCET problem

- After tree-level matching to $\mathrm{SCET_I}$ need matrix element of



$$\overset{\mathrm{SCET_I}}{\to} \int_0^1 du \left(C_9^{\rm eff}(u) + \frac{C_7^{\rm eff}}{u} \right) \bar{\chi}_{\rm hc}(\bar{u} p_\ell) \Gamma h_v \, \bar{\ell}_{\rm hc}(u p_\ell) \Gamma' \ell_{\overline{\rm hc}}(p_{\bar{\ell}})$$

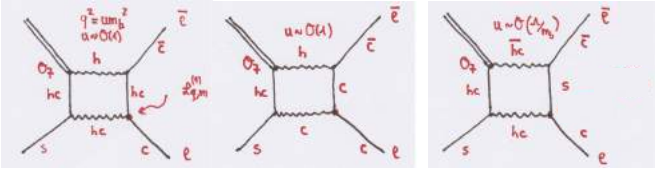
- Sum of hard-collinear and collinear loop in $\mathrm{SCET_{II}}$ gives a structure-dependent collinear logarithm $\ln(m_b\Lambda/m_\mu^2)$

- Endpoint (rapidity) divergence for $u \to 0$ in $C_7^{\rm eff}$ term

SCET interpretation (II)

Endpoint divergence is cancelled by the one-loop matrix element of the $\mathrm{SCET_I}$ operator

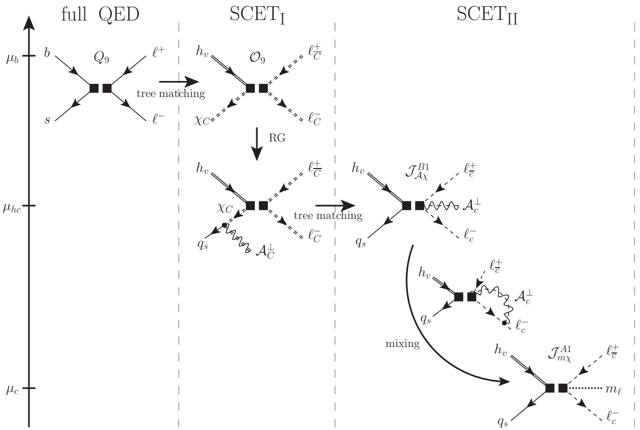
$$\bar{\chi}_{\rm hc}(p_\ell)\gamma_\perp^\mu h_v\,\mathcal{A}^\gamma_{{\rm hc},\perp\mu}(p_{\bar\ell}) \qquad \text{(third diagram below)}$$



- Involves power-suppressed SCET interactions and soft fermion (lepton) exchange
- Endpoint divergence results in another power of $\ln(m_b\Lambda/m_\mu^2)$. Fully calculable in perturbation theory, since the spectator quark is highly virtual (hard-collinear).

- Factorization and resummation of logs only understood for the Q_9 operator up to now. [BBS, 2019]

Matching, RGE, leading-(double) log resummation – sketch



- Q_9 operator only.

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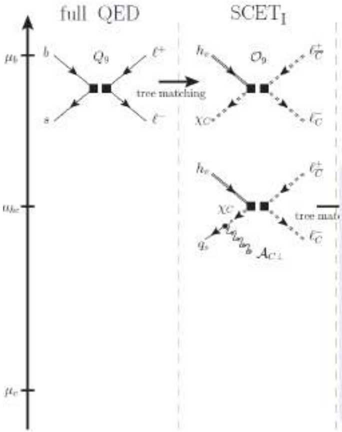


1) Matching QED → SCET$_1$ @ tree-level

→ yields operator $\mathcal{O}_9$ with

hard-collinear light quark χ_C

hard-collinear lepton ℓ^-_C

anti-hard-collinear anti-lepton $\ell^+_{\overline{C}}$

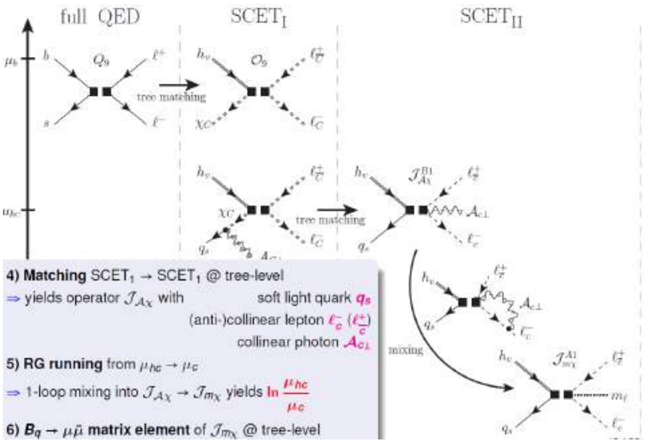
2) RG running from $\mu_b \to \mu_{hc}$

3) Matching SCET$_1$ → SCET$_2$ @ tree-level

change hard-coll. light quark χ_C → soft q_s

via power-suppressd $\mathcal{L}_{\xi q}$ → additional $A_{C\perp}$

Matching, RGE, leading-(double) log resummation – sketch

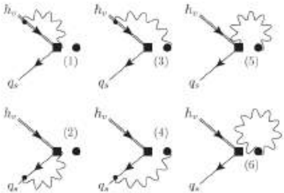


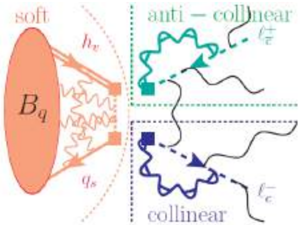
- Operator mixing in SCET$_{\mathrm{II}}$
  RGE with cusp anomalous dimension → double logarithms $\alpha \times \alpha^n_{(s)} \ln^{2n+1}$

SCET$_{\rm II}$ factorization and soft rearrangement

$$\widetilde{\mathcal{J}}^{B1}_{\mathcal{A}\chi}(v,t) = \bar{q}_s(vn_-)Y(vn_-,0)\frac{\not{n}_-}{2}P_L h_v(0)\left[Y_+^\dagger\, Y_-\right](0)\,\left[\bar{\ell}_c(0)(2\mathcal{A}_{c\perp}(tn_+)P_R)\ell_{\bar{c}}(0)\right] = \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}}$$

- s, c, $\bar{c}$ do not interact in SCET$_{\rm II}$. Sectors are factorized.
  Anomalous dimension should be separately well defined.
- But the anomalous dimension of the soft graphs is IR divergent.

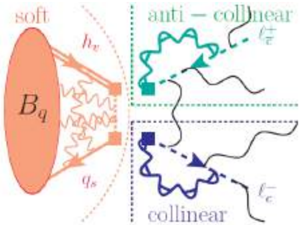


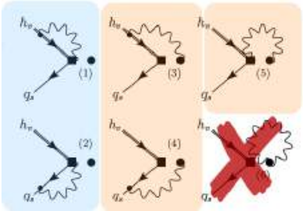


SCET$_{\rm II}$ factorization and soft rearrangement

$$\widetilde{\mathcal{J}}^{B1}_{\mathcal{A}\chi}(v,t) = \bar{q}_s(vn_-)Y(vn_-,0)\frac{\not{n}_-}{2}P_L h_v(0)\left[Y_+^\dagger\, Y_-\right](0)\,\left[\bar{\ell}_c(0)(2\mathcal{A}_{c\perp}(tn_+)P_R)\ell_{\bar{c}}(0)\right] = \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}}$$

- s, c, $\bar{c}$ do not interact in SCET$_{\rm II}$. Sectors are factorized.
  Anomalous dimension should be separately well defined.
- But the anomalous dimension of the soft graphs is IR divergent.





$$\langle 0|\left[Y_+^\dagger\, Y_-\right](0)|0\rangle \equiv R_+R_-$$

- Soft rearrangement $\quad \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}} = \dfrac{\widehat{\mathcal{J}}_s}{R_+R_-} \otimes R_+\widehat{\mathcal{J}}_c \otimes R_-\widehat{\mathcal{J}}_{\bar{c}}$

  Soft matrix element defines a generalized B-LCDA

Structure of the final result

Amplitude [evolved to μ_c]

$$
\begin{aligned}
i\mathcal{A}_9 \;=\;& e^{S_\ell(\mu_b,\,\mu_c)}\,T_+(\mu_c)\times\int_0^1 du\, e^{S_q(\mu_b,\,\mu_{hc})}\,2H_9(u;\mu_b)\int_0^\infty d\omega\, U_s^{\rm QED}(\mu_{hc},\mu_s;\omega)\, m_{B_q}F_{B_q}(\mu_{hc})\,\phi_+(\omega;\mu_{hc})\\
&\times\left[J_m(u;\omega;\mu_{hc})+\int_0^1 dw\, J_A(u;\omega,w;\mu_{hc})\left(M_A(w;\mu_c)-\frac{Q_\ell\overline{w}}{\beta_{0,\rm em}}\ln\eta_{\rm em}\right)\right]\\
\equiv\;& e^{S_\ell(\mu_b,\,\mu_c)}\times A_9\,[\bar{u}_c(1+\gamma_5)v_{\bar{c}}]
\end{aligned}
$$

– defines the non-radiative amplitude A_9. QED+QCD Logs between m_b and μ_c summed.

Decay rate [including ultrasoft photon radiation]

$$
\begin{aligned}
\Gamma[B_q\to\mu^+\mu^-](\Delta E) \;=\;& \underbrace{\frac{m_{B_q}}{8\pi}\beta_\mu\left(\left|A_{10}+A_9+A_7\right|^2+\beta_\mu^2\left|A_9+A_7\right|^2\right)}_{\text{non-radiative rate}}\times\underbrace{\left|e^{S_\ell(\mu_b,\,\mu_c)}\right|^2\mathcal{S}(v_\ell,v_{\bar{\ell}},\Delta E)}_{\text{ultrasoft radiation}}\\
=\;& \Gamma^{(0)}[B_q\to\mu^+\mu^-]\left(\frac{2\Delta E}{m_{B_q}}\right)^{-\frac{2\alpha}{\pi}\left(1+\ln\frac{m_\mu^2}{m_{B_q}^2}\right)}
\end{aligned}
$$

$$
\mathcal{S}(v_\ell,v_{\bar{\ell}},\Delta E)=\sum_{X_s}\left|\langle X_s|S_{v_\ell}^\dagger(0)S_{v_{\bar{\ell}}}(0)|0\rangle\right|^2\theta(\Delta E-E_{X_s})
$$

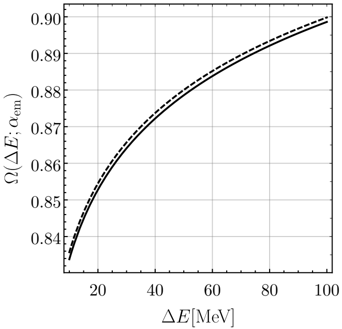
Ultrasoft function

Size of (structure-dependent) leading logarithms

- Once the final-state virtual Sudakov logs $\left|e^{S_\ell(\mu_b,\,\mu_c)}\right|^2$ are combined with the ultrasoft function, the remaining structure-dependent logarithms are small.

  $\Rightarrow$ justifies the naive treatment $\Lambda \to m_B$ *a posteriori*

- Reduces the enhanced QED correction by 20% – almost exlusively due to mixed QED + QCD logs.
- The energy resolution logarithms give a large correction to the radiative branching fraction.



Can sum leading logs, and calculate all QED effects between scale m_b and a few times Λ_{QCD}.

BUT: matching of SCET_{II} to the ultrasoft theory of point-like hadrons at a scale $\mu_c \sim \Lambda_{\text{QCD}}$ must be done non-perturbatively.

$B_s \to \mu^+\mu^-\gamma$

2008.12494, with C. Bobeth and Y. Wang

Basic features of $B_s \to \mu^+\mu^-\gamma$

- Same final state before, but consider energetic photon, $E_\gamma > 1.5\,\mathrm{GeV} \sim m_B/2$
- Very rare, branching fraction $10^{-10} - 10^{-8}$ depending on the $q^2 = m_{\mu^+\mu^-}$ bin.
  Not yet observed. Only LHCb can reach these small BRs.
- First calculation with systematic factorization methods.
  Want: QCD at NLO at LP in $\Lambda_{\mathrm{QCD}}/E_\gamma$ and $\Lambda_{\mathrm{QCD}}/m_b$, and LO at NLP,
  no QED corrections

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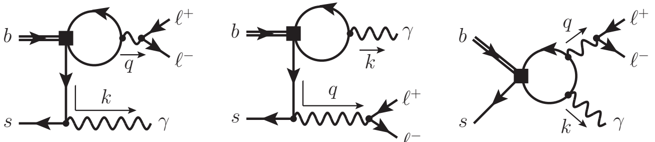


A-type B-type

- Theoretically shares features with $B \to \ell\nu\gamma$ [Descotes-Genon, Sachrajda, 2002; Lunghi, Pirjol, Wyler, 2002; Bosch et al., 2003] ($\to$ B-LCDA at LP) and $B \to K^{(*)}\ell\ell$ [MB, Feldmann, Seidel] (charmonium resonances, stay below $q^2 = 6\,\text{GeV}^2$)
- Standard SCET calculation, except for light-meson resonances in the B-type contribution.

Structure of the theoretical result

LP amplitude

$$\overline{\mathcal{A}}_{\text{type}-A} = ie\,\frac{\alpha_{\rm em}}{4\pi}\,\mathcal{N}_{\rm ew}\,\epsilon^\star_\mu \left\{ \left(V_9^{\rm eff}(q^2) + \frac{2\,\overline{m}_b\, m_{B_q}}{q^2}\, V_7^{\rm eff}(q^2)\right) L_{V,\nu} + V_{10}^{\rm eff}(q^2) L_{A,\nu} \right\} \mathcal{T}^{\mu\nu}(k)$$

$$\overline{\mathcal{A}}_{\text{type}-B} = ie\,\frac{\alpha_{\rm em}}{4\pi}\,\mathcal{N}_{\rm ew}\,\epsilon^\star_\mu\,\frac{4\,\overline{m}_b E_\gamma}{q^2}\, V_7^{\rm eff}(k^2=0) L_{V,\nu}\,\mathcal{T}^{\mu\nu}(q)$$

SCET_I correlation function of electromagnetic and flavour-changing current

$$\mathcal{T}^{\mu\nu}(r) \equiv \int d^4x\, e^{irx}\,\langle 0|\mathrm{T}\{j^\mu_{f,\,\mathrm{SCET}_I}(x),\ [\bar{q}_{\rm hc}\gamma^{\nu\perp} P_L h_v](0)\}|\overline{B}_q\rangle$$

$$\overset{\text{match to SCET}_\text{II}}{=} \underbrace{(g_\perp^{\mu\nu} + i\varepsilon_\perp^{\mu\nu})}_{\text{photon left-handed}}\ \frac{Q_q F_{B_q} m_{B_q}}{4} \int_0^\infty d\omega\, \phi_+(\omega)\, \frac{J(n\cdot r, r^2, \omega)}{\omega - r^2/n\cdot r - i0+}\,.$$

Structure of the theoretical result

LP amplitude

$$\overline{\mathcal{A}}_{\text{type}-A} = ie\,\frac{\alpha_{\text{em}}}{4\pi}\,\mathcal{N}_{\text{ew}}\,\epsilon^{\star}_{\mu}\left\{\left(V_9^{\text{eff}}(q^2)+\frac{2\,\overline{m}_b\,m_{B_q}}{q^2}\,V_7^{\text{eff}}(q^2)\right)L_{V,\nu}+V_{10}^{\text{eff}}(q^2)L_{A,\nu}\right\}\mathcal{T}^{\mu\nu}(k)$$

$$\overline{\mathcal{A}}_{\text{type}-B} = ie\,\frac{\alpha_{\text{em}}}{4\pi}\,\mathcal{N}_{\text{ew}}\,\epsilon^{\star}_{\mu}\,\frac{4\,\overline{m}_b E_\gamma}{q^2}\,V_7^{\text{eff}}(k^2=0)L_{V,\nu}\,\mathcal{T}^{\mu\nu}(q)$$

SCET$_{\text{I}}$ correlation function of electromagnetic and flavour-changing current

$$\mathcal{T}^{\mu\nu}(r) \equiv \int d^4x\,e^{irx}\,\langle 0|\text{T}\{j^{\mu}_{f,\,\text{SCET}_I}(x),\;[\bar{q}_{\text{hc}}\gamma^{\nu\perp}P_L h_v](0)\}|\overline{B}_q\rangle$$

$$\stackrel{\text{match to SCET}_{\text{II}}}{=} \underbrace{(g_{\perp}^{\mu\nu}+i\varepsilon_{\perp}^{\mu\nu})}_{\text{photon left-handed}}\;\frac{Q_q F_{B_q} m_{B_q}}{4}\int_0^\infty d\omega\,\phi_+(\omega)\,\frac{J(n\cdot r,r^2,\omega)}{\omega-r^2/n\cdot r-i0+}\,.$$

Resonance amplitude [Do no show other NLP contributions]

$$\overline{\mathcal{A}}_{\text{res}} = -ie\,\frac{\alpha_{\text{em}}}{4\pi}\,\mathcal{N}_{\text{ew}}\,\epsilon^{\star}_{\mu}\,(g_{\perp}^{\mu\nu}+i\varepsilon_{\perp}^{\mu\nu})\,\frac{m_{B_q}}{2}\,\frac{4\overline{m}_b E_\gamma}{q^2}\,V_7^{\text{eff}}(0)L_{V,\nu}\,\frac{c_V f_V m_V T_1^{B_q\to V}(0)}{m_V^2-im_V\Gamma_V-q^2}$$

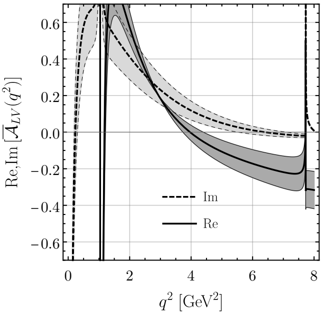
Corresponds to $B_s \to V[\to \mu^+\mu^-]\gamma$
Resonances $\phi(1020)$, $\phi(1680)$, $\phi(2170)$ with widths 4.249(12), 150(50), 104(20) MeV

Global duality violation and form factors

- The resonance contribution to the differential branching fraction is formally $\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$ but dominates any q^2 bin, in which it is contained, if its width is small [MB, Buchalla, Neubert, Sachrajda, 2009]

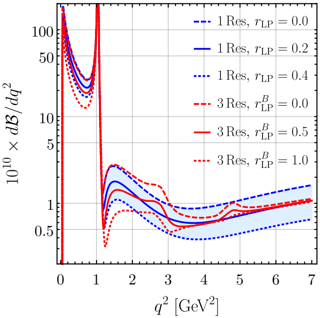
$$R \equiv \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma_{\rm res}}{dq^2} \Big/ \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma_{\rm LP}^{\rm type-B}}{dq^2} \approx 4\pi \left(\frac{c_V \lambda_{B_q} T_1^{B_q\to V}(0)}{Q_q F_{B_q}}\right)^2 \times \frac{f_V^2}{m_V \Gamma_V} \times \frac{1}{\ln \frac{q^2_{\rm max}}{q^2_{\rm min}}}$$

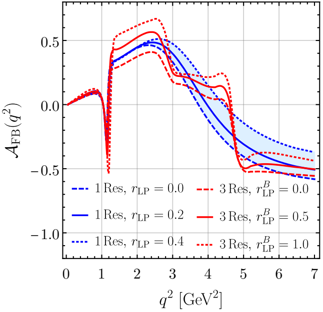
$$\approx 57 \quad \text{for } \phi(1020)$$



- Zero of real part implies forward-backward asymmetry $\propto \cos\theta_\ell$, but its observation requires B tagging $\to$ not observable at LHCb.

Rate predictions





q^2 bin	LP		NLP			uncertainty of "NLP all"			
[GeV^2]	LO	NLO	loc	loc + A	all	$\mu_{h,hc}$	$\lambda_{B_s}, \hat{\sigma}^{(q)}_{B_s}$	$r_{\rm LP}$	total
				$B_s \to \gamma\mu\bar{\mu}$					
$[4m_\mu^2, 6.0]$	2.32	2.96	3.81	4.03	12.43	$^{+0.11}_{-0.56}$	$^{+3.56}_{-1.42}$	$^{+1.39}_{-1.19}$	$^{+3.83}_{-1.93}$
$[2.0, 6.0]$	0.40	0.34	0.31	0.36	0.30	$^{+0.01}_{-0.04}$	$^{+0.21}_{-0.08}$	$^{+0.14}_{-0.11}$	$^{+0.25}_{-0.14}$
$[3.0, 6.0]$	0.30	0.22	0.19	0.22	0.21	$^{+0.01}_{-0.03}$	$^{+0.18}_{-0.07}$	$^{+0.10}_{-0.08}$	$^{+0.20}_{-0.10}$
$[4.0, 6.0]$	0.22	0.15	0.12	0.15	0.15	$^{+0.01}_{-0.02}$	$^{+0.14}_{-0.05}$	$^{+0.07}_{-0.05}$	$^{+0.16}_{-0.08}$
$[4m_\mu^2, 8.64]$	2.77	3.24	4.05	4.34	12.74	$^{+0.14}_{-0.60}$	$^{+3.85}_{-1.50}$	$^{+1.54}_{-1.31}$	$^{+4.15}_{-2.08}$

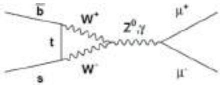
Bins above $q^2 > 2\,\mathrm{GeV}^2$ are theoretically on more solid ground but have branching fractions below 10^{-9}.

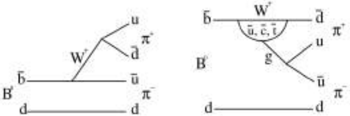
Charmless hadronic B two-body decays ($B \to \pi K$, ...)

2008.10615 and in preparation, with P. Böer, J. Toelstede and K. Vos

Charmless decays, $B \to \pi^+\pi^-$ vs. $\mu^+\mu^-$

- Same kinematics, charges, composite pions instead of elementary leptons. QED effects similar, identical for ultrasoft photons.
- But QCD dynamics is very different.





$B \to \mu^+\mu^-$
$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}\rangle$

$B \to \pi^+\pi^-$
$\langle \pi^+\pi^-|Q_i|\bar{B}\rangle$

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V^*_{pD}\Big(C_1\mathcal{O}_1^p + C_2\mathcal{O}_2^p + \sum_{i=(\mathrm{EW})\mathrm{pen},\ \mathrm{mag}} C_i\mathcal{O}_i\Big)$$

$$\mathcal{O}^p_{1,2} = (\bar{p}\Gamma b)\,(\bar{D}\Gamma' p) \qquad \mathcal{O}_{i,\mathrm{QCD\ pen}} = (\bar{D}\Gamma b) \sum_{q=u,d,s,c,b} (\bar{q}\Gamma' q)$$

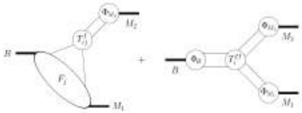
- Different CKM amplitudes, strong rescattering in $\langle \pi^+\pi^-|Q_i|\bar{B}\rangle \Rightarrow$ (direct) CP violation, determination of CKM angles, search for new physics
- Branching fractions 10^{-5}, first measured by CLEO in the late 1990s, now $\mathcal{O}(50-100)$ different two-body final states M_1M_2 measured.

QCD theory

"QCD factorization" [MB, Buchalla, Neubert, Sachrajda, 1999-2001], later understood and formulated as a $\mathrm{SCET_{II}}$ problem:

$$\mathrm{QCD} \quad \xrightarrow{\text{remove h}} \quad \mathrm{SCET_I} \quad \xrightarrow{\text{remove hc}} \quad \mathrm{SCET_{II}}(c,\bar{c},s)$$

$$\langle M_1 M_2|Q_i|\bar{B}\rangle = \underbrace{F^{BM_1}(0)}_{\text{form factor}} \int_0^1 du\, T_i^I(u)\Phi_{M_2}(u)$$



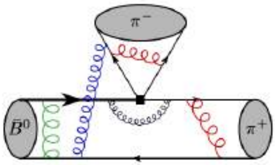
$$+ \int_0^1 dz du\, H_i^{\mathrm{II}}(z,u) \int_0^\infty d\omega \int_0^1 dv\, J(\omega,u,v)\, \underbrace{\Phi_B(\omega)\Phi_{M_1}(v)\Phi_{M_2}(u)}_{\text{LCDAs}}$$

- Rigorous at leading power in $\Lambda_{\mathrm{QCD}}/m_b$
- Strong rescattering phases are $\delta \sim \mathcal{O}(\alpha_s(m_b), \Lambda/m_b)$. $\mathrm{SCET_I}$ matching coefficients only. Direct CP asymmetry is calculable at LP

$$A_{\mathrm{CP}}(M_1 M_2) = \underbrace{a_1\alpha_s}_{1999} + \underbrace{a_2\alpha_s^2}_{2020} + \ldots + \mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$$

Theory of including QED effects is conceptually similar to $B_s \to \mu^+\mu^-$. More detailed slides than the following, see [Böer, Vos, talk at CERN, 16.10.2020 `https://indico.cern.ch/event/953761/`]

Including virtual QED effects into the factorization theorem



SCET$_\mathrm{I}$ operators

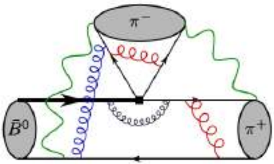
$$\mathcal{O}^\mathrm{I}(t) = [\bar\chi_{\bar C}(tn_-)\not{n}_-\gamma_5\chi_{\bar C}]\,[\bar\chi_C h_v]$$

$$\mathcal{O}^\mathrm{II}(t,s) = \underbrace{[\bar\chi_{\bar C}(tn_-)\not{n}_-\gamma_5\chi_{\bar C}]}_{\pi^-}\,\underbrace{[\bar\chi_C \not{\mathcal{A}}_{C,\perp}(sn_+)h_v]}_{B\to\pi^+}$$

QCD Factorization Formula

$$\langle M_1 M_2|Q_i|B\rangle = F^{B\to M_1}(q^2=0)\int_0^1 du\,\mathbf{T}_i^\mathrm{I}(u)\,f_{M_2}\phi_{M_2}(u)$$
$$+\int_0^\infty d\omega\int_0^1 du\,dv\,\mathbf{T}_i^\mathrm{II}(u,v,\omega)\,f_{M_1}\phi_{M_1}(v)\,f_{M_2}\phi_{M_2}(u)\,f_B\phi_B(\omega)$$

Including virtual QED effects into the factorization theorem



SCET$_\mathrm{I}$ operators

$$\mathcal{O}^{\mathrm{I}}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}^{\dagger(Q_{M_2})}_{n_+}h_v]$$

$$\mathcal{O}^{\mathrm{II}}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{A}_{C,\perp}(sn_+)\,\mathbf{S}^{\dagger(Q_{M_2})}_{n_+}h_v]$$

$$S^{(q)}_{n_\pm} = \exp\left\{-iQ_q e\int_0^\infty ds\,n_\pm A_s(sn_\pm)\right\}$$

QCD×QED Factorization Formula

$$\langle M_1M_2|Q_i|\bar{B}\rangle|_{\text{non-rad.}} = \mathcal{F}^{B\to M_1}_{Q_2}(q^2=0)\int_0^1 du\,\mathbf{T}^{\mathrm{I}}_{i,Q_2}(u)\,\mathcal{F}_{M_2}\Phi_{M_2}(u)$$
$$+\int d\omega\int_0^1 du\,dv\;\mathbf{T}^{\mathrm{II}}_{i,\otimes}(u,v,\omega)\,\mathcal{F}_{M_1}\Phi_{M_1}(v)\,\mathcal{F}_{M_2}\Phi_{M_2}(u)\,\mathcal{F}_{B,\otimes}\Phi_{B,\otimes}(\omega)$$

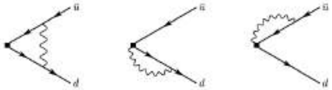
- Formula retains its form, but the hadronic matrix elements are generalized. They become process-dependent through the directions and charges of the *other* particles.
- Computation of $\mathcal{O}(\alpha_{\mathrm{em}})$ corrections to the h and hc short-distance coefficient (all poles cancel).

LCDA of a charged pion in QCD×QED

$$R_c \langle \pi^- | \bar{\chi}_{\bar{c}}^{(d)}(tn_-) \frac{\not{n}_-}{2} \gamma_5 \chi_{\bar{c}}^{(u)}(0) | 0 \rangle = -iE \int_0^1 du\, e^{iu\hat{t}} \mathscr{F}_{\pi^-} \Phi_{\pi^-}(u)$$

Renormalization/evolution kernel for the (anti-)collinear operator well-defined after soft rearrangement

$$\gamma(u,v) = -\frac{\alpha_{\rm em} Q_{M_2}}{\pi}\,\delta(u-v)\left(Q_d \ln\frac{\mu}{2Eu} - Q_u \ln\frac{\mu}{2E(1-u)} + \frac{3Q_{M_2}}{4}\right)$$
$$-\left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\rm em}}{\pi} Q_u Q_d\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\,\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\,\theta(u-v)\right]_+$$



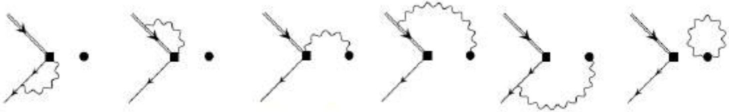
- The endpoint logarithms $\ln u$, $\ln(1-u)$ and energy dependence are a remnant of the soft physics.
- Gegenbauer polynomials are no longer eigenfunctions, asymptotic behaviour $\Phi_\pi(u,\mu) \stackrel{\mu\to\infty}{\to} 6u(1-u)$ no longer holds. QED evolution is asymmetric and endpoint behaviour changes from linear.

B-LCDA alias soft function in QCD×QED

Soft Function for $\bar{B}^0 \to M_1^+ M_2^-$

$$i m_B \int \mathrm{d}\omega\, e^{-i\omega t} \mathscr{F}_{B,+-} \Phi_{B,+-}(\omega) = \frac{1}{R_c R_{\bar{c}}} \langle 0 |\, \bar{q}_s^{(d)}(t n_-)[t n_-, 0]^{(d)}_{n_-} \frac{\not{n}_-}{2} h_v \,(S_{n_+}^{\dagger, Q_2} S_{n_-}^{Q_2})\, |\bar{B}^0\rangle$$

- $B \to M_1 M_2$ decays: four different soft functions for various charge assignments
- different objects compared to standard B meson LCDA in QCD
  $\to$ final-state rescattering, different support properties, …



- coupling of soft photon/gluon to incoming b quark with $n_- p_b = m_b \to \infty$
  $\to$ $\omega \in [0, \infty)$
- coupling of soft photon to outgoing anti-coll. π^- with $n_- q = m_b \to \infty$
  $\to$ $\boxed{\text{QED } B \text{ LCDA has support } \omega \in (-\infty, \infty) \text{ if anti-coll. meson is charged}}$

Slide from [Böer, Vos, talk at CERN, 16.10.2020 https://indico.cern.ch/event/953761/]

B-LCDA alias soft function in QCD×QED (II)

Anomalous Dimension for $\Phi_\pm$

$$\Gamma_>(\omega,\omega';\mu) = \left(\frac{\alpha_{\rm em}}{4\pi}Q_d^2 + \frac{\alpha_s C_F}{4\pi}\right)\left\{\delta(\omega-\omega')\left(2\log\frac{\mu^2}{\omega^2}-5\right) - 4F_>(\omega,\omega')\right\}$$
$$-\frac{\alpha_{\rm em}}{4\pi}2Q_dQ_2\left\{\delta(\omega-\omega')2\log\frac{\mu^2}{\omega^2} - 2G_>(\omega,\omega')\right\} - \frac{\alpha_{\rm em}}{\pi}Q_2^2\delta(\omega-\omega')i\pi$$

$$\Gamma_<(\omega,\omega';\mu) = \left(\frac{\alpha_{\rm em}}{4\pi}Q_d^2 + \frac{\alpha_s C_F}{4\pi}\right)\left\{\delta(\omega-\omega')\left(2\log\frac{\mu^2}{\omega^2}-5\right) - 4F_<(\omega,\omega')\right\}$$
$$-\frac{\alpha_{\rm em}}{4\pi}2Q_dQ_2\left\{\delta(\omega-\omega')2\log\frac{\mu^2}{-\omega^2} - 2G_<(\omega,\omega')\right\} - \frac{\alpha_{\rm em}}{\pi}Q_2^2\delta(\omega-\omega')i\pi$$

- contains plus-distributions and generalized plus-distributions, e.g.

$$G_> = \omega\left[\frac{\theta(\omega'-\omega)\theta(\omega)}{\omega'(\omega'-\omega)}\right]_+ + \left[\frac{\theta(\omega'-\omega)}{\omega'-\omega}\right]_\otimes \quad \text{with} \quad [\ldots]_\otimes f(\omega) \to [\ldots]\Big(f(\omega)-\theta(\omega)f(\omega')\Big)$$

Slide from [Böer, Vos, talk at CERN, 16.10.2020 https://indico.cern.ch/event/953761/]

Numerical estimate of QED effects for πK final states

Up to now virtual corrections to the non-radiative amplitude.
Add (ultra)soft photon radiation.

- Electroweak scale to m_B: QED corrections to Wilson coefficients included
- m_B to μ_c: $\mathcal{O}(\alpha_{\rm em})$ corrections to short-distance kernels included.
  QED effects in form factors and LCDA not included.
- Ultrasoft photon radiation included (same formalism as for $\mu^+\mu^-$ with $m_\mu \to m_\pi, m_K$)

$$U(M_1M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2+Q_{M_1}^2\left[1+\ln\frac{m_{M_1}^2}{m_B^2}\right]+Q_{M_2}^2\left[1+\ln\frac{m_{M_2}^2}{m_B^2}\right]\right)}\,.$$

$$\begin{aligned}
U(\pi^+K^-) &= 0.914 \\
U(\pi^0K^-) &= U(K^-\pi^0) = 0.976 \\
U(\pi^-\bar{K}^0) &= 0.954 \\
U(\bar{K}^0\pi^0) &= 1 \qquad \text{[for } \Delta E = 60\,\text{MeV]}
\end{aligned}$$

Isospin-protected ratios / sum rules

Consider ratios / sums where some QCD uncertainties drop out.

[MB, Neubert, 2003]

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \,\mathrm{Re}\,\delta_E + \delta_U$$

$$R_L^{\mathrm{QCD}} - 1 \approx (1 \pm 2)\% \qquad \delta_E \approx 0.1\% \qquad \delta_U = 5.8\%$$

QED correction larger than QCD and QCD uncertainty, but short-distance QED negligible.

[Gronau, Rosner, 2006]

$$\Delta(\pi K) \equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)$$

$$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\% \qquad \delta_\Delta(\pi K) \approx -0.4\%$$

QED correction of similar size but small.

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- QED factorization is more complicated than QCD due to charged external states. SCET applies and we now understand how to systematically include QED effects, but it requires new non-perturbative matrix elements, generalizing the familiar hadronic matrix elements.

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II) For $B_s \to \mu^+\mu^-$ there is a power-enhanced virtual electromagnetic correction.

- More long-distance QCD than f_B
- Effect of the same order as the non-parametric uncertainty, larger than previously estimated QED uncertainty

III) For charmless hadronic decays the QCD $\times$ QED factorization formula takes a similar form as in QCD alone, but the generalized pion (etc.) and B-meson LCDA exhibit novel properties (asymmetric evolution, soft rescattering phases in the B-LCDA)

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V) Comparison to experiment now requires precise statements how QED effects are treated in the analysis. Ideally compare theoretically well-defined and calculable *radiative* branching fractions and use Monte Carlo generators only to estimate efficiencies.