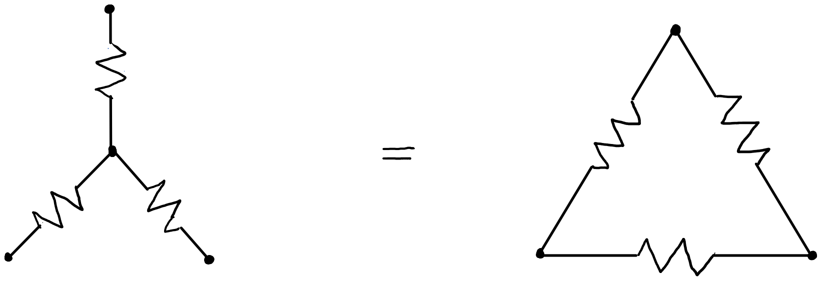
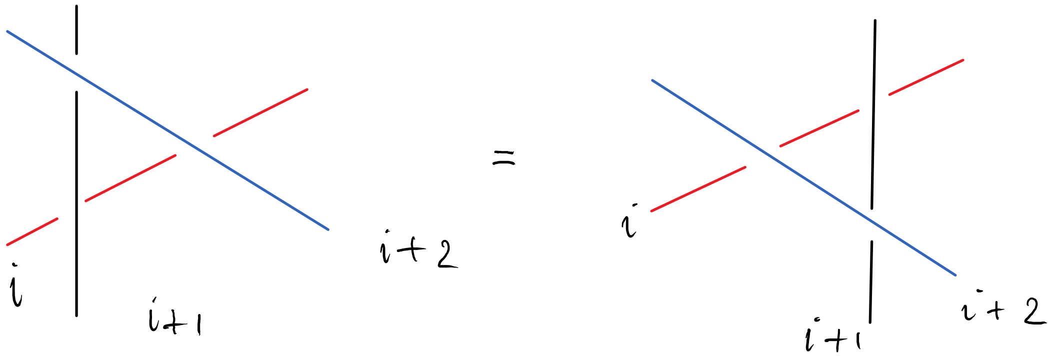
YANG-BAXTER EQ.: HISTORY [Ref 10]

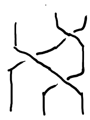
W.R.T. REFS OF 1611.00592

Y – Δ : TRANSFORM OF ELECTRICAL CIRCUITS [KENNELLY 1899]



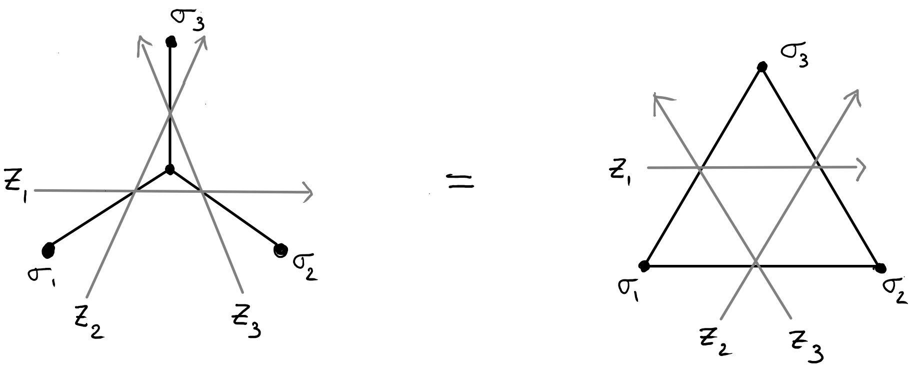
REIDEMEISTER MOVE : EQUIVALENCE OF KNOTS [1926]

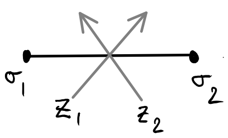


BRAID :  : $b_i \, b_{i+1} \, b_i = b_{i+1} \, b_i \, b_{i+1}$: 

☆ – Δ RELATION : ISING [ONSAGER 1944]

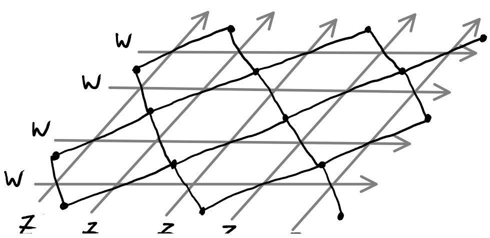
SPECTRAL PARAMETER

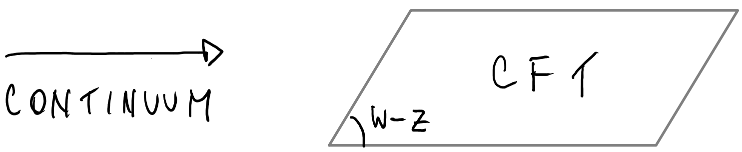


WITH 

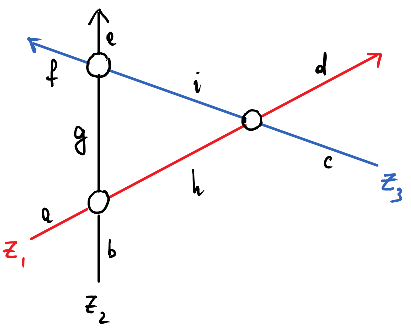
$$= 1 + \left(e^K - 1\right) \delta_{\sigma_1 \sigma_2} \, , \quad \frac{e^K - 1}{\sqrt{2}} = \frac{\sin\left(\frac{1}{4}(z_1 - z_2)\right)}{\sin\left(\frac{1}{4}\left(\pi - (z_1 - z_2\right)\right)}$$

Rem : CRITICALITY REQUIRES z = ANGLE IN EMBEDDED GRAPH :
(ALSO: POSITIVE WEIGHTS)

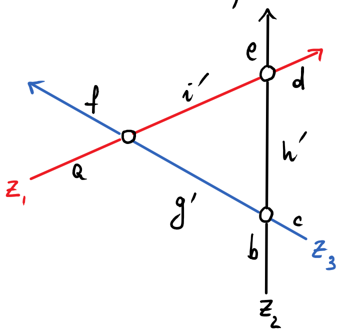




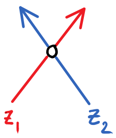
VERTEX MODELS [BAXTER 1971, SEE 9605187 FOR MORE]



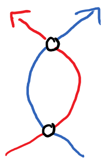
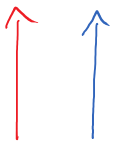
$=$

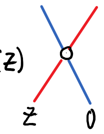


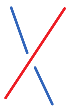
,

R - MATRIX : $R(z_1 - z_2) =$ 

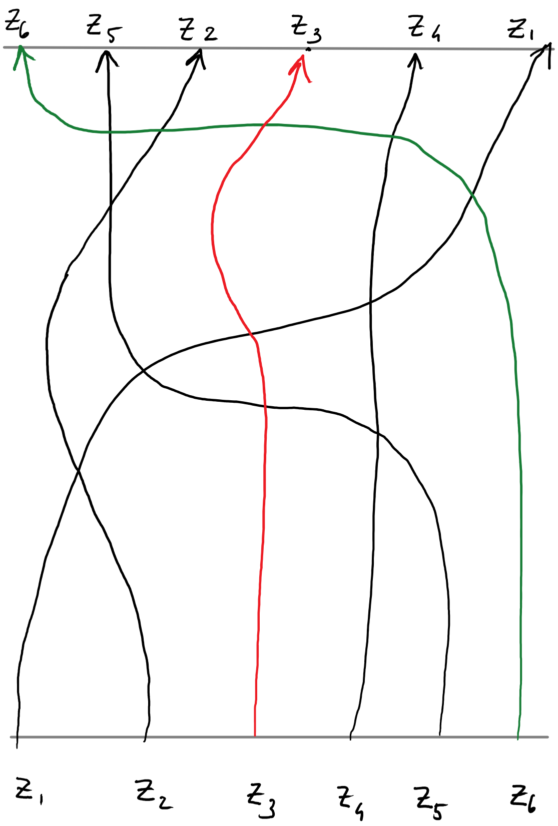
$\in \mathrm{End}(V_1 \otimes V_2)$, $V = G$ - REP.

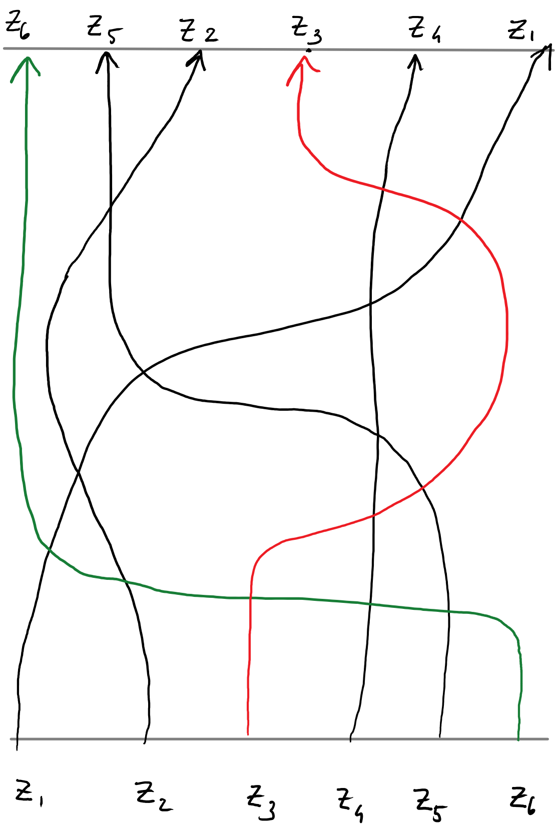
INVERSION REL :  $=$  : $R(-z) = R(z)^{-1}$.

Rem : $\lim_{z \to \infty} f(z)$ 

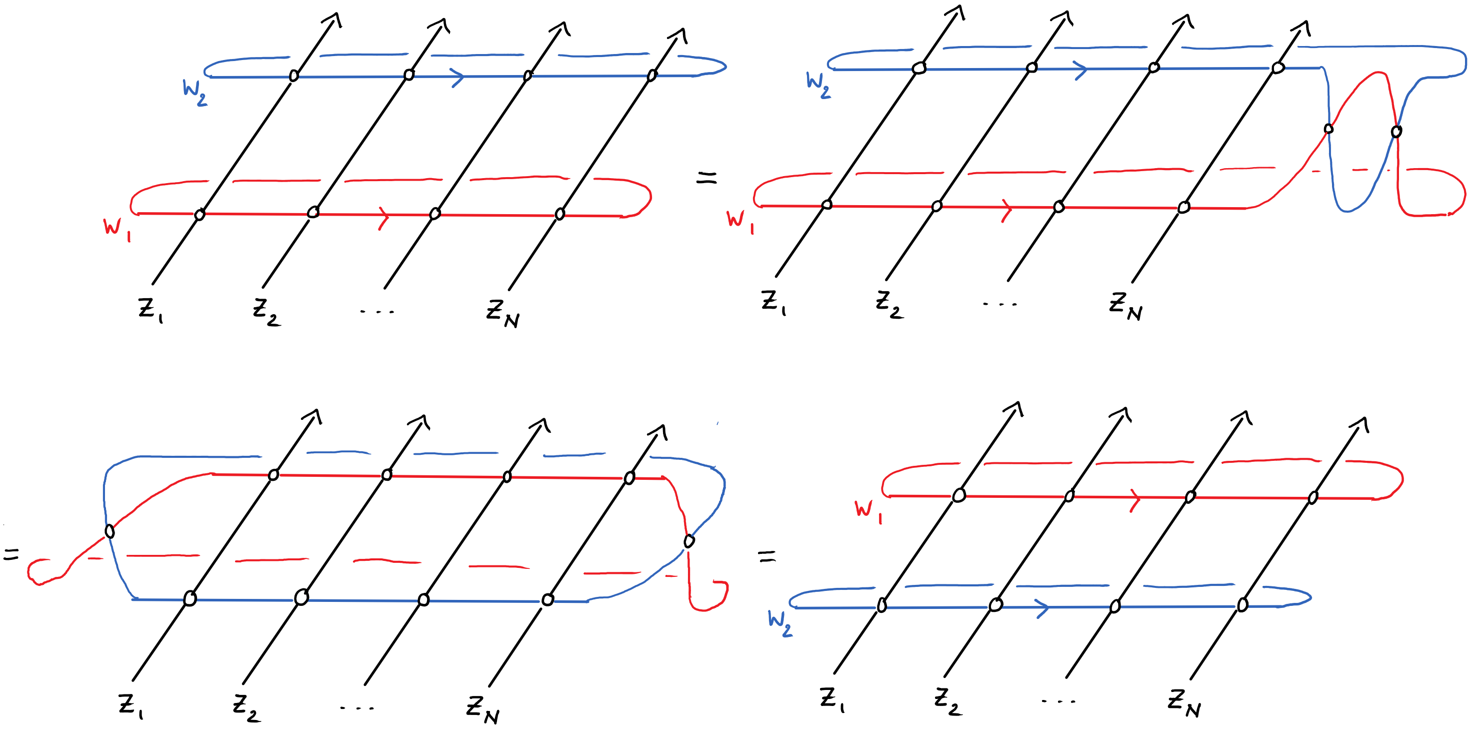
$=$  $\longrightarrow$ KNOT INVARIANT. [Ref : 9]

Z - INVARIANCE





QUANTUM INTEGRABILITY = COMMUTING TRANSFER MATRICES

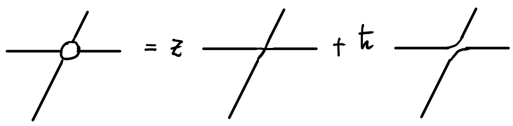


GENERATING FUNCTION OF CONSERVED QUANTITIES (HAMILTONIAN, TRANSLATION, ...)

EXAMPLE : XXX ("RATIONAL". CASE $N=2$: $P_{ij} = \frac{1}{2} + 2\,\vec{S}_i \cdot \vec{S}_j$)

QUANTUM PARAMETER

$$V_i = \mathbb{C}^N \,, \qquad R(z) = z + \hbar P \qquad \Rightarrow \qquad [R(z),\, g \otimes g] = 0 \,, \quad g \in GL_N$$



CHECK YBE $\quad R_{12}(z)\, R_{13}(z+w)\, R_{23}(w) = R_{23}(w)\, R_{13}(z+w)\, R_{12}(z)$

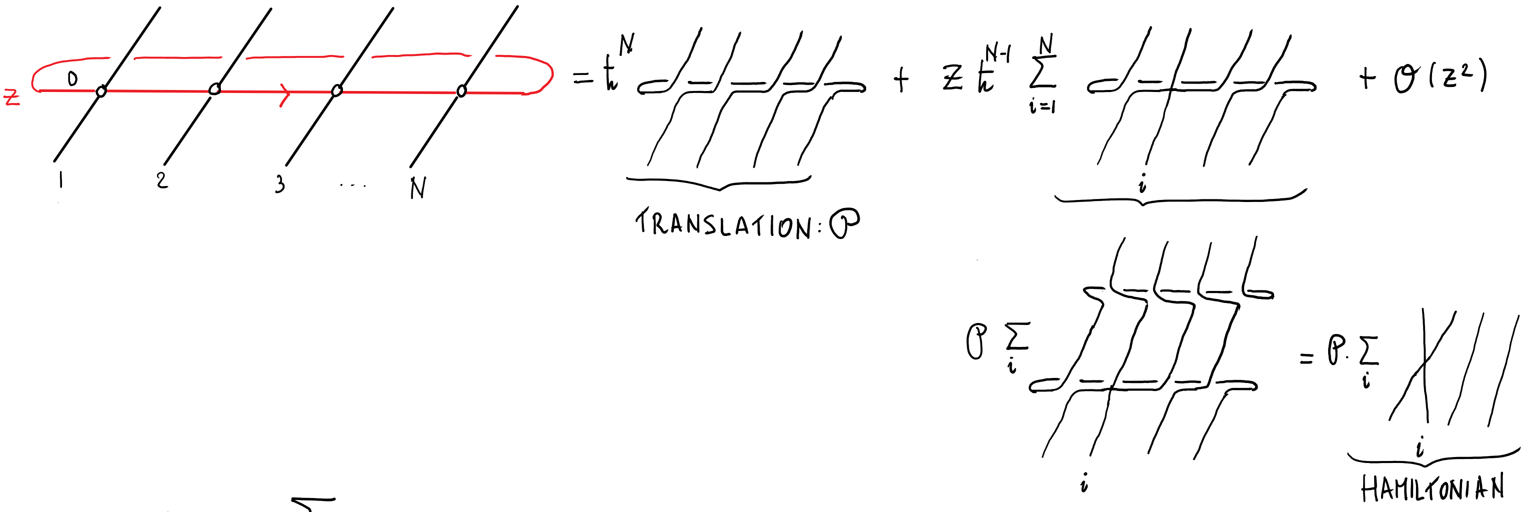
$$(z + \hbar P_{12})(z+w + \hbar P_{13})(w + \hbar P_{23}) = (w + \hbar P_{23})(z+w + \hbar P_{13})(z + \hbar P_{12})$$

$\hbar^0$: ✓ , $\hbar^1$: ✓ ,

$\hbar^2$: $w P_{12} P_{13} + (z+w) P_{12} P_{23} + z\, P_{13} P_{23} = z\, P_{23} P_{13} + (z+w) P_{23} P_{12} + w\, P_{13} P_{12}$ ✓

$\hbar^3$: $P_{12} P_{13} P_{23} = P_{23} P_{13} P_{12}$ ✓ , USE $P_{ij} P_{jk} = P_{jk} P_{ik}$, $P_{ij}^2 = 1$.

HEISENBERG CHAIN:



$\rightarrow \quad H = \sum_i \sigma_i^1\sigma_{i+1}^1 + \sigma_i^2\sigma_{i+1}^2 + \sigma_i^3\sigma_{i+1}^3$

YANGIAN: SOLUTIONS TO YBE, $\exists\ \forall\ V, \mathfrak{g}$. [Refs 16,17]

PROOF [DRINFELD] OF $\exists$ UNIVERSAL R-MATRIX

$\mathcal{R}(z) = 1 + \sum_{n\geq 0} (\hbar/z)^{n+1}\, \mathcal{R}_n \in Y(\mathfrak{g})\otimes Y(\mathfrak{g})$, WITH

$Y(\mathfrak{g})$ EXTENSION OF $\mathfrak{g}$ AND ALL SOLUTIONS $= (\rho_V\otimes\rho_V)\,\mathcal{R}(z)$.

EX: XXZ $\quad R_{ij}(z) = q^{-\frac{1}{2}\sigma_i^3\sigma_j^3} - z^2 q^{\frac{1}{2}\sigma_i^3\sigma_j^3} - 2z\,(q-q^{-1})(\sigma_i^1\sigma_j^1 + \sigma_i^2\sigma_j^2)$.

$\rightarrow \quad H = \sum_i \sigma_i^1\sigma_{i+1}^1 + \sigma_i^2\sigma_{i+1}^2 + \frac{1}{2}(q-q^{-1})\,\sigma_i^3\sigma_{i+1}^3$.

NO SU_2-INV., ONLY U_1. ("TRIGONOMETRIC")

$\rightarrow$ QUANTUM GROUPS $\mathcal{U}_q(\hat{sl}_2)$