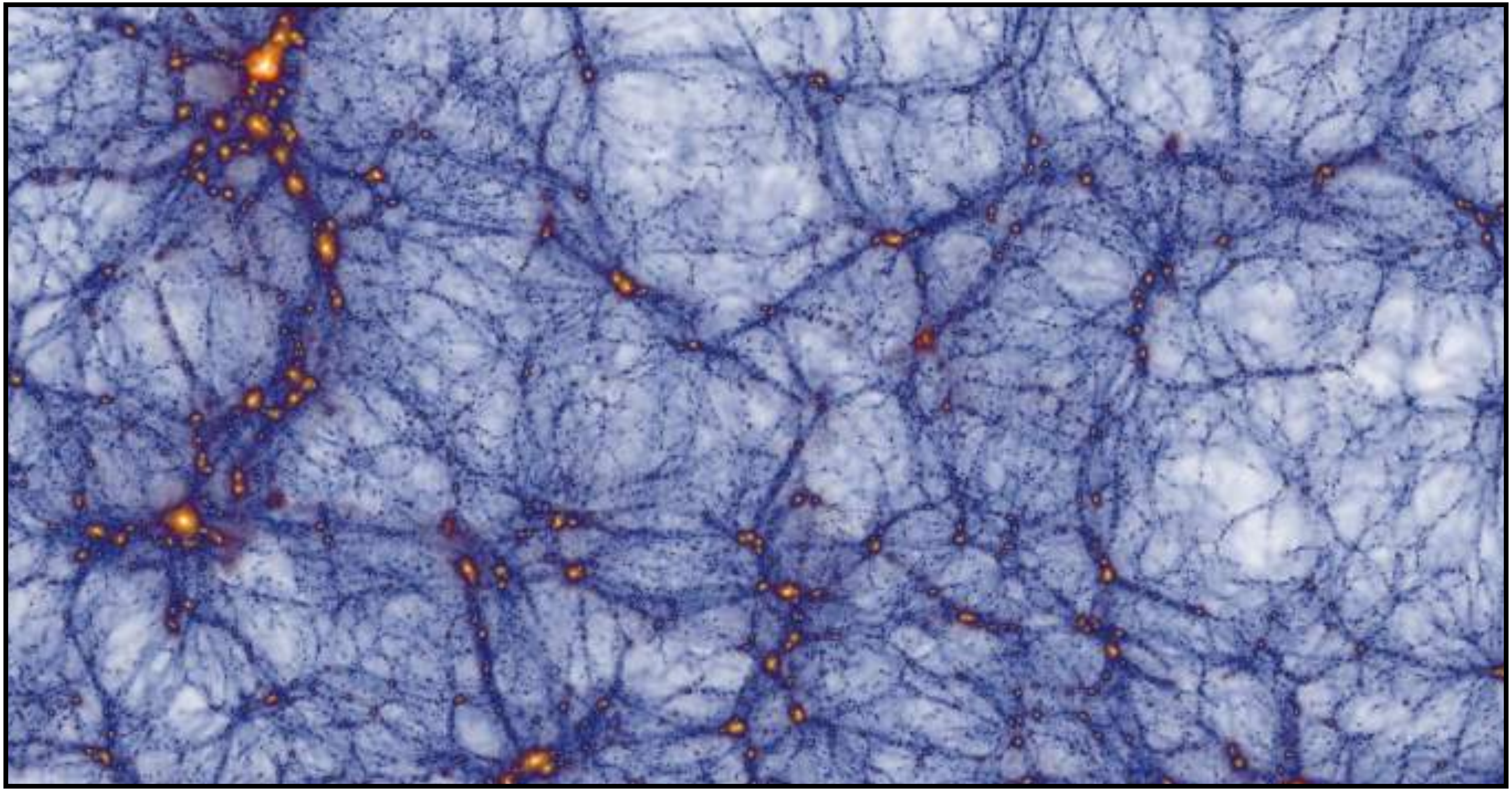
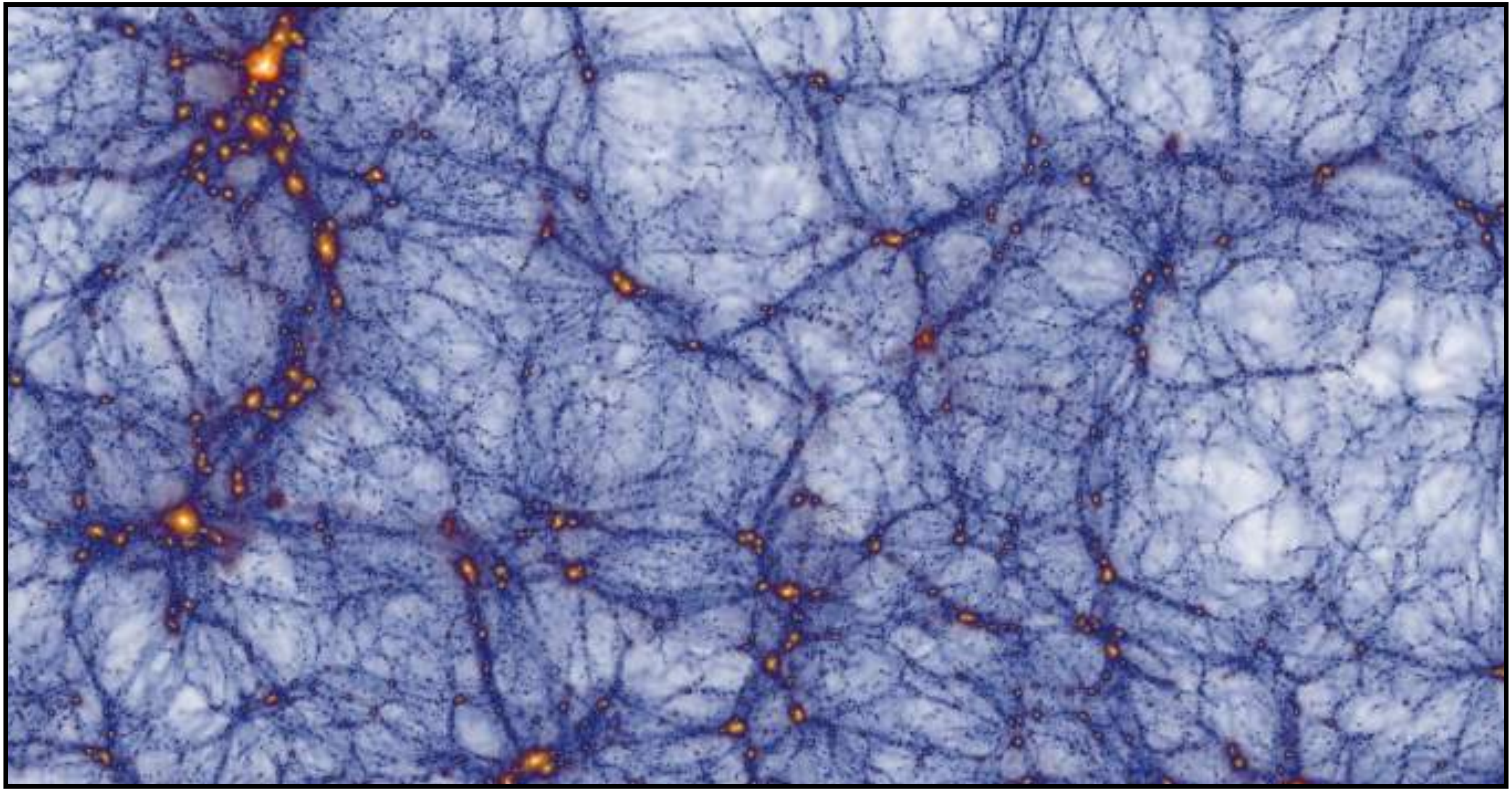


The Cosmological Bootstrap

Daniel Baumann
University of Amsterdam

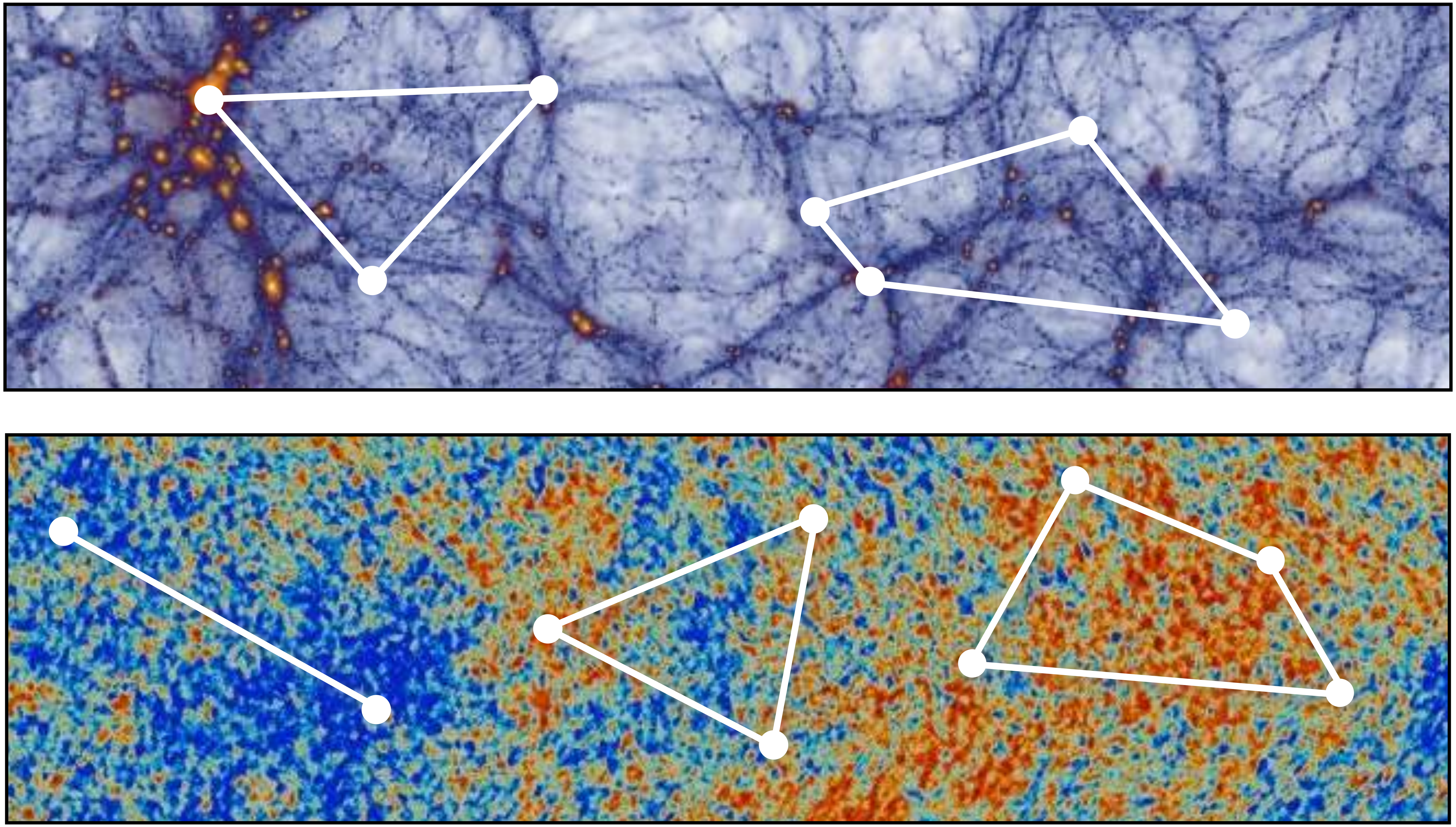
Imperial / Oxford Seminar,
28 April 2020



Based on work with

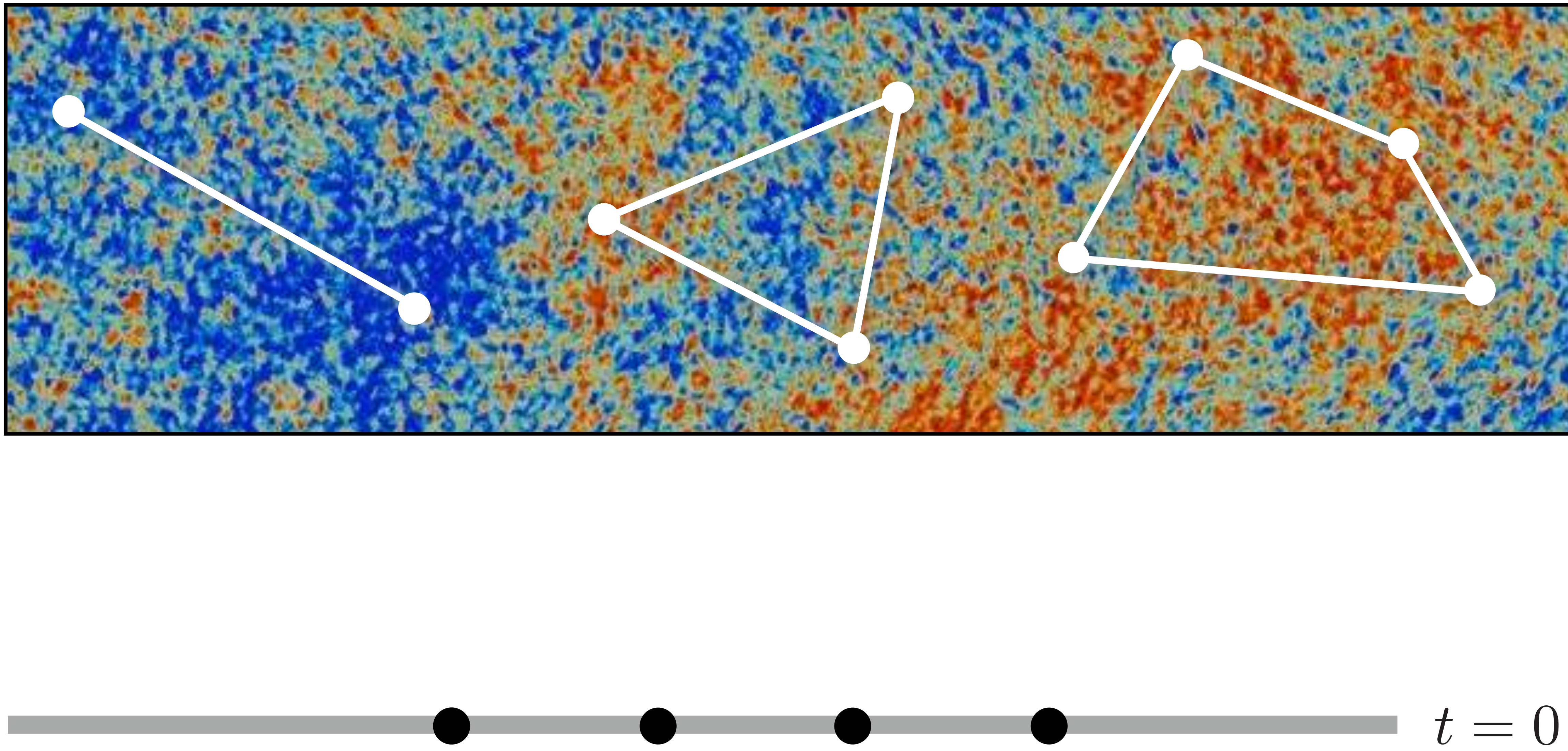
Nima Arkani-Hamed, Hayden Lee, Guilherme Pimentel,
Carlos Duaso Pueyo and Austin Joyce

The physics of the early universe is encoded in **spatial correlations** between cosmological structures at late times:



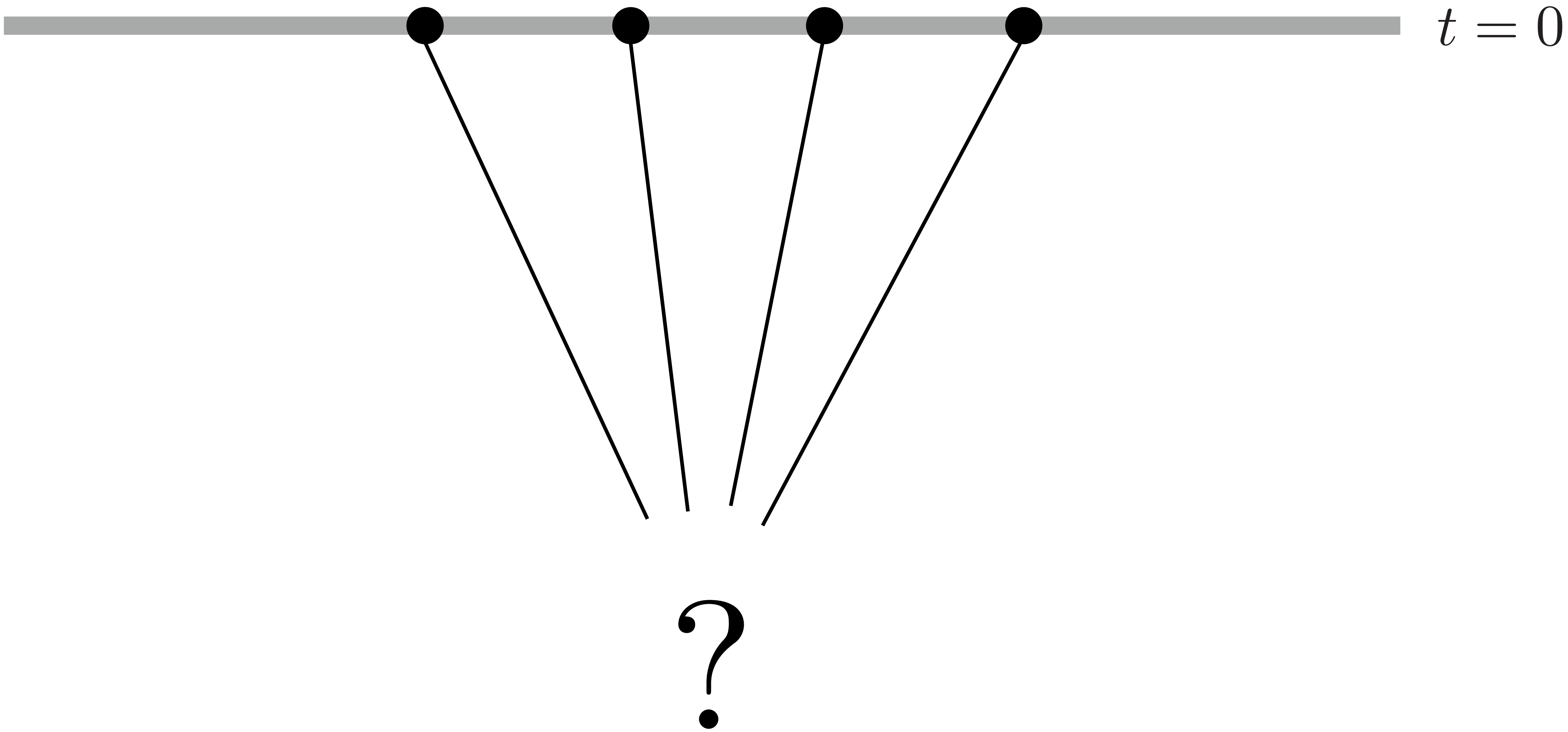
A central challenge of modern cosmology is to construct a **consistent history** of the universe that explains these correlations.

The correlations can be traced back to **primordial correlations** at the beginning of the hot big bang.



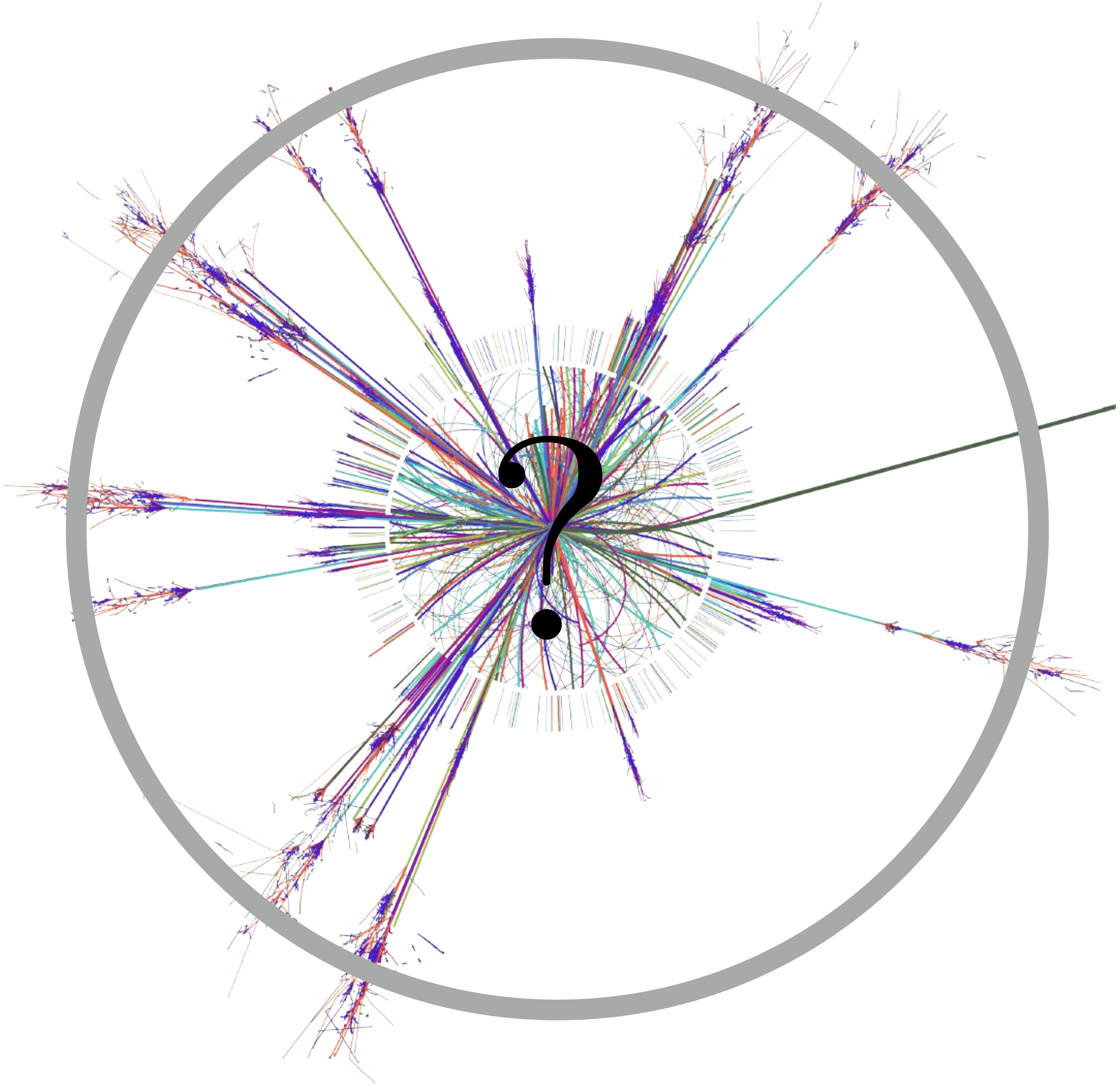
To explain the observed fluctuations in the CMB, these fluctuations must be created **before the hot big bang**!

What is the space of consistent histories?



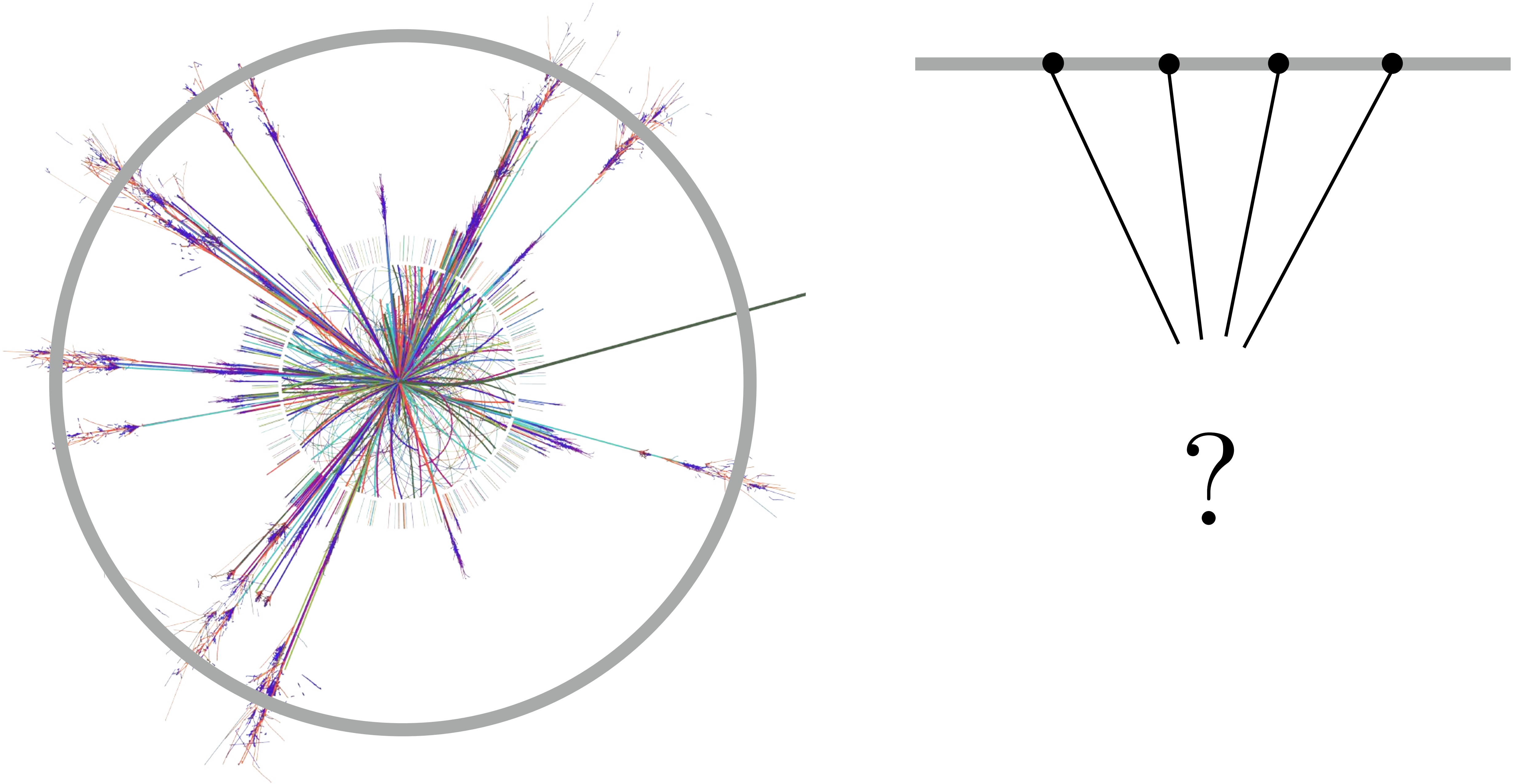
- What are the rules that consistent correlators have to satisfy?
- How are these rules encoded in the boundary observables?

Similar questions have been asked for **scattering amplitudes**:



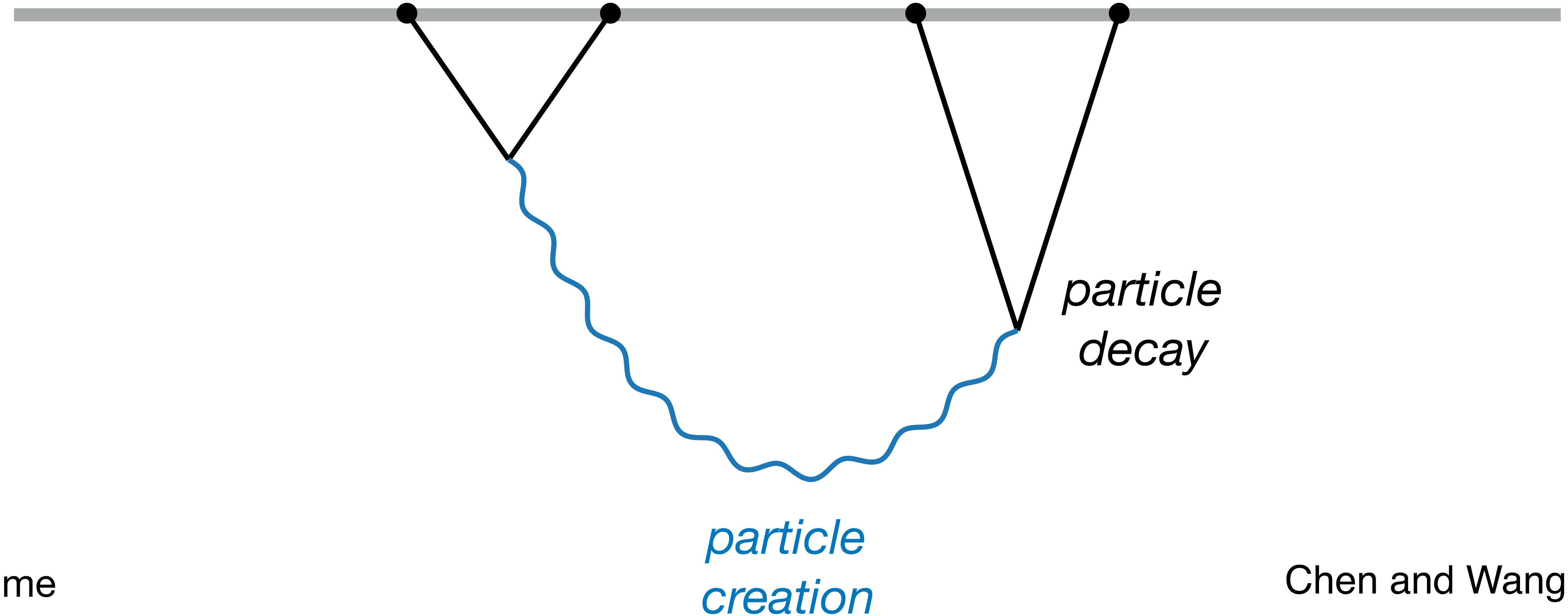
In that case, the rules of **quantum mechanics** and **relativity** are very constraining.

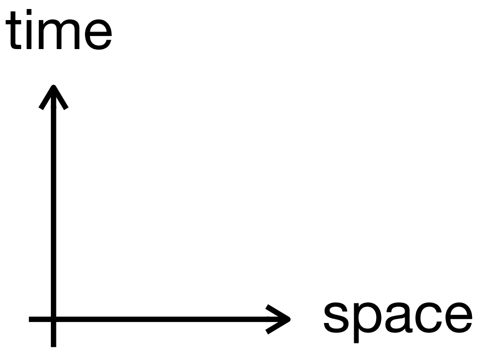
Does a similar **rigidity** exist for cosmological correlators?



Goal: Develop an understanding of cosmological correlators that parallels our understanding of flat-space scattering amplitudes.

The connection to scattering amplitudes is also relevant because the early universe was like a giant **cosmological collider**:

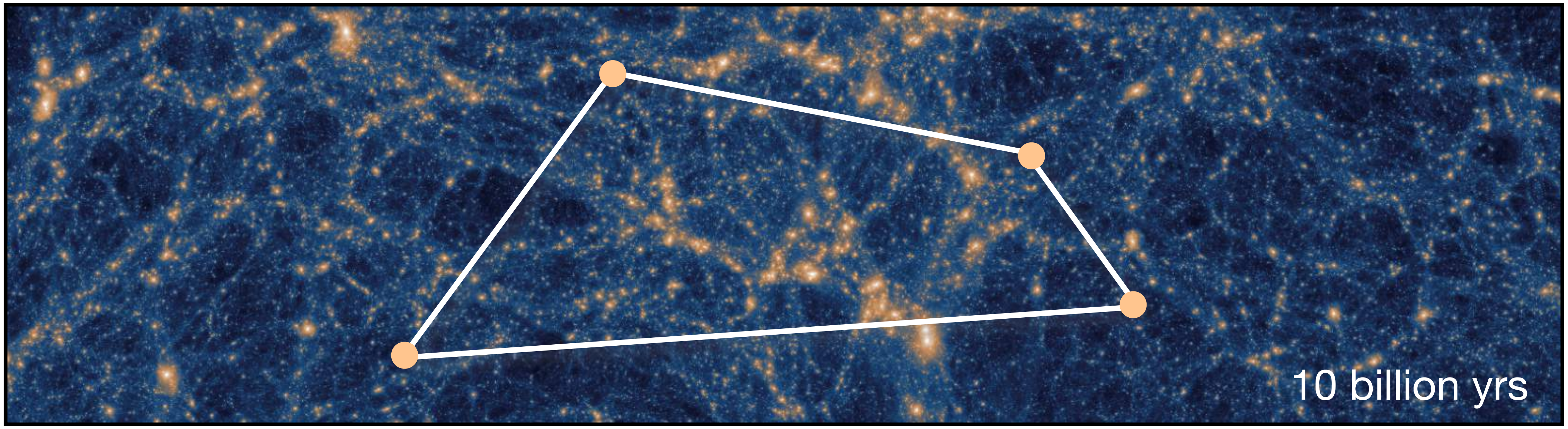


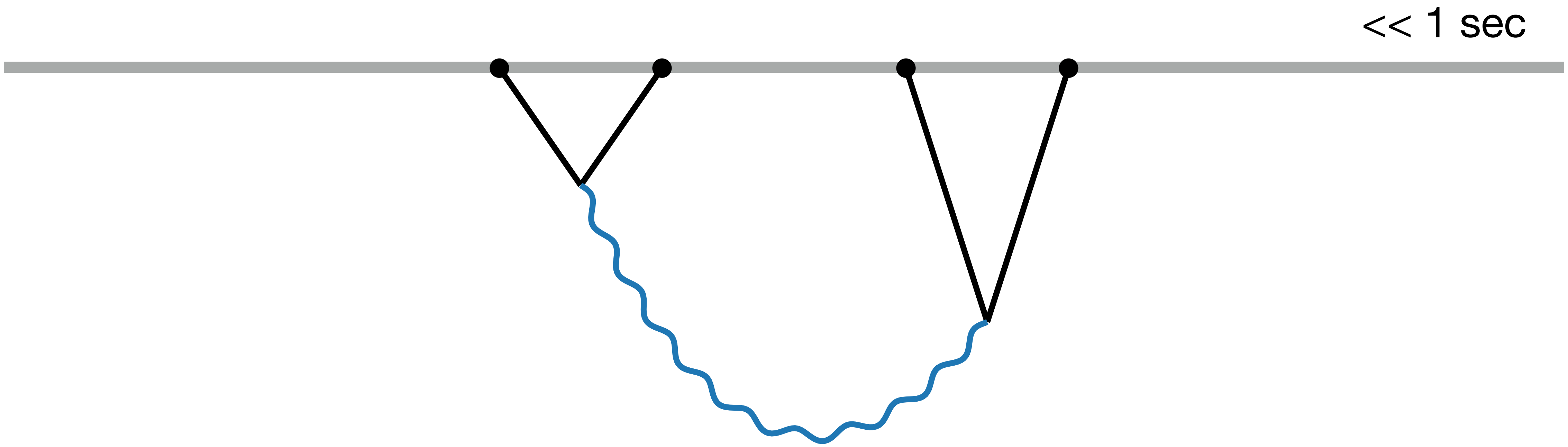


Chen and Wang [2009]
DB and Green [2011]
Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

During inflation, the rapid expansion can produce very **massive particles** ($\sim 10^{14}$ GeV) whose decays lead to nontrivial correlations.

- These correlations are tracers of the inflationary dynamics.
- They leaving an imprint in the distribution of galaxies.





Goal: Develop a systematic way to predict these signals.

Any Questions?

Outline

I. S-matrix Bootstrap

II. Cosmological Bootstrap

III. New Developments

I.

S-matrix Bootstrap

Bootstrap Philosophy

Physical Principles → Lagrangian / equations of motion / spacetime evolution / Feynman diagrams → **Observables**

(Lorentz, locality, …)

Lagrangian

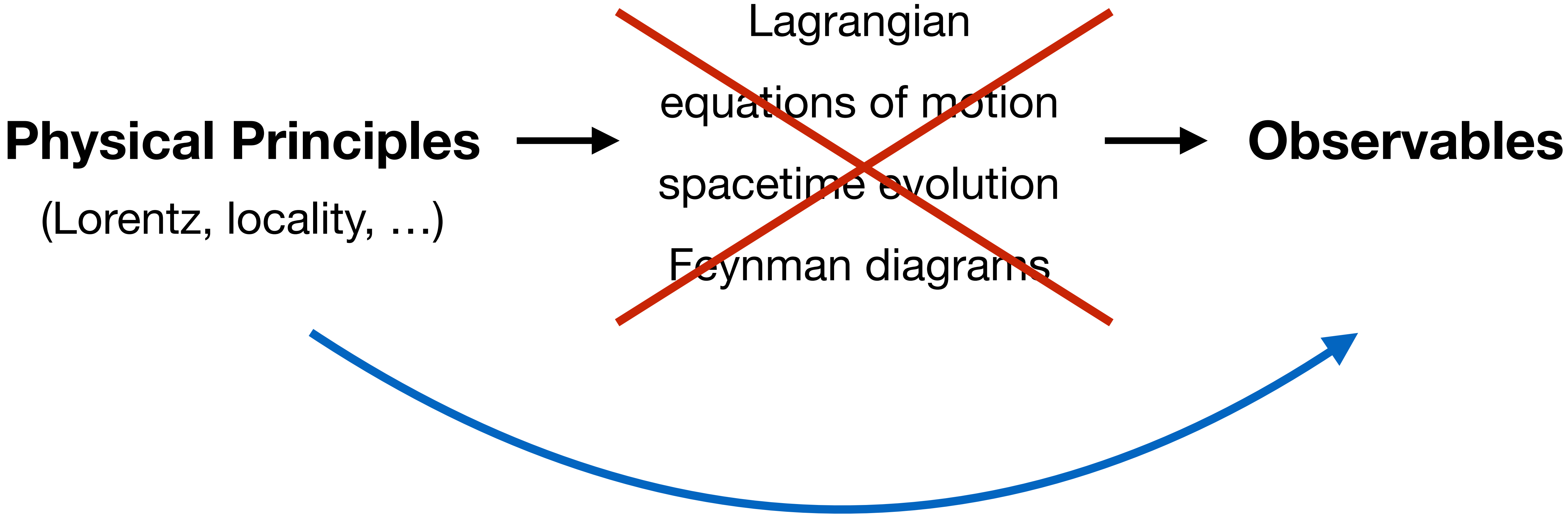
equations of motion

spacetime evolution

Feynman diagrams

$$S = \int \underset{\underset{\text{locality}}{\uparrow}}{\mathrm{d}^4x}\, \mathcal{L}\left[\frac{1}{2}\underset{\underset{\text{Lorentz}}{\uparrow}}{\partial_\mu\phi\partial^\mu\phi} - \frac{1}{2}\underset{\underset{\text{parameters}}{\uparrow}}{m^2}\phi^2 + \cdots\right]$$

Bootstrap Philosophy



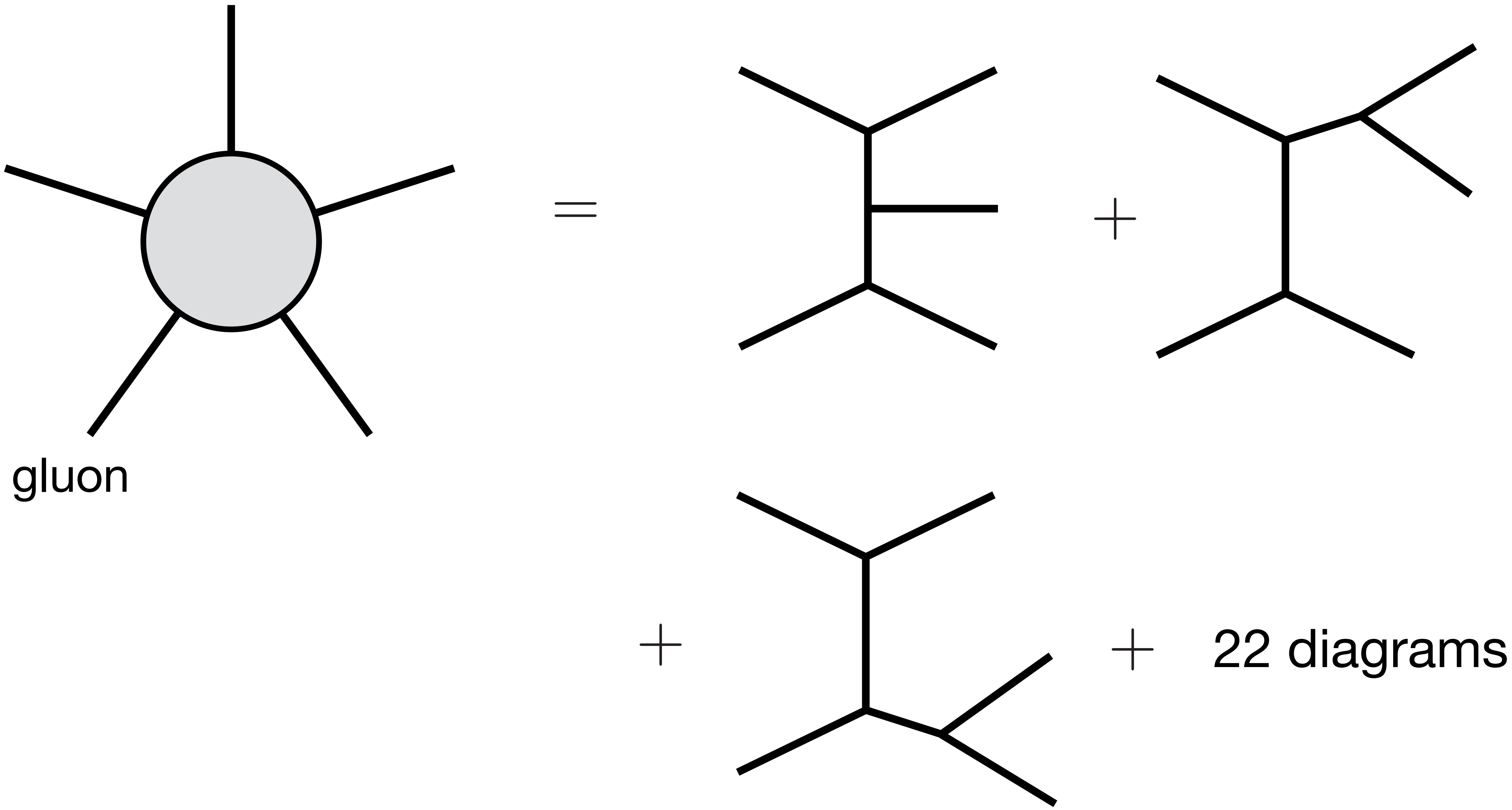
Physical Principles → ~~Lagrangian~~ ~~equations of motion~~ ~~spacetime evolution~~ ~~Feynman diagrams~~ → **Observables**

(Lorentz, locality, …)

Modern scattering
amplitudes programme

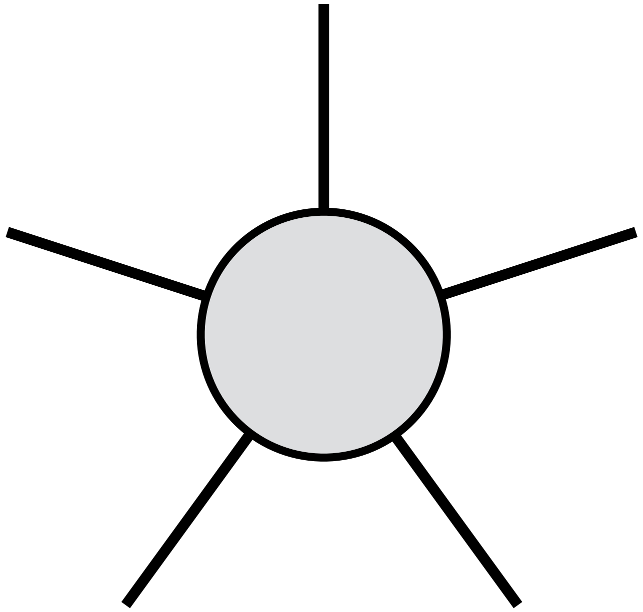
Why Bootstrap?

- Computations using Feynman diagrams are complicated.

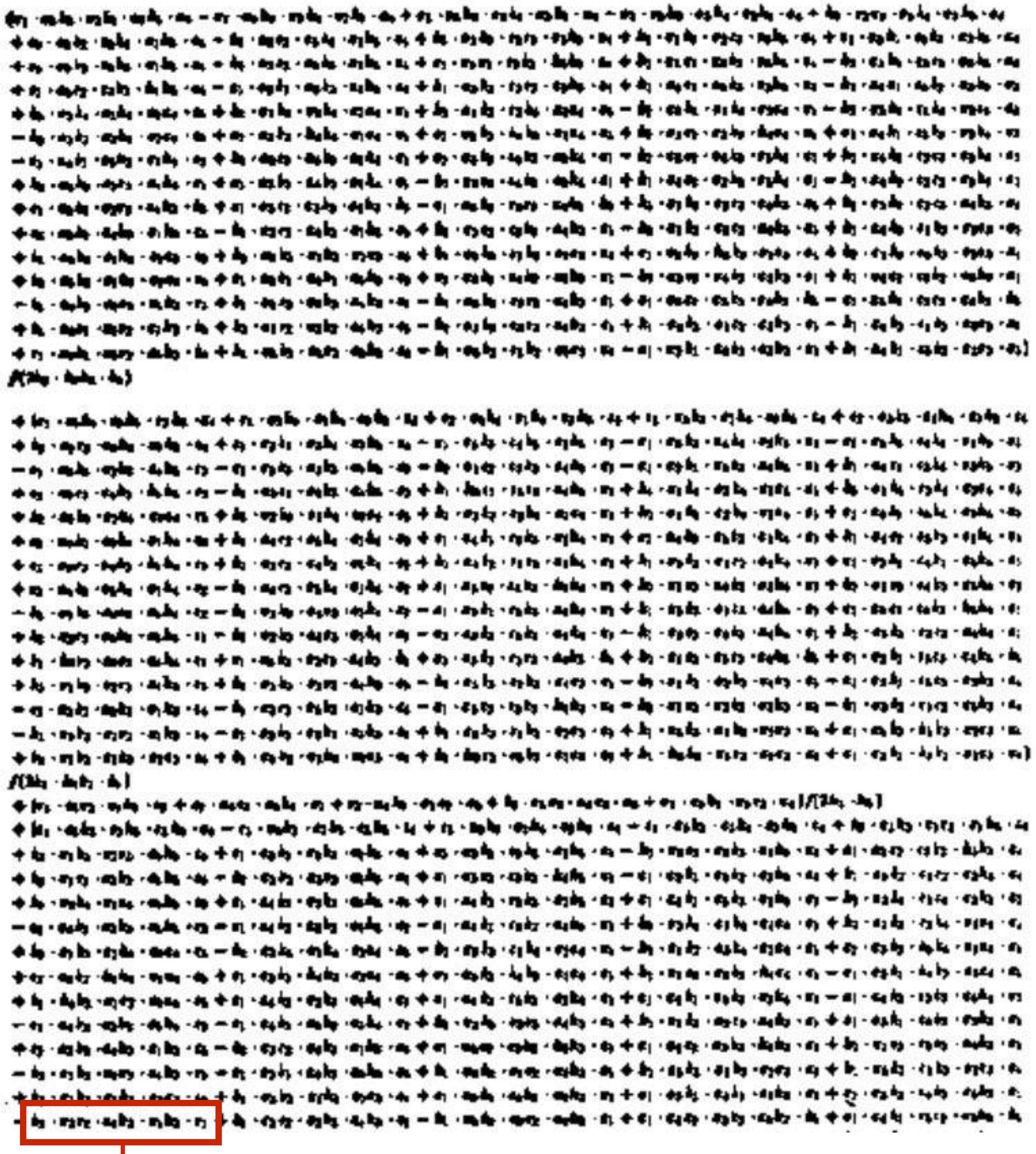


Why Bootstrap?

- Computations using Feynman diagrams are complicated.



$$A(1^{h_1}2^{h_2}3^{h_3}4^{h_4}5^{h_5}) \; =$$

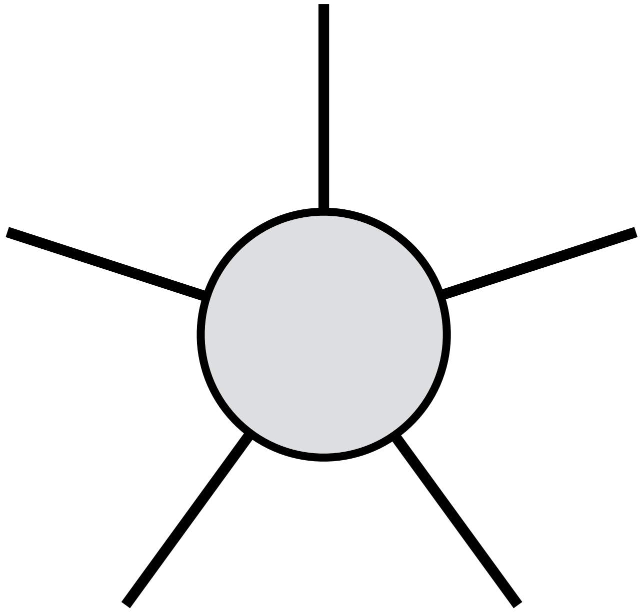


$p_1 \cdot p_4 \;\; \epsilon_2 \cdot p_1 \;\; \epsilon_1 \cdot \epsilon_3 \;\; \epsilon_4 \cdot \epsilon_5$

$\times$ 24 pages

Why Bootstrap?

- Physical answers are simple.



$$A(1^+2^+3^+4^+5^+) = 0$$

$$A(1^-2^+3^+4^+5^+) = 0$$

$$A(1^-2^-3^+4^+5^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

[spinor helicity variables]

$$p_i^\mu = \sigma^\mu_{a\dot{a}}\lambda_i^a\bar{\lambda}_i^{\dot{a}}$$

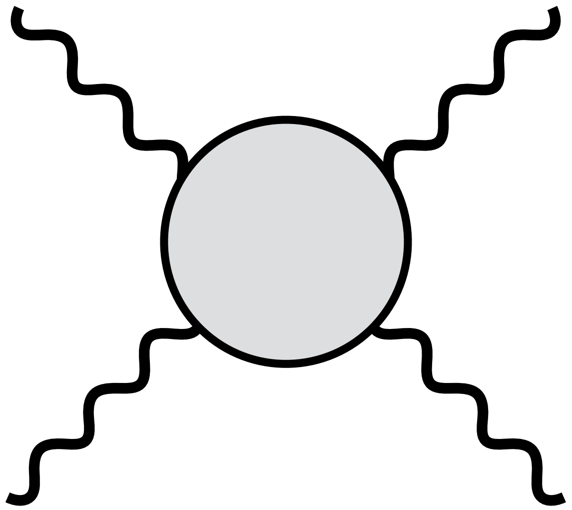
$$\langle ij\rangle = \lambda_i^a\lambda_j^b\epsilon_{ab}$$

$$[ij] = \bar{\lambda}_i^{\dot{a}}\bar{\lambda}_j^{\dot{b}}\epsilon_{\dot{a}\dot{b}}$$

Parke and Taylor [1985]

Why Bootstrap?

- Physical answers are simple.



$=$ thousands of diagrams, each involving hundreds of terms

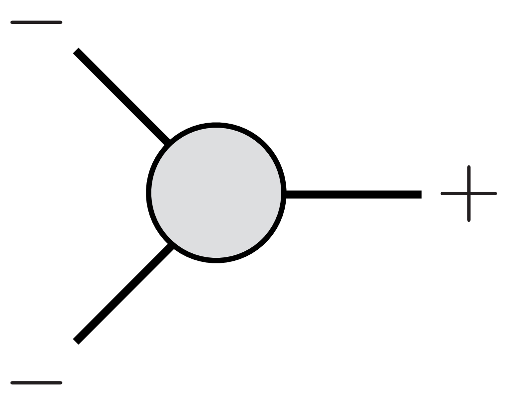
graviton

$$M(1^-2^-3^+4^+) \;=\; \frac{\langle 12\rangle^4[34]^4}{stu}$$

DeWitt [1967]

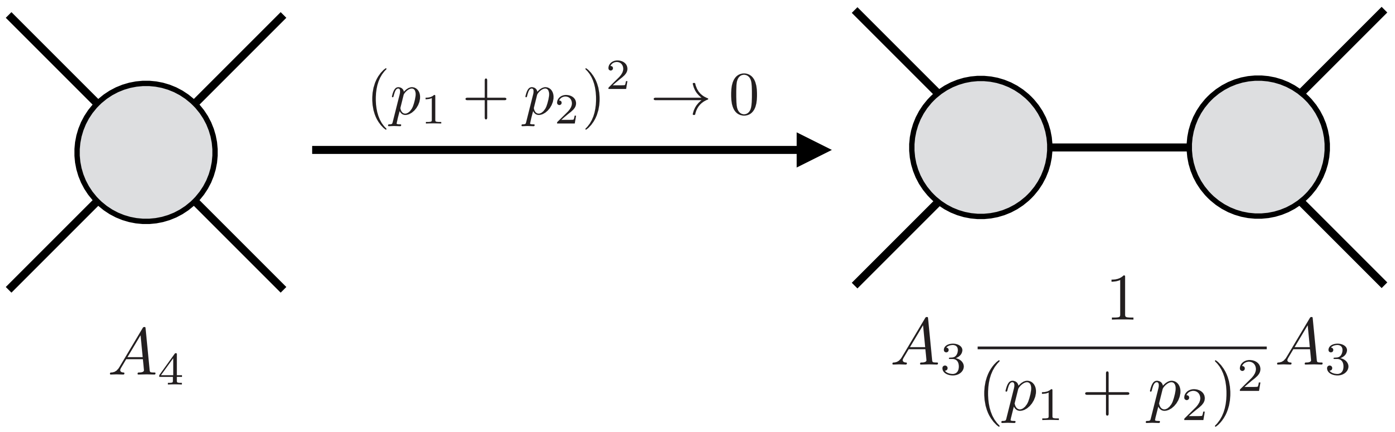
S-matrix Bootstrap

- Bootstrap methods are very powerful.
  - Massless 3pt amplitudes are fixed by **Lorentz invariance**:



$$= \left(\frac{\langle 12 \rangle^3}{\langle 13 \rangle \langle 23 \rangle} \right)^S$$

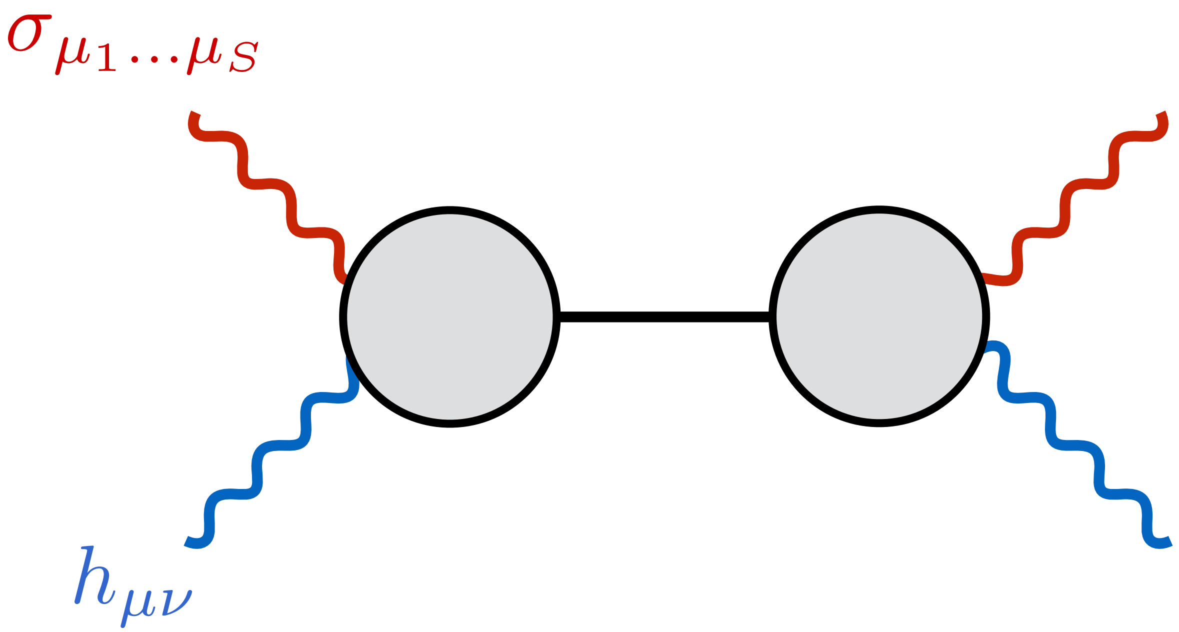
  - Higher-point amplitudes are constrained by **locality**:



Benincasa and Cachazo [2007]

S-matrix Bootstrap

- Bootstrap methods are very powerful.
  - Consistent factorisation is a nontrivial constraint:



  - Only consistent for spins $S = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$

YM SUSY GR

Benincasa and Cachazo [2007]
McGady and Rodina [2010]

Any Questions?

II.

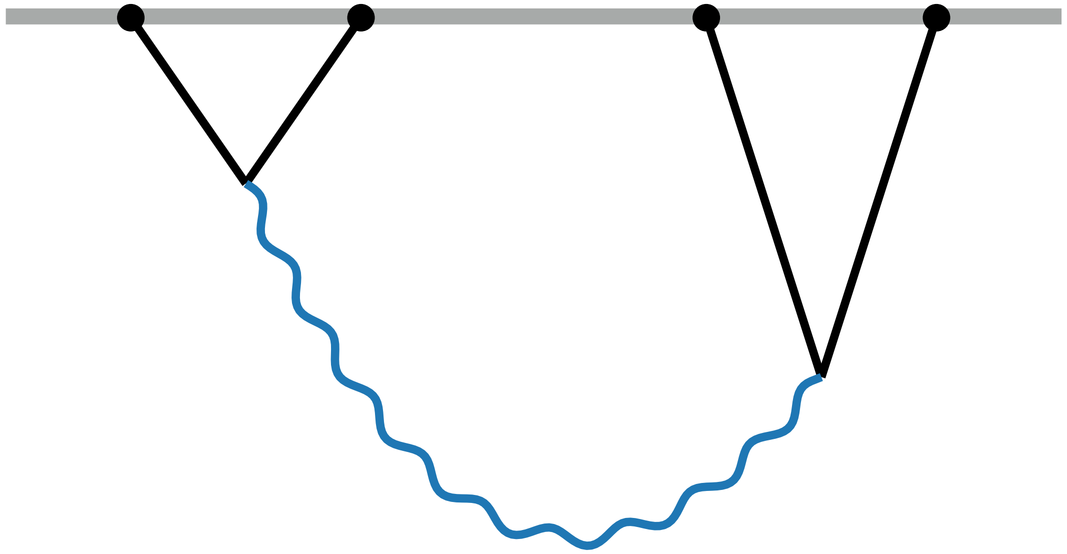
Cosmological Bootstrap

The Challenge

Cosmological correlators are hard to compute.

I. Scalar correlators

No analytic results, even for tree-level exchange.



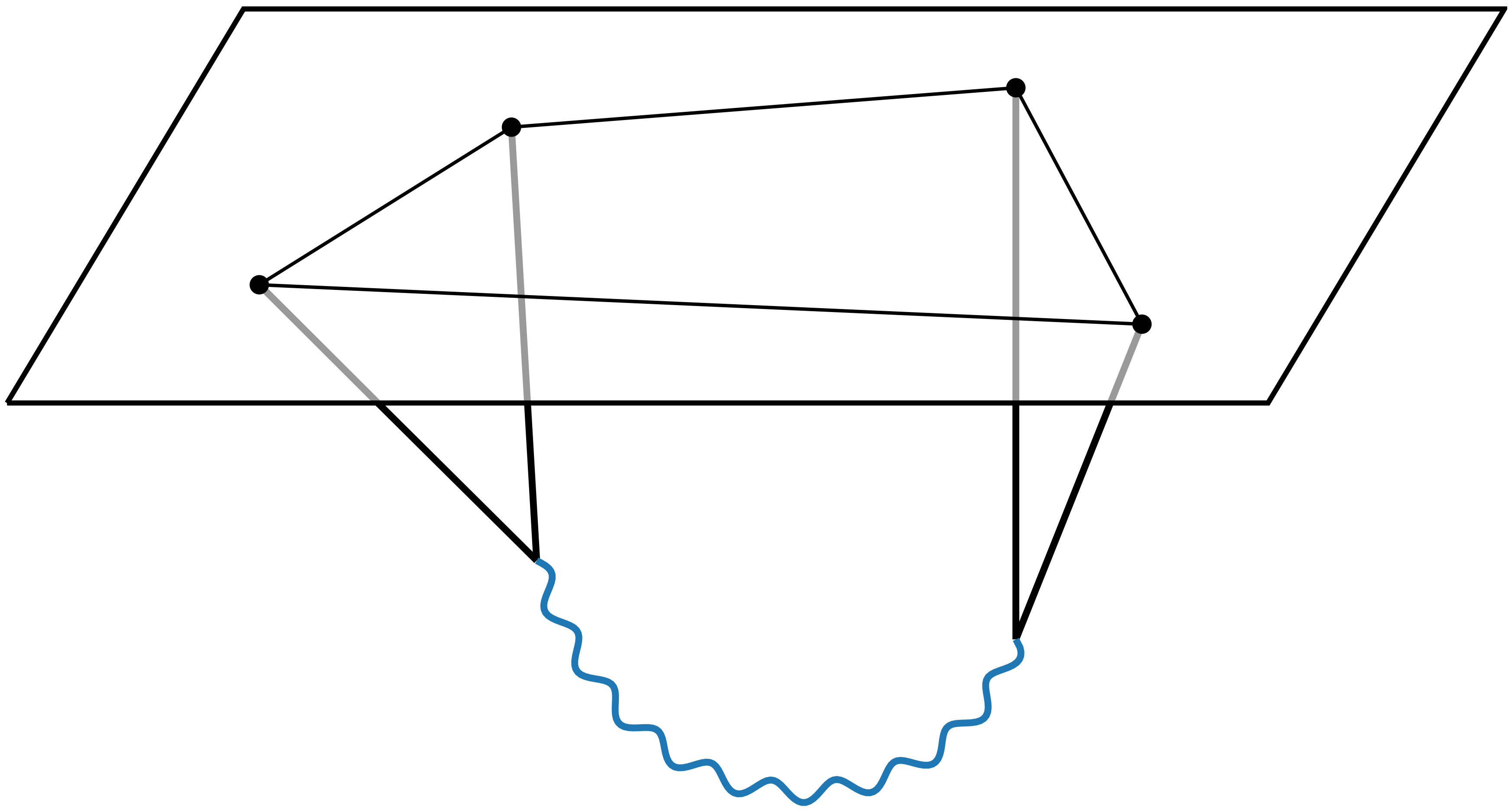
$$= \quad ?$$

II. Tensor correlators

No results beyond three-point functions.

Inflation → De Sitter

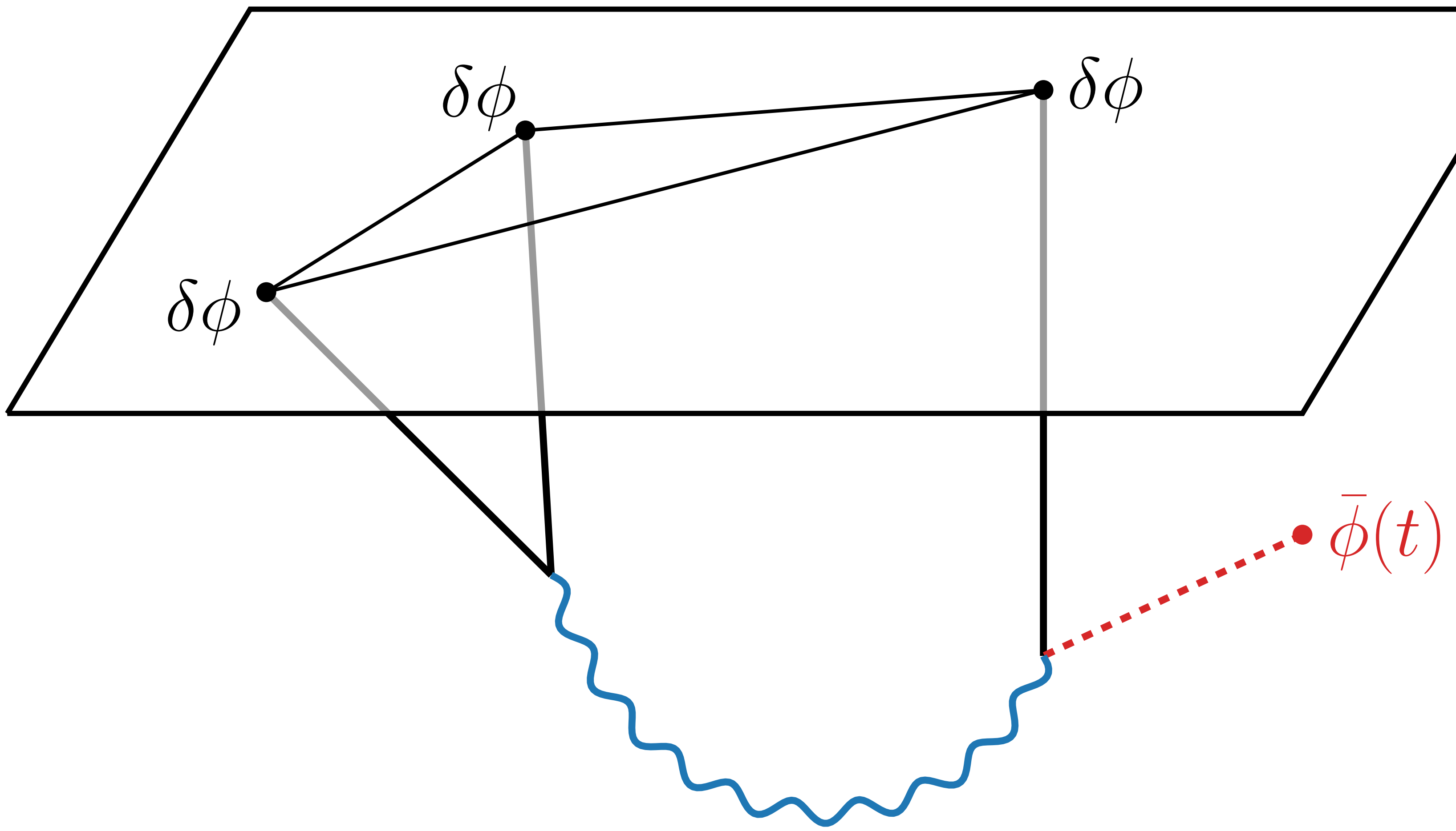
If inflation is correct, then all primordial correlations live on the boundary of an approximate de Sitter spacetime:



- Isometries of dS become conformal symmetries on the boundary.
- This constrains the correlations of weakly interacting particles.

De Sitter → Inflation

Inflationary three-point functions are obtained from de Sitter four-point functions by evaluating one of the external legs on the background:

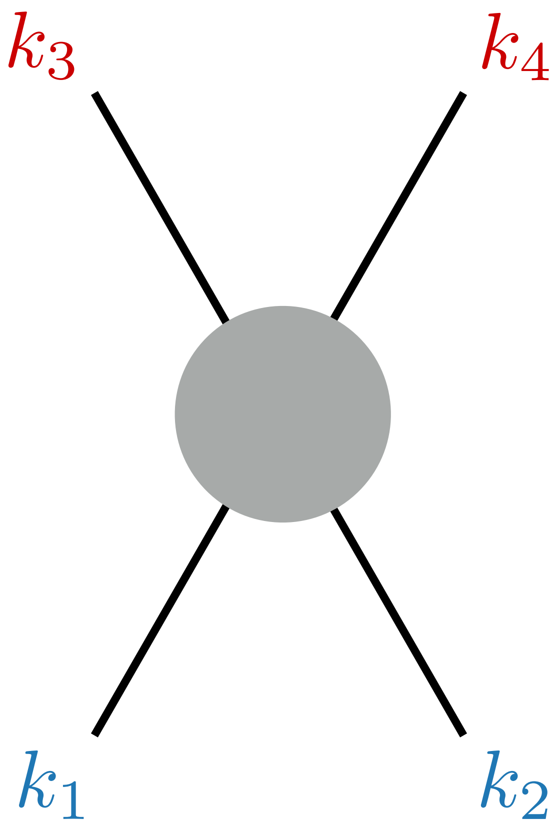


We can therefore study de Sitter four-point functions as the fundamental building blocks of inflationary correlators.

Kinematics

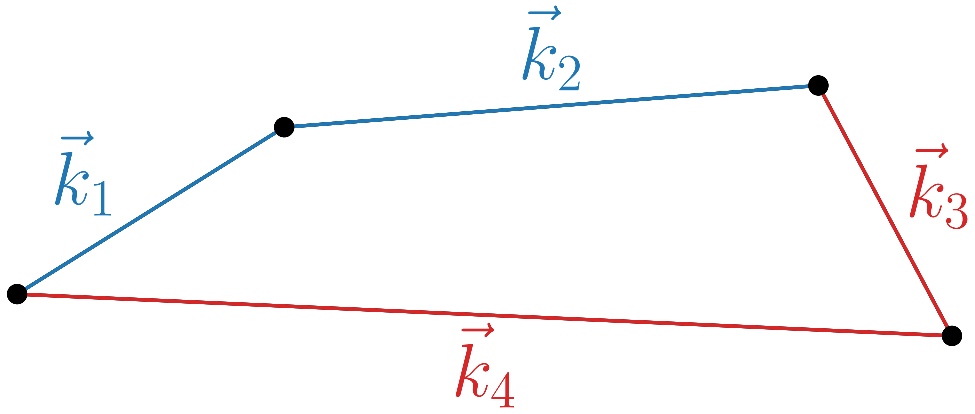
The kinematical data of correlators and amplitudes is similar:

$$A \equiv$$



$$\propto \quad \delta_D\left(E \equiv \sum |\vec{k}_a|\right)$$

$$F \equiv$$



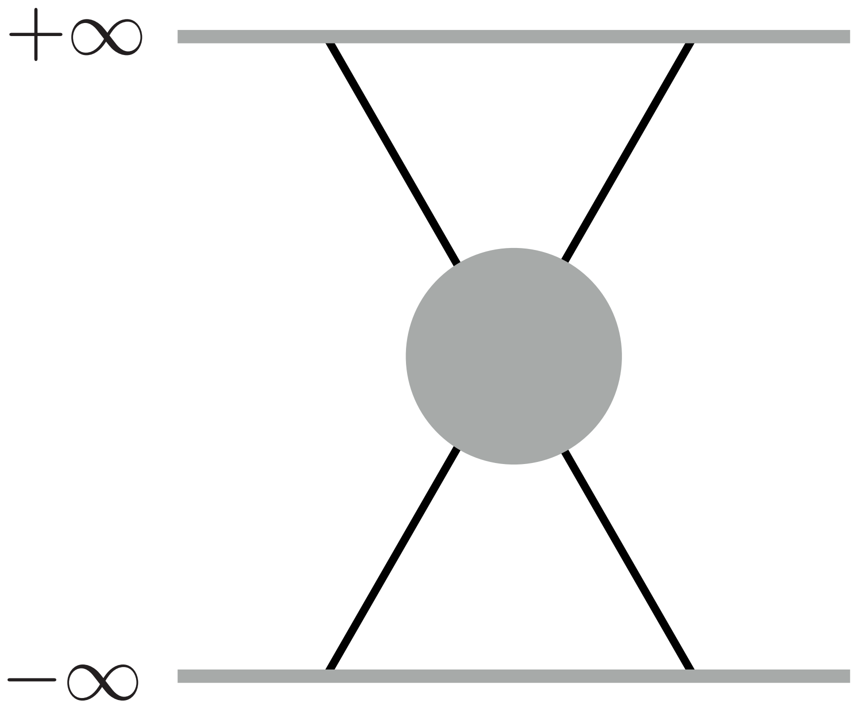
AMPLITUDE

$$\propto \quad \frac{A}{E^n} + \cdots$$

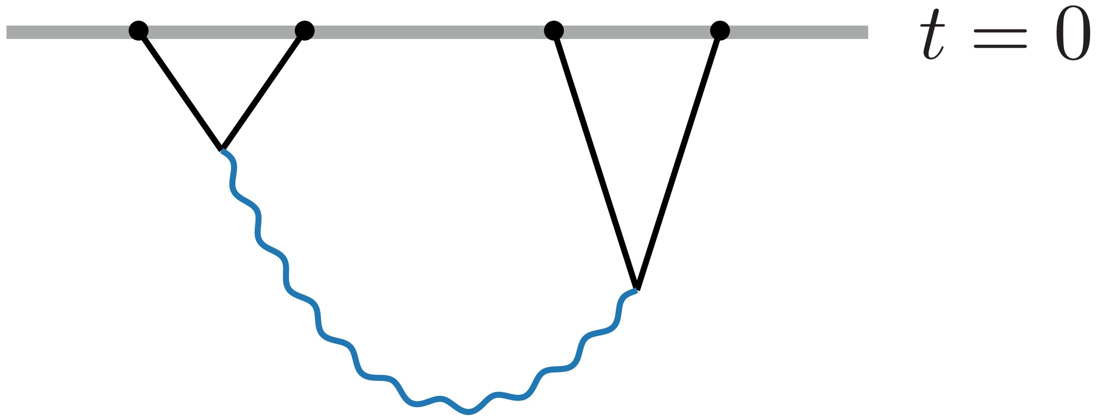
Raju [2012]
Maldacena and Pimentel [2011]

A Flat-Space Limit

The total-energy singularity is a flat-space limit:



$$= \int_{-\infty}^{+\infty} \mathrm{d}t\, e^{iEt} f(t, \vec{k}_n) \;\propto\; \delta_D(E)$$

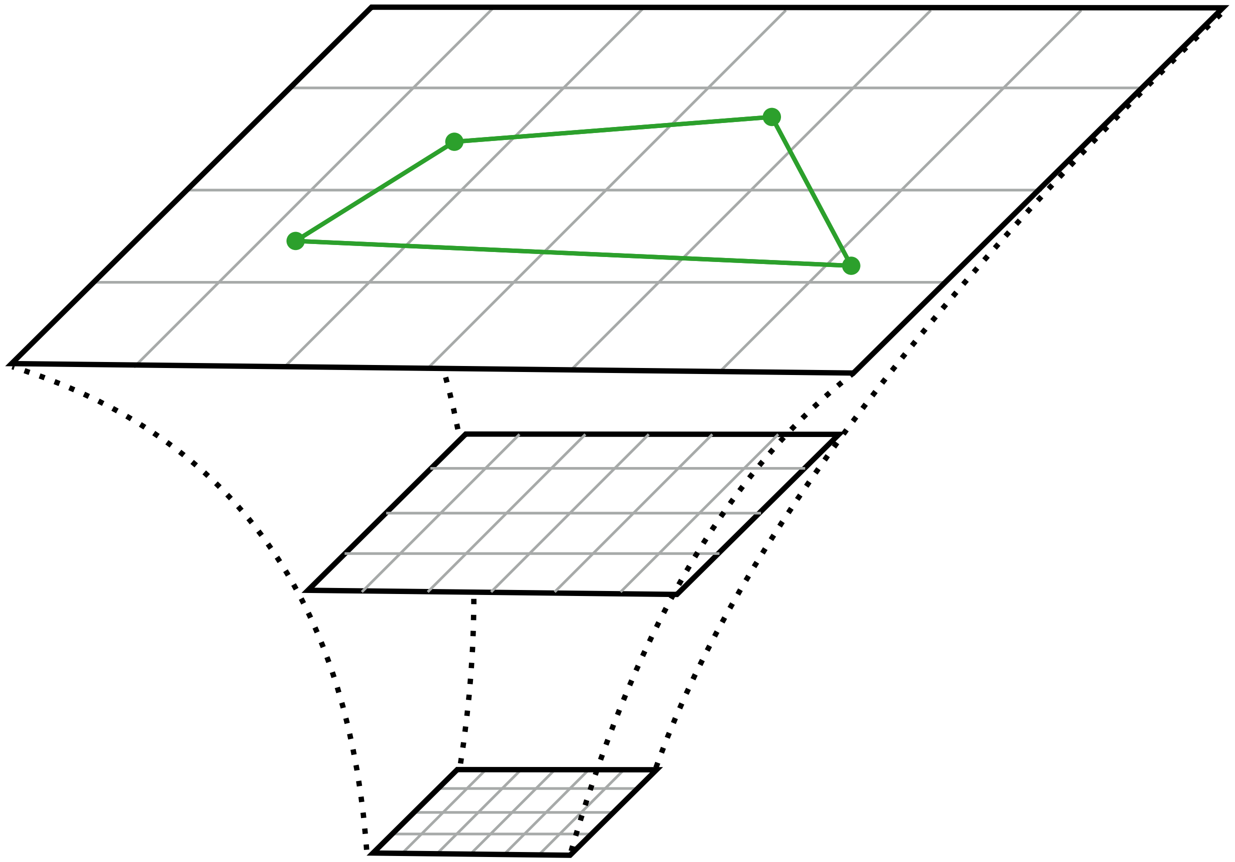


$$= \int_{-\infty}^{0} \mathrm{d}t\, e^{iEt} f(t, \vec{k}_n) \;\propto\; \frac{A}{E^n}$$

Note that this limit cannot be accessed for physical momenta.

Symmetries

If the couplings between particles are weak, then the primordial correlations inherit the symmetries of the quasi-de Sitter spacetime:



$$\boxed{\mathrm{d}s^2 = \frac{-\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2}}{(Ht)^2}}$$

1) **Dilatations**

$$t \to \lambda t$$
$$\vec{x} \to \lambda \vec{x}$$

2) **Special Conformal Transformations**

$$t \to \left(1 - \vec{b}\cdot\vec{x}\right) t$$
$$\vec{x} \to \left(1 - 2\vec{b}\cdot\vec{x}\right)\vec{x} + \left(x^2 - t^2\right)\vec{b}$$

Ward Identities

Invariance under **dilatations** and **SCTs** imply the following **Ward identities:**

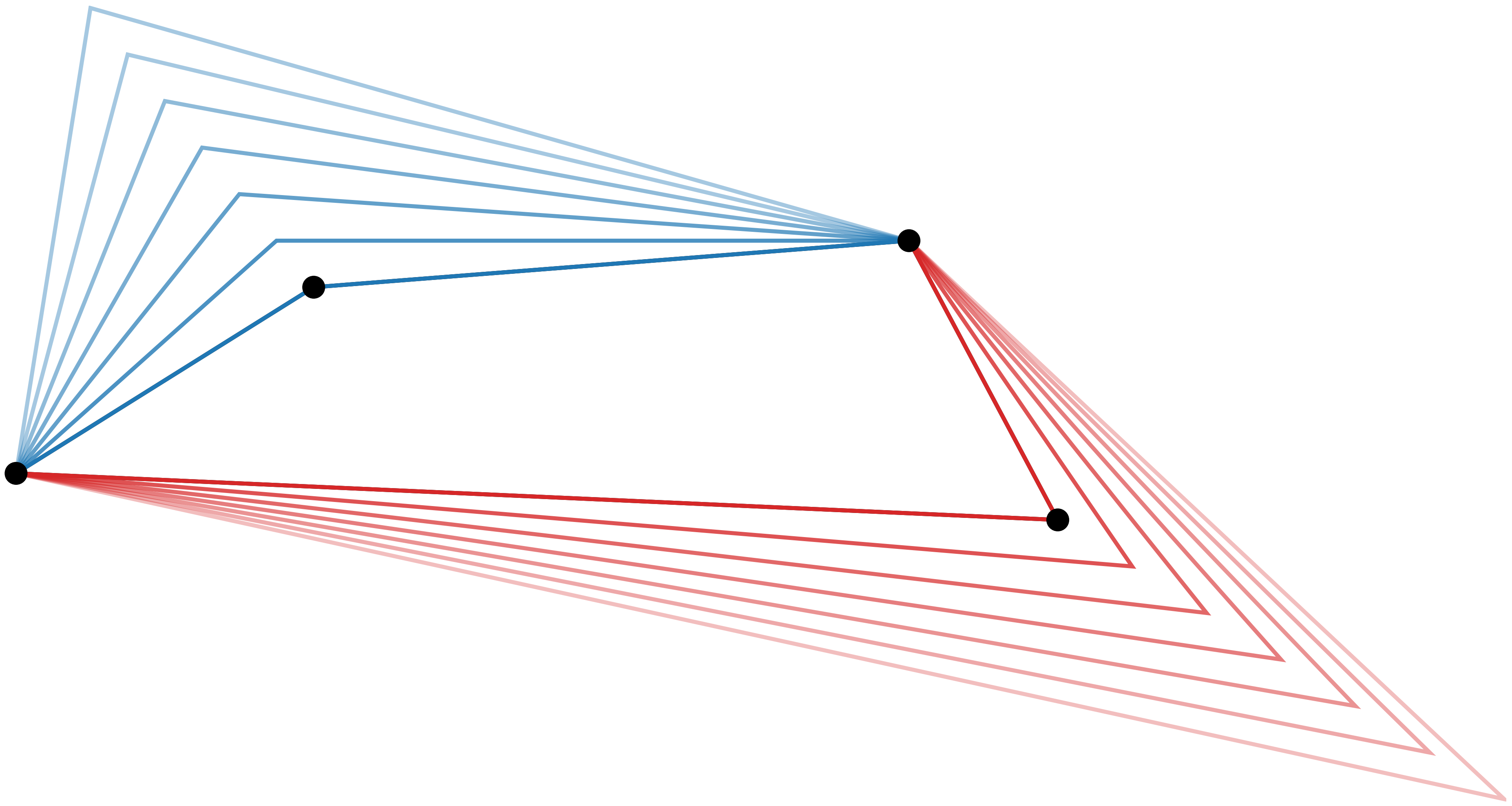
$$0 = \left[9 - \sum_{n=1}^{4}\left(\Delta_n - \vec{k}_n \cdot \partial_{\vec{k}_n}\right)\right] F$$

$$0 = \sum_{n=1}^{4}\left[(\Delta_n - 3)\partial_{\vec{k}_n} - (\vec{k}_n \cdot \partial_{\vec{k}_n})\partial_{\vec{k}_n} + \frac{\vec{k}_n}{2}(\partial_{\vec{k}_n} \cdot \partial_{\vec{k}_n})\right] F$$

Bzowski, McFadden and Skenderis [2014]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Ward Identities

These Ward identities dictate how the strength of the correlations changes as we change the external momenta:

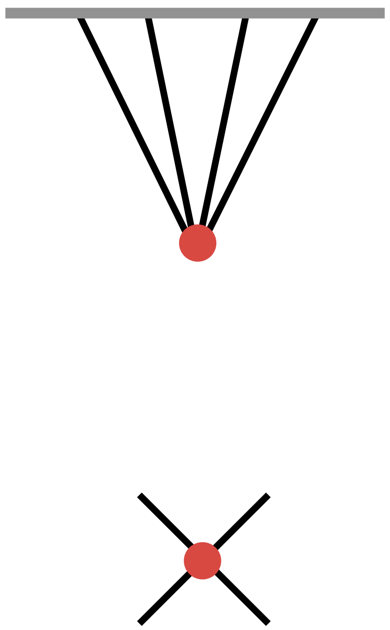


Bzowski, McFadden and Skenderis [2014]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Singularities

The solutions to the Ward identities can be classified by their **singularities**:

$\lim_{E\to 0}$



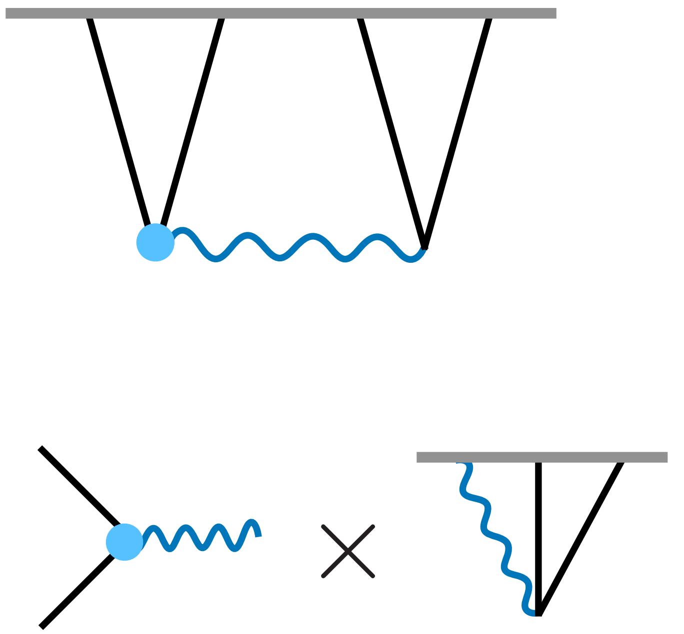
$=$

E^n

EFT
EXPANSION

Contact solutions only have total-energy poles.

$\lim_{E_L\to 0}$



$=$

$(E_L)^m$

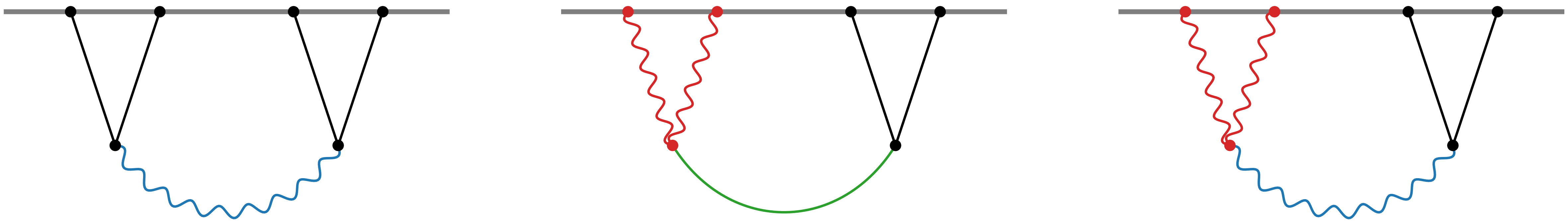
SPIN
EXCHANGE

Exchange solutions have additional partial-energy poles.

Arkani-Hamed, DB, Lee and Pimentel [2018]

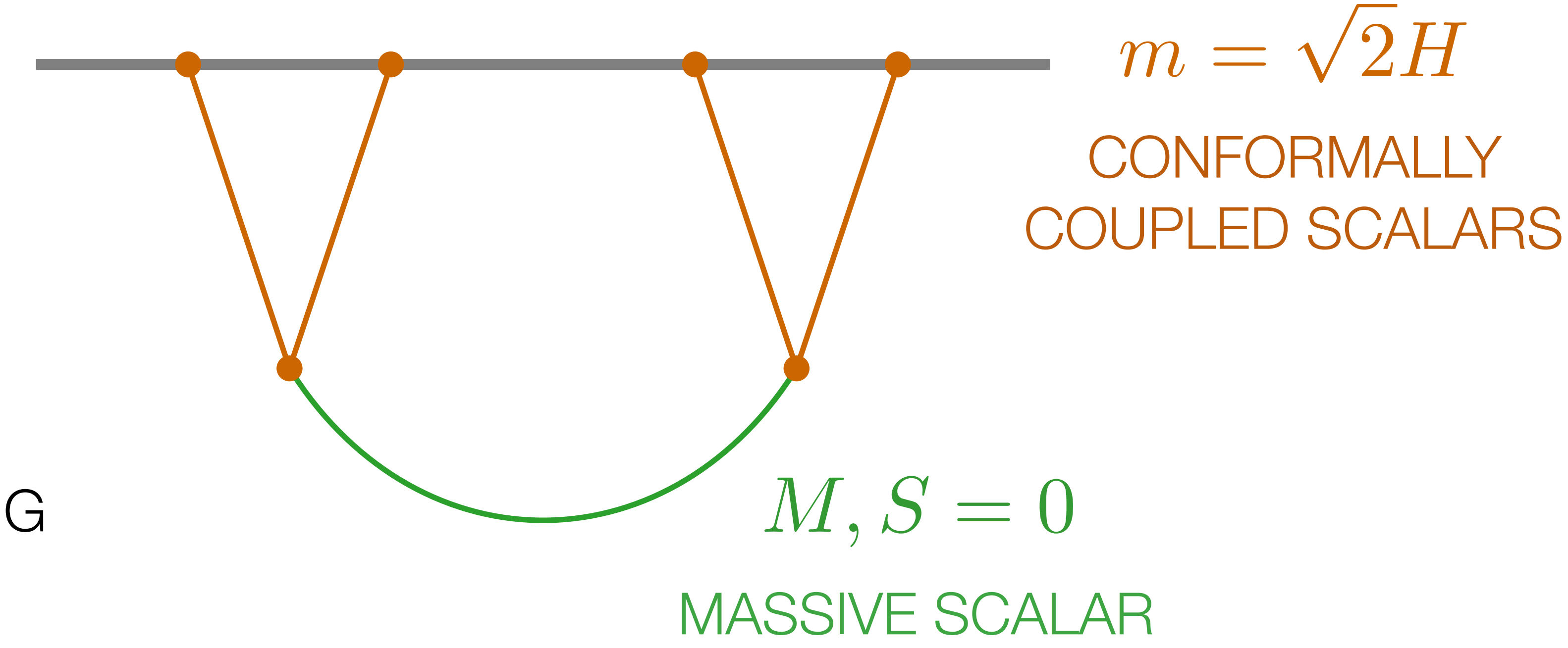
Exchange Solutions

There are **distinct solutions** for distinct microscopic processes during inflation:



$$= \mathcal{D}_n$$

WEIGHT-SHIFTING OPERATORS



Remarkably, all solutions can be reduced to a **unique building block**.

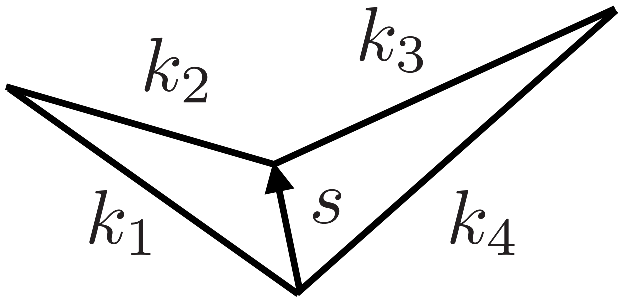
Arkani-Hamed, DB, Lee and Pimentel [2018]

Seed Solution

The explicit solution for the seed function is

$$\boxed{F = \underbrace{\sum_{m,n} c_{mn}(M) u^{2m} \left(\frac{u}{v}\right)^{2n}}_{\text{ANALYTIC}} + \underbrace{e^{-\pi M}\left(e^{iM} g(u,v) + \text{c.c.}\right)}_{\text{NON-ANALYTIC}}}$$

where $u \equiv s/(k_1 + k_2)$ and $v \equiv s/(k_3 + k_4)$.



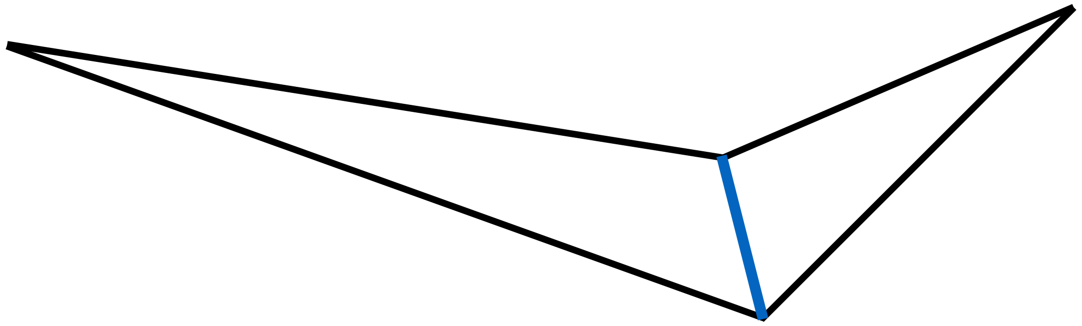
More complicated correlators are generated by **weight-shifting**.

Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2019]

The Collapsed Limit

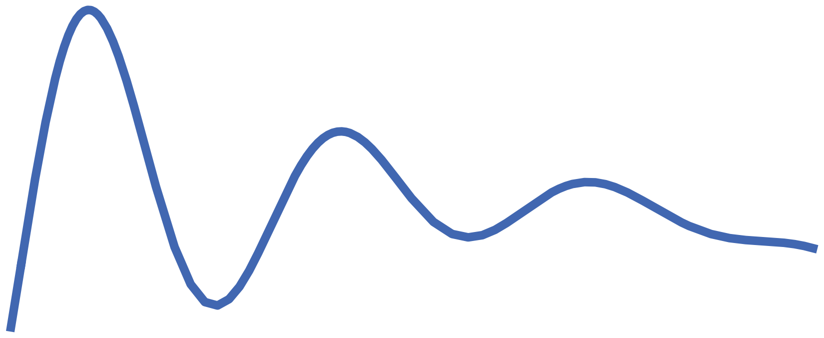
In the collapsed limit, the solution oscillates:

$$\lim_{s\to 0}$$



$$= \sin[M \log(s)]$$

$$=$$

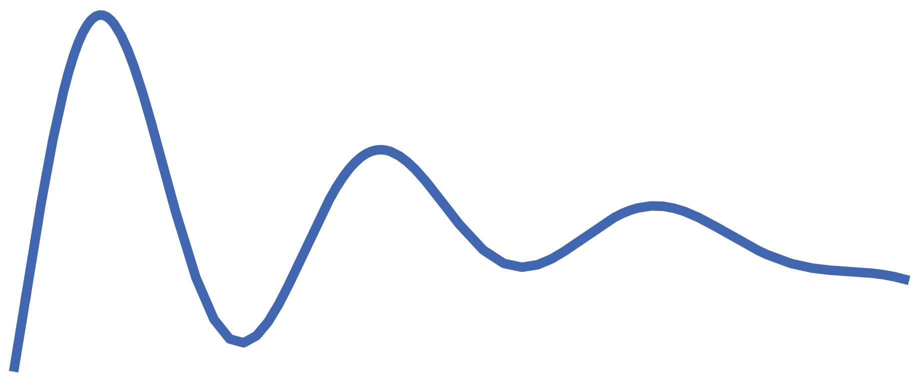


Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Particle Production

These oscillations are a key signature of **particle production** during inflation:

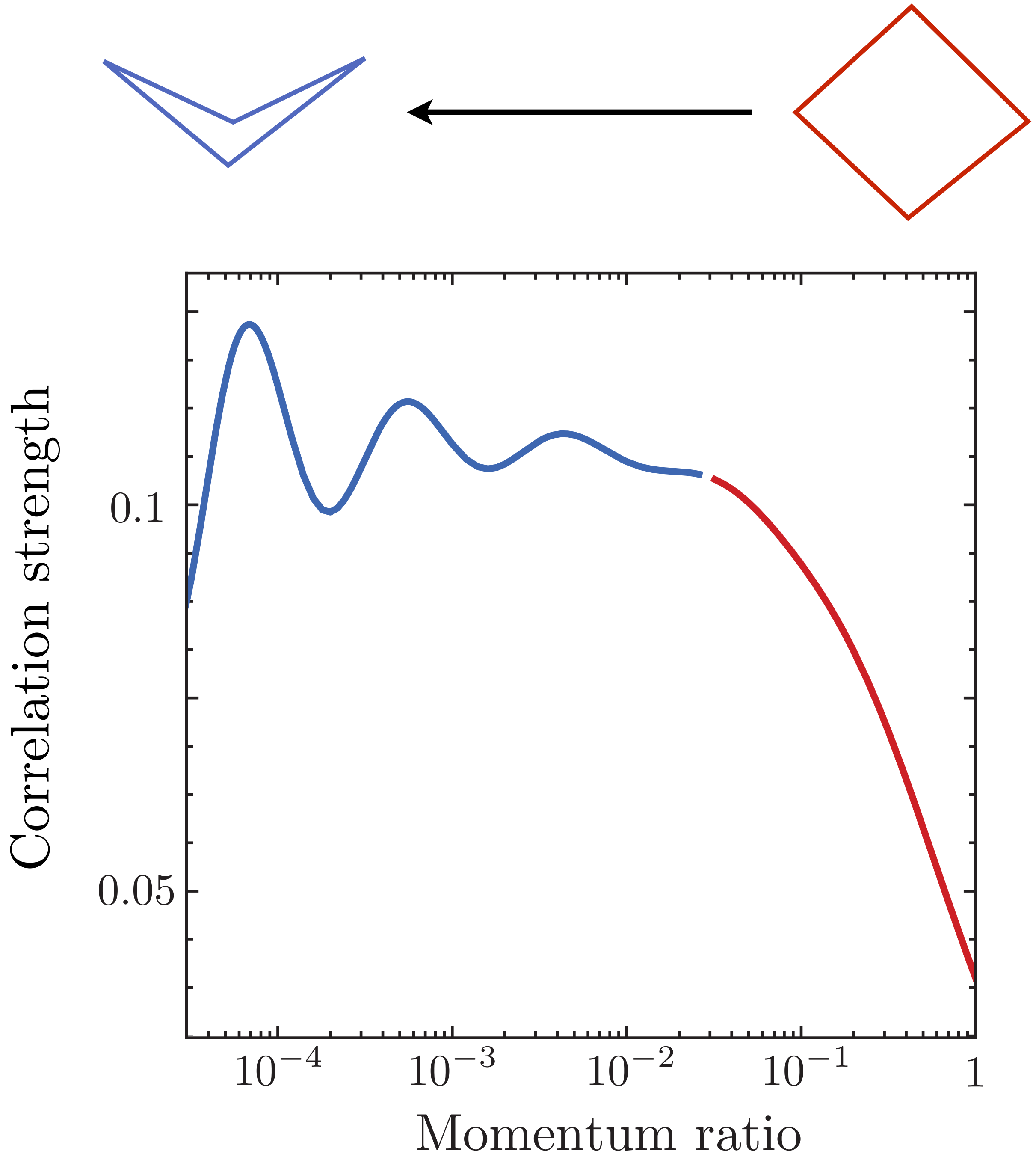
$$e^{iMt} \Rightarrow$$

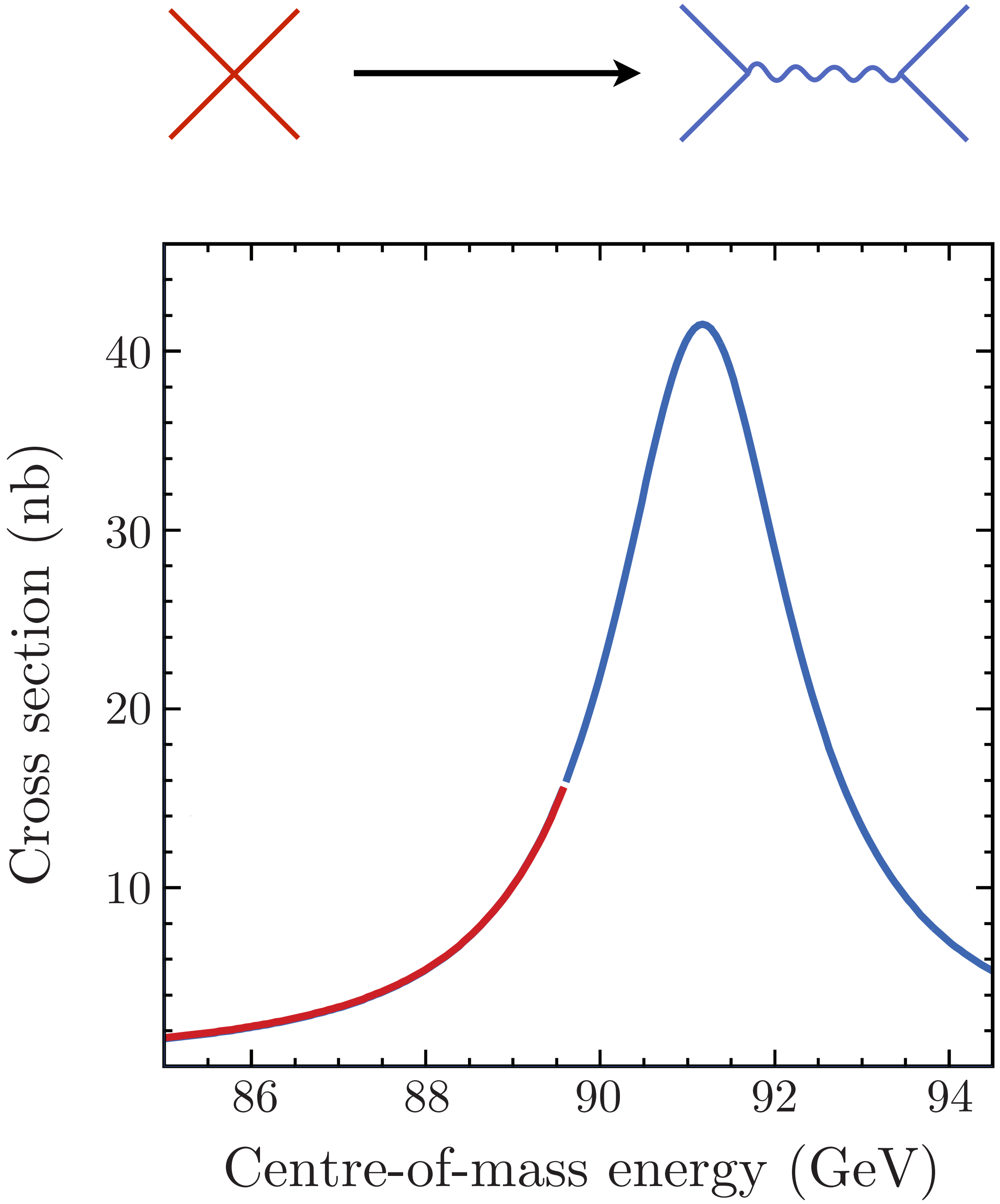


Oscillations in the superhorizon evolution become oscillations in the boundary correlations at late times.

Cosmological Collider Physics

This signal is the analog of **resonances** in collider physics:





Any Questions?

III. New Developments

So far, we have studied the correlations of scalar fields.

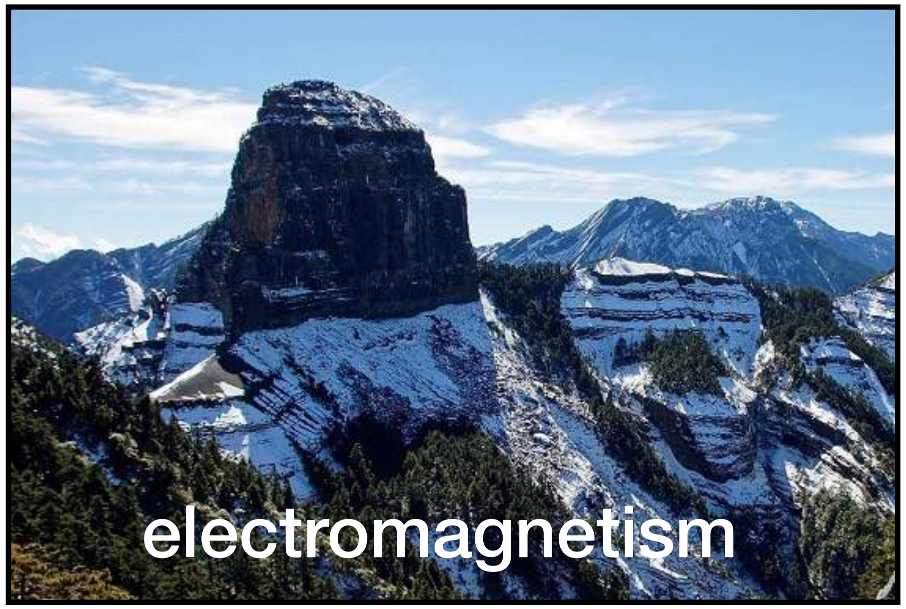
Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2019]

Now, we would like to extend the bootstrap to **spinning correlators**, especially to **massless** fields with spin.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Massless Particles in Flat Space

- Massless bosons mediate long-range forces:



- The interactions of massless particles are highly constrained:

spin 2 = **GR**

spin 1 = **YM**

Weinberg [1964]
Benincasa and Cachazo [2007]
McGady and Rodina [2010]

Massless Particles in Inflation

- Fluctuations of all massless fields are amplified during inflation.
- Every inflationary model has two massless modes:

$\boxed{\phi}$

scalar

$\boxed{\gamma_{ij}}$

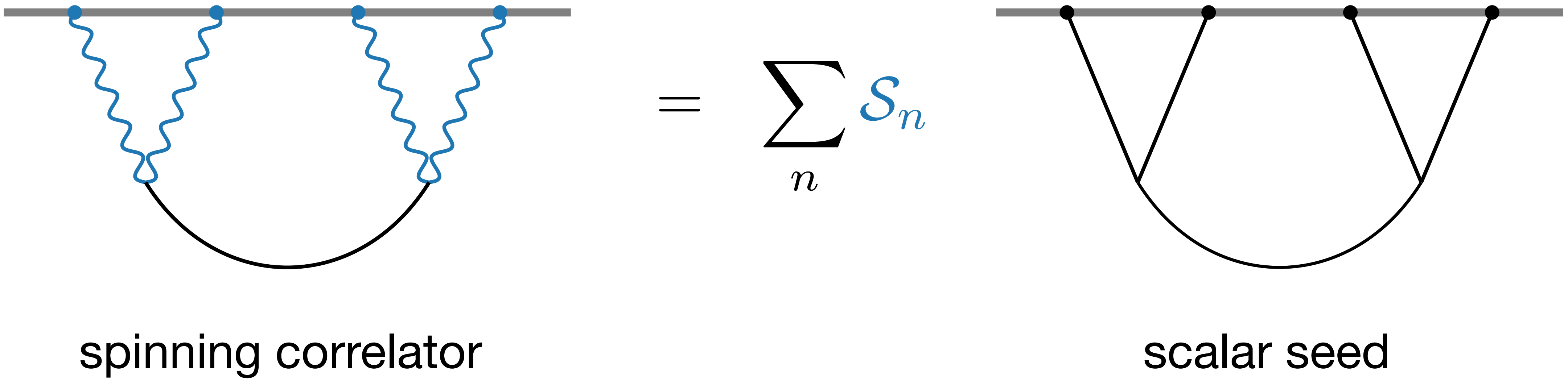
tensor

- Not much is known about tensor correlators beyond 3pt functions.
- Direct computations of spinning correlators are very complicated.
- Bootstrap methods are a necessity, not a luxury.

Two Approaches

In our new paper, we derive a large class of spinning correlators in de Sitter space. We use two different approaches:

1) **Spin-raising operators**



$$= \sum_n \mathcal{S}_n$$

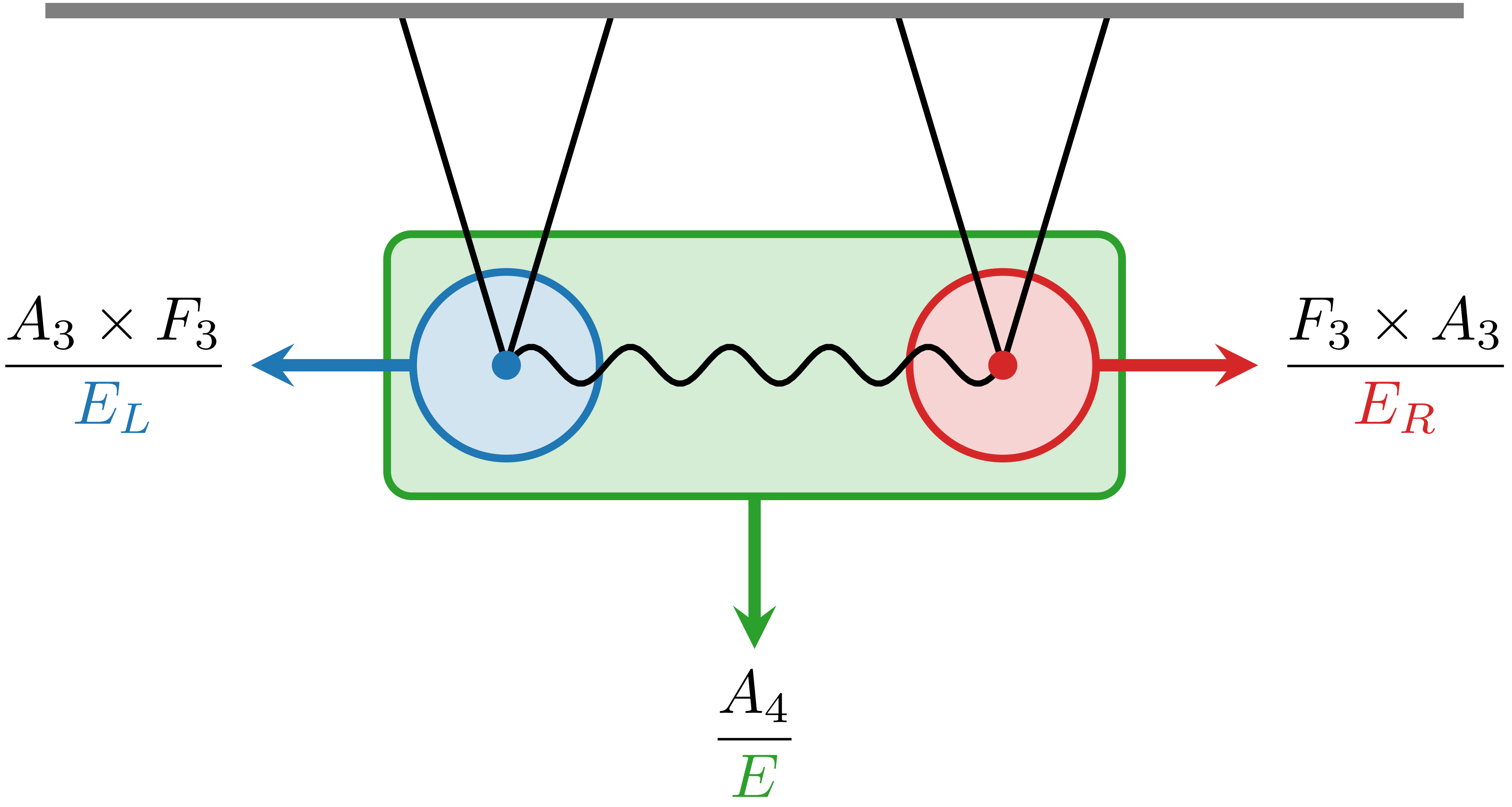
2) **Singularities**

In the following, I will describe the second approach.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Singularities of Cosmological Correlators

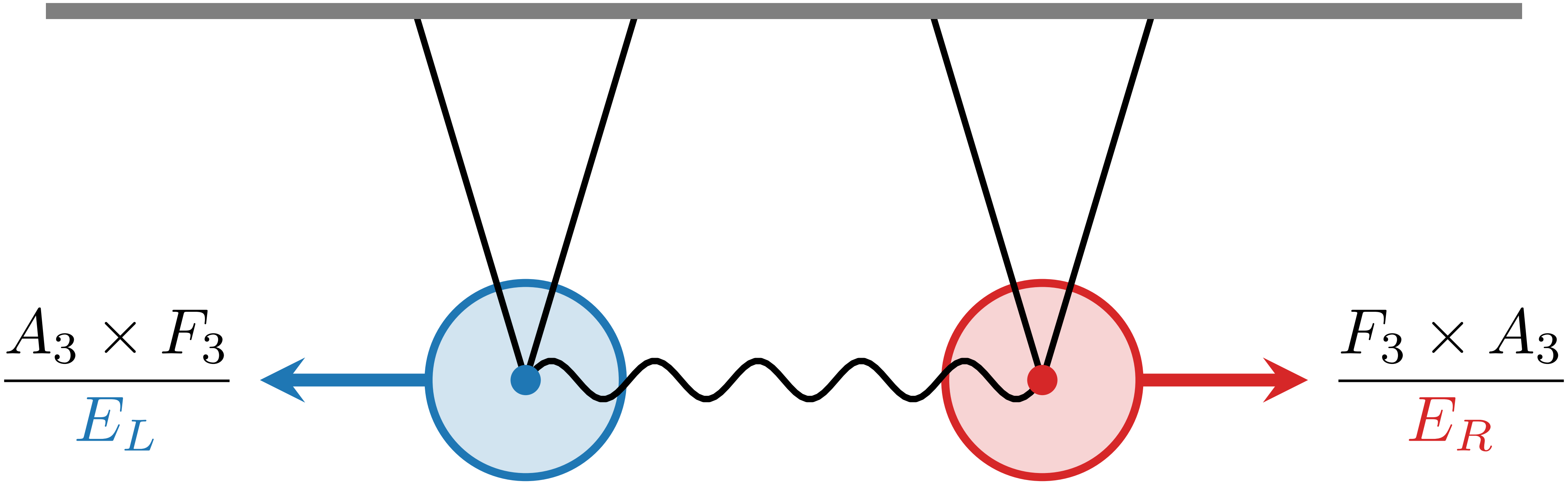
The four-point function is controlled by **three** singularities:



Raju [2012]
Maldacena and Pimentel [2011]
Arkani-Hamed, Benincasa, and Postnikov [2017]

Singularities of Cosmological Correlators

The four-point function is controlled by **three** singularities:



- Correlators of massless spinning particles can be constructed by gluing together these factorisation channels.

cf. BCFW [2005]

- Not all theories will be consistent with locality.

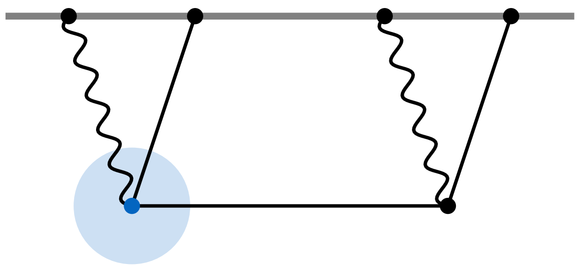
Benincasa and Cachazo [2007]
McGady and Rodina [2014]

A Simple Example

Consider **Compton scattering** in de Sitter space.

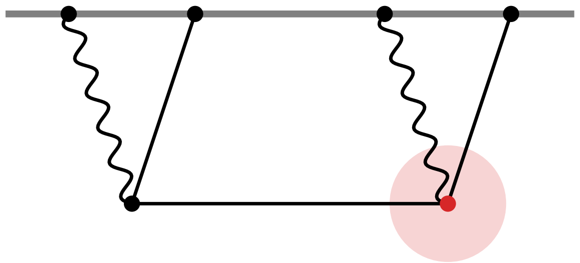
- The factorisation limits of the s-channel are

$$\lim_{E_L\to 0}$$

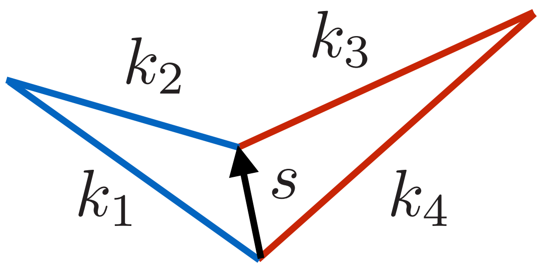


$$= \frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L}\,\frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R(k_{34}-s)}$$

$$\lim_{E_R\to 0}$$



$$= \frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R}\,\frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L(k_{12}-s)}$$



$$E_L \equiv k_{12}+s$$
$$E_R \equiv k_{34}+s$$
$$E \equiv k_{12}+k_{34}$$

- The unique solution that is consistent with these limits is

$$\boxed{\langle J\phi J\phi\rangle_s = \frac{(\vec{\xi}_1\cdot\vec{k}_2)(\vec{\xi}_3\cdot\vec{k}_4)}{E_L E_R E}}$$

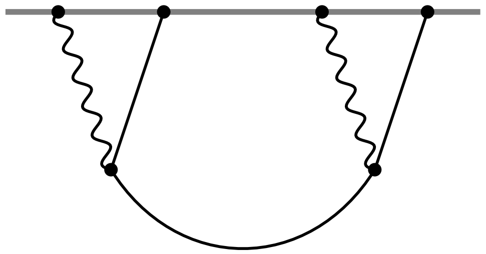
- The total energy singularity has the correct residue.

$$E_L E_R \xrightarrow{E\to 0} S$$

A More Complicated Example

Consider Compton scattering of **gravitons**.

- The solution in the s-channel is

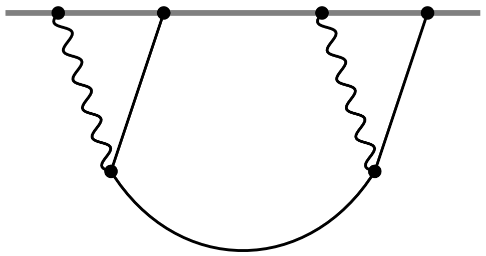


$$
\begin{aligned}
= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \Bigg[& \frac{1}{E_L^2 E_R^2} \overbrace{\left(\frac{2 s k_1 k_3}{E^2} + \frac{2 k_1 k_3 + E_L k_3 + E_R k_1}{E} \right)}^{\text{fixed by factorisation}} \\
& \frac{1}{E_L E_R} \left(\underbrace{\frac{2 k_1 k_3}{E^3}}_{\text{fixed by total energy singularity}} + \underbrace{\frac{k_{13}}{E^2} + \frac{1}{E}}_{\text{fixed by conformal symmetry}} \right) \Bigg]
\end{aligned}
$$

A More Complicated Example

Consider Compton scattering of **gravitons**.

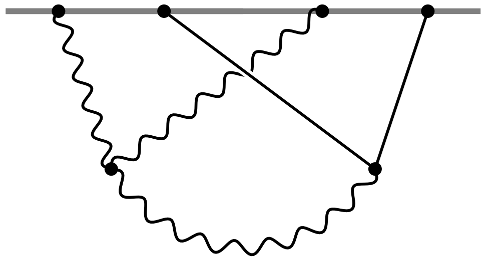
- The solution in the s-channel is



$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2} \left(\frac{2s k_1 k_3}{E^2} + \frac{2k_1 k_3 + E_L k_3 + E_R k_1}{E} \right) \right.$$

$$\left. \frac{1}{E_L E_R} \left(\frac{2k_1 k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E} \right) \right]$$

- The solution in the u-channel is



$$= \frac{1}{E_L^2 E_R^2} \overbrace{\left(\frac{2k_1 k_3}{E^2} + \frac{E_L}{E} \right) \mathcal{N}(\vec{\xi}_1, \vec{\xi}_3, \vec{k}_2, \vec{k}_4)}^{\text{fixed by factorisation}}$$

$$+ \frac{1}{E_L E_R} \left(\underbrace{\frac{2k_1 k_3}{E^3}}_{\text{fixed by total energy singularity}} + \underbrace{\frac{k_{13}}{E^2} + \frac{1}{E}}_{\text{fixed by conformal symmetry}} \right) \mathcal{M}(\vec{\xi}_1, \vec{\xi}_3, \vec{k}_2, \vec{k}_4)$$

One Channel Is Not Enough

So far, we have constructed the individual channels separately.
But, these channels are **not** physical (like Feynman diagrams).

- The sum of all channels is constrained by

1) **Gauge invariance**

$$\boxed{q_i \langle A^i_{\vec{q}} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle = \sum_{a=2}^{4} e_a \langle \phi_{\vec{k}_a + \vec{q}} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle}$$

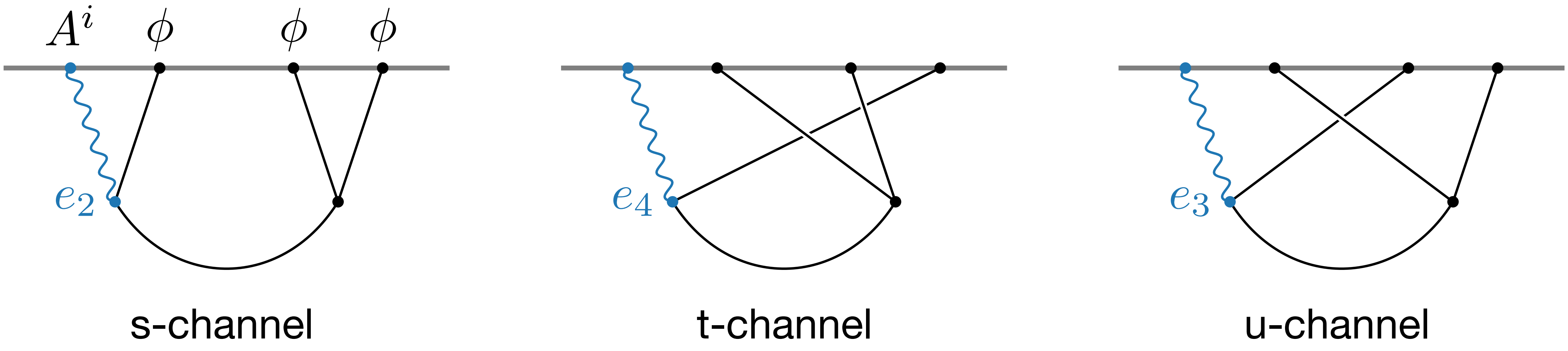
current conservation = **Ward-Takahashi identity**

2) **Lorentz symmetry**

Conformal invariance of the correlator implies Lorentz invariance of the total energy singularity. Neither is automatic!

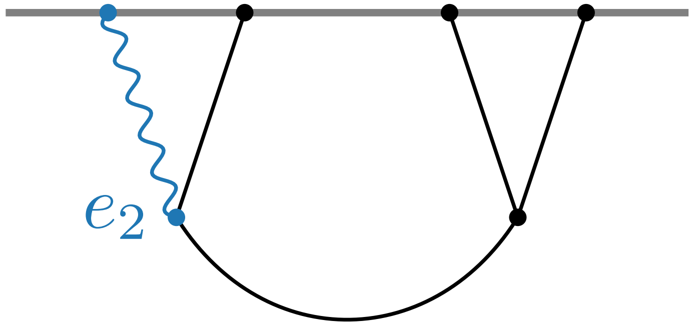
Charge Conservation

Consider the correlator of one photon and three scalars:



s-channel t-channel u-channel

- The flat-space limit of the s-channel is not Lorentz-invariant:



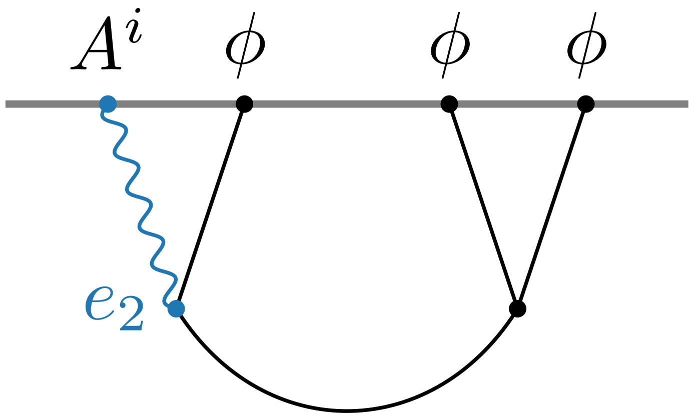
$$\xrightarrow{E\to 0} \frac{e_2}{E}\left(\frac{\langle 12\rangle\langle\bar{2}\bar{4}\rangle\langle 41\rangle}{ST} - \frac{\langle 14\rangle\langle\bar{4}1\rangle}{2k_1}\frac{1}{T}\right)$$

flat-space amplitude

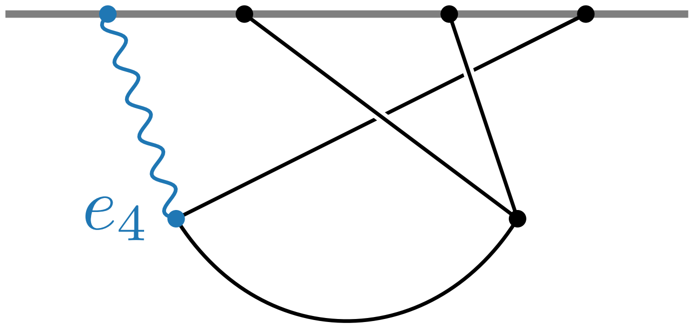
not Lorentz-invariant

Charge Conservation

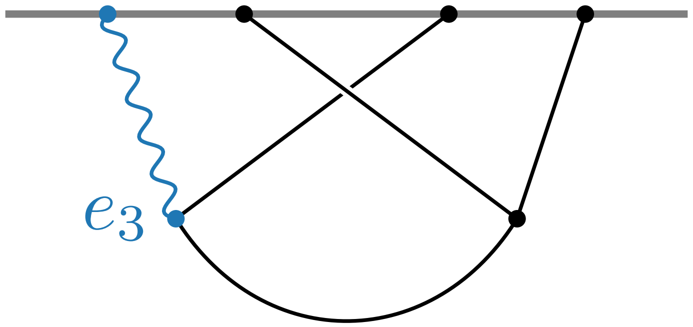
Consider the correlator of one photon and three scalars:



s-channel



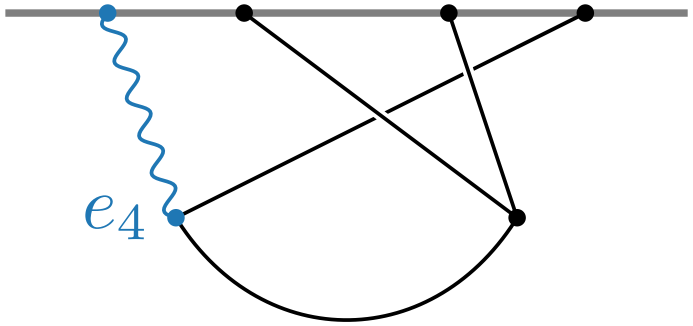
t-channel



u-channel

- Adding the t-channel, we get

$+$

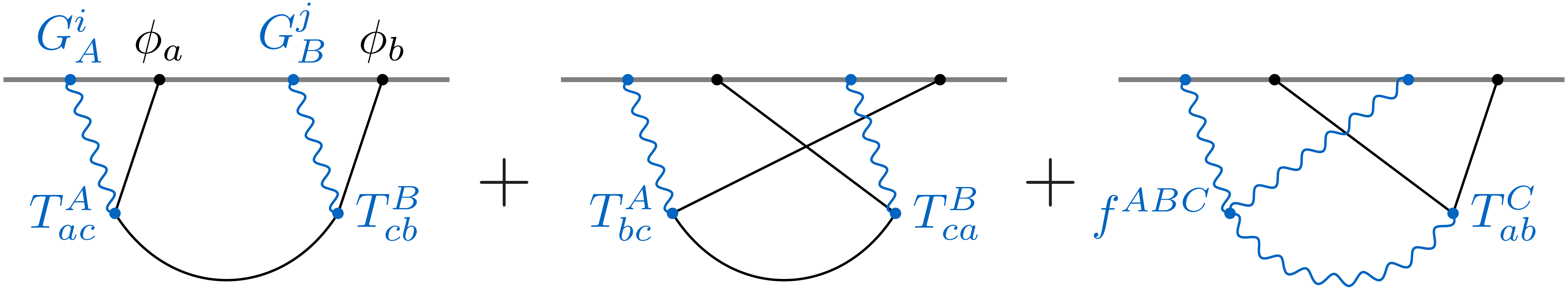


$$\xrightarrow{E\to 0} \frac{1}{E}\left(e_2\frac{\langle 12\rangle\langle\bar{2}\bar{4}\rangle\langle 41\rangle}{ST} - (e_2+e_4)\frac{\langle 14\rangle\langle\bar{4}1\rangle}{2k_1}\frac{1}{T}\right)$$

Lorentz-violation disappears when $\boxed{e_2+e_4=0}$ **charge conservation**

Discovering Yang-Mills (without gauge symmetry)

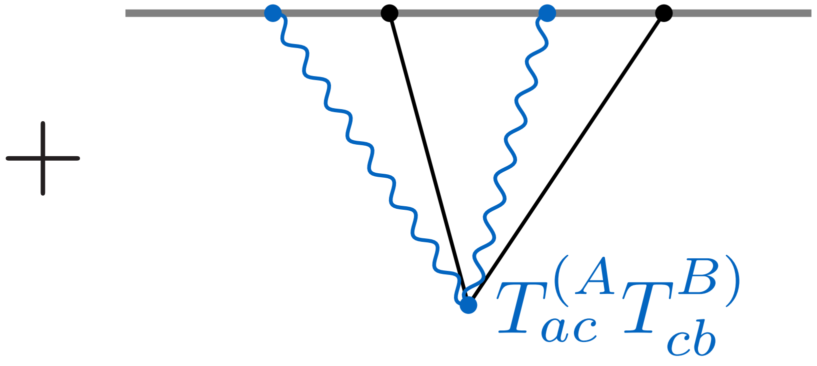
Consider two gluons and two scalars:



s-channel t-channel u-channel

- The sum of all channels is only consistent if the couplings satisfy the **Lie algebra**:

$$\boxed{[T^A, T^B]_{ab} = f^{ABC} T^C_{ab}}$$

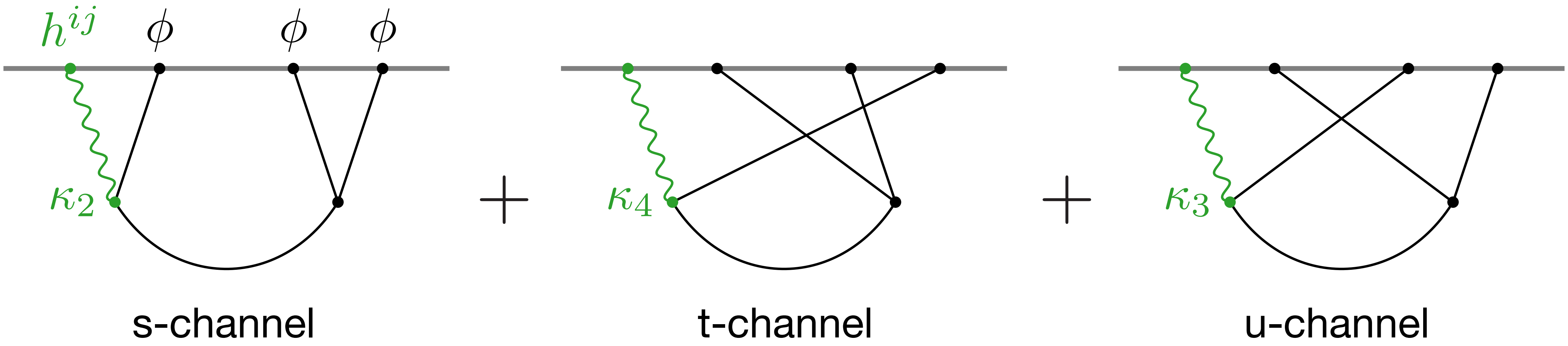


contact

- Consistency also fixes the contact term required by gauge invariance.

Equivalence Principle (without falling elevators)

Consider one graviton and three scalars:

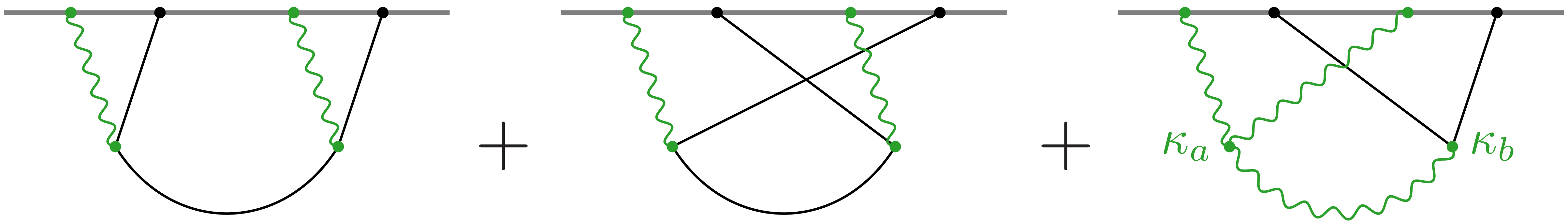


- The individual channels are not consistent.
- The sum of all channels is consistent if and only if

$$\boxed{\kappa_2 = \kappa_3 = \kappa_4}$$

Equivalence Principle (without falling elevators)

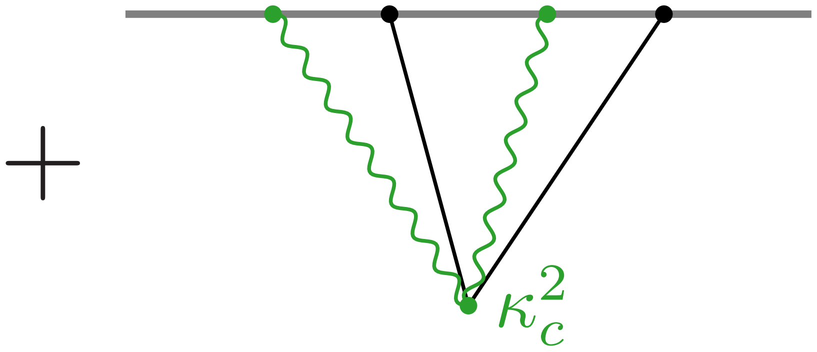
Consider two gravitons and two scalars:



s-channel t-channel u-channel

- The sum of all channels is only consistent if all gravitational couplings are **universal**:

$$\boxed{\kappa_a = \kappa_b = \kappa_c}$$

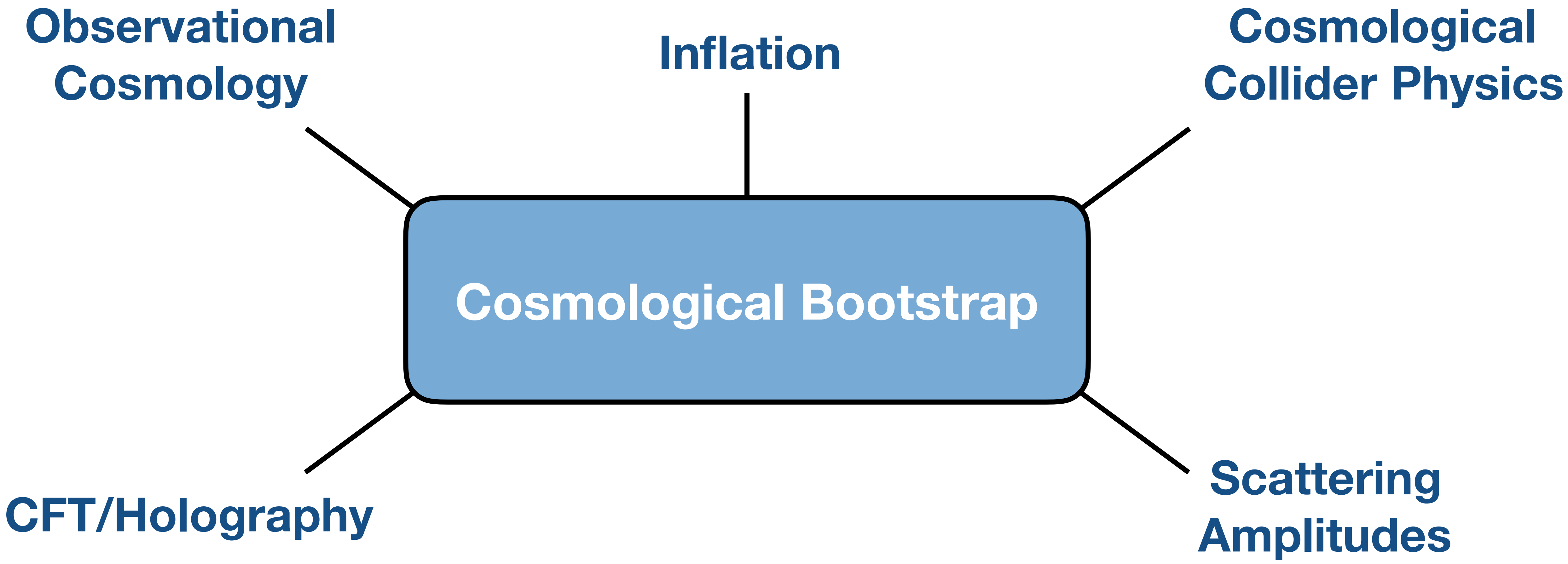


contact

Any Questions?

Conclusions

We have only scratched the surface of a fascinating subject:



Much more remains to be discovered.

Thank you for your attention!