$$\{\chi_j, \chi_k\} = \delta_{jk}$$

$$N \gg 1$$

$$\lambda = \beta J$$

$\lambda \gg 1$ – holographic

$$H = \frac{1}{4!}\sum_{jklm} J_{jklm}\,\chi_j\chi_k\chi_l\chi_m$$

$$\frac{1}{3!}\sum_{k,l,m} \overline{J_{jklm}^2} = J^2$$

AdS basic (Wikipedia)

$$-X_1^2 - X_2^2 + \sum_{i=3}^{n+1} X_i^2 = -\alpha^2$$

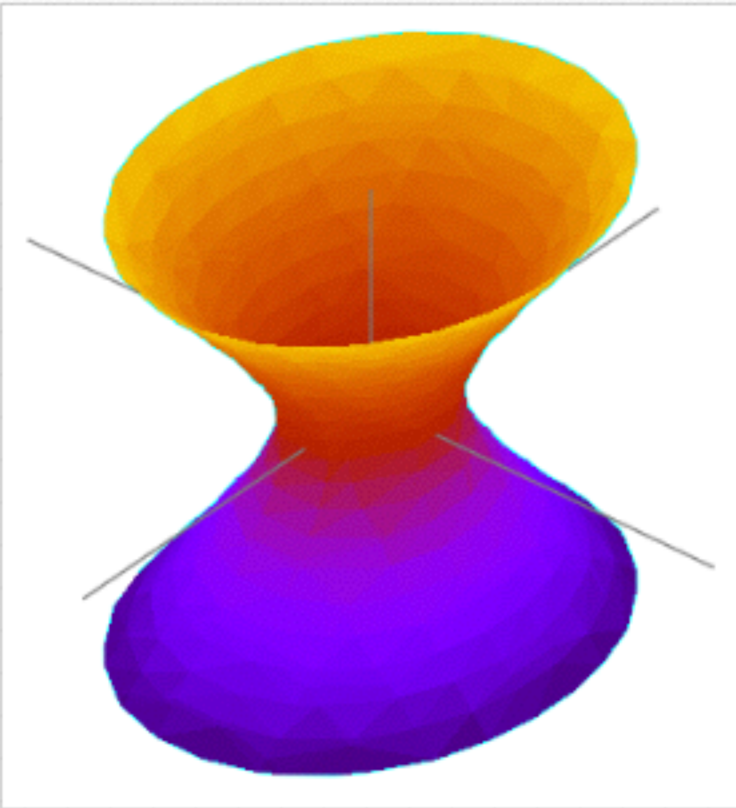


Image of (1 + 1)-dimensional anti-de Sitter space embedded in flat (1 + 2)-dimensional

Global coordinates [edit]

AdS_n is parametrized in global coordinates by the parameters $(\tau, \rho, \theta, \varphi_1, \cdots, \varphi_{n-3})$

$$\begin{cases} X_1 = \alpha \cosh\rho \cos\tau \\ X_2 = \alpha \cosh\rho \sin\tau \\ X_i = \alpha \sinh\rho \, \hat{x}_i \qquad \sum_i \hat{x}_i^2 = 1 \end{cases}$$

where $\hat{x}_i$ parametrize a S^{n-2} sphere. i.e. we have $\hat{x}_1 = \sin\theta \sin\varphi_1 \ldots \sin\varphi_{n-3}$, $\hat{x}_2 = \sin\theta \sin\varphi_1 \ldots \cos\varphi_{n-3}$ etc. The AdS_n metric in these coordinates is:

$$ds^2 = \alpha^2(-\cosh^2\rho \, d\tau^2 + d\rho^2 + \sinh^2\rho \, d\Omega_{n-2}^2)$$

where $\tau \in [0, 2\pi]$ and $\rho \in \mathbb{R}^+$. Considering the periodicity of time τ and in order to avoid closed timelike curves (CTC), one should take the universal cover $\tau \in \mathbb{R}$. In the limit $\rho \to \infty$ one can approach to the boundary of this space-time usually called AdS_n conformal boundary.

With the transformations $r \equiv \alpha \sinh\rho$ and $t \equiv \alpha\tau$ we can have the usual AdS_n metric in global coordinates:

$$ds^2 = -f(r)\, dt^2 + \frac{1}{f(r)}\, dr^2 + r^2 \, d\Omega_{n-2}^2$$

where $f(r) = 1 + \frac{r^2}{\alpha^2}$

$$N \gg 1$$

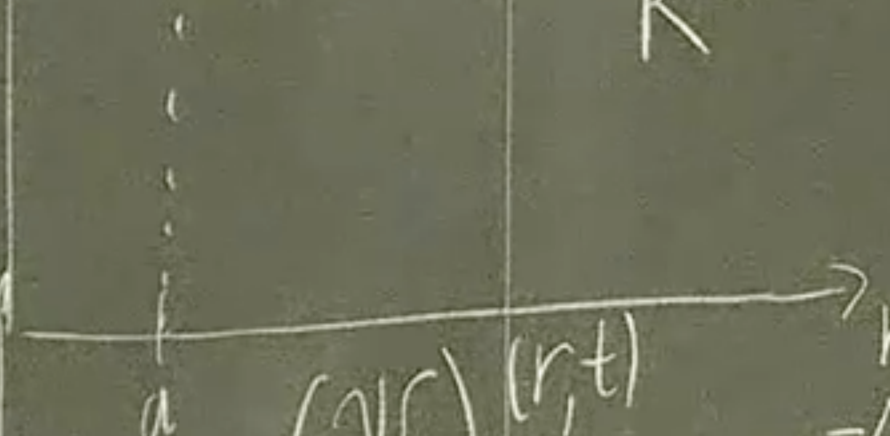
$$\lambda = \beta J$$

$$\lambda \gg 1 - \text{holographic}$$

$$\sum_{klm} J_{jklm}\, \chi_k \chi_l \chi_m = \mathcal{O}_j$$

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + \cancel{r^2 d\Omega^2}$$

$$f(r) = \frac{r(r-a)}{R^2} \sim r(r-1)$$



$$\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}(r,t) \sim r^{-\Delta}\begin{pmatrix}1 \\ 1\end{pmatrix}\mathcal{O}(t)$$

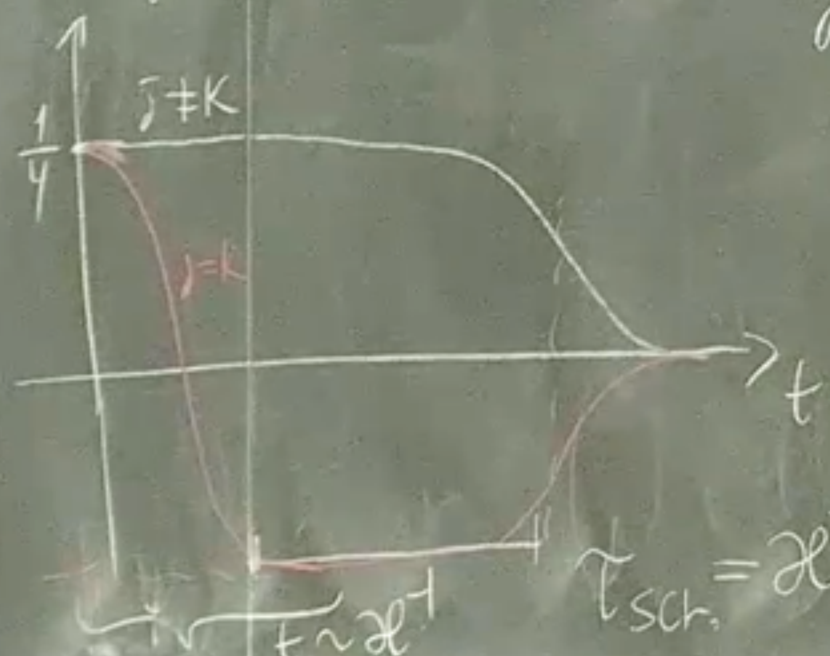
$$A_{\mathcal{O},\mathcal{O}}(w) = \int e^{iwt}\{\mathcal{O}(t),\mathcal{O}(0)\}\,dt$$

$$A^{(hor)}_{\psi,\psi}(w) = \text{const}$$

$$A_{\mathcal{O}\mathcal{O}}(w) \sim \cosh\frac{\beta w}{2}\left|\Gamma\left(\frac{3}{4} - \frac{i\beta w}{2\pi}\right)\right|^2$$

Fourier Im $G(\tau_1,\tau_2) = -\frac{a}{\sqrt{2\beta}}\left(\sin\frac{\pi(\tau_1-\tau_2)}{\beta}\right)^{-1/2}$

$$\langle \chi_j(t)\,\chi_k(0)\,\chi_j(t)\,\chi_k(0)\rangle \qquad j \neq k$$



$$\tau_{sch.} = \varkappa^{-1} \ln N$$

$$\varkappa = \frac{2\pi}{\beta} \quad \text{if} \quad \lambda \gg 1$$

$$\varkappa \sim J \quad \text{if} \quad \lambda \ll 1$$

$$\langle D(t)\,C(0)\,B(t)\,A(0)\rangle - \langle CA\rangle\langle DB\rangle$$

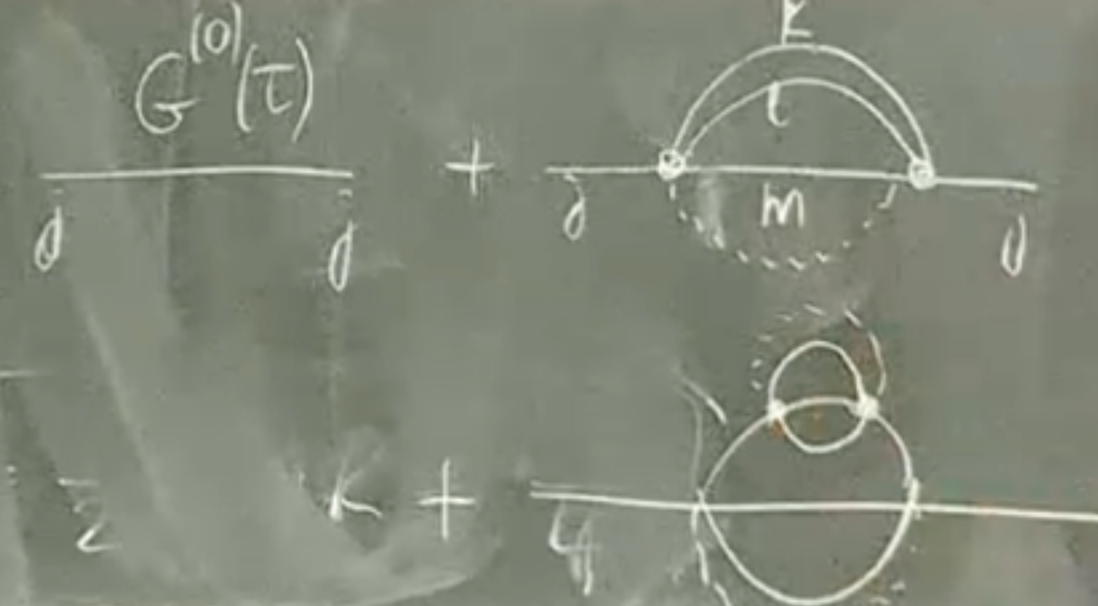
$$\sim \frac{1}{N} e^{\varkappa t} \quad \text{for} \quad \varkappa^{-1} \ll t < \varkappa^{-1} \ln N$$

$G_{jj}(\tau) = -\langle T\,\chi_j(\tau)\,\chi_j(0)\rangle \qquad \tau\in[0,\beta]$

Free mode: $G^{(0)}(\tau) = -\frac{1}{2}\,\mathrm{sgn}\,\tau$

$S = \int\left(\frac{i}{2}\chi_j\dot{\chi}_j - H\right)dt$

$G_{jj}(\tau) =$



$G(i\omega_n)^{-1} = i\omega_n - \Sigma(i\omega_n) \qquad \omega_n = \frac{2\pi}{\beta}\left(n+\frac{1}{2}\right)$

$\Sigma(\tau) = J^2\,G(\tau)^3$

$G(+0) = -\frac{1}{2}, \quad G(-0) = +\frac{1}{2}$

$$G\Sigma = 1$$

$$\begin{cases} \int G(\tau_1,\tau_2)\,\Sigma(\tau_2,\tau_3)\,d\tau_2 = \delta(\tau_1-\tau_3) \\ \Sigma(\tau_1,\tau_2) = J^2\, G(\tau_1,\tau_2)^3 \end{cases}$$

$$G(\tau_1,\tau_2) \longrightarrow G(f(\tau_1), f(\tau_2)) \cdot f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta}$$

$$\Sigma(\tau_1,\tau_2) \longrightarrow \Sigma(f(\tau_1), f(\tau_2)) \cdot f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta}$$

$$\Delta = \frac{3}{4}$$

$$f(\tau) = e^{2\pi i \tau/\beta}$$

$$G(\tau_1,\tau_2) = \tilde{G}(f(\tau_1), f(\tau_2)) \times f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta}$$

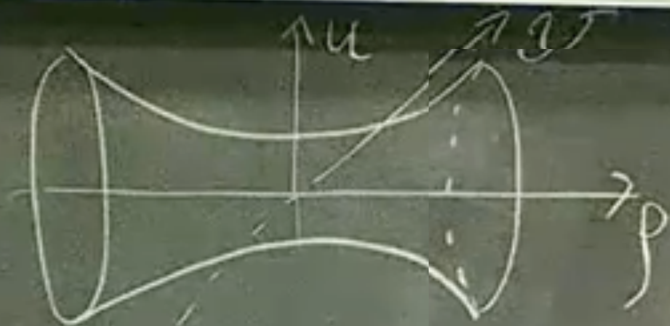
$$\tilde{G}(z_1,z_2) = c\,(z_1-z_2)^{2(\Delta-1)}$$

$$G(\tau_1,\tau_2) = -\frac{a}{\sqrt{2\beta}}\left(\sin\frac{\pi(\tau_1-\tau_2)}{\beta}\right)^{-1/2}$$

$$z = e^{\frac{2\pi}{\beta}t}$$

$$PSL(2,R)$$

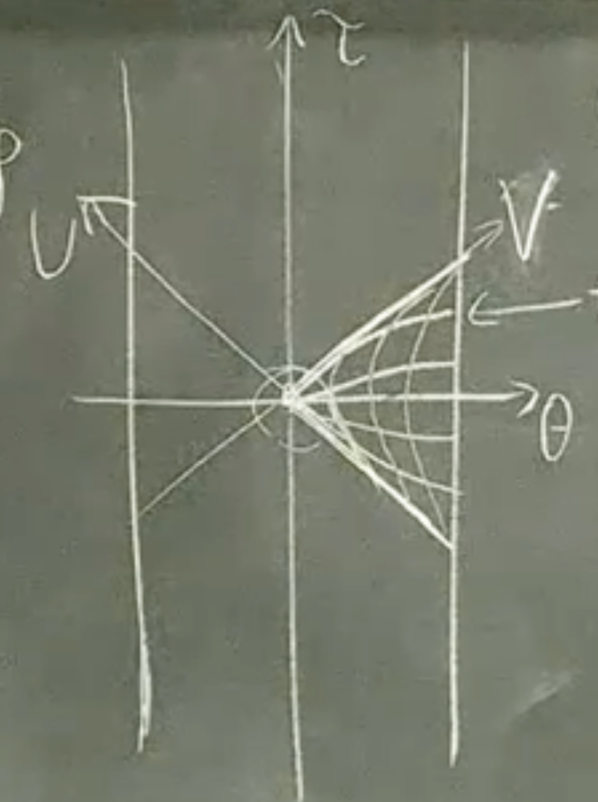
$$\simeq SO(2,1)$$



$$\rho = \tanh\theta$$

$$u = \rho\cos\tau$$

$$v = \rho\sin\tau$$

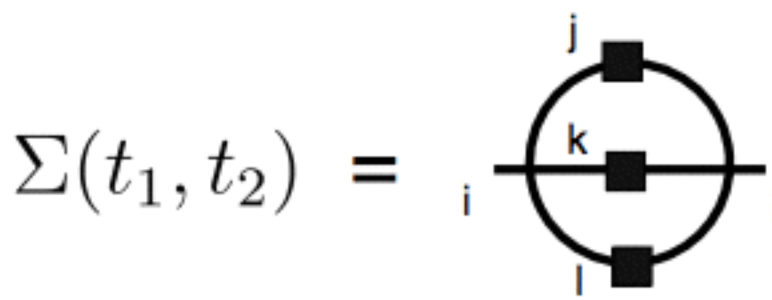


$$ds^2 = \frac{-d\tau^2 + d\theta^2}{\cos^2\theta}$$

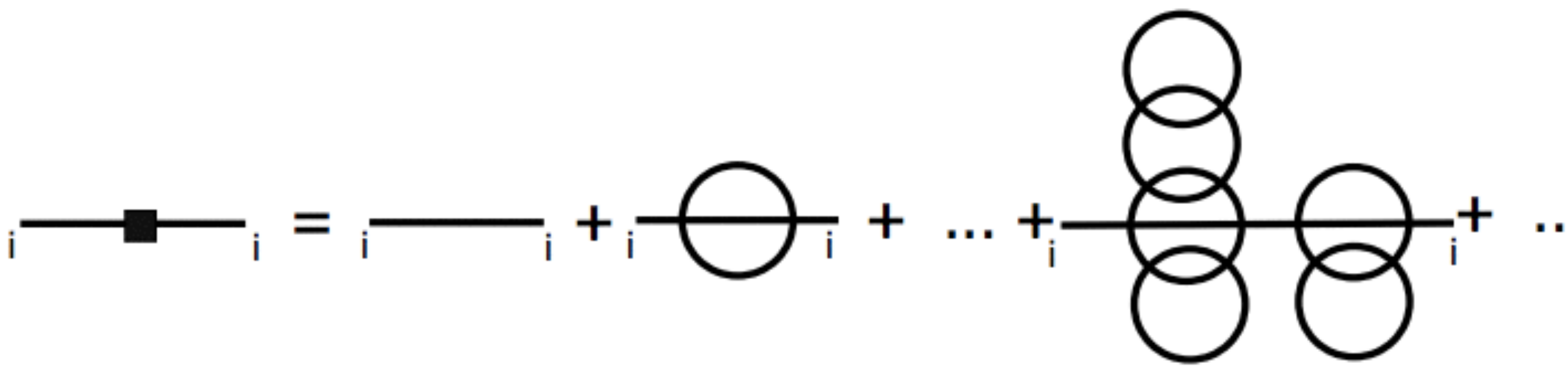
$$z \to (1+\xi)z$$

$$z \to z + \xi$$

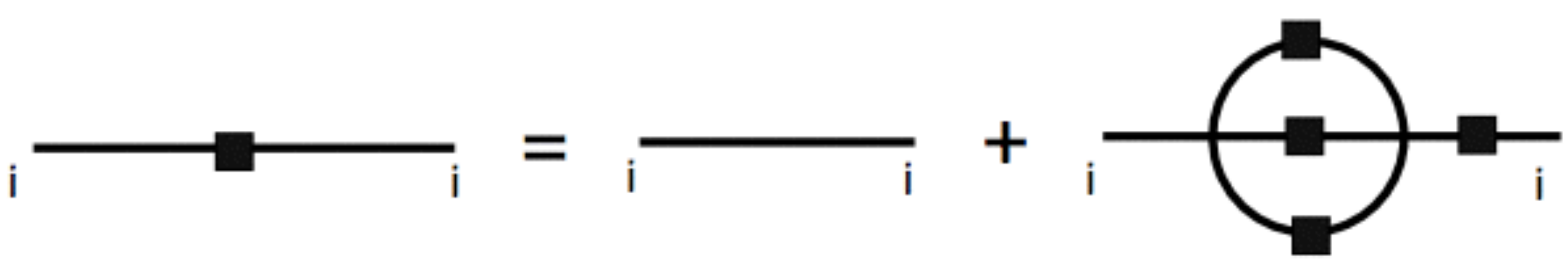
$$z^{-1} \to z^{-1} + \xi$$



(a)



(b)



$$\langle\phi(z,\bar{z})\phi(z',\bar{z}')\rangle = (z-z')^{-2h}(\bar{z}-\bar{z}')^{-2\bar{h}}. \tag{2.1}$$

If ϕ is a primary operator, under the conformal mapping $w=f(z)$ the correlation function transforms according to

$$\langle\phi(z,\bar{z})\phi(z',\bar{z}')\rangle = (f'(z))^{h}\left(\overline{f'(z)}\right)^{\bar{h}}(f'(z'))^{h}\left(\overline{f'(z')}\right)^{\bar{h}}\langle\phi(w,\bar{w})\phi(w',\bar{w}')\rangle. \tag{2.2}$$

Choosing $f(z)=(l/2\pi)\ln z$ and using (2.1), we obtain the correlation function in the strip:

$$\langle\phi(w,\bar{w})\phi(w',\bar{w}')\rangle = \frac{(\pi/l)^{2x}}{(\sinh\pi(w-w')/l)^{2h}(\sinh\pi(\bar{w}-\bar{w}')/l)^{2\bar{h}}}. \tag{2.3}$$