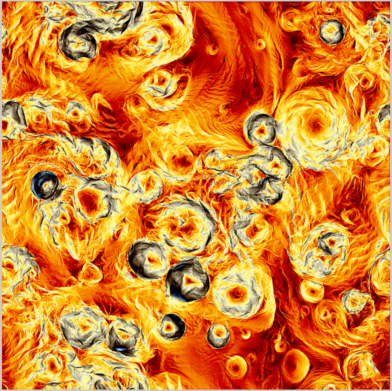
Reflection-driven turbulence in the super-Alfvénic solar wind



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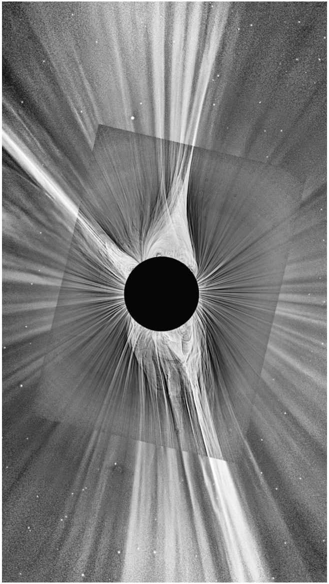

Isothermal hydrostatic equilibrium

- $\frac{dP}{dr} = -G\rho$
- Ideal gas: $P = nkT$
- $\Rightarrow$ Scale height: $H = \frac{kT}{m}$

The scale height predicted is 1,000x smaller than observed!!!

So the solar atmosphere is either:

- Made of a new form of matter, 1,000x lighter than hydrogen (“Coronium”, 1905).
- 1,000x hotter than the surface .

A corona this hot is unstable

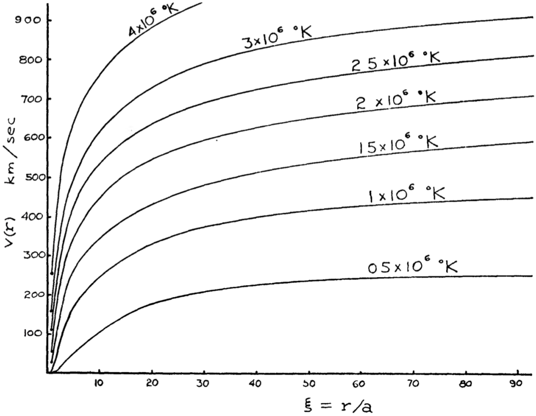

FIG. 1 —Spherically symmetric hydrodynamic expansion velocity $v(r)$ of an isothermal solar corona with temperature T_0 plotted as a function of r/a, where a is the radius of the corona and has been taken to be 10^{11} cm

Figure: Parker ApJ 1958

Leading model for the origin of the solar wind

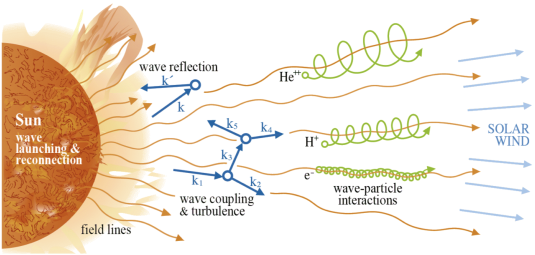

Figure: Image courtesy of B. Chandran

- The Sun launches Alfvén waves, which transport energy outwards
- The waves become turbulent, which causes energy to “cascade” from long wavelengths to short wavelengths
- Short-wavelengths fluctuations dissipates, heating the plasma. This increases the thermal pressure, which, accelerates the solar wind.

Expanding Box Model

We consider the turbulence dynamics in a frame co-moving with a spherically expanding flow.

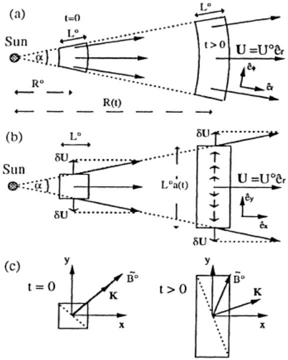

Figure: Grappin et al. 1993

Simplest model imaginable:

- Fluctuations are transverse, non compressible
- Radial background magnetic field
- $k_{\perp}\rho_i \ll 1$
- U is radial, constant and $\gg V_A$
- All fields are 3D periodic

Expanding Box Model

Simplest questions imaginable:

- How fast various types of energy decay?
- How the outer scale evolves?

RMHD Expanding Box Model

$$\dot{a}\frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial a} \mp v_A \frac{\partial \tilde{\mathbf{z}}^{\pm}}{\partial z} + \frac{1}{a^{3/2}}\left(\tilde{\mathbf{z}}^{\mp} \cdot \nabla_{\perp} \tilde{\mathbf{z}}^{\pm} + \frac{\nabla_{\perp} p}{\rho}\right) = -\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^{\mp}$$

$$a(t) = \frac{R(t)}{R_0} = 1 + \dot{a}t, \qquad \tilde{\mathbf{z}}^{\pm} \doteq \sqrt{\frac{L_z}{2\pi v_A}}\mathbf{z}^{\pm}$$

RMHD equations with two modifications :

- additional linear terms coupling counter-propagating Alfvénic perturbations: $-\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^{\mp}$
- modified expression for the gradients accounting for the increasing lateral stretching of the plasma with distance: $\nabla_{\perp} \rightarrow \nabla_{\perp}/a$

RMHD Expanding Box Model

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Ideal Invariants

- Individual wave action Elsasser energies are **not conserved.**
- Wave action cross helicity $\tilde{E}^c = \tilde{E}^+ - \tilde{E}^-$ is **conserved.**

Reflection terms can act as a source or a sink of wave action power $\epsilon = -\dot{a}\left\langle \tilde{\mathbf{z}}^{\pm}\cdot\tilde{\mathbf{z}}^{\mp}\right\rangle/a$ depending on the sign of the correlation between the Elsasser fields.

Conservation Laws & Characteristic time

Due to conservation of mass and magnetic flux:

- $\rho = \rho_0/a^2$
- $B = B_0/a^2$
- $v_A = v_{A0}/a$

Characteristic time-scales

- Expansion $\tau_{exp} = a/\dot{a}$
- Alfvén time $\tau_A = (k_z v_A)^{-1}$
- Non-linear $\tau_{nl}^{\pm} \sim a^{-3/2}(k_\perp \tilde{z}^{\mp})^{-1}$

$$\Gamma \equiv \dot{a}\frac{L_z}{v_{A0}} = \text{const.}$$

To match solar wind condition we considered $\Gamma = 1/10$.

Linear Dynamics

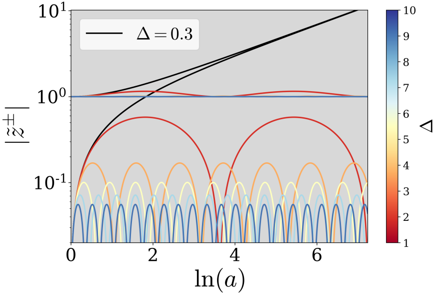
Using the transformation $\alpha = \ln(a)$ and defining $\Lambda = \sqrt{1/4 - \Delta^2}$ with $\Delta = \tau_{exp}/\tau_A = k_z v_{A0}/\dot{a}$,

$$\frac{\partial}{\partial \alpha}\begin{pmatrix} \tilde{z}^+ \\ \tilde{z}^- \end{pmatrix} = \begin{pmatrix} -i\Delta & -1/2 \\ -1/2 & i\Delta \end{pmatrix}\begin{pmatrix} \tilde{z}^+ \\ \tilde{z}^- \end{pmatrix}$$

$$\tilde{z}^{\pm}(\alpha) = \tilde{z}_0^{\pm}\cosh(\Lambda\alpha) + \frac{(\tilde{z}_0^{\mp} - 2i\Delta\tilde{z}_0^{\pm})\sinh(\Lambda\alpha)}{2\Lambda}$$

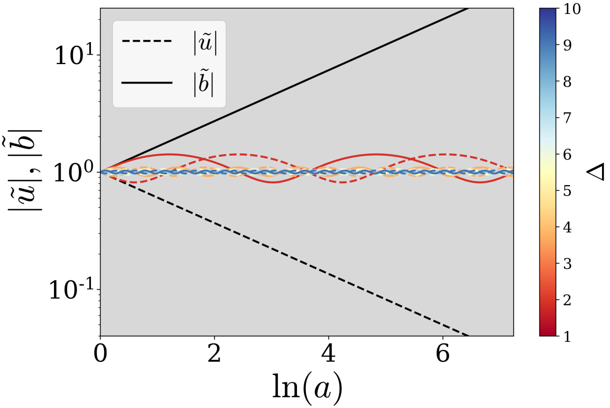
- $\Delta > 1/2 \Rightarrow \tilde{z}^{\pm} \propto a^{1/2}$
- $\Delta < 1/2 \Rightarrow \tilde{z}^{\pm} \propto a^{0}$

Elsassers fields do no necessarily propagate in opposite directions.



Linear Dynamics

- $\Delta < 1/2 \Rightarrow \tilde{u} \propto a^{-1/2}, \tilde{b} \propto a^{1/2}$
- $\Delta > 1/2 \Rightarrow \tilde{u}, \tilde{b} \propto a^{0}$



Non-linear scaling theory

Premises

- $\tilde{z}^+_{rms} \gg \tilde{z}^-_{rms}$
- $\chi^- = \tau_A/\tau^-_{nl} > 1$
- Anomalous coherence

Anomalous coherence

The weaker field is generated by the reflection of the stronger one. The weaker field can therefore distort the stronger field in a time-coherent way.

Non-linear scaling theory

Premises

- $\tilde{z}^+_{rms} \gg \tilde{z}^-_{rms}$
- $\chi^- = \tau_A/\tau^-_{nl} > 1$
- Anomalous coherence

$$\boxed{\frac{1}{a^{3/2}}\left(\tilde{\mathbf{z}}^+ \cdot \nabla_\perp \tilde{\mathbf{z}}^- + \frac{\nabla_\perp p}{\rho}\right) = -\frac{\dot{a}}{2a}\tilde{\mathbf{z}}^+} \Rightarrow \tilde{z}^-_{rms} \propto \frac{\dot{a}a^{1/2}}{2}\lambda^+$$

$$\lambda^+ = \frac{2\pi}{\tilde{E}^r}\int dk_\perp \frac{\langle \tilde{z}^+ \cdot \tilde{z}^- \rangle}{k_\perp} \sim \lambda^+_0 a^\beta$$

β is a free parameter to be determined empirically.

Non-linear scaling theory

Premises

- $\tilde{z}^+_{rms} \gg \tilde{z}^-_{rms}$
- $\chi^- = \tau_A/\tau^-_{nl} > 1$
- Anomalous coherence

$$\begin{cases} \dot{a}\dfrac{\partial \tilde{\mathbf{z}}^+}{\partial a} + \dfrac{1}{a^{3/2}}\left(\tilde{\mathbf{z}}^- \cdot \nabla_\perp \tilde{\mathbf{z}}^+ + \dfrac{\nabla_\perp p}{\rho}\right) = -\dfrac{\dot{a}}{2a}\tilde{\mathbf{z}}^- \\ \tilde{z}^-_{rms} \propto \dfrac{\dot{a}a^{1/2}}{2}\lambda^- \end{cases}$$

$$\frac{\partial \ln \tilde{E}^+}{\partial a} \sim -2\frac{a^{-3/2}}{\dot{a}}\frac{\tilde{z}^-_{rms}}{\lambda^+} \propto -\frac{\lambda^-}{\lambda^+}\frac{\partial \ln a}{\partial a}$$

Assuming $\lambda^-/\lambda^+ \equiv \gamma \propto a^0 \Rightarrow \tilde{E}^+ \propto a^{-\gamma}$

Transition toward magnetically dominated states

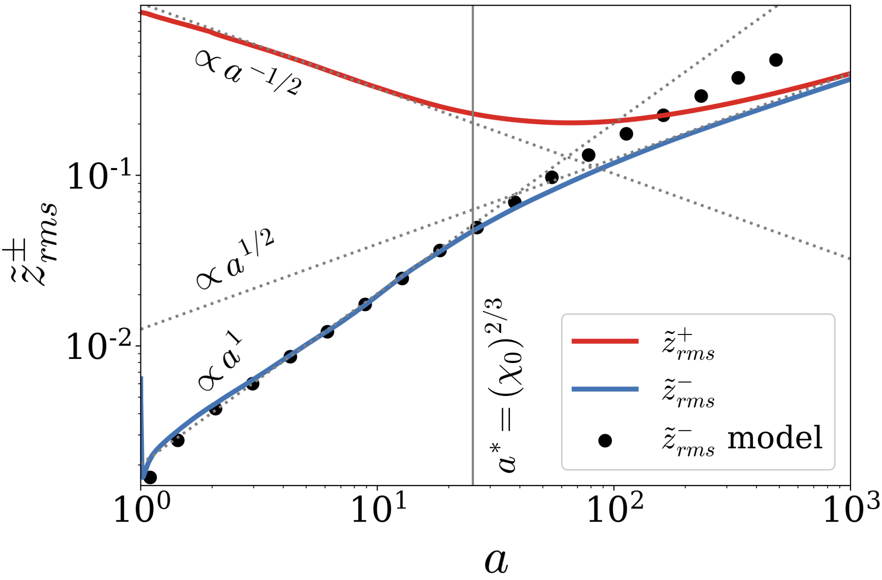
Reflection driven turbulence drives itself into a state that no longer satisfies the premises on which the theory is based.

$$\begin{cases} \lambda^+ \propto \lambda_0^+ a^\beta \\ \tilde{E}^+ \propto \tilde{E}_0^+ a^{-1} \end{cases} \quad \Rightarrow \chi^- \propto a^{-1-\beta} \chi_0^-$$

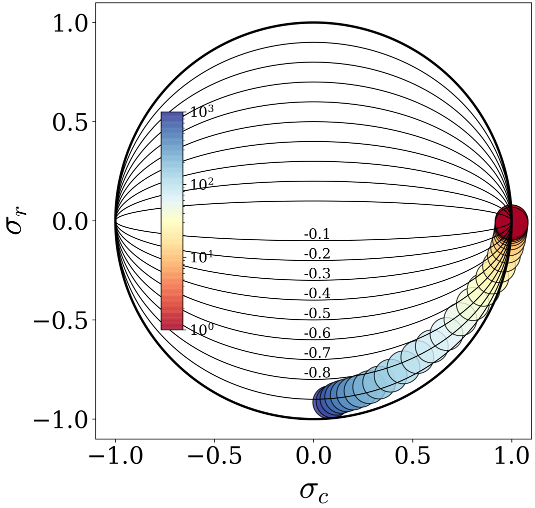
$$\chi^-(a^*) = 1 \Rightarrow a^* = \left(\chi_0^-\right)^{1/(1+\beta)}$$

For $a > a^*$, anomalous coherence break down, the system become quasi linear, and transit toward a magnetically dominated 2D states.

Numerical Results



Numerical Results



- $\sigma_c = \dfrac{E^+ - E^-}{E^+ + E^-}$
- $\sigma_r = \dfrac{E^u - E^b}{E^u + E^b}$

Anomalous conservation of anastrophy

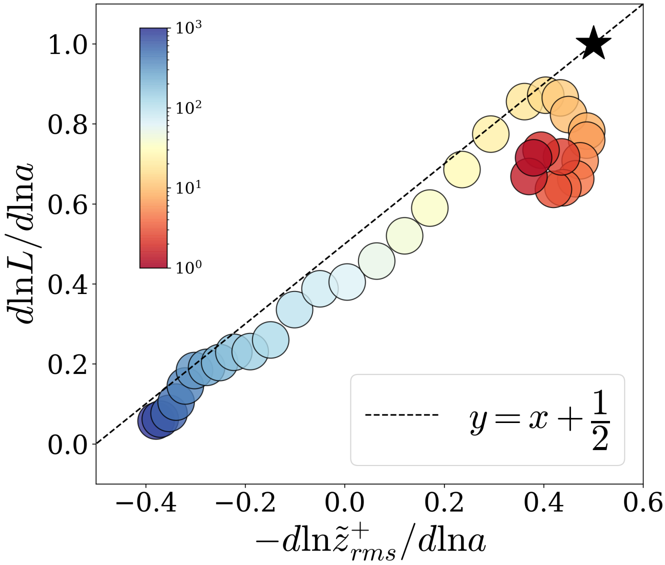
In the nonlinear regime, the weaker field is born coherent and is short-lived. It doesn't propagate against but with the stronger field.

The decay process through coalescence of magnetic structures which characteristic size L increasing according to

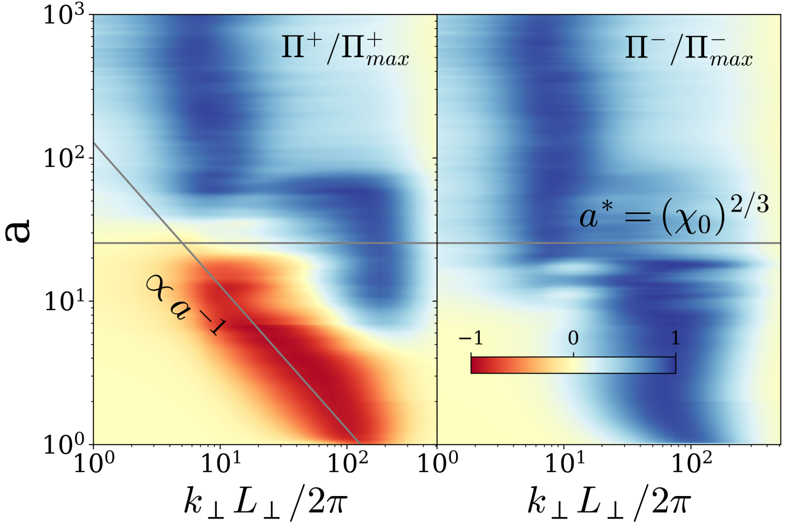
$$\left\langle \psi^2 \right\rangle \sim a^{-1} \tilde{E}^+ L^2 \sim \text{const} \Rightarrow L \propto a^{(\gamma+1)/2}$$

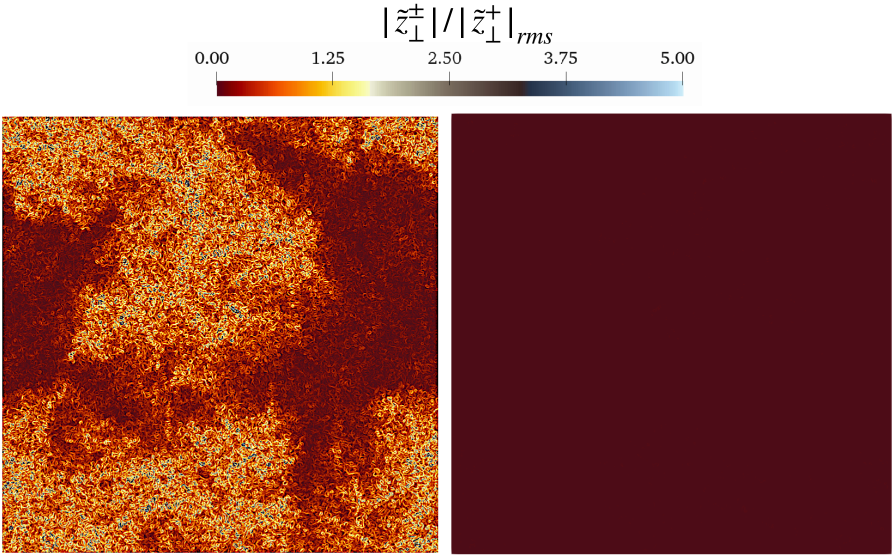
with ψ the out-of-plane component of the magnetic vector potential.

Anomalous conservation of anastrophy

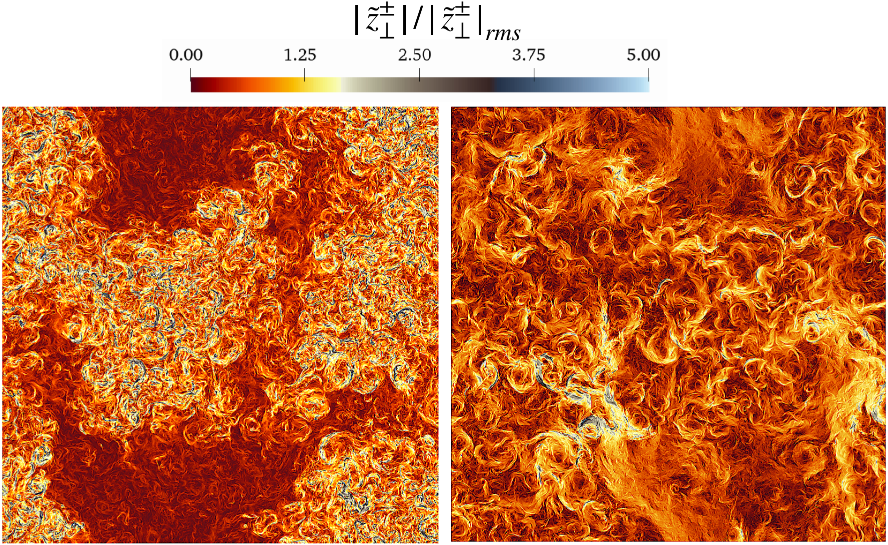


Bi-directional Elsasser cascades

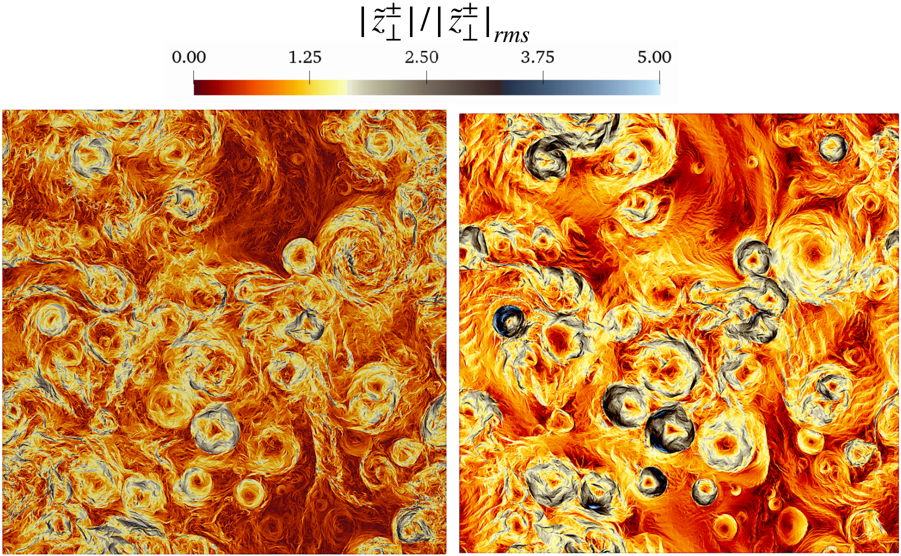




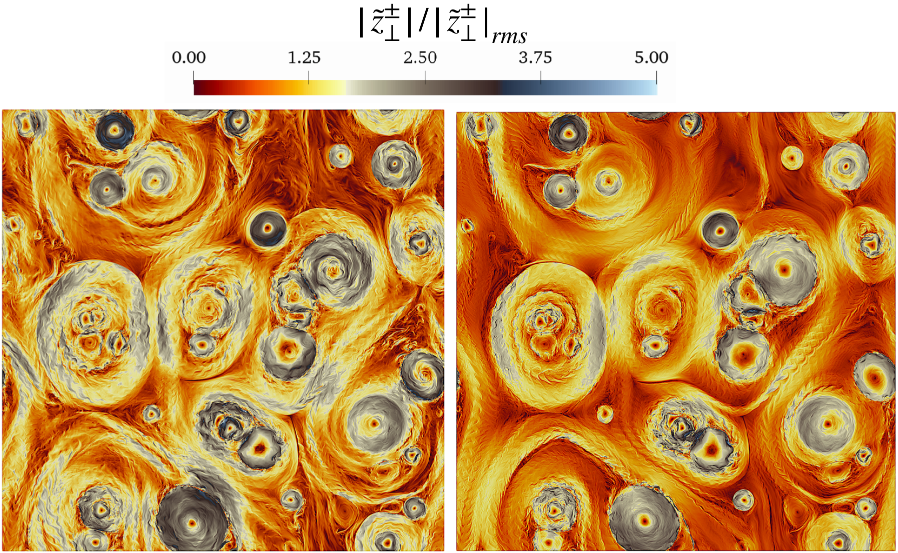
$$a = 1, \quad 1 - \sigma_c = 1e - 4 \tag{1}$$



$$a = 5, \quad 1 - \sigma_c = 1e - 3 \qquad (2)$$

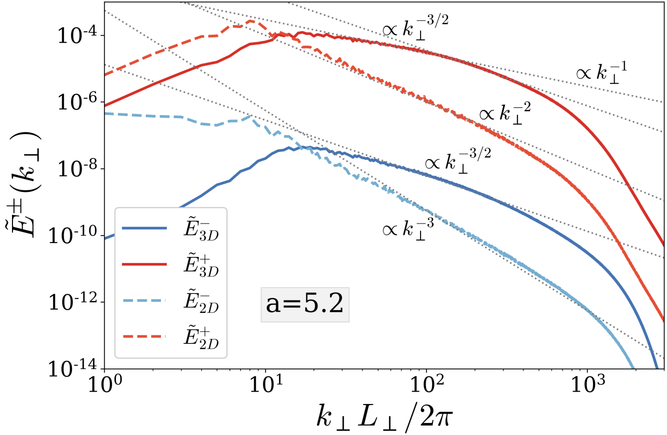


$$a = 50, \quad 1 - \sigma_c = 0.25 \tag{3}$$



$$a = 250, \quad 1 - \sigma_c = 0.75 \qquad (4)$$

Power Spectra



Reflection-driven turbulence gives rise to a $k_\perp^{-1}$ spectrum as observed in the solar wind.

Conclusions

In its simplest form, the reflection-driven turbulence may explain essential features of the solar wind:

- Double power law at intermediate and large scales, with power indices -3/2 and -1 respectively
- Bi-directional Elsasser cascades in highly imbalance streams.
- Formation of Alfvén vortices.
- Generation of high negative residual energy states.