Collisional transport of alpha particles in stellarators

Thomas E. Foster

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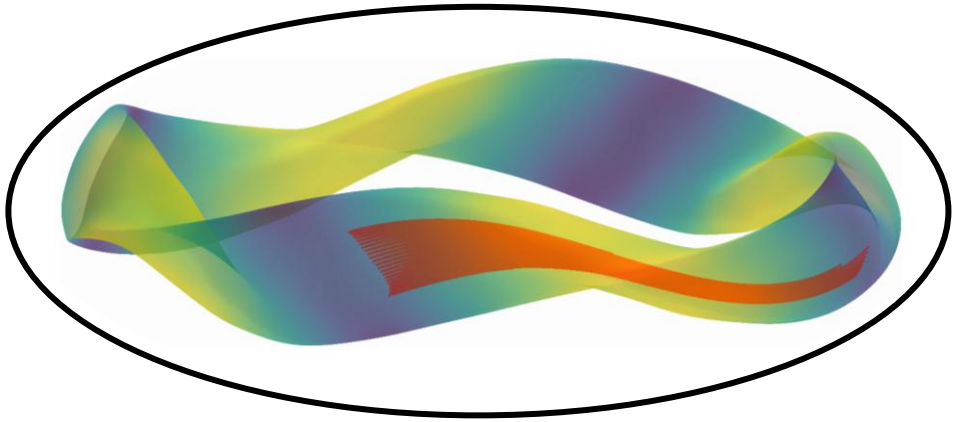

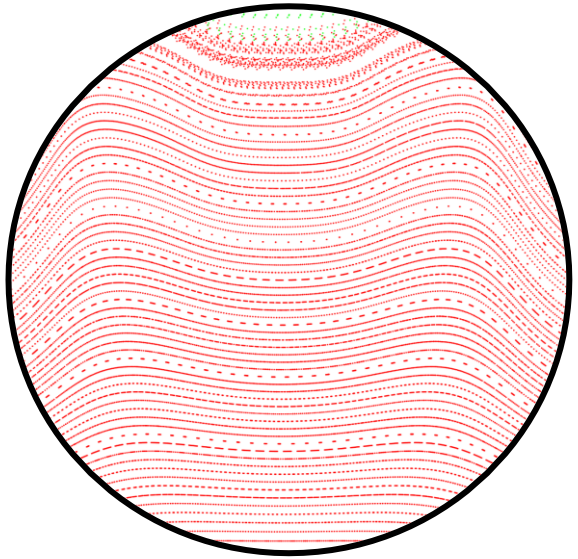

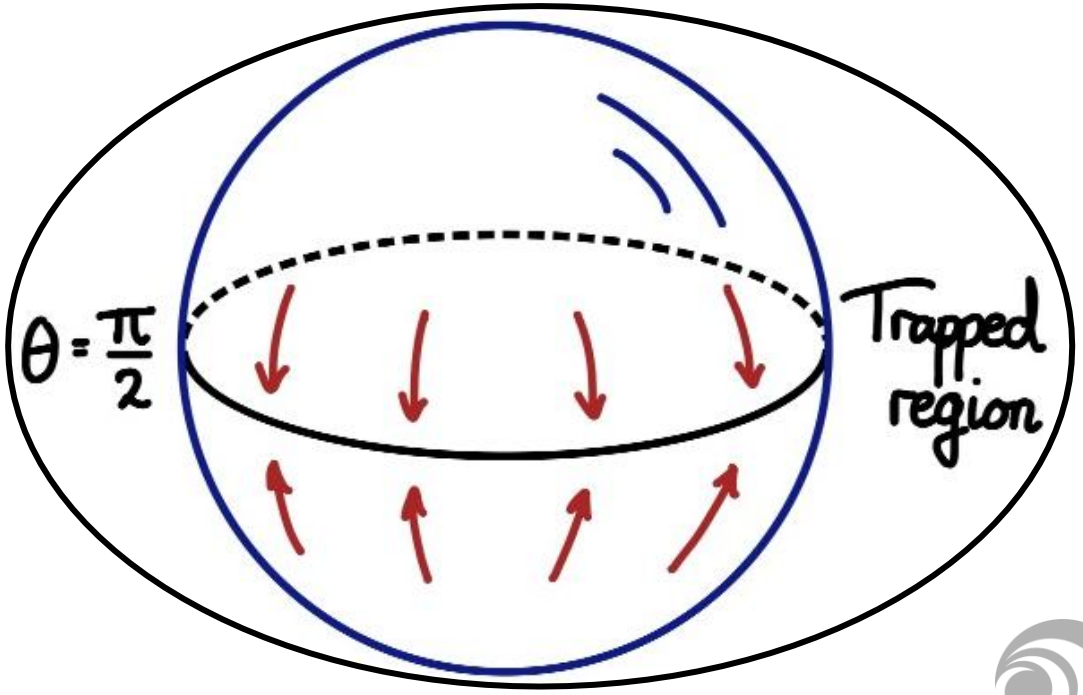

Overview

1. Motivation
2. Key physics
3. Unoptimised large-aspect-ratio stellarators
4. Optimised large-aspect-ratio stellarators

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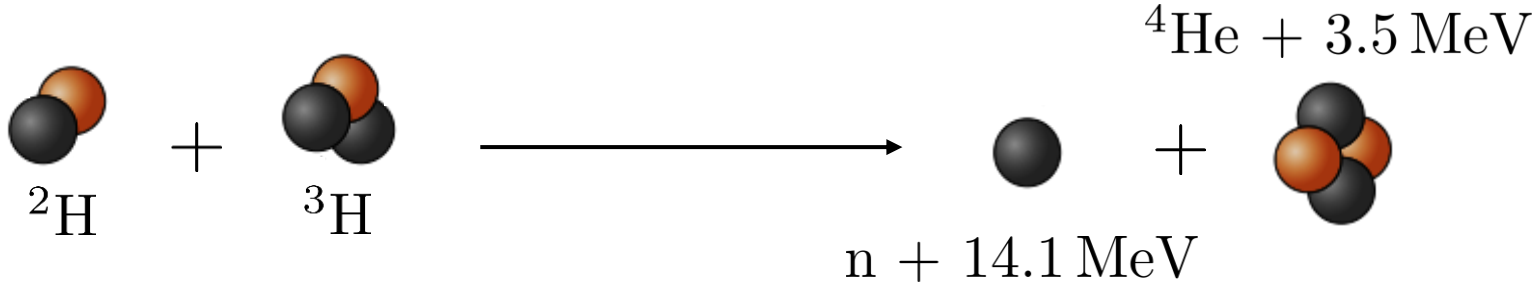
Motivation

- D-T fusion produces 3.5-MeV alpha particles that must be confined



- In ITER:

  (i) $< 30\%$ alpha energy losses needed to reach ignition

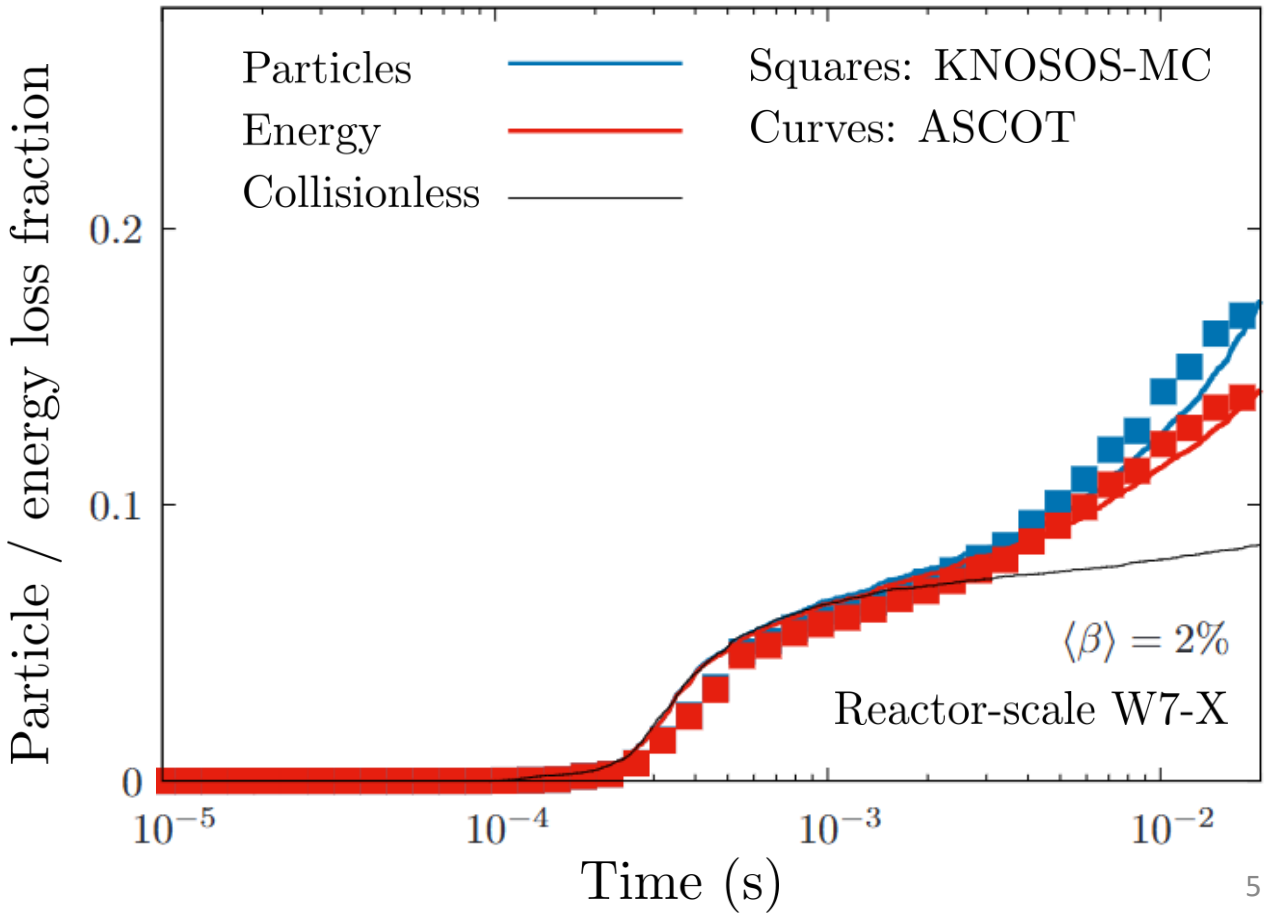
  (ii) $< 5\%$ alpha energy losses to ensure damage to wall is tolerable

📖 ITER Physics Expert Group on Energetic Particles, Heating and Current Drive and ITER Physics Basis Editors 1999

Motivation

- D-T fusion produces 3.5-MeV alpha particles that must be confined
- Alpha confinement in stellarator designs is often assessed by simulating many alpha particles and plotting losses over time
- Alpha speed is $\sim 10^7\,\mathrm{m\,s^{-1}}$, slowing-down time is $\tau_\mathrm{s} \sim 0.1\,\mathrm{s}$
- **But** to determine losses and alpha distribution, collisions are essential

Aim: Construct analytical theory for collisional alpha-particle losses



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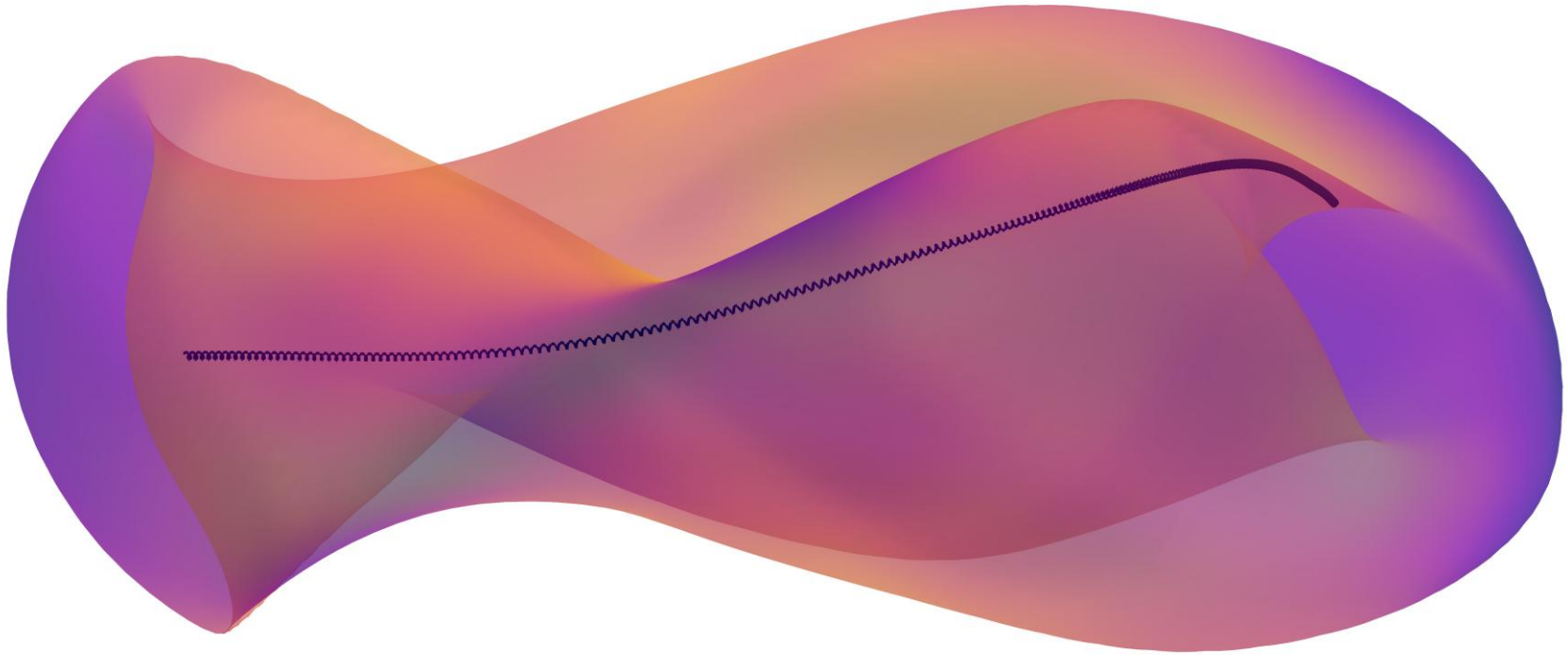
Key physics

- At 3.5 MeV: $\boxed{\rho_\star \sim 0.03}$ $\Longrightarrow$ can use drift kinetics.



Key physics

- At 3.5 MeV: $\rho_\star \sim 0.03 \implies$ can use drift kinetics.
- Drift-kinetic equation: $\partial_t f + v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f + \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f = C[f] + S_\alpha$

Magnetic drifts – $\boldsymbol{E} \times \boldsymbol{B}$ drift negligible

$$\boldsymbol{v}_\mathrm{d} = \frac{1}{\Omega} \hat{\boldsymbol{b}} \times \left(v_\parallel^2 \boldsymbol{\kappa} + \mu \boldsymbol{\nabla} B \right)$$

Key physics

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Sourcing: isotropic, sharply peaked around 3.5 MeV

$$S_\alpha(r, v, \vartheta) = \frac{S_0(r)}{4\pi v_0^2}\, \delta(v - v_0)$$

Key physics

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Collisions: when $v_{\mathrm{th},i} \ll v \ll v_{\mathrm{th},e}$
slowing-down and pitch-angle scattering

$$C_\parallel[f] = \frac{1}{\tau_\mathrm{s}} \frac{1}{v^2} \partial_v \left((v^3 + v_\mathrm{c}^3) f \right)$$

$$C_\perp[f] = \frac{1}{2\tau_\mathrm{s}} \frac{m_i}{m_\alpha} \frac{v_\mathrm{c}^3}{v^3} \frac{1}{\sin\vartheta} \partial_\vartheta (\sin\vartheta \, \partial_\vartheta f)$$

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ALPHA-PARTICLE CONFINEMENT AND HELIUM ASH ACCUMULATION IN STELLARATOR REACTORS

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Abstract—The effect of local magnetic wells produced by the external helical windings of a stellarator reactor on α-particle confinement is investigated by using the Fokker–Planck equation. It is found that α-particles can deposit most of their energy into the background plasma before they scatter into the trapped region in velocity space and are lost. An estimate is made on the steady-state, low-energy (thermal) α-particle concentration. The result shows that this concentration will be less than a few per cent of the background plasma density.

1. INTRODUCTION

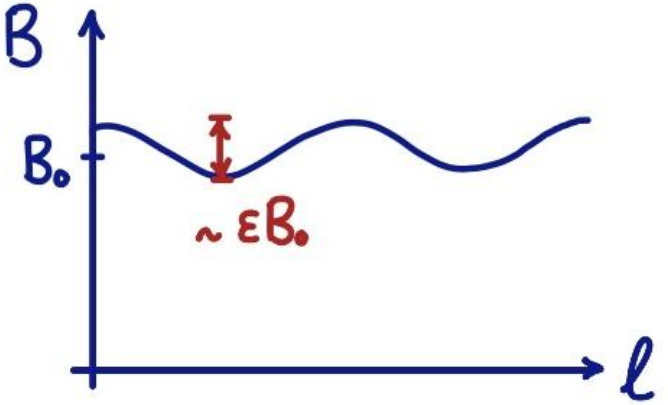
STELLARATORS have the attractive feature of operating without a toroidal current since the rotational transform is provided by external helical windings. Besides providing steady-state operation, the current-free nature of the stellarator should prevent major disruptions. These advantages make the stellarator an attractive candidate for a future high performance reactor. However, stellarators have a major drawback. This arises from the fact that the external helical windings produce local magnetic wells (henceforth, these local magnetic wells are referred to as helical wells) in which trapped particles are badly confined. This is a direct consequence of the lack of toroidal symmetry. The enhanced neoclassical transport produced by particles trapped in helical wells is strongest in the high-temperature, low-collisionality regime (i.e. an untrapped particle can go around the torus many times before it is entrapped by collisions). This is the regime in which reactors are likely to operate. Thus, the nature of neoclassical transport resulting from nonaxisymmetry plays a critical role in determining the confinement in stellarator reactors.

Unoptimised large-aspect-ratio stellarators

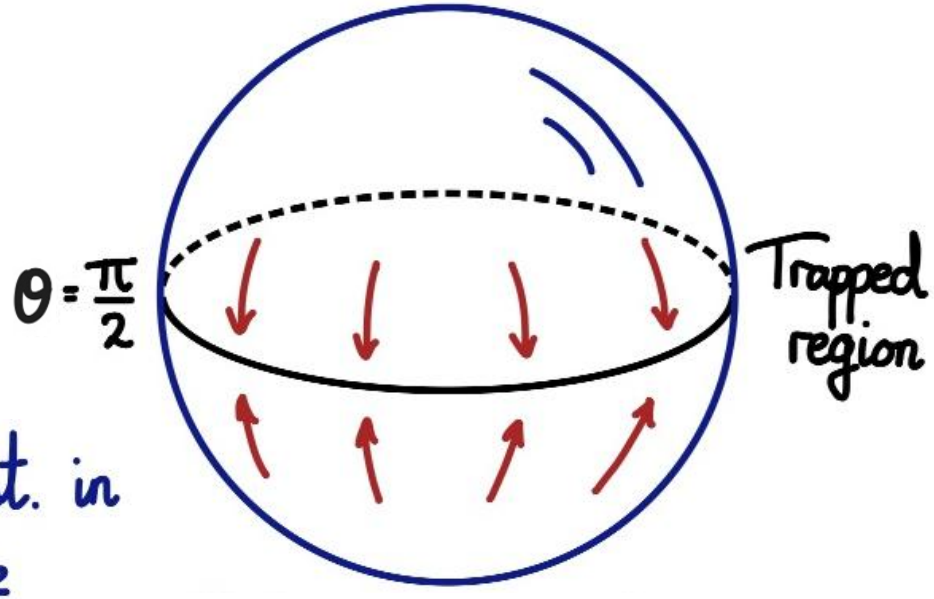
- 'Unoptimised': all trapped particles promptly lost
- 'Large aspect ratio': $\epsilon := a/R \ll 1$,

$$B(r,\theta,\zeta) = B_0 + B_1(r,\theta,\zeta)$$

const. $\quad \sim \epsilon B_0$



- Trapped particles have pitch angles $\vartheta \sim \sqrt{\epsilon}$
- Passing particle pitch-angle scatter into the trapped region 'loss cone'



Sphere v = const. in velocity space

Energy loss estimate

Slowing down:

$$\mathrm{d}v \sim -\frac{v}{\tau_\mathrm{s}}\,\mathrm{d}t$$

Pitch-angle scattering:

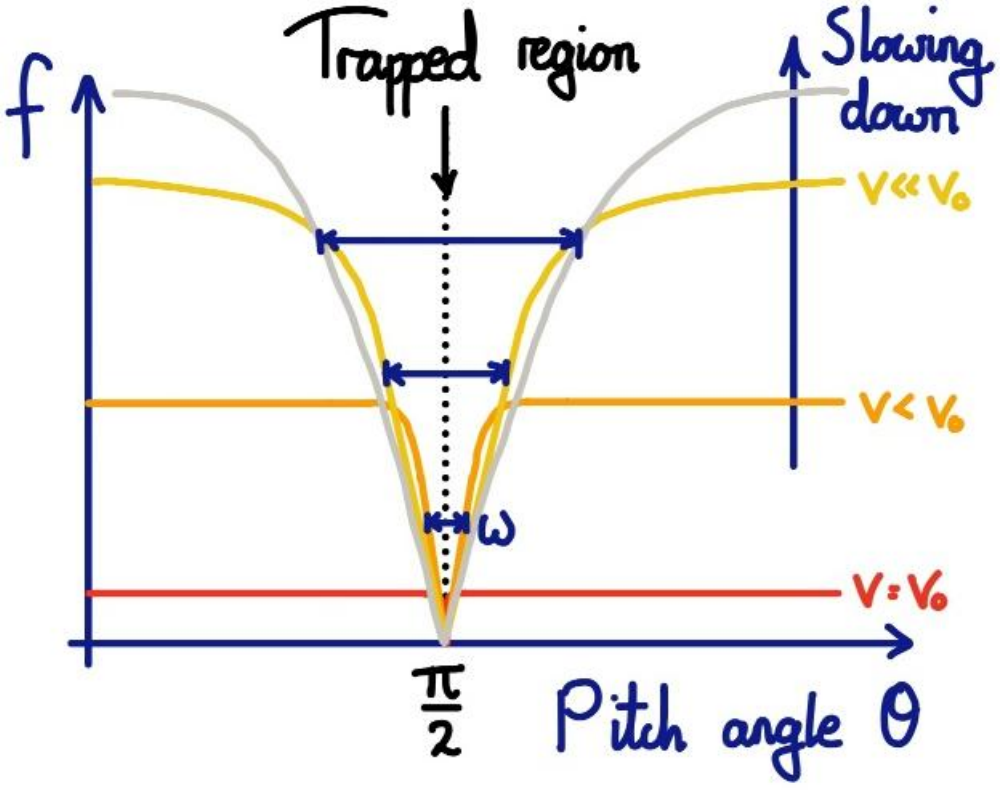
$$\mathrm{d}(w^2) \sim \nu_\perp \mathrm{d}t \sim \frac{1}{\tau_\mathrm{s}}\frac{v_\mathrm{c}^3}{v^3}\,\mathrm{d}t$$

$$\mathrm{d}(w^2) \sim -\frac{v_\mathrm{c}^3}{v^4}\,\mathrm{d}v$$

$$\Longrightarrow \quad w \sim \left(\frac{v_\mathrm{c}^3}{v^3} - \frac{v_\mathrm{c}^3}{v_0^3}\right)^{1/2}$$

$$\mathrm{d}f_E \sim \left(\frac{v^2}{v_0^2}\right)\mathrm{d}w$$

$$f_E \sim \int_0^{v_0} \left(\frac{v}{v_0}\right)^2 \mathrm{d}w \sim \left(\frac{v_\mathrm{c}}{v_0}\right)^{3/2}$$



Kinetic equation for passing particles

- Drift-kinetic equation: $\cancel{\partial_t f} + v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f + \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f = C[f] + S_\alpha$
- Expand $f = f_0 + f_1 + \ldots$ Leading order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_0 = 0$, so f_0 depends on r, $\mathcal{E}$, μ only
- First order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_1 + (\boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} r)\, \partial_r f_0 = C[f_0] + S_\alpha$
- To eliminate the term containing f_1, take flux-surface average:

$$0 = \left\langle \frac{BC[f_0]}{|v_\parallel|} \right\rangle_\mathrm{fs} + \left\langle \frac{BS}{|v_\parallel|} \right\rangle_\mathrm{fs} \qquad \langle \ldots \rangle_\mathrm{fs} = \frac{1}{V'} \int_0^{2\pi} \int_0^{2\pi} \frac{(\ldots)}{\boldsymbol{B} \cdot \boldsymbol{\nabla} \zeta} \mathrm{d}\theta \, \mathrm{d}\zeta$$

$$\downarrow$$

$$\frac{1}{\tau_\mathrm{s}} \left[\frac{1}{2} \frac{m_i}{m_\alpha} \frac{v_\mathrm{c}^3}{v^3} \frac{1}{\sin\vartheta} \partial_\vartheta (\sin\vartheta \, \partial_\vartheta f_0) + \frac{1}{v^2} \partial_v \left((v^3 + v_\mathrm{c}^3) f_0 \right) \right] = -\frac{S_0}{4\pi v_0^2} \delta(v - v_0)$$

Heat equation on a sphere

$$g = \left(\frac{v^3 + v_c^3}{v_c^3}\right) f_0 \qquad \tau = \frac{1}{6}\frac{m_i}{m_\alpha} \log\left(\frac{v_0^3}{v_0^3 + v_c^3}\frac{v^3 + v_c^3}{v^3}\right)$$

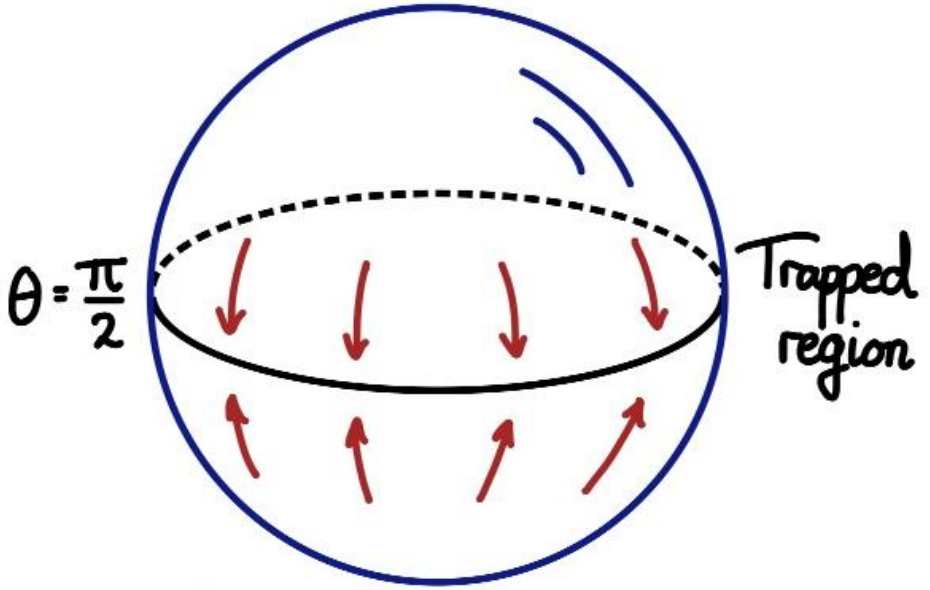
$$\partial_\tau g = \frac{1}{\sin\vartheta}\partial_\vartheta(\sin\vartheta\,\partial_\vartheta g)$$

Boundary conditions:

(i) $g = 0$ at $\vartheta = \pi/2$,

(ii) $\partial_\vartheta g = 0$ as $\vartheta \to \pm\pi/2$

$$\frac{1}{\tau_s}\left[\frac{1}{2}\frac{m_i}{m_\alpha}\frac{v_c^3}{v^3}\frac{1}{\sin\vartheta}\partial_\vartheta(\sin\vartheta\,\partial_\vartheta f_0) + \frac{1}{v^2}\partial_v\left((\iota\right.\right.$$



Solution and energy loss

$$f_0(v,\vartheta) = \frac{S_0\tau_\mathrm{s}}{2\pi(v^3+v_\mathrm{c}^3)} \sum_{l\ \mathrm{odd}} \frac{(-1)^{\frac{l-1}{2}}(l+\frac{1}{2})(l-1)!}{2^l\,(\frac{l-1}{2})!(\frac{l+1}{2})!}\, P_l(\cos\vartheta)\, \left(\frac{v_0^3}{v_\mathrm{c}^3+v_0^3}\,\frac{v^3+v_\mathrm{c}^3}{v^3}\right)^{-\frac{m_i}{6m_\alpha}l(l+1)}$$

Limits:

$v \ll v_\mathrm{c}$ $\qquad f_0(v,\vartheta) \simeq \dfrac{3S_0\tau_\mathrm{s}}{8\pi v_\mathrm{c}^3}\cos\vartheta\left(\dfrac{v}{v_\mathrm{c}}\right)^{m_i/m_\alpha}$ $\qquad$ No ash!

$v_\mathrm{c} \ll v$ $\qquad f_0(v,\vartheta) \simeq \dfrac{S_0\tau_\mathrm{s}}{2\pi v^3}\,\mathrm{erf}\left[\sqrt{\dfrac{3m_\alpha}{2m_i}}\left(\dfrac{v_\mathrm{c}^3}{v^3}-\dfrac{v_\mathrm{c}^3}{v_0^3}\right)^{1/2}\cos\vartheta\right]$

Energy loss fraction using asymptotic expansion in $v_\mathrm{c}/v_0 \ll 1$:

Surprisingly small!
For $v_0/v_\mathrm{c} = 3.3$,
about 12%

$$f_E \simeq \frac{3^{1/2}\Gamma(\frac{1}{3})^3}{2^{11/6}\pi^{3/2}}\left(\frac{m_i}{m_\alpha}\right)^{1/2}\left(\frac{v_\mathrm{c}}{v_0}\right)^{3/2}$$

Dominated by
$v \gg v_\mathrm{c}$ alphas

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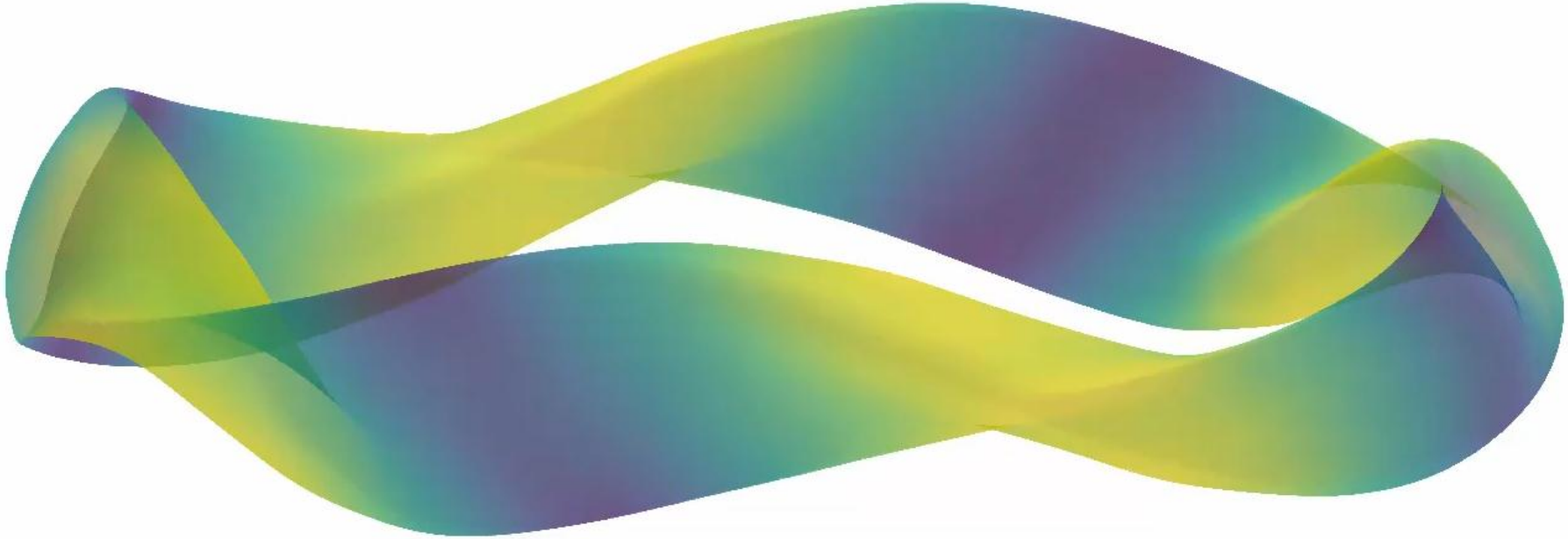
Optimised large-aspect-ratio stellarators

- What do we mean by 'optimised'?



- In a stellarator, trapped particles conserve: $J(r,\alpha) = \int v_{\parallel}\,\mathrm{d}l$

'Second adiabatic invariant'

- To confine trapped particles, want: $J(r,\alpha) = J(r)$ ⟵ 'Omnigeneous'

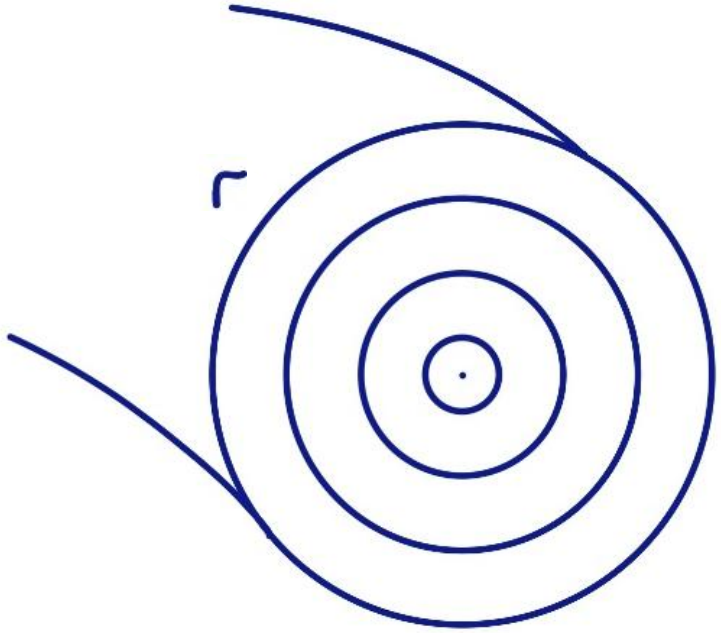
Optimised large-aspect-ratio stellarators

- What do we mean by 'optimised'?
- Before, we had:

$$B(r,\theta,\zeta) = B_0 + B_1(r,\theta,\zeta)$$

B_0: const. $\quad B_1$: $\sim \epsilon B_0$

$$\epsilon \ll 1$$



- Now we also assume:

$$B_1(r,\theta,\zeta) = B_1^{[0]}(r,\theta,\zeta) + B_1^{[1]}(r,\theta,\zeta)$$

$B_1^{[0]}$: omnigeneous $\quad B_1^{[1]}$: $\sim \delta B_1^{[0]}$

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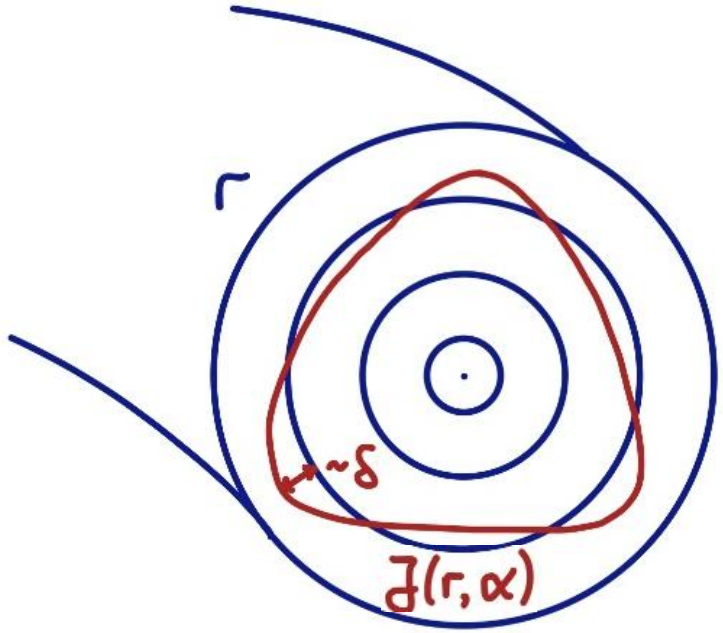

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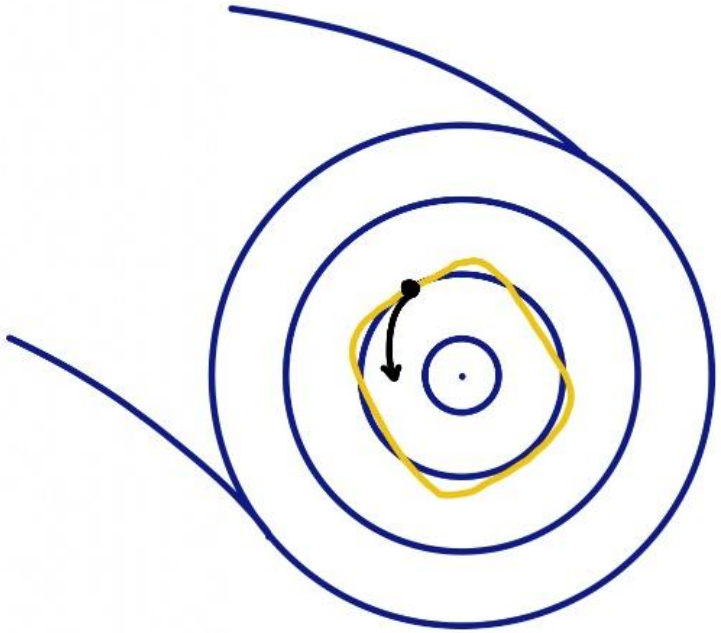

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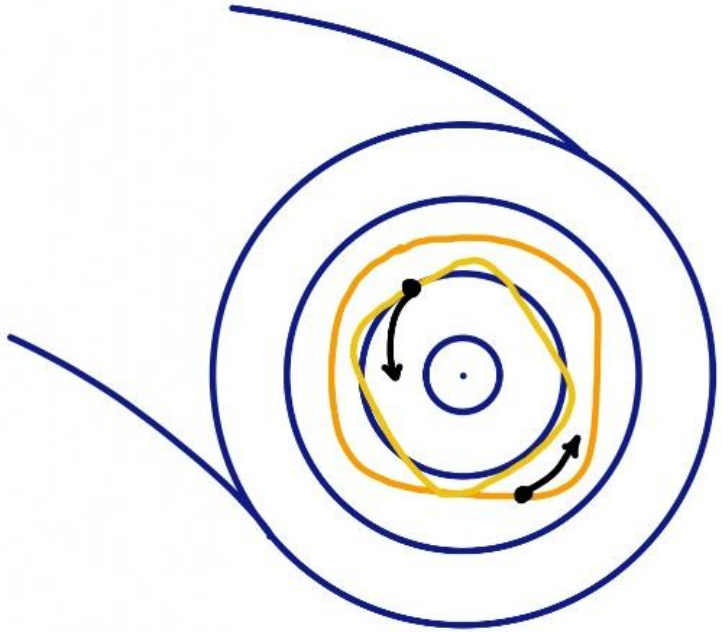

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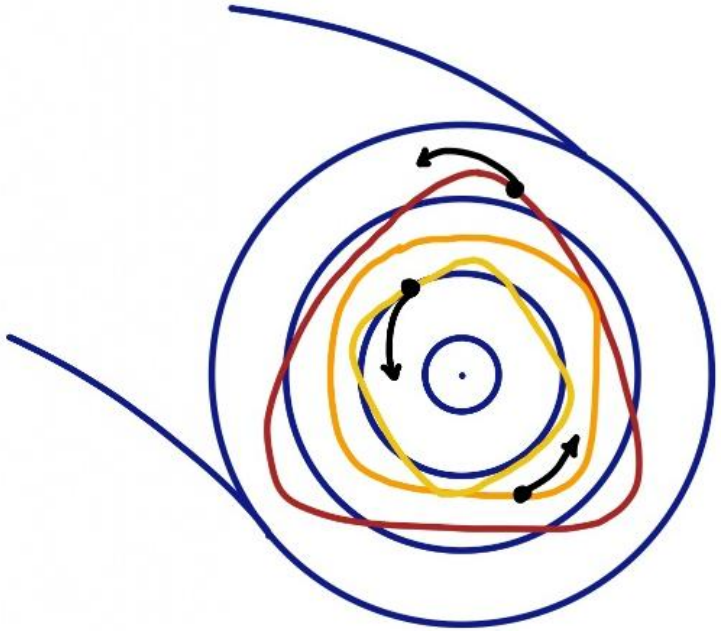

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$$\delta \ll 1$$

Effect of optimisation on alpha losses

📖 Catto 2019, Catto, Tolman & Parra 2023, Catto 2025

Very optimised

Trapped particles take $t \gg \tau_s$ to diffuse to edge

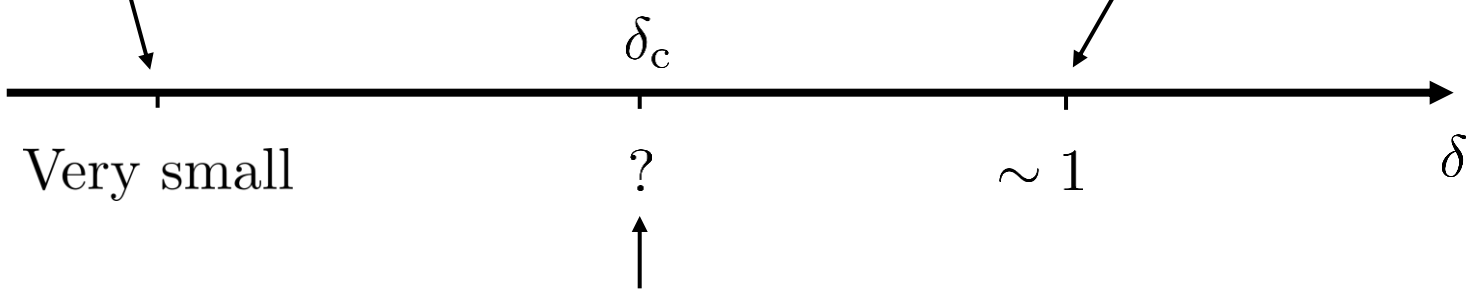
Distribution close to slowing-down:

$$f(v) \simeq \frac{S\tau_s}{4\pi(v_c^3 + v^3)}$$

Unoptimised

Trapped particles reach edge in $t \ll \tau_s$

Anisotropic loss-cone distribution from before



Intermediate

Trapped particles diffuse to edge in $t \sim \tau_s$

Estimating critical delta

- Can we estimate δ_c?

Pitch-angle-scattering flux: $\Gamma \sim \nu_\perp a n_\alpha$

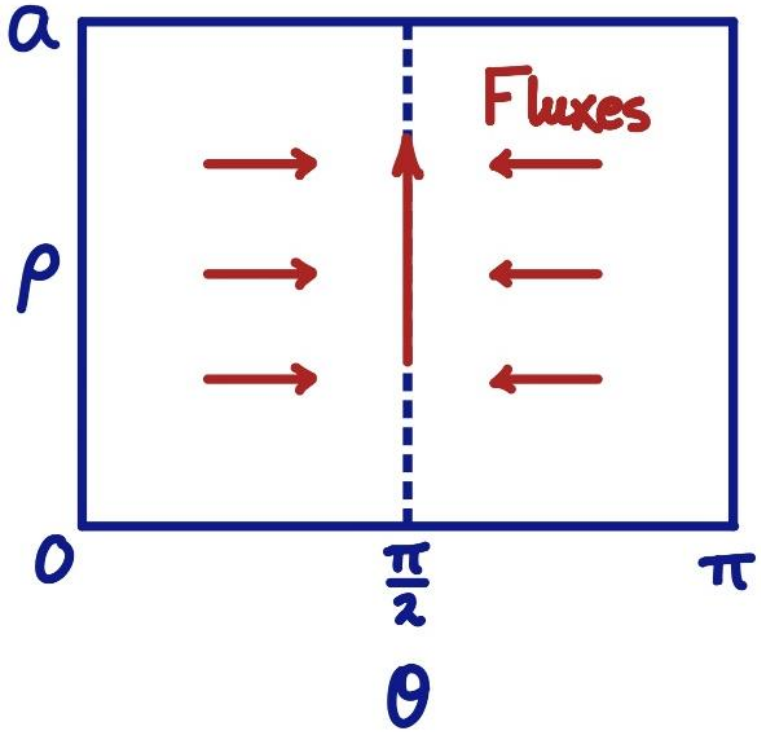
Radial flux: $\Gamma \sim \sqrt{\epsilon}\left(\frac{\nu_\perp}{\epsilon}\right)(\delta a)^2\left(\frac{n_\alpha}{a}\right)$

Size of trapped region

Enhanced collision frequency

Step size in radial random walk

$\Longrightarrow \quad \delta_c \sim \epsilon^{1/4}$



Low-collisionality kinetic equation

- Drift-kinetic equation: $\cancel{\partial_t f} + \underbrace{v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f}_{\sim 1} + \underbrace{\boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f}_{\sim \rho_\star} = \underbrace{C[f] + S}_{\sim \nu_\star}$ with respect to vf/L

- Assume $\nu_\star \sim \rho_\star^2$. Expand $f = f_0 + f_1 + \ldots$ Leading order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_0 = 0$, so f_0 depends on ψ, α, $\mathcal{E}$, μ only.
- First order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_1 + \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f_0 = 0$
- To eliminate the term containing f_1, we bounce average:

$$\underbrace{\langle \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} \psi \rangle_\mathrm{b} \, \partial_\psi f_0 + \langle \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} \alpha \rangle_\mathrm{b} \, \partial_\alpha f_0}_{= \langle \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} \alpha \rangle_\mathrm{b} \left. \dfrac{\partial f_0}{\partial \alpha} \right|_J} = 0 \qquad \langle \ldots \rangle_\mathrm{b} = \frac{1}{\tau_\mathrm{b}} \oint \frac{(\ldots)}{|v_\parallel|} \, \mathrm{d}l \, .$$

so f_0 is a function of v, λ and J only.

Low-collisionality kinetic equation

- Drift-kinetic equation: $\cancel{\partial_t f} + \underbrace{v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f}_{\sim 1} + \underbrace{\boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f}_{\sim \rho_\star} = \underbrace{C[f] + S}_{\sim \nu_\star}$ with respect to vf/L

- Assume $\nu_\star \sim \rho_\star^2$. Expand $f = f_0 + f_1 + \ldots$ Leading order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_0 = 0$, so f_0 depends on r, α, $\mathcal{E}$, μ only.
- First order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_1 + \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f_0 = 0$
- Second order is $v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} f_2 + \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} f_1 = C[f_0] + S_\alpha$
- To eliminate the term containing f_2, we bounce average:

$$\langle \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} \alpha \rangle_\mathrm{b} \left. \frac{\partial f_1}{\partial \alpha} \right|_J = \langle C[f_0] \rangle_\mathrm{b} + \langle S_\alpha \rangle_\mathrm{b}$$

integrate at constant J

- To eliminate the term containing f_1, we 'drift' average:

$$0 = \langle \langle C[f_0] \rangle_\mathrm{b} \rangle_\mathrm{d} + \langle \langle S \rangle_\mathrm{b} \rangle_\mathrm{d} \qquad \text{where} \quad \langle \ldots \rangle_\mathrm{d} = \frac{1}{\tau_\mathrm{d}} \oint \frac{(\ldots)}{|\langle \boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\nabla} \alpha \rangle_\mathrm{b}|} \mathrm{d}\alpha$$

Averaging the collision operator

- Equation for f_0: $\langle\langle C[f_0]\rangle_\mathrm{b}\rangle_\mathrm{d} + \langle\langle S\rangle_\mathrm{b}\rangle_\mathrm{d} = 0$

$$C[f_0] = \frac{1}{\tau_\mathrm{s}}\left[\frac{1}{2}\frac{m_i}{m_\alpha}\frac{v_\mathrm{c}^3}{v^3}\frac{1}{\sin\vartheta}\partial_\vartheta(\sin\vartheta\,\partial_\vartheta f_0) + \frac{1}{v^2}\partial_v\left((v^3+v_\mathrm{c}^3)f_0\right)\right]$$

- After taking the averages, the pitch-angle scattering term takes the form

$$\langle\langle C_\perp[f_0]\rangle_\mathrm{b}\rangle_\mathrm{d} = \frac{1}{\mathcal{J}}\begin{pmatrix}\partial_\lambda & \partial_J\end{pmatrix}\begin{pmatrix}M_{\lambda\lambda} & M_{\lambda J}\\ M_{J\lambda} & M_{JJ}\end{pmatrix}\begin{pmatrix}\partial_\lambda\\ \partial_J\end{pmatrix} f_0 \qquad \lambda = \frac{\mathcal{E}}{\mu}$$

coefficients containing geometry

- The $\delta \ll 1$ and $\epsilon \ll 1$ expansions simplify these coefficients:

$$\langle\langle C[f_0]\rangle_\mathrm{b}\rangle_\mathrm{d} \simeq \frac{1}{\hat{\mathcal{J}}}\begin{pmatrix}\partial_\lambda & \partial_r\end{pmatrix}\begin{pmatrix}\hat{M}_{\lambda\lambda} & \hat{M}_{\lambda r}\\ \hat{M}_{r\lambda} & \hat{M}_{rr}\end{pmatrix}\begin{pmatrix}\partial_\lambda\\ \partial_r\end{pmatrix} f_0 + \frac{1}{\tau_\mathrm{s}}\frac{1}{v^2}\partial_v\left((v^3+v_\mathrm{c}^3)f_0\right)$$

Two-dimensional diffusion equation

Result: $$\partial_\tau g = \frac{1}{\sin\vartheta}\partial_\vartheta(\sin\vartheta\,\partial_\vartheta g)$$

with boundary condition:

involves average of the $\hat{M}$s over λ

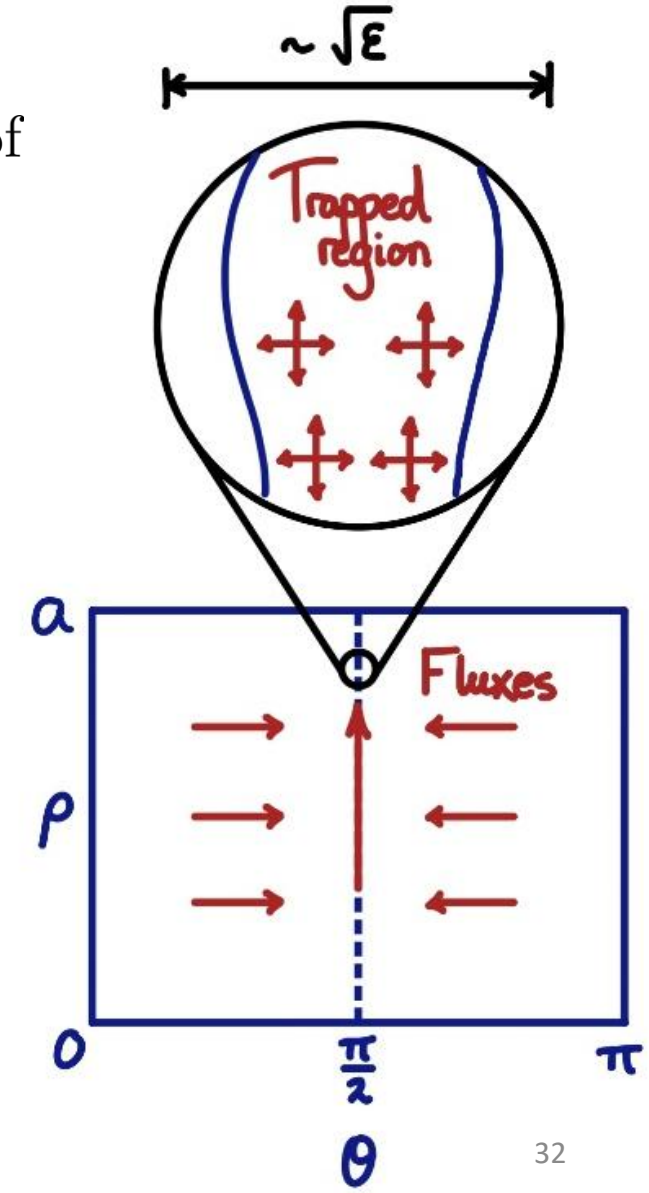
$$|d(r)\,\partial_\vartheta g|_{\vartheta=\pi/2}| = \partial_r\left(D(r)\,\partial_r g\right)|_{\vartheta=\pi/2}$$

where

$$d(r) = \frac{V'}{\tau_s}\frac{B_0^2}{\pi N_{fp}}$$

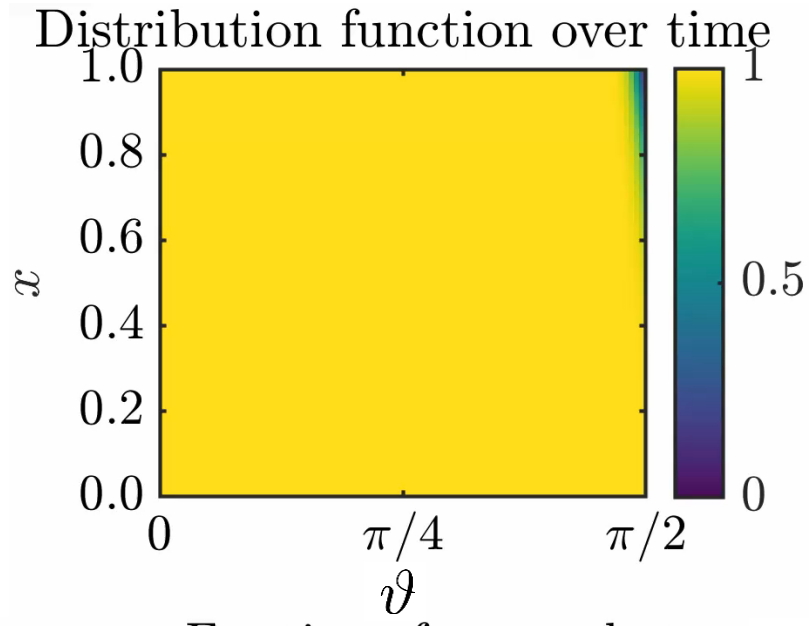
$$D(r) = \frac{1}{\tau_s}\int_{\lambda_{min}}^{\lambda_{max}}\frac{J_0}{v}\left(\left\langle\left[\partial_\lambda\left(\frac{J_1 - J_{1,max}}{\partial_r J_0}\right)\right]^2\right\rangle_\alpha - \left\langle\partial_\lambda\left(\frac{J_1 - J_{1,max}}{\partial_r J_0}\right)\right\rangle_\alpha^2\right)d\lambda$$

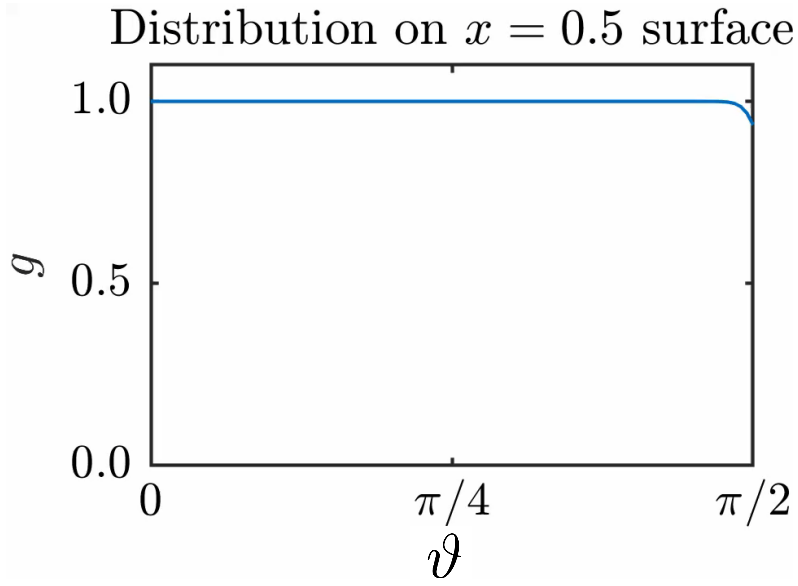
$$J(r,\alpha) = J_0(r) + \underbrace{J_1(r,\alpha)}_{\sim\delta J_0}, \qquad J_{1,max}(r) = \max_\alpha J_1(r,\alpha)$$

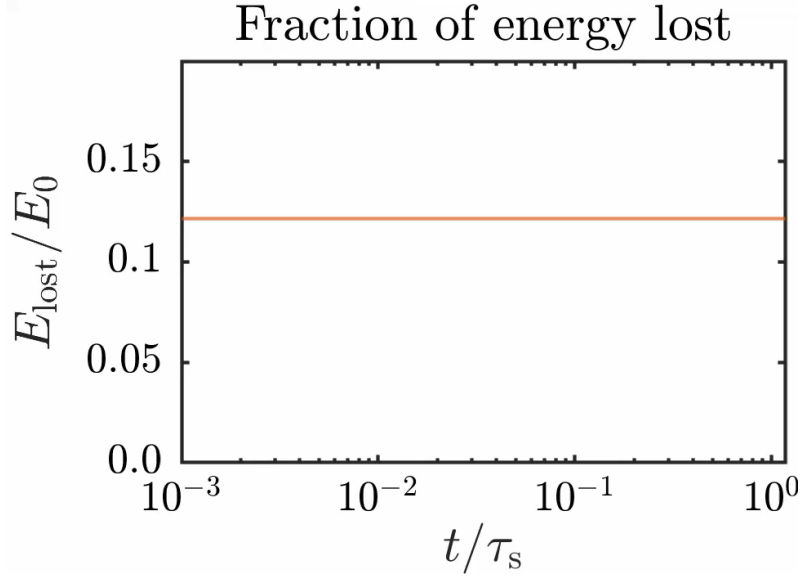


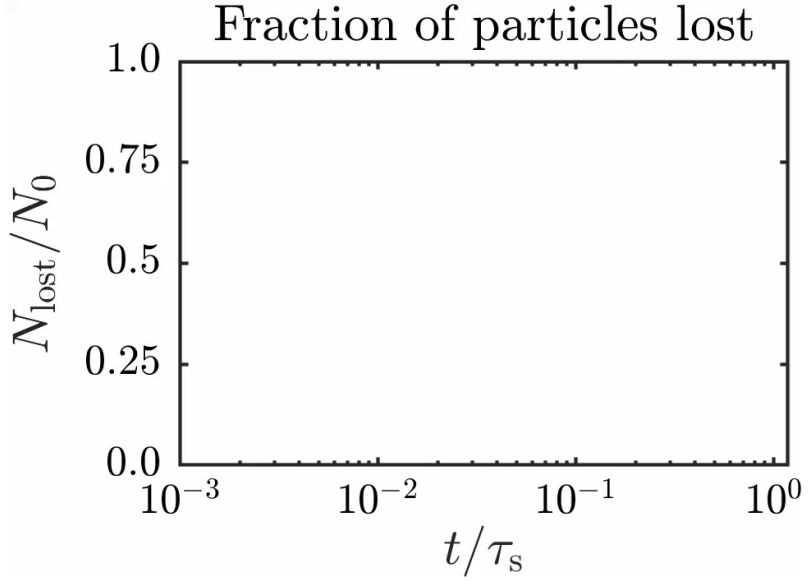
Simulations of diffusion equation

using $D(x) = 1$



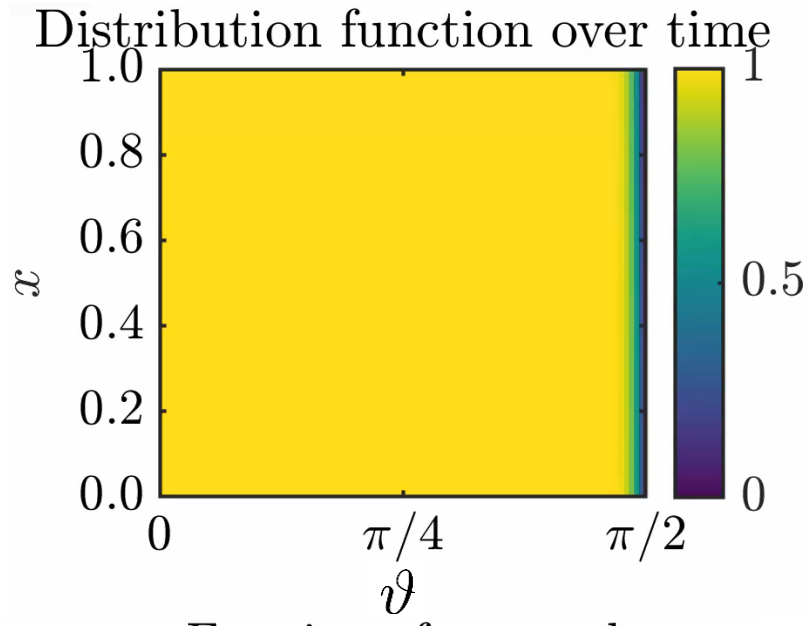


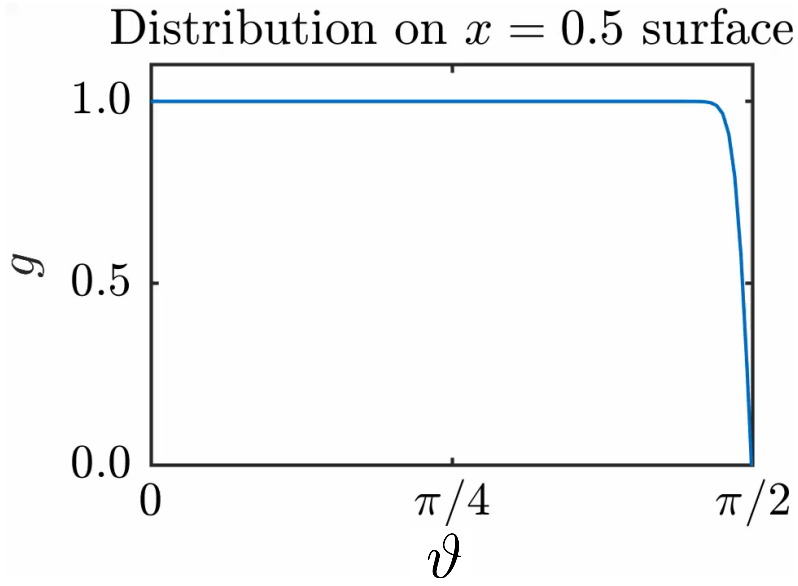


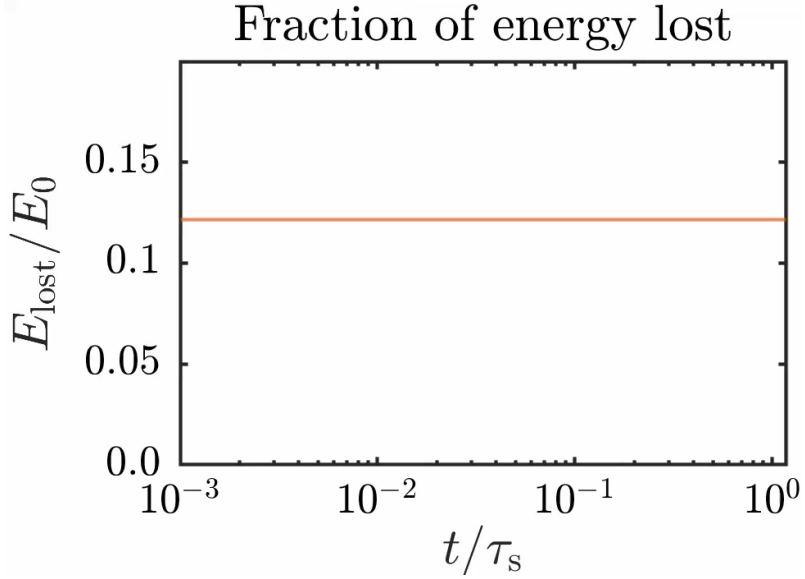


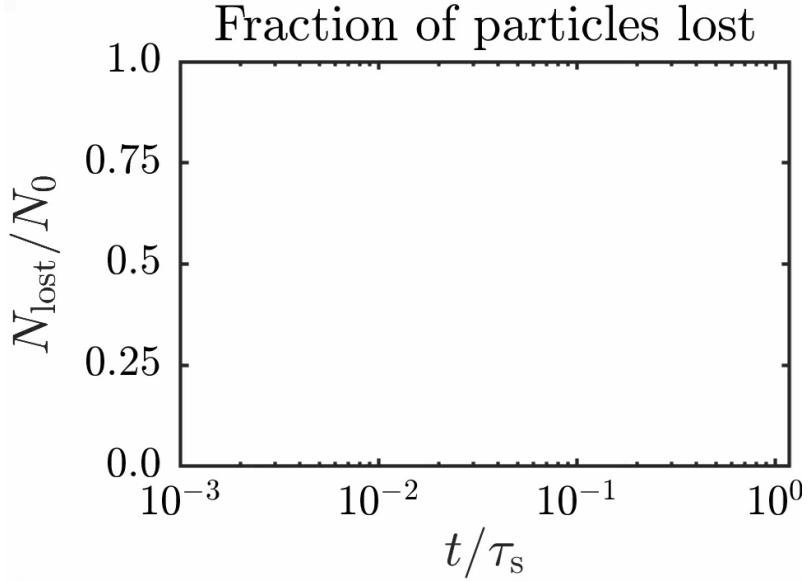
Simulations of diffusion equation

using $D(x) = 100$









Conclusions

1. Alpha energy loss fraction from unoptimised, large-aspect-ratio stellarator is $\sim (v_c/v_0)^{3/2}$. Surprisingly small!

2. Collisions eventually remove all alphas — good for avoiding ash accumulation

3. We found an ordering in which the leading-order alpha distribution f_0 departs from slowing-down due to radial diffusion in the trapped region: $\delta \sim \epsilon^{1/4}$

4. Can use resulting two-dimensional diffusion equation to predict losses over time and perhaps optimise stellarators